A Tour of Sage

This is a tour of Sage that closely follows the tour of Mathematica that is at the beginning of the Mathematica Book.

Sage as a Calculator

The Sage command line has a `sage:` prompt; you do not have to add it. If you use the Sage notebook, then put everything after the `sage:` prompt in an input cell, and press shift-enter to compute the corresponding output.

```
sage: 3 + 5
8
```

The caret symbol means “raise to a power”.

```
sage: 57.1 ^ 100
4.60904368661396e175
```

We compute the inverse of a 2×2 matrix in Sage.

```
sage: matrix([[1,2], [3,4]])^(-1)
[  -2    1]
[ 3/2 -1/2]
```

Here we integrate a simple function.

```
sage: x = var('x')   # create a symbolic variable
sage: integrate(sqrt(x)*sqrt(1+x), x)
1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1)/x + 1) - 1/8*log(sqrt
```

This asks Sage to solve a quadratic equation. The symbol `==` represents equality in Sage.

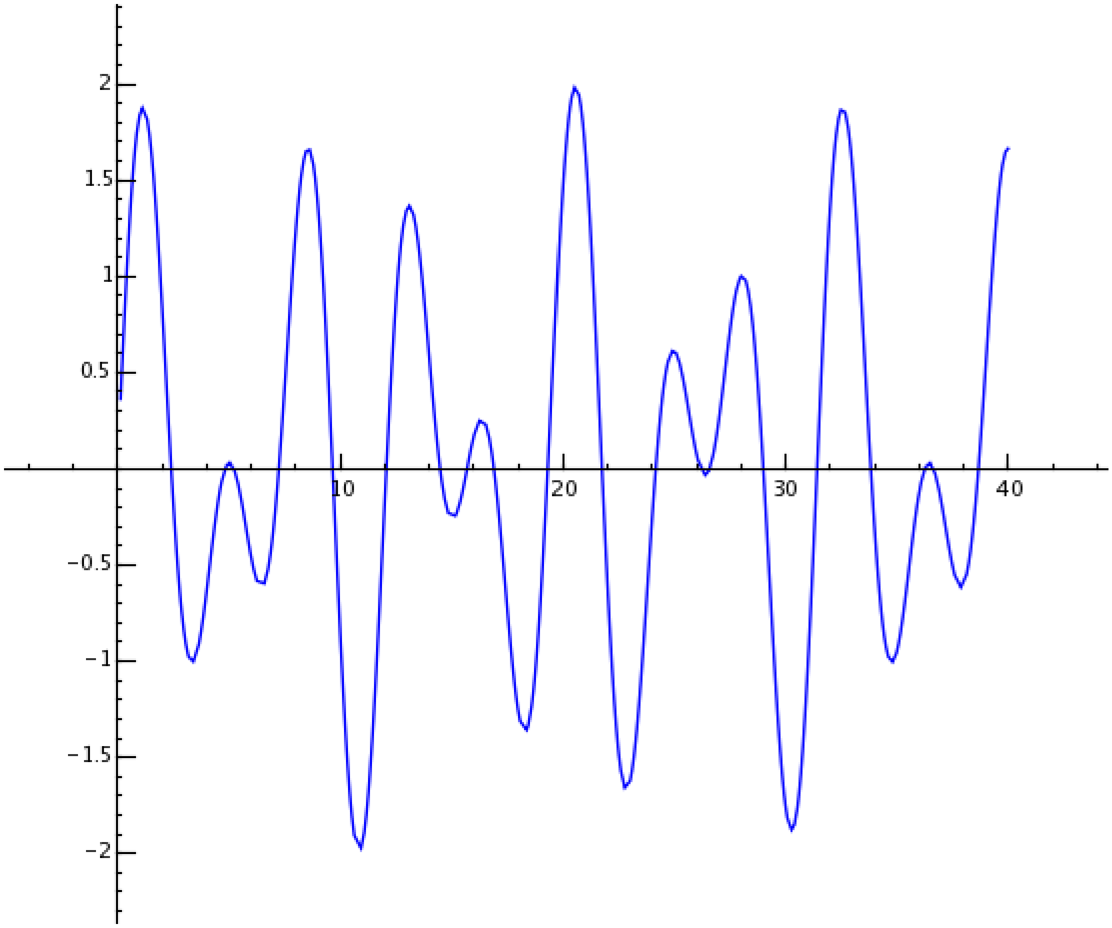
```
sage: a = var('a')
sage: S = solve(x^2 + x == a, x); S
[x == -1/2*sqrt(4*a + 1) - 1/2, x == 1/2*sqrt(4*a + 1) - 1/2]
```

The result is a list of equalities.

```
sage: S[0].rhs()
-1/2*sqrt(4*a + 1) - 1/2
```

Naturally, Sage can plot various useful functions.

```
sage: show(plot(sin(x) + sin(1.6*x), 0, 40))
```



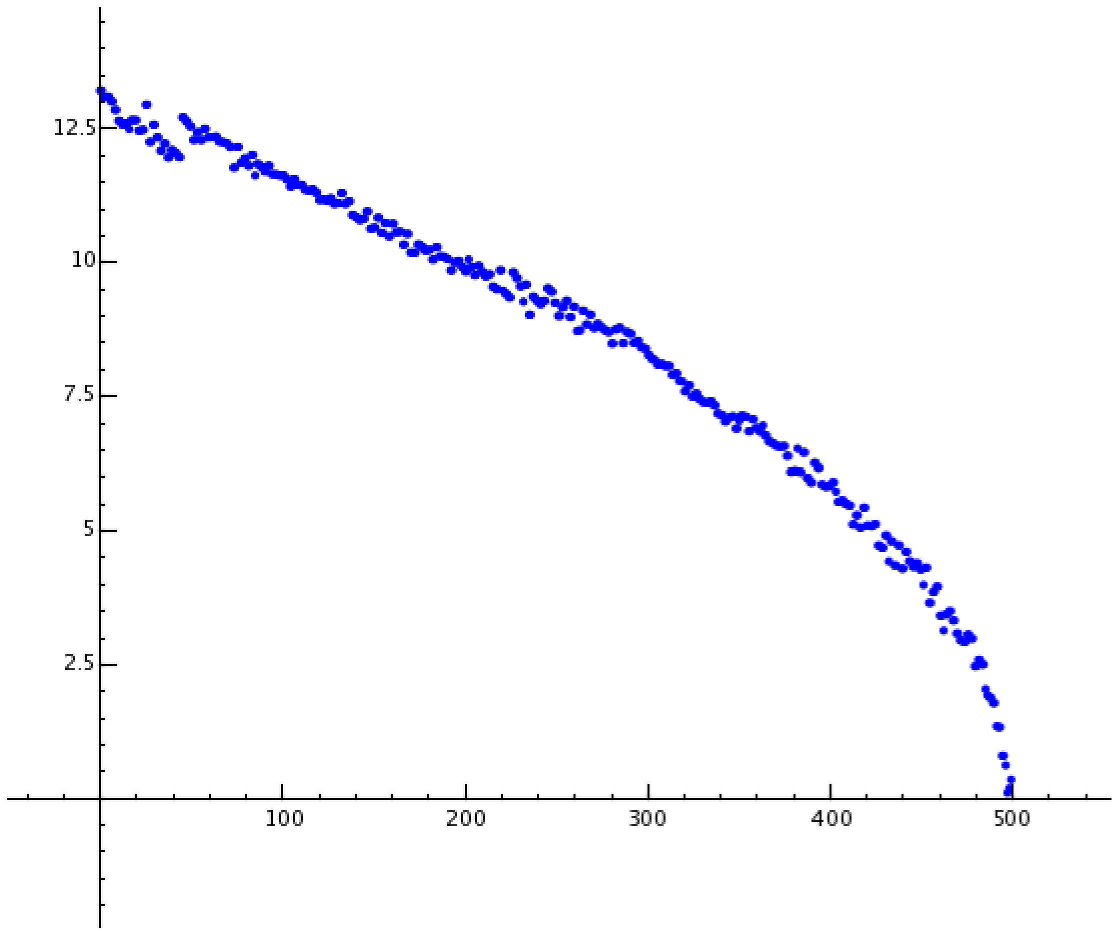
Power Computing with Sage

First we create a 500×500 matrix of random numbers.

```
sage: m = random_matrix(RDF,500)
```

It takes Sage a few seconds to compute the eigenvalues of the matrix and plot them.

```
sage: e = m.eigenvalues()  #about 2 seconds
sage: w = [(i, abs(e[i])) for i in range(len(e))]
sage: show(points(w))
```



Thanks to the GNU Multiprecision Library (GMP), Sage can handle very large numbers, even numbers with millions or billions of digits.

```
sage: factorial(100)
933262154439441526816992388562667004907159682643816214685929638952175999932299156089414639761565182
sage: n = factorial(1000000)  #about 2.5 seconds
```

This computes at least 100 digits of π.

```
sage: N(pi, digits=100)
3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211 7
```

This asks Sage to factor a polynomial in two variables.

```
sage: R.<x,y> = QQ[]
sage: F = factor(x^99 + y^99)
sage: F
(x + y) * (x^2 - x*y + y^2) * (x^6 - x^3*y^3 + y^6) *
(x^10 - x^9*y + x^8*y^2 - x^7*y^3 + x^6*y^4 - x^5*y^5 +
 x^4*y^6 - x^3*y^7 + x^2*y^8 - x*y^9 + y^10) *
(x^20 + x^19*y - x^17*y^3 - x^16*y^4 + x^14*y^6 + x^13*y^7 -
 x^11*y^9 - x^10*y^10 - x^9*y^11 + x^7*y^13 + x^6*y^14 -
 x^4*y^16 - x^3*y^17 + x*y^19 + y^20) * (x^60 + x^57*y^3 -
 x^51*y^9 - x^48*y^12 + x^42*y^18 + x^39*y^21 - x^33*y^27 -
 x^30*y^30 - x^27*y^33 + x^21*y^39 + x^18*y^42 - x^12*y^48 -
```

```
 x^9*y^51 + x^3*y^57 + y^60)
sage: F.expand()
x^99 + y^99
```

Sage takes just under 5 seconds to compute the numbers of ways to partition one hundred million as a sum of positive integers.

```
sage: z = Partitions(10^8).cardinality() #about 4.5 seconds
sage: str(z)[:40]
'1760517045946249141360373894679135204009'
```

Accessing Algorithms in Sage

Whenever you use Sage you are accessing one of the world's largest collections of open source computational algorithms.