Feedback linearization approach to distributed feedback manipulation

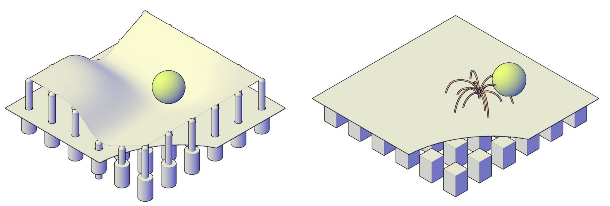
Zdeněk Hurák and Jiří Zemánek

***Abstract*— This report formulates the problem of a distributed planar manipulation realized by shaping a spatially continuous force field and it suggests a control strategy based on feedback linearization. Force fields derived from potential fields are considered. The potentials are "shaped" by a set of spatially discrete "actuators" such as electrodes in the case of dielectrophoresis, electromagnets in the case of planar magnetic manipulators, or linear piezoelectric actuators in the case of deformable flat surfaces. Distinguished feature of such force fields is that the contribution from an individual actuator usually affects the situation in the neighbouring zones too, but usually not in too remote zones. As an idealization, the spatial domain is considered unbounded, which enables examination of asymptotic behavior of the manipulation scheme.**

I. Motivation and State of the Art

A. Motivation

Consider an ensamble of objects placed freely on top of an *intelligent surface* which can exert force on these objects and initiate their planar motion. An example of mechanically deformable surface with a ball on top of it is visualized in Fig.1(a). In fact, the surface need not be mechanically deformable, the concept of potential carries over to other physical phenomena than gravity. For instance, electric potential in electrophoresis, gradient of squared magnitude of electric field in dielectrophoresis or magnetic potential for magnetic manipulators; see Fig. 1(b) for a simple sketch.



(a) Potential field shaped by deforming the compliant surface. (b) Potential field shaped by controlling electric or magnetic field.

Fig. 1. Visualization of a planar motion by shaping the potential field.

The task is to shape the force field in such a way that the objects move as required: following reference trajectories for individual objects or achieving some collective characteristics, possibly in a stochastic sense. Open-loop or feedback control schemes can be considered.

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In order to make things notationally and conceptually simpler, consider a single particle and a one-dimensional spatial domain as in Fig.2. The notation is explained in a later section, nevertheless the general principle reflected in the figure is that the total potential field is composed from a countable number of potential fields induced by local (zonal) "actuators" whose force effects overlap into the neighbouring zones. The force is then derived as a gradient of the total potential. The obvious restriction is that only potential or force field for which the principle of superposition works can be considered.

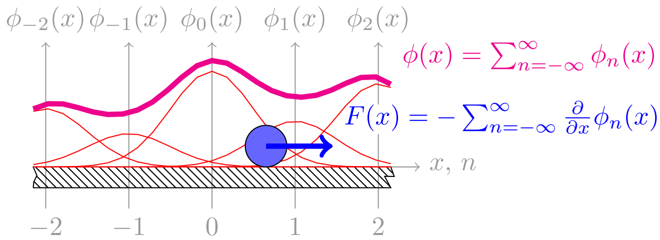


Fig. 2. One-dimensional restriction of planar manipulation by shaping a potential field.

B. State of the art

Research in the area of distributed planar manipulation by shaping force or potential fields was initiated in 1990s with the anticipation of MEMS-based arrays of microactuators. One of the first researchers who systematically investigated use of arrays of (micro)actuators is K. F. Böhringer. The paper initiating a long series of papers is [1]. In that paper, Böhringer not only described a novel MEMS-based array of so-called motion pixels flipping around a fixed axis but also gives a first formulation of orientation of an object using a sequence of simple preprogrammed force fields. He took an inspiration from [2] and [3], who studied the problem of orienting an object using a parallel jaw gripper without any sensoric feedback. In Böhringer's work, the role of grippers is taken over by a discrete force field. The theoretical concept of *programmable (vector) force field (PFF)* was further extended in [4], where a radial force field was added to the previously introduced squeeze fields. Quite a few more papers described how the work of Böhringer and his colleagues proceeded. For instance, [5] describes a novel CMOS ciliary array as a tool to manipulate small objects. A culmination of this work on discrete MEMS actuator arrays is summarized in a book [6].

Böhringer's research triggered interest of many other researchers. Among the interesting contributions is [7], where

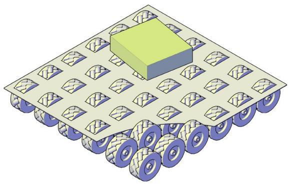


Fig. 3. Array of omnidirectional wheels for planar manipulation.

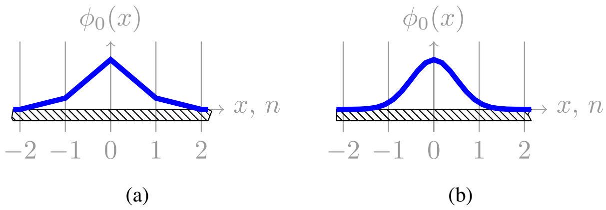


Fig. 4. Some unit potential functions: (a) piece-wise affine, (b) Gaussian.

they enhance the PFF concept with some more complex shapes/actions of the field, using force fields derived from elliptic potential fields. [8] increased the complexity of the field (and the associated analysis) even further by segmenting the field into polygonal regions and invoking some results from computational geometry. Their research on intelligent motion surfaces (term coined in [9]) has fancy photos, videos and animations on the website `http://www.isi.edu/mass`. The application domain for array manipulators is vast, the discrete MEMS arrays are not the only technology. [10], [11] and [12], [13] and many others describe use of arrays of air jets to manipulate objects hovering above the plane.

In the work of Böhringer and his colleagues and followers, dense discrete actuator arrays are modeled using continuum models. This approximation seems only justified by the denseness of the MEMS arrays they considered. As soon as the array is not very dense, discreteness issues must be studied. Which is what Luntz did in his research, major results presented in [14]. The particular technology that he considers, namely manipulation of parcels on an array of (inline-skates-like) wheels, was described earlier in [15].

Luntz continued in this work through the work of his student [16] and [17] studying certain class of force fields that derive from quadratic potential fields. The advantage of such force fields is that the requirements on density of the actuator array are lower and the methodology is especially useful for naturally existing phenomena such as airflows.

Discreteness issues tackled by Luntz were also studied by Murphey and Burdick [18]. They based their work on the assumption of a quasistatic (with no significant effect of inertia) motion and the concept of power dissipation models introduced by [19] and later extended in [20] for multi-wheeled vehicles. Their methodology shows that continuous PFF concept can lead to instabilities when translating and rotating an object to a required position and orientation without a feedback. Introduction of local feedback is thus justified.

This paper aims at contributing to further progress of research in feedback control in the case of a spatially continuous force field generated by a spatially discrete array of “actuators”.

II. Modeling the System

Consider a single object to be manipulated over a one-dimensional spatial domain. As a few examples consider a bead moving along a string, cart traveling on rails, microparticle transported in a microfluidic channel, etc. Response of a single particle constrained to a horizontal motion in one dimension is described by the second law of Newton

$$m\ddot{x}(t) = \sum_{n=-\infty}^{\infty} F_n(x(t)) - b\dot{x}(t), \tag{1}$$

where F_n is a force contributed by the individual zone indexed by n, m is mass of the particle and b is the coefficient of viscous friction. Apparently, F_n is a function of the position $x(t)$ of the object. At first we consider an infinite number of zones so that the spatial domain is unbounded and some stability considerations can be carried out (in future research) not only for the temporal but also for the spatial domain. The model also includes a damping frictional term corresponding to the velocity.

Assuming that each zone exhibits identical force, a *unit influence function* $f(x)$ (spatial analogue to impulse response) can be introduced, which, assuming equidistant spatial sampling h, can be used to write the motion equation as

$$m\ddot{x}(t) = \sum_{n=-\infty}^{\infty} u_n(t) \cdot f(x(t) - nh) - b\dot{x}(t), \tag{2}$$

where $\{u_n\}$ is a sequence of weights that are used to scale the individual local contributions to the global force field. These weights constitute the control vector and generally they vary in time.

Although not immediately useful in the one-dimensional case, we should anticipate the more notationally clumsy two- or three-dimensional case and consider the force fields that are derived from (scalar) potential fields. Working with the scalar quantity will make things at least notationally simpler

$$m\ddot{x}(t) = \sum_{n=-\infty}^{\infty} -u_n(t) \cdot \frac{d}{dx}\varphi(x(t) - nh) - b\dot{x}(t). \tag{3}$$

More generally, the spatial derivative with respect to x will be replaced by gradient

$$m\ddot{x}(t) = \sum_{n=-\infty}^{\infty} -u_n(t) \cdot \nabla\varphi(x(t) - nh) - b\dot{x}(t). \tag{4}$$

The unit influence function can be then represented by the equivalent unit potential function. A few basic shapes of the potential can be seen in Fig. 4.

But for the time being, back to explicitly written force fields. The state-space model characterizing dynamics of a single object is

$$\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x(t)\\ v(t)\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\sum_{n=-\infty}^{\infty} u_n(t)\cdot f(x(t)-nh)\end{bmatrix}. \tag{5}$$

Equivalently

$$\begin{bmatrix}\dot{x}(t)\\ \dot{v}(t)\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ 0 & -\frac{b}{m}\end{bmatrix}}_{A}\begin{bmatrix}x(t)\\ v(t)\end{bmatrix} + \underbrace{\frac{1}{m}\begin{bmatrix}\cdots & 0 & 0 & 0 & \cdots\\ \cdots & f(x+h) & f(x) & f(x-h) & \cdots\end{bmatrix}}_{B(x(t))}\underbrace{\begin{bmatrix}\vdots\\ u_{-1}(t)\\ u_0(t)\\ u_1(t)\\ \vdots\end{bmatrix}}_{u(t)}, \tag{6}$$

from which the structure of a state-dependent model characterized by two matrices A and B resembling a classical linear state-space model can be deduced

$$\dot{x}(t) = A\cdot x(t) + B(x(t))\cdot u(t), \tag{7}$$

where we abuse the notation temporarily and use the symbol x for the full state vector. Of course this is a nonlinear model. The class of nonlinear models with the structure

$$\dot{x}(t) = A(x)\cdot x(t) + B(x)\cdot u(t). \tag{8}$$

is called *state-dependent models* and their structural similarity with linear state-space models gives rise to the optimal control approach named *state-dependend Riccati equations (SDRE)*, see [21], [22], [23], [24], [25].

III. Control design problem formulation

Given the initial position of the object $x(t_0) = x_0$, bring the object into the origin $x(t) = 0$ as fast as possible by applying as cheap the distributed control $u_n(t)$ as possible. We summarize for convenience, that the control $u_n(t)$ shapes the potential field $\phi(x,t)$, which induces the derived force field $F(x,t)$. This suggests the classical control trade-off of LQ optimal control style.

The *control effort* part of the criterion should be discussed. It may not be immediately clear if the cost function should be based on the spatially continuous force (or potential) field or the spatially discrete array of command signals.

For the latter the proposed optimality cost might be

$$J = \int_{t_0}^{t_f}\left[x^{\mathrm{T}}(t)Qx(t) + \sum_{n=-\infty}^{\infty} r u_n^2(t)\right]\mathrm{d}t, \tag{9}$$

where $Q\in\mathbb{R}^{2\times 2}$, $r\in\mathbb{R}$, and we again abused the notation here by using the symbol x for the full state vector. Or simply

$$J = \int_{t_0}^{t_f}\left[x^{\mathrm{T}}(t)Qx(t) + \underbrace{\sum_{n=-\infty}^{\infty} u_n^2(t)}_{\|u(t)\|_2^2}\right]\mathrm{d}t, \tag{10}$$

where $u(t)$ denotes a *frozen-time* (corresponding to a given time t) infinite sequence of controls. Obviously, this sequence needs to be square-summable in order for integral to be well-defined. Formally, $u(t)\in\ell_2(\mathbb{Z})$. Of course this is automatically satisfied if the spatial domain is bounded, i.e., it has a finite number of zones (actuators).

For the former the proposed optimality cost might be

$$J = \int_{t_0}^{t_f}\left[x^{\mathrm{T}}(t)Qx(t) + \underbrace{\int_{-\infty}^{\infty}\phi^2(t,y)\mathrm{d}y}_{\|\phi(t)\|_2^2}\right]\mathrm{d}t, \tag{11}$$

or perhaps better

$$J = \int_{t_0}^{t_f}\left[x^{\mathrm{T}}(t)Qx(t) + \int_{-\infty}^{\infty}\left(\frac{\partial\phi(t,y)}{\partial y}\right)^2\mathrm{d}y\right]\mathrm{d}t, \tag{12}$$

In both situations the finiteness of control horizont can be relaxed, in which case $t_f = \infty$ as traditional in the LQ optimal control. Of course other than quadratic criteria can be considered, we opt for the more conventional cost function in this initial treatment, in which we do not actually tackle the proposed problem, only by formulating it we specify the general expectations from the control scheme.

Recalling the intuitive example of a mechanically deformable surface, we could perhaps include the $\left\|\frac{\partial^2\phi(t,x)}{\partial x^2}\right\|_2$ term in the cost function which roughly corresponds to the deformation of the surface at a given time. Penalization of the second-order derivative of the spatially continuous potential field reminds of the classical *minimal surface* problem (also known as *Plateau's problem*) studied in the classical variational calculus, which is to find the shape of a soap film stretched across a wire frame. It is known that the shape assumed by the soap film minimizes energy corresponding to the surface tension. Mathematically, the optimal shape minimizes curvature of the surface.

IV. Feedback linearization

With the model being nonlinear, a control design technique that immediately pops up is that of feedback linearization. The control $u(t)$ in (7) is chosen such that the product $B(x)\cdot u(t)$ is linear. A simple solution is to include in $u(t)$ the inverse of $B(x)$. The product term from (6) is

$$B(x) \cdot u(t) =$$

$$\frac{1}{m} \begin{bmatrix} \cdots & 0 & 0 & 0 & \cdots \\ \cdots & f(x+h) & f(x) & f(x-h) & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ u_{-1}(t) \\ u_0(t) \\ u_1(t) \\ \vdots \end{bmatrix}.$$

In order to avoid technical troubles with infinite summation, consider temporarily just a finite spatial domain, that is the summation is carried over a finite number of actuators only. Then the control sequence $u(t)$ chosen as

$$u(t) = \begin{bmatrix} u_{-N}(t) \\ \vdots \\ u_{-1}(t) \\ u_0(t) \\ u_1(t) \\ \vdots \\ u_N(t) \end{bmatrix} = \begin{bmatrix} 1/f(x+Nh) \\ \vdots \\ 1/f(x+h) \\ 1/f(x) \\ 1/f(x-h) \\ \vdots \\ 1/f(x-Nh) \end{bmatrix} \bar{u}(t) \qquad (13)$$

renders the system linear, with a new scalar control $\bar{u}(t)$.

The crucial trouble with this approach is that it attempts to equalize the contribution from the individual actuators: if the object to be manipulated resides in the k-th zone, the normalized contribution of the k-th reflected by the force $f(x-kh)$ is far greater then the contribution of, say, the $(k-10)$-th actuator reflected by the force $f(x-(k+10)h)$. What is the point in trying to do the job by this remote actuator with a very low influence? And that is exactly what feedback linearization atempts at.

A different strategy is proposed. We need to enforce into the control scheme the rule that the “actuators” with the greates influence on the object are used to manipulation. Typically these are the closest ones. Hence, we need to include some weights, certainly dependent on the state and time. Finiteness of the sequences is not necessary.

$$u(t) = \begin{bmatrix} \vdots \\ u_{-1}(t) \\ u_0(t) \\ u_1(t) \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \vdots \\ w_{-1}(x,t) \\ w_0(x,t) \\ w_1(x,t) \\ \vdots \end{bmatrix}}_{W(x,t)} \bar{u}(t). \qquad (14)$$

A natural candidate for such weights is the vector

$$W(x) = \begin{bmatrix} \cdots & f(x+h) & f(x) & f(x-h) & \cdots \end{bmatrix}^{\mathrm{T}}, \qquad (15)$$

which turns the $B(x) \cdot u(t)$ product into

$$B(x) \cdot u(t) = \sum_{n=-\infty}^{\infty} \left(f(x-nh)\right)^2 \cdot \bar{u}(t) \qquad (16)$$

with a scalar control $\bar{u}(t)$. The term is still nonlinear in x, although it is spatially *periodic* in x with the period of h. The boundedness of the sum is equivalent to the specification that f is square-integrable as a function of x, that is, $f(x) \in \mathcal{L}_2$. Its spatially sampled version is square-summable, that is, the infinite sequence of function values is $f(x-nh) \in \ell_2$ for arbitrary value of x. This assumption is justifiable from a practical viewpoint as the influence of a given actuator usually decays very fast with distance. It is only now that the feedback linearization makes sense. The ultimate control u is then

$$u(x,t) = \begin{bmatrix} \vdots \\ f(x+h) \\ f(x) \\ f(x-h) \\ \vdots \end{bmatrix} \frac{1}{\sum_{n=-\infty}^{\infty} \left(f(x-nh)\right)^2} \tilde{u}(t), \qquad (17)$$

which turns out to be a function of x and in general also t (through the $\tilde{u}(t)$ scalar term). Hence, it is a feedback control (but not state-feedback). The infinite sum $\sum_{n=-\infty}^{\infty} \left(f(x-nh)\right)^2$ in (16) simplifies to a constant term for a *triangular potential*, a special case of the piecewise affine potential visualized at Fig.5.

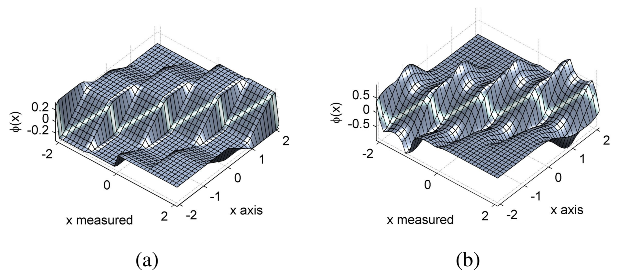


Fig. 5. Potential field in dependence on actual position x for (a) triangular potential, (b) Gaussian potential.

V. Parallel manipulation

Now let us consider simultaneous manipulation with several objects in the same force field. To investigate a control scheme for parallel manipulation, we can focus at first on the basic case of manipulation with two objects. Dynamics of such system is described using the state-space model

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{v}_1(t) \\ \dot{x}_2(t) \\ \dot{v}_2(t) \end{bmatrix} = \underbrace{\left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & -\frac{b_1}{m_1} & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{b_2}{m_2} \end{array}\right]}_{A} \underbrace{\begin{bmatrix} x_1(t) \\ v_1(t) \\ x_2(t) \\ v_2(t) \end{bmatrix}}_{x(t)}$$

$$+ \underbrace{\left[\begin{array}{cc|cc} \vdots & \vdots & \vdots & \vdots \\ f(x_1+h)/m_1 & 0 & f(x_2+h)/m_2 & 0 \\ f(x_1)/m_1 & 0 & f(x_2)/m_2 & 0 \\ f(x_1-h)/m_1 & 0 & f(x_2+h)/m_2 & 0 \\ \vdots & \vdots & \vdots & \vdots \end{array}\right]^{\mathrm{T}}}_{B(x(t))} \underbrace{\begin{bmatrix} \vdots \\ u_{-1}(t) \\ u_0(t) \\ u_1(t) \\ \vdots \end{bmatrix}}_{u(t)}. \qquad (18)$$

The major task during parallel manipulation is to control individual objects independently, which brings concerns about controllability of the system. Let us begin with introducing an individual input for each object (subsystem). This could be done using a straightforward extension of the weights (17); both inputs will have corresponding weighting vector

$$u(x,t) = \begin{bmatrix} \vdots & \vdots \\ f(x_1+h)/sf_1 & f(x_2+h)/sf_2 \\ f(x_1)/sf_1 & f(x_2)/sf_2 \\ f(x_1-h)/sf_1 & f(x_2-h)/sf_2 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \end{bmatrix},$$

$$sf_i = \sum_{n=-\infty}^{\infty} \left(f(x_i - nh)\right)^2 . \quad (19)$$

This leads to

$$\dot{x}(t) = A(x)\cdot x(t) + \tilde{B}(x)\cdot \begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \end{bmatrix} \quad (20)$$

where the new input matrix $\tilde{B}(x)$ is in the form

$$\tilde{B}(x) = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}. \quad (21)$$

The elements b_{12} and b_{21} describe coupling between one input and the subsystem corresponding to the other input. The system can be further decoupled by introducing the inverse of the matrix $\tilde{B}(x)$.

VI. Numerical simulations

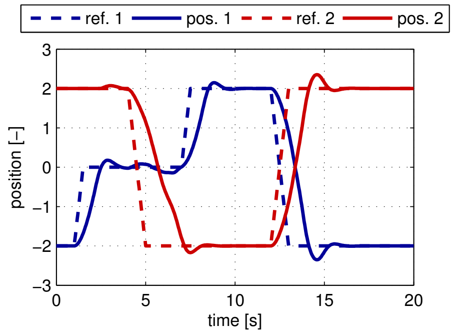
The proposed control scheme is demonstrated using simulations. We considered an array of 11 actuators generating a local Gaussian potential field. The width of the Gaussian was 0.7, that is, the actuator influences significantly the neighbouring zones only. The suggested scheme was tested on a feedback system consisting of the state-space model (6) describing an object in a potential field, feedback linearization (17) and a SISO controller. The simulations revealed coupling during parallel manipulation but also a positive effect of the decoupling as seen in Fig. 6.

Total potential shaped by the set of actuators is visualised in Fig. 7. Notice how the controller sets the object in motion and also how it decelerates it and holds another object in position at the same time.

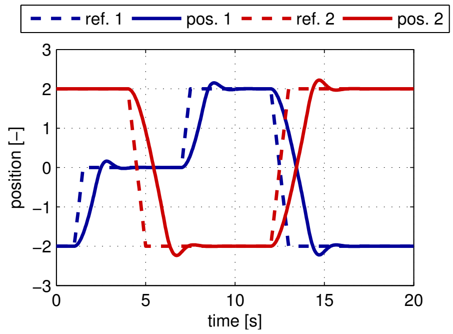
VII. Experimental motivation

One particular physical principle for a distributed planar manipulator is dielectrophoresis (DEP). DEP is motion of electrically neutral but polarizable particles in nonuniform electric field. Thanks to its significant selectivity, DEP is widely used for separation and detection of various objects in biology and medicine, but it can also be exploited for transportation and characterization. If a spherical particle with radius r is placed into a harmonic electric field $\mathbf{E}$ with angular frequency ω, immersed in a liquid with permittivity ε_m then the field exerts the average force

$$\langle \mathbf{F} \rangle_{\mathrm{DEP}} = \pi \varepsilon_m r^3 \Re\left\{K(\omega)\right\} \nabla \left\|\mathbf{E}\right\|^2 . \quad (22)$$

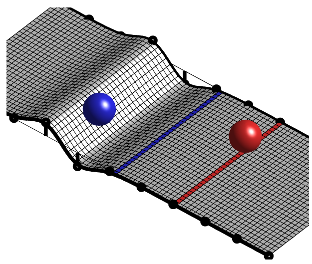


(a)

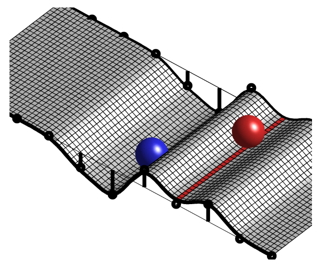


(b)

Fig. 6. Simulation of parallel manipulation with two objects in the potential field created using 11 actuators. Each actuator acts as a source of local a Gaussian potential. a) Without decoupling - position of object 1 is influenced by motion of object 2 (around t=5 s), b) with decoupling.



(a)



(b)

Fig. 7. Visualization of the total potential field created by deformation of surface. The field is created using 11 actuators (visible as circles). Two objects put in the field are represented by spheres and their reference positions as lines. a) Object is approaching its final position, t=1.7 s. b) Object is in the final position. The second object is held in position as a consequence of decoupling, t=2.7 s.

The force depends on gradient of square of the magnitude of the electric field. Therefore, a non-uniform field is necessary for DEP. Coefficient K contained in the expression is known as *Clausius-Mossotti factor* (CM factor) describing frequency dependent behaviour of the particle.

Computation of the DEP force can only be carried out analytically for simple electrode layouts. For more complex structures it is necessary to use numerical methods. The influence function can be then approximated based on the numerical data. The electric field can be composed from individual contribution of the electrodes [26]. Example of the final solution showing the DEP force above parallel electrodes is visualized in Fig. 8. Voltage is applied to one of the electrode, which creates a local force field.

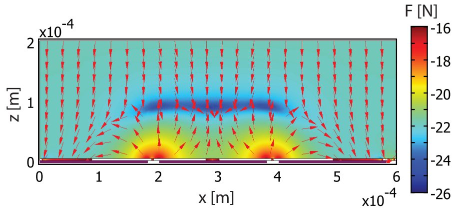
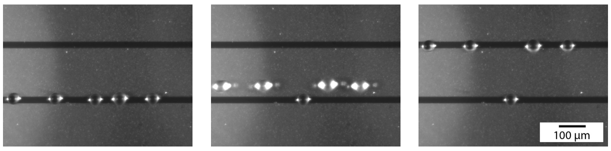


Fig. 8. Numerical solution for the total force acting on particles above interdigitated electrodes during DEP. The electrode in the middle is energized and creates local force field. Colour stands for logarithm of magnitude of the force, arrows for direction of the force.

Theoretical expectations raised in the simulations can also be approved experimentally. Motion of some micro particles is shown in a few snapshots in Fig. 9. The particles move away from energized electrode because of local force field.



(a) (b) (c)

Fig. 9. Motion of the particles (50micron polystyrene beads)caused by DEP. (a) Bottom electrode is energized with voltage 1 MHz, 24 Vpp and phase, the others grounded, t=0 s. (b) The particles move away from the edge, because there is a high gradient of the electric field. (c) The particles sink to a groove between the electrodes, t=1.2 s.

Exploration of applicability of the proposed results to DEP-based micromanipulation is the content of a continuating research of the authors.

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