

Forward Propagation

$$\text{net}_{h1} = W_1 i_1 + W_2 i_2 + b_1 \times 1$$

$$= 0.15 \cdot 0.05 + 0.2 \cdot 0.1 + 0.35 \cdot 1$$

$$\boxed{\text{net}_{h1} = 0.3775}$$

$$\text{out}_{h1} = \frac{1}{1 + e^{-x}} = \frac{1}{1 + e^{-0.3775}}$$

$$\boxed{\text{out}_{h1} = 0.593269992}$$

$$\text{net}_{h2} = W_3 i_1 + W_4 i_2 + b_1 \times 1$$

$$= 0.25 \cdot 0.05 + 0.3 \cdot 0.1 + 0.35 \cdot 1$$

$$\boxed{\text{net}_{h2} = 0.3925}$$

$$\text{out}_{h2} = \frac{1}{1 + e^{-x}} = \frac{1}{1 + e^{-0.3925}}$$

$$\boxed{\text{out}_{h2} = 0.596884378}$$

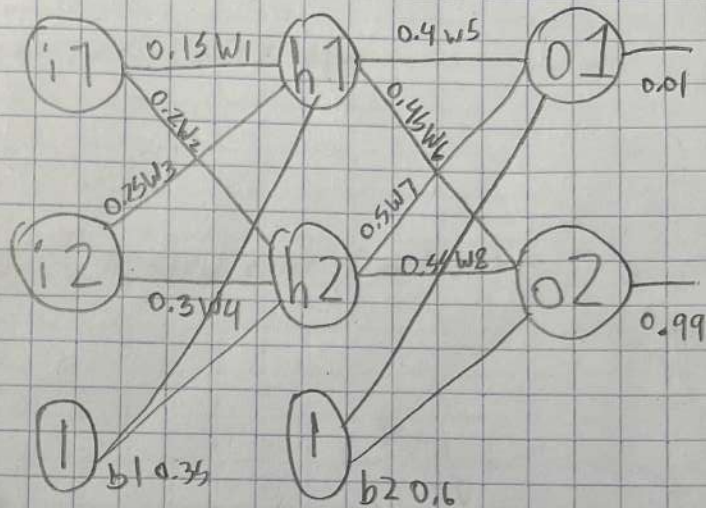
$$\text{net}_{o1} = W_5 (\text{out}_{h1}) + W_6 (\text{out}_{h2}) + b_2 \times 1$$

$$= 0.4 (0.593269992) + 0.45 (0.596884378) + 0.6 \times 1$$

$$\boxed{\text{net}_{o1} = 1.105905967}$$

$$\text{out}_{o1} = \frac{1}{1 + e^{-1.105905967}} = \frac{1}{1 + 0.330910951997}$$

$$\boxed{\text{out}_{o1} = 0.75136507}$$



$$\begin{aligned} \text{net}_{o2} &= W_7(\text{out}_{h1}) + W_8(\text{out}_{h2}) + b_2 \times 1 \\ &= 0.5(0.593269992) + 0.55(0.596884378) + 0.6 \times 1 \end{aligned}$$

$$\boxed{\text{net}_{o2} = 1.2249214039}$$

$$\text{out}_{o2} = \frac{1}{1 + e^{-1.2249214039}} = \frac{1}{1 + 0.29378078944}$$

$$\boxed{\text{out}_{o2} = 0.772928465}$$

Step 2: Calculating the total error (Squared error function)

$$E_{\text{total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$E_{o1} = \frac{1}{2} (0.01 - 0.75136507)^2$$

$$\boxed{E_{o1} = 0.274811083}$$

$$E_{o2} = \frac{1}{2} (0.99 - 0.772928465)^2$$

$$\boxed{E_{o2} = 0.023560026}$$

∴ The total error for the NN is:

$$E_{\text{total}} = E_{o1} + E_{o2}$$

$$= 0.274811083 + 0.023560026$$

$$\boxed{E_{\text{total}} = 0.298371109}$$

Backward Propagation

$$E_{\text{total}} = \frac{1}{2}(\text{target}_{01} - \text{out}_{01})^2 + \frac{1}{2}(\text{target}_{02} - \text{out}_{02})^2$$

$$\frac{\partial E_{\text{total}}}{\partial E_{\text{out}_{02}}} = \cancel{2} \frac{1}{\cancel{2}} (\text{target}_{02} - \text{out}_{02}) \cdot (0-1) + 0$$

$$= -(\text{target}_{02} - \text{out}_{02})$$

$$= -(0.99 - 0.772928465)$$

$$\boxed{\frac{\partial E_{\text{total}}}{\partial E_{02}} = -0.217071535}$$

$$\text{out}_{02} = \frac{1}{1 + e^{-\text{net}_{02}}}$$

$$\frac{\partial \text{out}_{02}}{\partial \text{net}_{02}} = \text{out}_{02}(1 - \text{out}_{02})$$

$$= 0.772928465(1 - 0.772928465)$$

$$\boxed{= 0.175510052993}$$

$$\frac{\partial \text{net}_{02}}{\partial w_7} = \frac{\partial}{\partial w_7} (w_7 \cdot \text{outh}_1) + \frac{\partial}{\partial w_7} (w_8 \cdot \text{outh}_2) + \frac{\partial}{\partial w_7} (b_2)$$

$$= \text{outh}_1 \cdot \frac{\partial w_7}{\partial w_7} + 0 + 0$$

$$= \text{outh}_1 \times 1$$

$$\boxed{= 0.593264992}$$

$$\frac{\partial E_{\text{total}}}{\partial w_7} = -0.217071535 \times 0.175510052993 \times 0.593264992$$

$$\boxed{= -0.0226025405295}$$

$$W_{\text{new}} = W_{\text{old}} - \alpha * \frac{\partial E_{\text{total}}}{\partial W_{\text{old}}}$$

$$W_7 = W_7 - \alpha * \frac{\partial E_{\text{total}}}{\partial W_7}$$

$$= 0,5 - 0,5 * -0,0226625403295$$

$$W_{7_{\text{new}}} = 0,511301270269$$

$$\frac{\partial E_{\text{total}}}{\partial W_3} = \frac{\partial E_{\text{total}}}{\partial \text{outh}_2} \times \frac{\partial \text{outh}_2}{\partial \text{net}_2} \times \frac{\partial \text{net}_2}{\partial W_3}$$

$$\frac{\partial E_{\text{total}}}{\partial \text{outh}_2} = \frac{\partial E_{01}}{\partial \text{outh}_2} + \frac{\partial E_{02}}{\partial \text{outh}_2}$$

$$\frac{\partial E_{01}}{\partial \text{outh}_2} = \frac{\partial E_{01}}{\partial \text{net}_{01}} \times \frac{\partial \text{net}_{01}}{\partial \text{outh}_2}$$

$$\frac{\partial E_{01}}{\partial \text{net}_{01}} = \frac{\partial E_{01}}{\partial \text{out}_{01}} \times \frac{\partial \text{out}_{01}}{\partial \text{net}_{01}}$$

$$= \frac{\partial}{\partial \text{out}_{01}} \left[\frac{1}{2} (\text{target}_{01} - \text{out}_{01})^2 \right] \times 0,186815602$$

$$= 1 \times \frac{1}{2} (\text{target}_{01} - \text{out}_{01}) \times \frac{\partial}{\partial \text{out}_{01}} (\text{target}_{01} - \text{out}_{01}) \times 0,186815602$$

$$= (\text{target}_{01} - \text{out}_{01}) \times (0-1) \times 0,186815602$$

$$\frac{\partial E_{01}}{\partial \text{net}_{01}} = 0,74136507 \times -0,186815602$$

$$= -0,138498562$$

$$\frac{\partial \text{net}_{o1}}{\partial \text{out}_{h2}} = \frac{\partial}{\partial \text{out}_{h2}} (w_5(\text{out}_{h1}) + w_6(\text{out}_{h2}) + b_2 \times 1)$$

$$= 0 + w_6 \times 1 + 0$$

$$= w_6 = 0.45$$

$$\therefore \frac{\partial E_{o1}}{\partial \text{out}_{h2}} = 0.138498562 \times 0.45$$

$$= 0.0623243529$$

$$\frac{\partial E_{o2}}{\partial \text{out}_{h2}} = \frac{\partial E_{o2}}{\partial \text{out}_{o2}} \times \frac{\partial \text{out}_{o2}}{\partial \text{net}_{o2}} \times \frac{\partial \text{net}_{o2}}{\partial \text{out}_{h2}}$$

$$= \frac{\partial}{\partial \text{out}_{o2}} \left[\frac{1}{2} (\text{target}_{o2} - \text{out}_{o2})^2 \right] \times \frac{\partial}{\partial \text{out}_{o2}} (1 - \text{out}_{o2})$$

$$\times \frac{\partial}{\partial \text{out}_{h2}} (w_7(\text{out}_{h1}) + w_8(\text{out}_{h2}) + b_2 \times 1)$$

$$= 2 \times \frac{1}{2} (\text{target}_{o2} - \text{out}_{o2}) \times (0-1)$$

$$= -(\text{target}_{o2} - \text{out}_{o2}) \times \text{out}_{o2}(1 - \text{out}_{o2}) \times w_8$$

$$= -(0.99 - 0.772928465) \times 0.772928465(1 - 0.772928465) \times 0.55$$

$$= -0.217071533 \times 0.175510053 \times 0.55$$

$$= -0.0209540299439$$

$$\therefore \frac{\partial E_{total}}{\partial E_{outhz}} = \frac{\partial E_{o1}}{\partial outhz} + \frac{\partial E_{o2}}{\partial outhz}$$

$$= 0.0623243529 + -0.0209540299439$$

$$= 0.041370322961$$

$$- \frac{\partial outhz}{\partial nethz}$$

$$\frac{\partial}{\partial nethz} \left(\frac{1}{1+e^{-nethz}} \right)$$

$$\left[\begin{aligned} f(x) &= \frac{1}{1+e^{-x}} \\ f'(x) &= f(x)(1-f(x)) \end{aligned} \right]$$

$$\frac{\partial outhz}{\partial nethz} = outhz(1-outhz) = 0.596884378(1-0.596884378)$$

$$= 0.2406134173$$

$$- \frac{\partial nethz}{\partial w_3}$$

$$\frac{\partial nethz}{\partial w_3} = \frac{\partial}{\partial w_3} (w_3 i_1 + w_4 i_2 + b_1 x_1)$$

$$= i_1 x_1 + 0 + 0$$

$$= i_1 = 0.5$$

$$\frac{\partial E_{total}}{\partial W_3} = \frac{\partial E_{total}}{\partial out_{hz}} \times \frac{\partial out_{hz}}{\partial net_{hz}} \times \frac{\partial net_{hz}}{\partial W_3}$$

$$= 0.0413703229561 \times 0.2406134173 \times 0.5$$

$$= 0.000497712739064$$

$$W_{new} = W_{old} - \alpha * \frac{\partial E_{total}}{\partial W_{old}}$$

$$W_{3new} = 0.25 - 0.5 * 0.000497712739064$$

$$W_{3new} = 0.24975114363$$