1. The probability of rolling a six on a dice is 1/6. On average, we need to roll the dice six times to get a six. This means we would need to pay a total of $12 in entry fees ($2 per roll). Assuming the game is fair, if we win, we will receive $12 in winnings, minus the $12 we paid in entry fees, leaving us with a net gain of $0. However, the probability of winning is only 1/6, so the expected value of playing the game is:

$$\mathrm{E(x)} = \sum x_i p(x_i)$$

$$\mathrm{E(x)} = \left(\frac{1}{6}\right)(12-2) - \left(\frac{5}{6}\right)(2)$$

$$\mathrm{E(x)} = \left(\frac{10}{6}\right) - \left(\frac{10}{6}\right)$$

$$\mathrm{E(x)} = 0$$

Therefore, the game is a truly fair game. I will play this game just for the thrill of it.

Q2

Other Online Courses

$P(Y) = \frac{8}{15}$

$P(N) = \frac{7}{15}$

$$\left.\begin{aligned} P(Pass|Y) &= \frac{5}{8} \\ P(Fail|Y) &= \frac{3}{8} \end{aligned}\right\} \text{Gini Index} = 1 - \left(\left(\frac{5}{8}\right)^2 + \left(\frac{3}{8}\right)^2\right) = 0.46875$$

$$\left.\begin{aligned} P(Pass|N) &= \frac{3}{7} \\ P(Fail|N) &= \frac{4}{7} \end{aligned}\right\} \text{Gini Index} = 1 - \left(\left(\frac{3}{7}\right)^2 + \left(\frac{4}{7}\right)^2\right) = 0.48980$$

$$\text{Gini Index for Other Online Courses} = \frac{8}{15}(0.46875) + \frac{7}{15}(0.48980) = \boxed{0.47857}$$

Student Background

$P(Maths) = \frac{7}{15}$

$P(CS) = \frac{4}{15}$

$P(other) = \frac{4}{15}$

$$\left.\begin{aligned} P(Pass|Maths) &= 4/7 \\ P(Fail|Maths) &= 3/7 \end{aligned}\right\} G.I. = 1 - \left(\left(\frac{4}{7}\right)^2 + \left(\frac{3}{7}\right)^2\right) = 0.48980$$

$$\left.\begin{aligned} P(Pass|CS) &= 4/4 \\ P(Fail|CS) &= 0 \end{aligned}\right\} G.I. = 1 - \left(\left(\frac{4}{4}\right)^2 + 0^2\right) = 0$$

$$\left.\begin{aligned} P(Pass|other) &= 0 \\ P(Fail|other) &= 4/4 \end{aligned}\right\} G.I. = 1 - \left(0^2 + \left(\frac{4}{4}\right)^2\right) = 0$$

$$\text{G.I. for Student Background } \frac{7}{15}(0.48980) + \frac{4}{15}(0) + \frac{4}{15}(0) = \boxed{0.22857}$$

Working Status

$P(NW) = 6/15$

$P(W) = 9/15$

$$\left.\begin{aligned} P(Pass|NW) &= 5/6 \\ P(Fail|NW) &= 1/6 \end{aligned}\right\} G.I. = 1 - \left(\left(\frac{5}{6}\right)^2 + \left(\frac{1}{6}\right)^2\right) = 0.27778$$

$$\left.\begin{aligned} P(Pass|W) &= 3/9 \\ P(Fail|W) &= 6/9 \end{aligned}\right\} G.I. \; 1 - \left(\left(\frac{3}{9}\right)^2 + \left(\frac{6}{9}\right)^2\right) = 0.44444$$

$$\text{G.I. for Working Status} = \frac{6}{15}(0.27778) + \frac{9}{15}(0.44444) = \boxed{0.37778}$$

$\therefore$ Student Background has the lowest Gini Index so it should be chosen for the root node.