

## Tutorial 1

Q1 Asymptotic Notations

Asymptotic Notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.  $\text{big } O$ ,  $\text{big } \Omega$ ,  $\text{big } \Theta$  are the different types of asymptotic notations.

Q2

$$\begin{array}{ll}
 2^0 & i=1 \\
 2^1 & i=2 \\
 2^2 & i=4 \\
 2^3 & i=8 \\
 2^4 & i=16
 \end{array}$$

$2^k$  (k times)  
for n values.

So  $2^k = n$

$$\log 2^k = \log n$$

$$k \log 2 = \log n$$

$$k = \frac{\log n}{\log 2}$$

$$k = \log_2 n$$

Hence the time complexity is  $O(\log n)$



Soln

$$T(n) = 3T(n-1) \quad \text{--- (1)} \quad T(0) = 1$$

$$\text{let } n = n-1$$

$$T(n-1) = 3T(n-1-1)$$

$$T(n-1) = 3T(n-2) \quad \text{--- (2)}$$

put (2) in (1)

$$T(n) = 3 \cdot 3T(n-2)$$

$$\text{put } n = n-2$$

$$T(n-2) = 3T(n-2-1)$$

$$T(n-2) = 3T(n-3) \quad \text{--- (3)}$$

put (3) in (1)

$$T(n) = 3 [3 \cdot 3T(n-3)] \quad \text{--- (3)}$$

So from above 3 equations we should obtain a gen.

$$T(n) = 3^k T(n-k)$$

$$\text{let } n-k = 0$$

$$\boxed{n = k}$$

$$T(n) = 3^k T(0)$$

$$\text{Here } T(0) = 1$$

$$\text{So, } T(n) = 3^k \cdot 1$$

$$= 3^n$$

$$\text{So time complexity is } 3^n = O(3^n)$$

Q4.  $T(n) = 2T(n-1) - 1 \quad \text{--- (1)}$

$$T(0) = 1$$

$$\text{let } n = n-1$$

$$\text{put } T(n-1) = 2T(n-1-1) - 1$$

$$T(n-1) = 2T(n-2) - 1$$



$$T(n) = 2[2T(n-2) - 1] - 1$$

$$T(n) = 4T(n-2) - 3 \quad \text{--- (2)}$$

$$n = n-2$$

$$T(n-2) = 2T(n-3) - 1$$

$$T(n) = 4[2T(n-3) - 1] \quad \text{--- (3)}$$

$$T(n) = 8T(n-3) - 4 \quad \text{--- (3)}$$

$$T(n) = 2^k [(n-k) - \{1 + 2^{k-1} + 2^{k-2} + \dots + 2^2 + 2^1 + 1\}]$$

$$\text{Let } n-k = 0$$

$$= 2^n T(0) - \{1 + 2 + 2^2 + \dots + 2^{k-1}\}$$

$$= 2^n \times 1 + 2^k + 1$$

$$= 2^n + 2^n + 1$$

$$= 2^n + 1$$

$$O(2^n)$$

is the given time complexity for given relation.

Soln. Here  $S_i = S_{i-1} + i$

the value of  $i$  increased by 1 for each iteration the value contained in 'S'

at the  $i$ th iteration is the sum

of the first  $i$  positive integers

If  $k$  is the total no. of iterations taken by program the loop like



$$1+2+3+\dots+k = \frac{k(k+1)}{2} > n$$

So,  $k = O(\sqrt{n})$   
 hence the time complexity is  $O(\sqrt{n})$   
 let

log	passes	let $n=16$
$i=1$	for $k=1$	1
$i=2$	for $k=2$	4
$i=3$	for $k=3$	9
$i=4$	for $k=4$	16
$\vdots$	$\vdots$	$\vdots$
$i=n$	for $k=n$	$n^2$

$$1 \quad 4 \quad 9 \quad 16 \quad \dots \quad n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= O(\log^2 1) + O(\log^2 2) + \dots + O(\log^2 n)$$

$$\leq C \cdot O(\log^2 n)$$

therefore time complexity  $= O(\log^2 n)$

Sol 7

(let  $n = 12$ )

i	j	k
6	1	1
7	2	2
8	4	4
9	8	8
8	16	16

(out of bound)

$$\text{So, } \left(\frac{n}{2}\right) \times (\log n) \times (\log n)$$

as const can be ignored.  
then for each value of  $i$  it iterates & checks the condition for  $k$ .

So, time complexity is like

$$= n \cdot (\log n \cdot \log n)$$

$$= O(n \log^2 n)$$

8

i	j	no. of times pass test
1	1	-1
2	2	-2
1	1	1
1	1	1
n	n	n time

$i = n$  times

$j = n$  times

$i \cdot j = n \cdot n$



$(n) \cdot (n)$  times

Here  $n = n-3$

$(n-3) \cdot (n-3)$

$O(n^2 + 9 - 6n)$

$= O(n^2)$  is time complexity

(9)

Let  $n = 12$

for  $i = 1$  to  $n$  {

for  $j = 1; j \leq n; j = j+1$

printf ("\* ");

}

$i=1, j=2, 3, 4, 5 \dots (n-1)$

$i=2, j=3, 4, 5, \dots (n-1)$

$i=3, j=4, 5, 6, \dots (n-1)$

$i=n, j=(n+1) \dots (n-1)$

for each value of  $i$  in  $n$ , it iterates through  $(n-1)$  times for  $n(n-1)$  time.

$$= n^2 - n$$

$$= O(n^2)$$

Hence the time complexity is  $O(n \log n)$