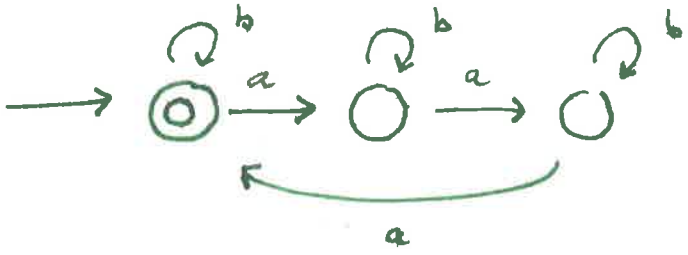
DFAs, NFAs, Reg. Exps.

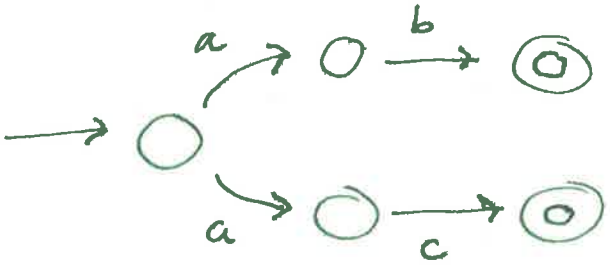
e.g.:



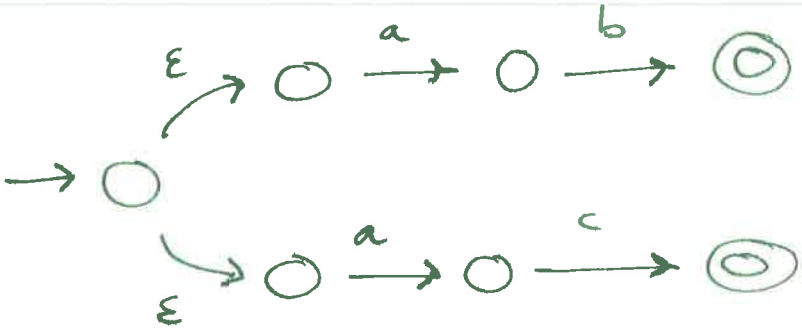
This Finite Automaton accepts strings (over $\Sigma = \{a, b\}$) with # a's divisible by 3.

Finite Automata
- → Deterministic (DFA)
- → Non-deterministic (NFA)

e.g. (NFA w/ Contradictory Trans.)



e.g. (NFA w/ ε-trans.)



defn a DFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

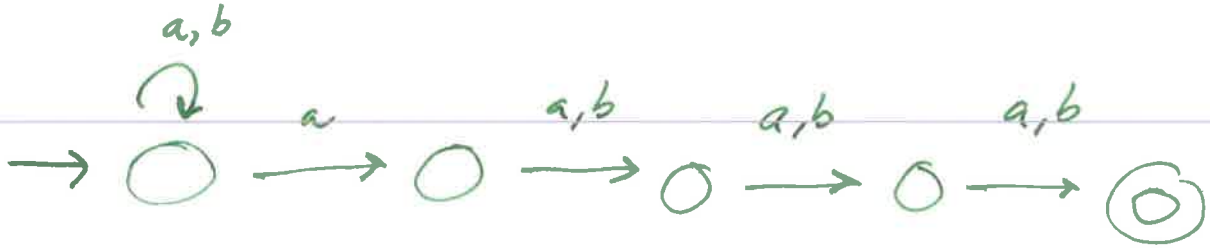
- Q is a finite set of states
- Σ is a finite set of symbols (alphabet)
- $\delta : Q \times \Sigma \to Q$ is a transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of final states.

defn An NFA is exactly the same except...

- $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \mathcal{P}(Q)$.
  is the non-det. trans. function

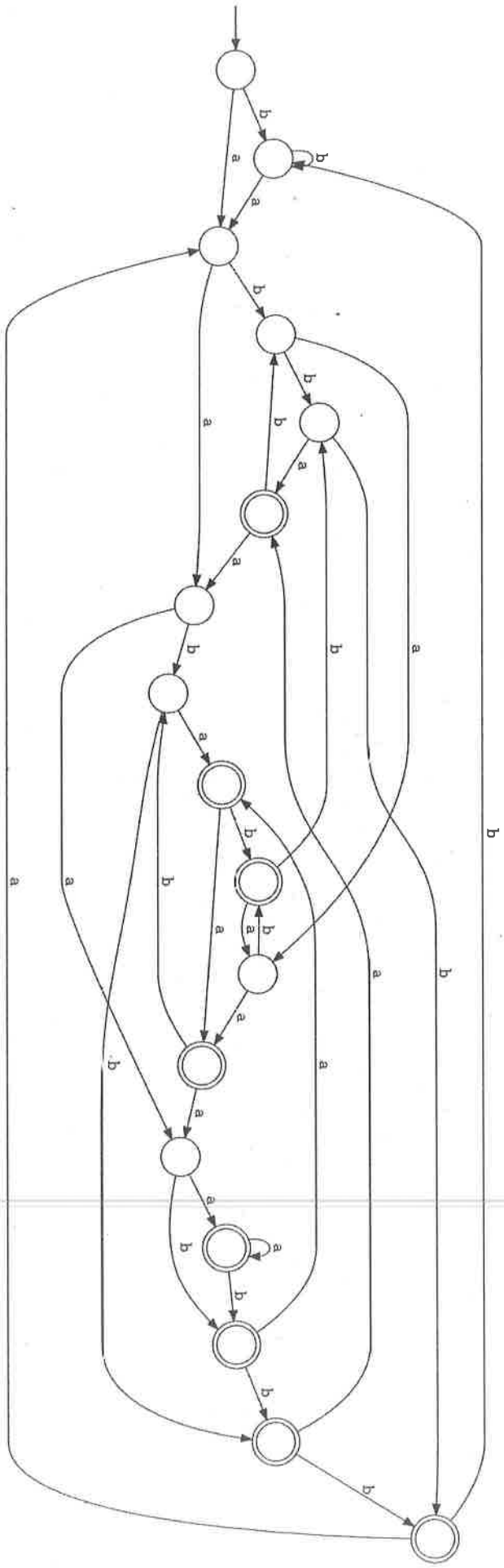
e.g. Write an NFA that accepts strings where the 4th to last character is an a (where $\Sigma = \{a, b\}$).

soln.

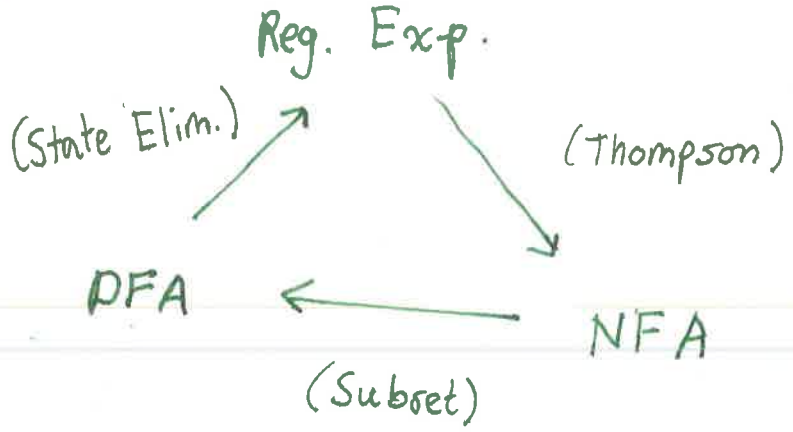


Question: Can we write a DFA for this?

Yes...

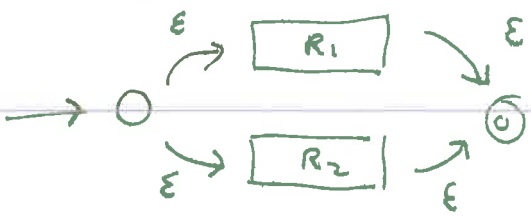
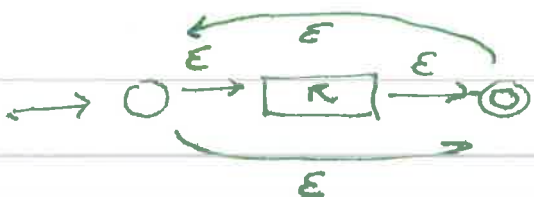


thm (Kleene's).

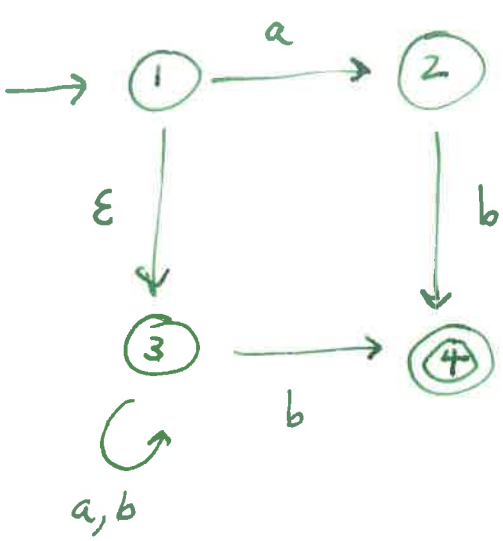


Reg. Exp. are equivalent to NFAs which are equivalent to DFAs.

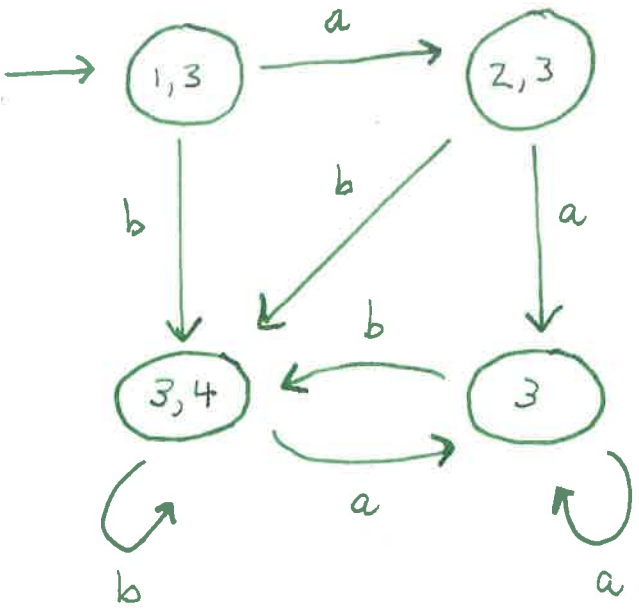
alg. (Thompson).

Reg Exp.	NFA
ε	
$a \in \Sigma$	
$R_1 R_2$	
$R_1 \mid R_2$	
R^*	

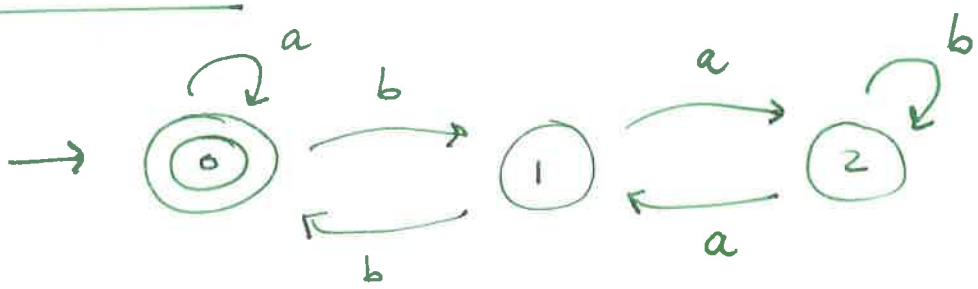
e.g. (subset)



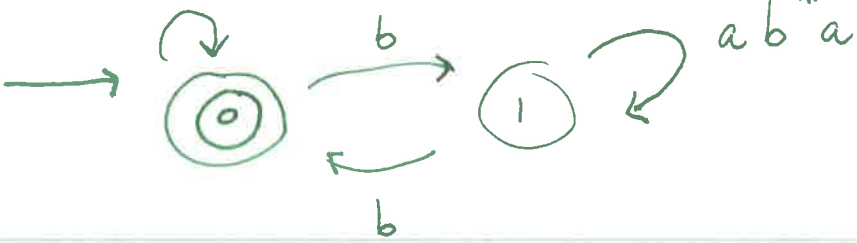
subset ⟹



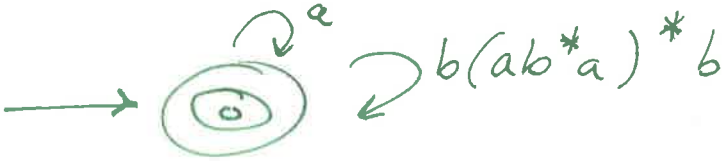
e.g. (State Elim.)



⇓ [elim 2]



⇓ [elim 1]



⇓

$$(a \mid (b(ab^*a)^*b))^*$$