**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:- since work beings 10 mins after the car is dropped, the time left to complete work is 50**

**Mins.**

**Probability that Service Manager can not meet his commitment**

**= p(x>50)**

**= 1-pr(x<=50)**

**Convert 50 to z\_scores**

**Standard normal variable Z=**

**P(x<=50)=p(z<=(50-45/8)=PR(z<=0.625)=0.73237=73.23%**

**Probability that service manager will not meet his commitment is 100-73.237=26.763%**

**= 0.2676**

**So the answer is B**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:- A)**  **Given: mean = 38**

**Standard deviation = 6**

**Z score = (value – mean)/sd**

**Z score for 44 = (44 – 38)/6 1 => 84.13%**

**=>People above 44 age = 100-84.13 = 15.87% ~ 63 out of 400**

**Z score for 38 = (38 -38)/6 = 0 => 50%**

**Hence people between 38 & 44 age = 84.13 – 50 = 34.13% ~ 137 out of 400**

**Hence more employees at the processing centre are older than 44 than between 38**

**and is FALSE**

B**) Z Score for 30 = (30-38)/6 = -1.33 = 9.15% ~ 36 out of 400**

**Hence A training program for employees under the age of 30 at the centre would be expected to attract about 36 emplloyees TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters

**Ans** **:-**  The difference between 2X1 And X1 +X2 is N(0,6σ^2)

According to the Central Limit Theorem, any large sum of independent identically distributed random variable is approximately normal.

The normal distribution is defined by two parameter, the mean ,µ, and the variance, σ^2 and written as X ~ N(µ,σ^2).

**Given** X1 ~ N(µ,σ^2) and X2 ~ N(µ , σ^2) are two independent identically distributed random variables.

From the properties of normal random variables,

If  X N( µ1 ,σ1^2,) and Y N ( µ1, σ2^2 , ) are two independent identically distributed random variable then

The sum of normal random variables is given by

X + Y ~ N(µ1 + µ2 , σ1 ^2+ σ2^2)

The difference of normal random variable is given by

X - Y ~ N(µ1 - µ2 , σ1 ^2+ σ2^2)

When Z = aX , the product of X is given by

Z ~ N(aµ1 ,a^2σ1^2,)

When Z = aX + bY , the linear combination of X and Y is given by

Z ~ N(aµ1 + b µ2, a^2 σ1 ^2+ b^2 σ2^2)

Given to find, 2X1

Thus, following the property of multiplication, we get

2 X1 ~ N(2µ, 2^2 σ^2) 🡺 2X1 ~ N (2 µ, 4 σ^2)

And the following the property of addition,

X1 + X2 ~ N(µ + µ , σ^2 + σ^2) ~ N(2 µ, 2 σ^2)

And the difference between the two is given by

2 X1 - (X1+ X2) ~ N(2 µ - 2 µ, 2 σ1 ^2 + 4 σ1 ^2) ~ N(0,6 σ^2)

The mean of 2 X1and X1 + X2 is same but the var(σ^2) of 2X1 is 2 times more then the variance of X1 + X2

The difference between the two says that the two given variables are identically and independently distributed,

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.

**Ans:-** Given:  p(a<x<b) = 0.99 ,m ean =100,standardDeviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                                         Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576)+100= 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year

**Ans:- Given that :** $1 = Rs. 45

Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

Thus, company’s profit :

P~ N(5+7,3^2 + 4^2)=N(12,5^2)

A):- 95% of the **probability** **lies**between 1.96 **standard deviations**of the **mean**.

Thus **range**is:

(12 - 1.96 \* 5,12 + 1.96 \* 5 )

= ($2.2M, $22.8M)

= (Rs. 99M, Rs. 1026M)

**B):- Fifth percentile**is calculated as: P(Z<= p-12/5) = 0.05

From p values of Z\_scores table, we get

p-12/5 = -1.644

p=12-8.22=3.78

thus at $3.78M dollars, or Rs 170.1M amount, 5th percentile of profit lies. Or 5th percentile of profit is Rs . 170.1M

**C**) :- Loss is when profit <0

Thus : p<0

The first division of company , thus have larger probability of making a loss in a given year.