code

March 28, 2024

[53]:
```
import pandas as pd
import numpy as np
from scipy.stats import shapiro, levene, f_oneway , chi2
from statsmodels.stats.multicomp import pairwise_tukeyhsd
import matplotlib.pyplot as plt
import seaborn as sns
from tabulate import tabulate
from scipy.stats import f
from scipy.stats import ttest_ind
from scipy.stats import kstest
import pingouin as pg
from statsmodels.stats.multitest import multipletests
```

1 Exam Performance

[54]:
```
data = pd.read_csv('exam_scores.csv')
```

Null Hypothesis (H0): Exam performance is not affected by type of schooling

Alternative Hypothesis (H1): Type of schooling affects exam performance

[55]:
```
group_stats = {
    'Groups': ['Home', 'Boarding', 'Regular'],
    'Count': [len(data['Home']), len(data['Boarding']), len(data['Regular'])],
    'Sum': [data['Home'].sum(), data['Boarding'].sum(), data['Regular'].sum()],
    'Average': [data['Home'].mean(), data['Boarding'].mean(), data['Regular'].
 ↪mean()],
    'Variance': [data['Home'].var(), data['Boarding'].var(), data['Regular'].
 ↪var()]
}
group_stats_df = pd.DataFrame(group_stats)
print(group_stats_df)
```

```
     Groups  Count   Sum    Average    Variance
0      Home     15  1182  78.800000  141.171429
1  Boarding     15  1078  71.866667   73.980952
2   Regular     15  1263  84.200000   50.457143
```

1.1 Check for Normality

[56]:
```
shapiro_normality_tests = {}
for col in data.columns:
    stat, p = shapiro(data[col])
    shapiro_normality_tests[col] = {'Shapiro-Wilk Statistic': stat, 'p-value':␣
↪p, 'Normality': p > 0.05}

lilliefors_normality_tests = {}
for col in data.keys():
    n = len(data[col])
    d, p = kstest(data[col], 'norm', args=(np.mean(data[col]), np.
↪std(data[col], ddof=1)))
    lilliefors_normality_tests[col] = {'Kolmogorov-Smirnov Statistic': d,␣
↪'p-value': p, 'Normality': p > 0.05}

print("Shapiro Normality Tests:")
print(pd.DataFrame(shapiro_normality_tests))
print("\n")
print("Kolmogorov-Smirnov Tests with Lilliefors Significance Correction:")
print(pd.DataFrame(lilliefors_normality_tests))
```

```
Shapiro Normality Tests:
                            Home  Boarding   Regular
Shapiro-Wilk Statistic  0.905427  0.969238   0.97489
p-value                 0.115225  0.846616  0.922709
Normality                   True      True      True


Kolmogorov-Smirnov Tests with Lilliefors Significance Correction:
                                  Home  Boarding   Regular
Kolmogorov-Smirnov Statistic  0.154949  0.114252  0.099591
p-value                       0.812204  0.976888  0.994779
Normality                         True      True      True
```

1.1.1 Check homogeneity of variances

[57]:
```
levene_mean = levene(data['Home'], data['Boarding'], data['Regular'],␣
↪center='mean')
levene_median = levene(data['Home'], data['Boarding'], data['Regular'],␣
↪center='median')
levene_trimmed_mean = levene(data['Home'], data['Boarding'], data['Regular'],␣
↪center='trimmed',proportiontocut=0.1)
levene_adjusted_df = levene(data['Home'], data['Boarding'], data['Regular'],␣
↪center='trimmed', proportiontocut=0.05)
print("\nHomogeneity of Variances Test:")
levene_test = pd.DataFrame({
```

```
    'Center': ['Mean', 'Median', 'Trimmed Mean','Adjusted df'],
    'Test-Statistic': [levene_mean.statistic, levene_median.statistic,␣
 ↪levene_trimmed_mean.statistic,levene_adjusted_df.statistic],
    'p-value': [levene_mean.pvalue, levene_median.pvalue, levene_trimmed_mean.
 ↪pvalue,levene_adjusted_df.pvalue]
})

print(levene_test)
if levene_mean.pvalue > 0.05:
    print("Variance is homogenous based on mean")
elif levene_median.pvalue > 0.05:
    print("Variance is homogenous based on median")
elif levene_trimmed_mean.pvalue > 0.05:
    print("Variance is homogenous based on trimmed mean")
elif levene_adjusted_df.pvalue > 0.05:
    print("Variance is homogenous based on adjusted df")
else:
    print("Variance is not homogenous")


if levene_mean.pvalue > 0.05:
    print("\nOne-Way ANOVA Test is chosen")
else:
    print("\nRobust Welch's ANOVA Test is chosen")
```

```
Homogeneity of Variances Test:
         Center  Test-Statistic   p-value
0          Mean        1.674937  0.199589
1        Median        1.647691  0.204693
2  Trimmed Mean        1.948518  0.157222
3   Adjusted df        1.674937  0.199589
Variance is homogenous based on mean

One-Way ANOVA Test is chosen
```

1.1.2 Check for sphericity of variances

[58]:
```
data = pd.DataFrame(
    {
        'Home': data['Home'],
        'Boarding': data['Boarding'],
        'Regular': data['Regular']
    }
)
mauchly_test = pg.sphericity(data)
print("\nMauchly's Test of Sphericity:")
```

```
print(mauchly_test)
statistic_value = mauchly_test[1]
p_value = mauchly_test[4]

if p_value > 0.05:
    print("Sphericity is assumed")
else:
    print("Sphericity is not assumed")
```

```
Mauchly's Test of Sphericity:
SpherResults(spher=True, W=0.9227681119421941, chi2=1.044905017549741, dof=2,
pval=0.5930642676095743)
Sphericity is assumed
```

2 One-way ANOVA

[59]:
```
Home = data['Home']
Boarding = data['Boarding']
Regular = data['Regular']

k = 3
N = len(Home) + len(Boarding) + len(Regular)
group_means = [np.mean(Home), np.mean(Boarding), np.mean(Regular)]
grand_mean = np.mean([np.mean(Home), np.mean(Boarding), np.mean(Regular)])
SSb = sum([len(Home) * (group_means[0] - grand_mean) ** 2,
           len(Boarding) * (group_means[1] - grand_mean) ** 2,
           len(Regular) * (group_means[2] - grand_mean) ** 2])
dfb = k-1
MSb = SSb / dfb
SSw = sum([(x - group_means[i]) ** 2 for i, data in enumerate([Home, Boarding,␣
 ↪Regular]) for x in data])
dfw = N-k
MSw = SSw / dfw
F_value = MSb / MSw
alpha = 0.05
F_crit = f.ppf(1 - alpha, dfb, dfw)
p_value = 1-f.cdf(F_value, dfb, dfw)


anova_table = [
    ["Between Groups", f"{SSb:.6f}", dfb, f"{MSb:.6f}", f"{F_value:.
 ↪6f}",f"{p_value:.6f}" ,f"{F_crit:.6f}"],
    ["Within Groups", f"{SSw:.6f}", dfw, f"{MSw:.6f}"],
    ["Total", f"{SSb+SSw:.6f}", dfb+dfw]
]
```

```
print("ANOVA Table")
print(tabulate(anova_table, headers=["Source of Variation", "SS", "df", "MS",␣
↪"F", "p-value","F crit"], tablefmt="pretty"))
```

```
ANOVA Table
+----------------------+-------------+----+------------+----------+----------+---
-------+
| Source of Variation |      SS     | df |     MS     |    F     | p-value  |  F
crit  |
+----------------------+-------------+----+------------+----------+----------+---
-------+
|   Between Groups    | 1146.711111 | 2  | 573.355556 | 6.475922 | 0.003537 |
3.219942 |
|    Within Groups    | 3718.533333 | 42 | 88.536508  |          |          |
|
|        Total        | 4865.244444 | 44 |            |          |          |
|
+----------------------+-------------+----+------------+----------+----------+---
-------+
```

[60]:
```
anova_result = f_oneway(data['Home'], data['Boarding'], data['Regular'])
print("\nOne-way ANOVA Test:")
print(f"F-statistic: {anova_result.statistic}")
print(f"p-value: {anova_result.pvalue}")

if anova_result.pvalue < 0.05:
    print("\nSince p-value < 0.05, there are significant differences between␣
↪groups. Using a one way ANOVA we observed that the schooling method has a␣
↪significant effect on exam  performance")
    print("Main efffect(F) is significant")
else:
    print("\nNo significant differences between groups.")
```

```
One-way ANOVA Test:
F-statistic: 6.475922406683641
p-value: 0.003536773789503349

Since p-value < 0.05, there are significant differences between groups. Using a
one way ANOVA we observed that the schooling method has a significant effect on
exam  performance
Main efffect(F) is significant
```

3 Effect size calculation

[61]:
```
Effect_size = SSb / (SSb + SSw)
print(f"Effect Size: {Effect_size:.6f}")
print(f"Type of schooling explains {Effect_size*100:.2f}% of the variance in␣
↪exam performance")
```

```
Effect Size: 0.235694
Type of schooling explains 23.57% of the variance in exam performance
```

We know there is difference between the groups,but which groups perform better or worse?

- Planned comparison (contrast) – prior to experiment (based on the literature)
- Regular schooling > (boarding or home school)
- Regular schooling – Control condition
- Boarding school – Experimental condition 1
- Home school – Experimental condition 2
- But as the no. of planned comparisons increase (>2 comparisons), the alpha level has to adjusted, again to avoid Type I error. This is done by dividing the alpha level by the no. of comparisons. This is called Bonferroni correction.

3.0.1 Post-hoc Bonferroni for group comparisons

[62]:
```
t_statistic_home_boarding, p_value_home_boarding = ttest_ind(Home, Boarding)
t_statistic_boarding_regular, p_value_boarding_regular = ttest_ind(Boarding,␣
↪Regular)
t_statistic_regular_home, p_value_regular_home = ttest_ind(Regular, Home)

alpha = 0.05
alpha_corrected = alpha / 3

print("alpha corrected: ", alpha_corrected)

p_value_home_boarding_corrected = p_value_home_boarding * 3
p_value_boarding_regular_corrected = p_value_boarding_regular * 3
p_value_regular_home_corrected = p_value_regular_home * 3

table_data = [
    ['Groupwise comparisons', 'T-test p-value', 'Bonferroni-corrected p-value'],
    ['Home vs Boarding', p_value_home_boarding,␣
↪p_value_home_boarding_corrected],
    ['Boarding vs Regular', p_value_boarding_regular,␣
↪p_value_boarding_regular_corrected],
    ['Regular vs Home', p_value_regular_home, p_value_regular_home_corrected]
]
print(tabulate(table_data, headers="firstrow", tablefmt="grid"))
```

```
alpha corrected:  0.016666666666666666
+--------------------------+------------------+--------------------------------+
| Groupwise comparisons    |   T-test p-value |   Bonferroni-corrected p-value |
+==========================+==================+================================+
| Home vs Boarding         |       0.07781    |                    0.23343     |
+--------------------------+------------------+--------------------------------+
| Boarding vs Regular      |       0.00019644 |                    0.000589321 |
+--------------------------+------------------+--------------------------------+
| Regular vs Home          |       0.142042   |                    0.426125    |
+--------------------------+------------------+--------------------------------+
```

3.1 Holm method for multiple comparisons

[63]:
```python
datasets = [('Home', data['Home']), ('Boarding', data['Boarding']), ('Regular',␣
↪data['Regular'])]
alpha = 0.05
p_values = []
for i in range(len(datasets)):
    for j in range(i + 1, len(datasets)):
        group1_name, group1_data = datasets[i]
        group2_name, group2_data = datasets[j]
        t_stat, p_value = ttest_ind(group1_data, group2_data)
        p_values.append(p_value)

table = []
table.append(['Group 1', 'Group 2', 'Significant Difference', 'p-corrected'])
reject, p_values_corrected, _, _ = multipletests(p_values, alpha=alpha,␣
↪method='holm')
index = 0
for i in range(len(datasets)):
    for j in range(i + 1, len(datasets)):
        group1_name, _ = datasets[i]
        group2_name, _ = datasets[j]
        if reject[index]:
            print(f"There is a significant difference between {group1_name} and␣
↪{group2_name} (p-corrected = {p_values_corrected[index]})\n")
            table.append([group1_name, group2_name, "Yes",␣
↪f"{p_values_corrected[index]}"])
        else:
            print(f"No significant difference between {group1_name} and␣
↪{group2_name} (p-corrected = {p_values_corrected[index]})\n")
            table.append([group1_name, group2_name, "No",␣
↪f"{p_values_corrected[index]}"])
        index += 1
print(tabulate(table, headers='firstrow', tablefmt='grid'))
```

```
No significant difference between Home and Boarding (p-corrected =
```

```
0.1556199753902239)

No significant difference between Home and Regular (p-corrected =
0.1556199753902239)

There is a significant difference between Boarding and Regular (p-corrected =
0.0005893211647290649)

+-----------+-----------+------------------------+---------------+
| Group 1   | Group 2   | Significant Difference |   p-corrected |
+===========+===========+========================+===============+
| Home      | Boarding  | No                     |   0.15562     |
+-----------+-----------+------------------------+---------------+
| Home      | Regular   | No                     |   0.15562     |
+-----------+-----------+------------------------+---------------+
| Boarding  | Regular   | Yes                    |   0.000589321 |
+-----------+-----------+------------------------+---------------+
```

3.1.1 Tukeýs HSD post-hoc test

[64]:
```python
if anova_result.pvalue < 0.05:
    data_melted = pd.melt(data)
    posthoc = pairwise_tukeyhsd(data_melted['value'], data_melted['variable'], alpha=0.05)
    print(posthoc)
    print(posthoc.q_crit)
    HSD = posthoc.q_crit*np.sqrt(MSw / len(data))
    print(f"HSD: {HSD:.6f}")
    print(f"The mean difference between any two samples must be more than {HSD:.6f} at alpha = 0.05 for the difference to be statistically significant")
```

```
  Multiple Comparison of Means - Tukey HSD, FWER=0.05
=======================================================
 group1   group2 meandiff p-adj   lower   upper  reject
-------------------------------------------------------
Boarding    Home   6.9333 0.1204  -1.414 15.2806  False
Boarding Regular  12.3333 0.0024   3.986 20.6806   True
    Home Regular      5.4  0.269 -2.9473 13.7473  False
-------------------------------------------------------
3.4358230206770175
HSD: 8.347306
The mean difference between any two samples must be more than 8.347306 at alpha
= 0.05 for the difference to be statistically significant
```

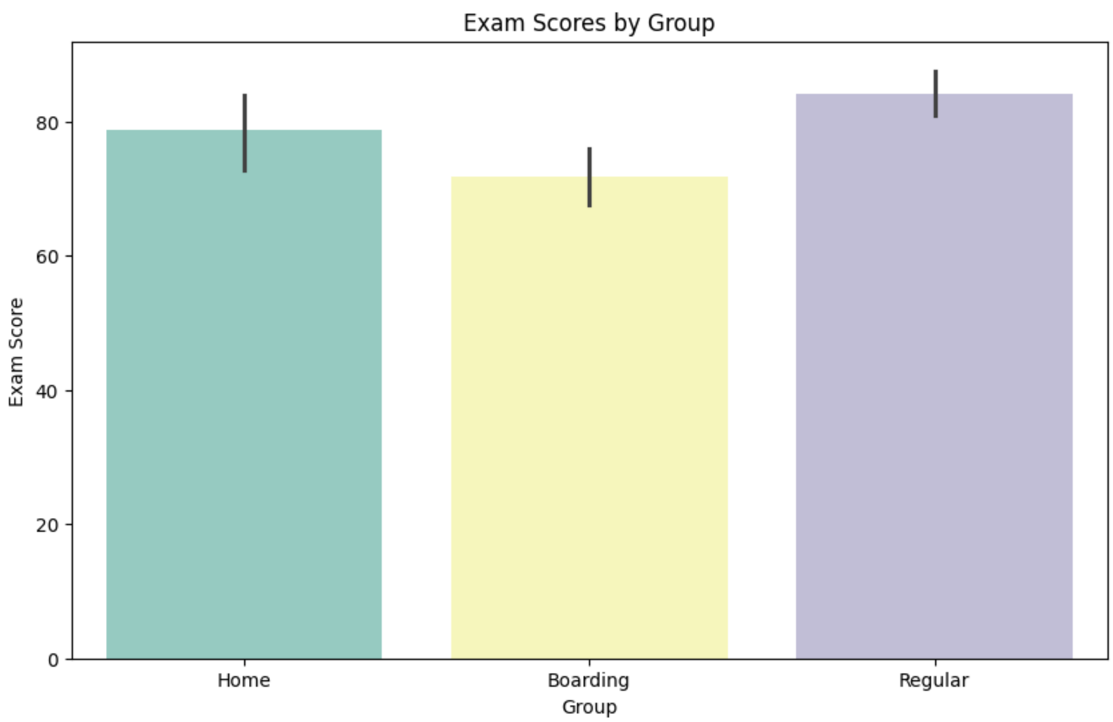
Using Bonferroni post-hoc test, we found that regular school resulted in better exam performance than boarding school (p<.001). There was no significant difference between the other groups

Using a one way ANOVA we observed that the schooling method has a significant effect on exam performance

Using Bonferroni post-hoc test, we found that regular school resulted in better exam performance than boarding school (p<.001). There was no significant difference between the other groups.

4 Plot Analyzed Data

[65]:
```
data_melted = pd.melt(data)
plt.figure(figsize=(10, 6))
sns.barplot(x='variable', y='value', data=data_melted, errorbar=('ci', 95),␣
 ↪hue='variable', palette="Set3")
plt.title('Exam Scores by Group')
plt.xlabel('Group')
plt.ylabel('Exam Score')
plt.show()
print("Error bars denote confidence intervals (CI) of 95%")
```




```
Error bars denote confidence intervals (CI) of 95%
```

[66]:
```
plt.figure(figsize=(10, 6))
```

```
sns.boxplot(x='variable', y='value', hue='variable', data=data.melt(),
    palette="Set3", legend=False)
plt.title('Exam Scores by Group')
plt.xlabel('Group')
plt.ylabel('Exam Score')
plt.show()
```