

2021101113

April 11, 2024

Regression Assignment  
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```
[58]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import warnings
warnings.filterwarnings("ignore")
from statsmodels.stats.diagnostic import het_breuschpagan
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
import statsmodels.api as sm
from scipy.stats import pearsonr
from scipy.stats import bartlett
from statsmodels.stats.diagnostic import het_breuschpagan
from statsmodels.compat import lzip
```

## 1 PART 1

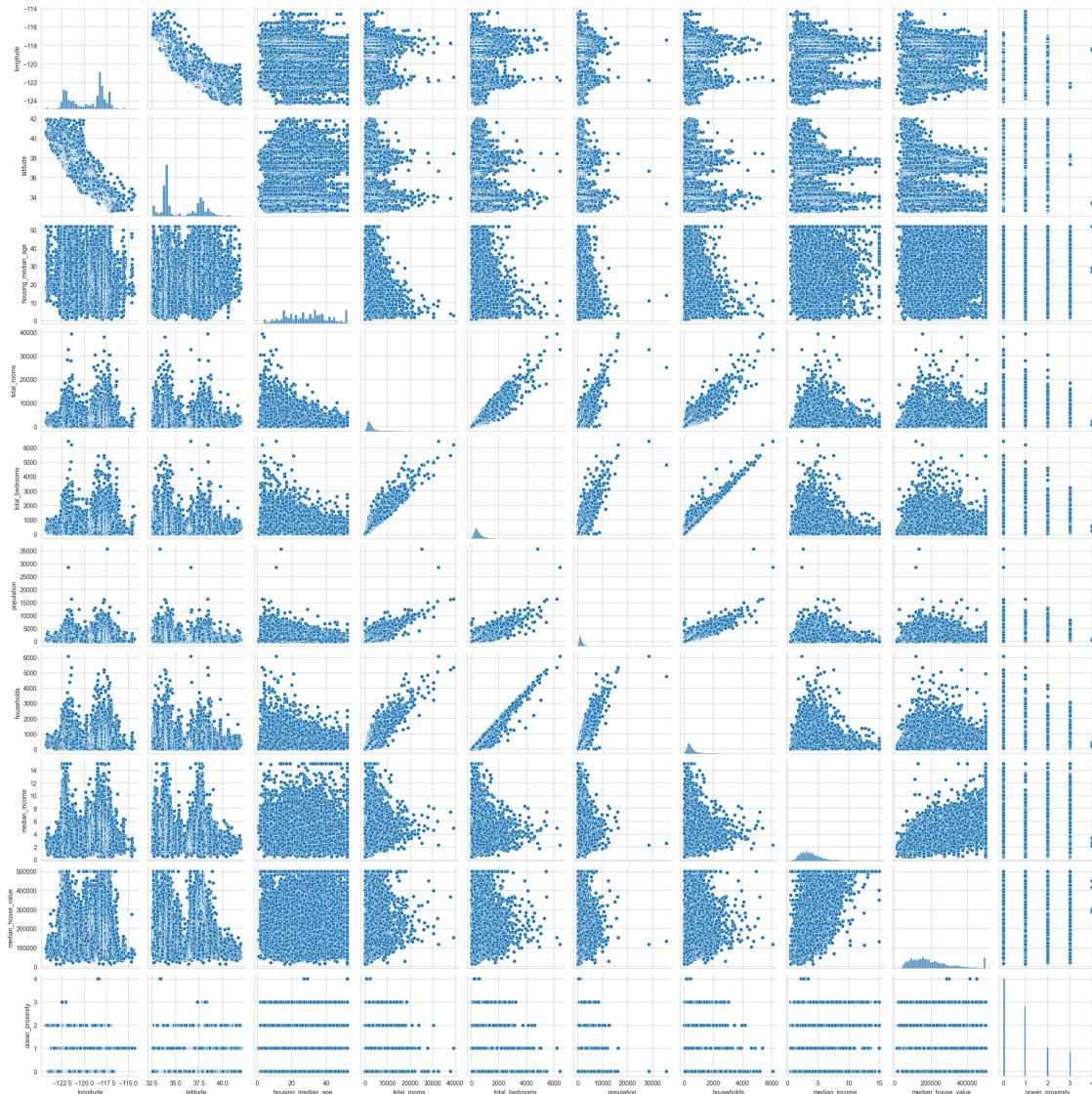
```
[59]: df = pd.read_csv("housing.csv")
counts = df['ocean_proximity'].value_counts()
```

We can see that ocean\_proximity is having string variables. Lets convert it to numericals before we perform the correlation analysis

```
[60]: # cleaning the data by removing the nan values and changing data to numerical
      ↪ variables
df['ocean_proximity'] = df['ocean_proximity'].map({'<1H OCEAN':0, 'INLAND':1,
      ↪ 'NEAR OCEAN':2, 'NEAR BAY':3, 'ISLAND':4})
df = df.dropna()
```

## 1.1 Visualize some correlations between variables in the data set

```
[61]: sns.pairplot(df.dropna())  
plt.show()  
plt.tight_layout()
```



<Figure size 640x480 with 0 Axes>

```
[62]: numeric_cols = df.columns  
num_rows = 5  
num_cols = 2  
fig, axes = plt.subplots(num_rows, num_cols, figsize=(15, 20))  
axes = axes.flatten()  
for i, col_name in enumerate(numeric_cols):
```

```
axes[i].hist(df[col_name], bins=50, color='purple')
axes[i].set_title(f'Histogram of {col_name}')
axes[i].set_xlabel(col_name)
axes[i].set_ylabel('Frequency')

plt.tight_layout()
plt.show()
```



```
[63]: df_1 = df['median_income']
df_1 = np.array(pd.DataFrame(df_1, columns=['median_income'])).reshape(-1, 1)
```

```

y = df['median_house_value']
df_2 = df.copy()
df_2 = df_2.drop('median_house_value', axis=1)

plt.figure(figsize=(12, 6))
sns.distplot(df['median_income'], bins=50, color='purple')
plt.title('Median Income Distribution')
plt.xlabel('Median Income')
plt.ylabel('Frequency')
plt.show()

```

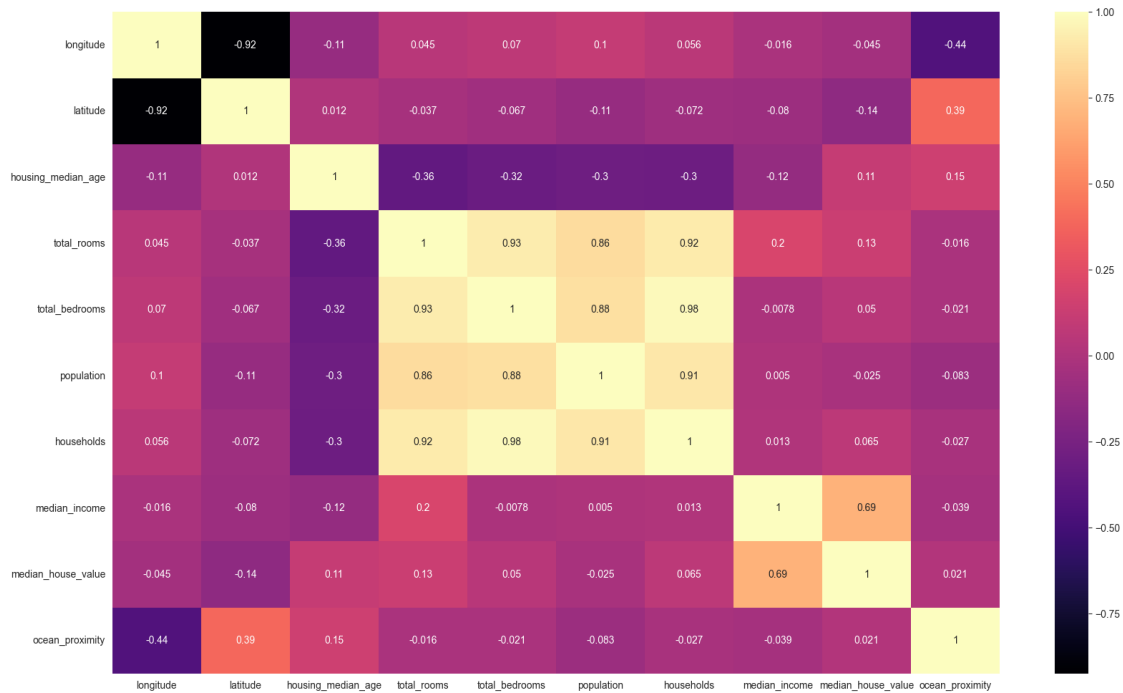


```

[64]: sns.set_style('whitegrid')
plt.figure(figsize=(20, 12))
sns.heatmap(df.corr(), annot=True, cmap='magma')

```

[64]: <Axes: >



We can clearly see some of the variables are highly correlated, now lets perform a correlation test to confirm the collinearity before building the model

```
[65]: def correlation_test(data1, data2, alternative):
    corr, p_value = pearsonr(data1, data2)
    print("Correlation coefficient:", corr)
    print("p-value:", p_value)
    print("Alternative hypothesis:", alternative)
    if alternative == "greater":
        if p_value/2 < 0.05:
            print("Reject the null hypothesis: There is a positive correlation_
↪between the two variables")
        else:
            print("Fail to reject the null hypothesis: There is no positive_
↪correlation between the two variables")
    elif alternative == "less":
        if p_value/2 < 0.05:
            print("Reject the null hypothesis: There is a negative correlation_
↪between the two variables")
        else:
            print("Fail to reject the null hypothesis: There is no negative_
↪correlation between the two variables")
    else:
        if p_value < 0.05:
```

```

        print("Reject the null hypothesis: There is a correlation between_
↪the two variables")
    else:
        print("Fail to reject the null hypothesis: There is no correlation_
↪between the two variables")

print("Correlation test for total_bedrooms and total_rooms:")
correlation_test(df['total_bedrooms'], df['total_rooms'], alternative="greater")
print("\nCorrelation test for households and population:")
correlation_test(df['households'], df['population'], alternative="greater")
print("\nCorrelation test for longitude and latitude:")
correlation_test(df['longitude'], df['latitude'], alternative="less")

```

Correlation test for total\_bedrooms and total\_rooms:

Correlation coefficient: 0.930377047611133

p-value: 0.0

Alternative hypothesis: greater

Reject the null hypothesis: There is a positive correlation between the two variables

Correlation test for households and population:

Correlation coefficient: 0.9071823610456953

p-value: 0.0

Alternative hypothesis: greater

Reject the null hypothesis: There is a positive correlation between the two variables

Correlation test for longitude and latitude:

Correlation coefficient: -0.9246131238737124

p-value: 0.0

Alternative hypothesis: less

Reject the null hypothesis: There is a negative correlation between the two variables

Based on the correlation tests conducted earlier, it's evident that whenever the p-value falls below 0.05, indicating a significant correlation, utilizing just one of the variables from the correlated pair is adequate for model construction.

We constructed three linear regression models by selecting only one variable from each highly correlated pair, effectively reducing the dimensions by three in each model. In the third model, we employed only two variables with notably high absolute correlation values. Notably, in all cases, the p-value was below 0.05, indicating a strong fit of the model to the data.

## 1.2 Pick 2 linear regression models to predict median house value

### 1.2.1 Method 1 : Model 1 - Linear Regression

```
[66]: f1 = 'median_house_value ~ longitude + housing_median_age + total_rooms +  
      ↪households + median_income + ocean_proximity'  
model = sm.formula.ols(formula=f1, data=df)  
result = model.fit()  
r1 = result  
print(result.summary())
```

```
OLS Regression Results  
=====
```

Dep. Variable:	median_house_value	R-squared:	0.538
Model:	OLS	Adj. R-squared:	0.538
Method:	Least Squares	F-statistic:	3968.
Date:	Fri, 12 Apr 2024	Prob (F-statistic):	0.00
Time:	01:28:33	Log-Likelihood:	-2.5926e+05
No. Observations:	20432	AIC:	5.185e+05
Df Residuals:	20425	BIC:	5.186e+05
Df Model:	6		
Covariance Type:	nonrobust		

```
=====
```

	coef	std err	t	P> t	[0.025
0.975]					
-----					
Intercept	-1.134e+05	3.63e+04	-3.122	0.002	-1.85e+05
-4.22e+04					
longitude	-546.5557	305.900	-1.787	0.074	-1146.144
53.033					
housing_median_age	1834.7140	47.477	38.644	0.000	1741.655
1927.773					
total_rooms	-18.3783	0.737	-24.940	0.000	-19.823
-16.934					
households	131.6558	4.062	32.412	0.000	123.694
139.618					
median_income	4.715e+04	328.365	143.605	0.000	4.65e+04
4.78e+04					
ocean_proximity	2810.3734	614.282	4.575	0.000	1606.331
4014.416					
-----					
Omnibus:	4186.301	Durbin-Watson:	0.903		
Prob(Omnibus):	0.000	Jarque-Bera (JB):	11124.625		
Skew:	1.107	Prob(JB):	0.00		
Kurtosis:	5.858	Cond. No.	2.30e+05		

```
=====
```



Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 2.3e+05. This might indicate that there are strong multicollinearity or other numerical problems.

### 1.2.2 Check for collinearity using VIF to remove highly correlated variables from the models

```
[67]: from statsmodels.stats.outliers_influence import variance_inflation_factor
features = ['longitude', 'housing_median_age', 'total_rooms', 'households',
           ↪ 'median_income', 'ocean_proximity']
X = df[features]
vif_data_f1 = pd.DataFrame()
vif_data_f1["Feature"] = X.columns
vif_data_f1["VIF"] = [variance_inflation_factor(X.values, i) for i in
           ↪ range(len(X.columns))]
print(vif_data_f1)
```

	Feature	VIF
0	longitude	17.077745
1	housing_median_age	7.320093
2	total_rooms	21.136321
3	households	21.646955
4	median_income	6.654595
5	ocean_proximity	1.889437

To address collinearity, we utilized the Variance Inflation Factor (VIF) to detect multicollinearity. A VIF value above 5 suggests significant multicollinearity within the model, indicating the need for further adjustments.

A VIF exceeding 5 presents a potential issue. Therefore, in our scenario, we could address multicollinearity by eliminating either ‘total\_rooms’ or ‘households’, as they exhibit high correlation with each other.

```
[68]: f1_modified = 'median_house_value ~ longitude + housing_median_age + households
           ↪ + median_income + ocean_proximity'
model_modified = sm.formula.ols(formula=f1_modified, data=df)
result_modified = model_modified.fit()
print(result_modified.summary())
```

#### OLS Regression Results

```
=====
Dep. Variable:    median_house_value    R-squared:                0.524
Model:                OLS              Adj. R-squared:           0.524
Method:             Least Squares      F-statistic:             4500.
Date:                Fri, 12 Apr 2024  Prob (F-statistic):        0.00
```

Time: 01:28:33 Log-Likelihood: -2.5957e+05  
 No. Observations: 20432 AIC: 5.192e+05  
 Df Residuals: 20426 BIC: 5.192e+05  
 Df Model: 5  
 Covariance Type: nonrobust

```
=====
=====
              coef      std err          t      P>|t|      [0.025
0.975]
-----
-----
Intercept      -1.094e+05    3.69e+04     -2.967    0.003   -1.82e+05
-3.71e+04
longitude      -575.7428    310.513     -1.854    0.064   -1184.373
32.888
housing_median_age  2064.8016    47.275     43.676    0.000    1972.139
2157.464
households       37.5906     1.531     24.557    0.000     34.590
40.591
median_income   4.338e+04    295.852    146.636    0.000    4.28e+04
4.4e+04
ocean_proximity 1748.4443    622.051     2.811    0.005     529.175
2967.714
=====
Omnibus:            4220.055   Durbin-Watson:           0.832
Prob(Omnibus):      0.000   Jarque-Bera (JB):       10450.912
Skew:               1.142   Prob(JB):                0.00
Kurtosis:           5.657   Cond. No.                4.21e+04
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 4.21e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
[69]: features_modified = ['longitude', 'housing_median_age', 'households',
    ↪ 'median_income', 'ocean_proximity']
X_modified = df[features_modified]
vif_data_f1_modified = pd.DataFrame()
vif_data_f1_modified["Feature"] = X_modified.columns
vif_data_f1_modified["VIF"] = [variance_inflation_factor(X_modified.values, i)
    ↪ for i in range(len(X_modified.columns))]
print(vif_data_f1_modified)
```

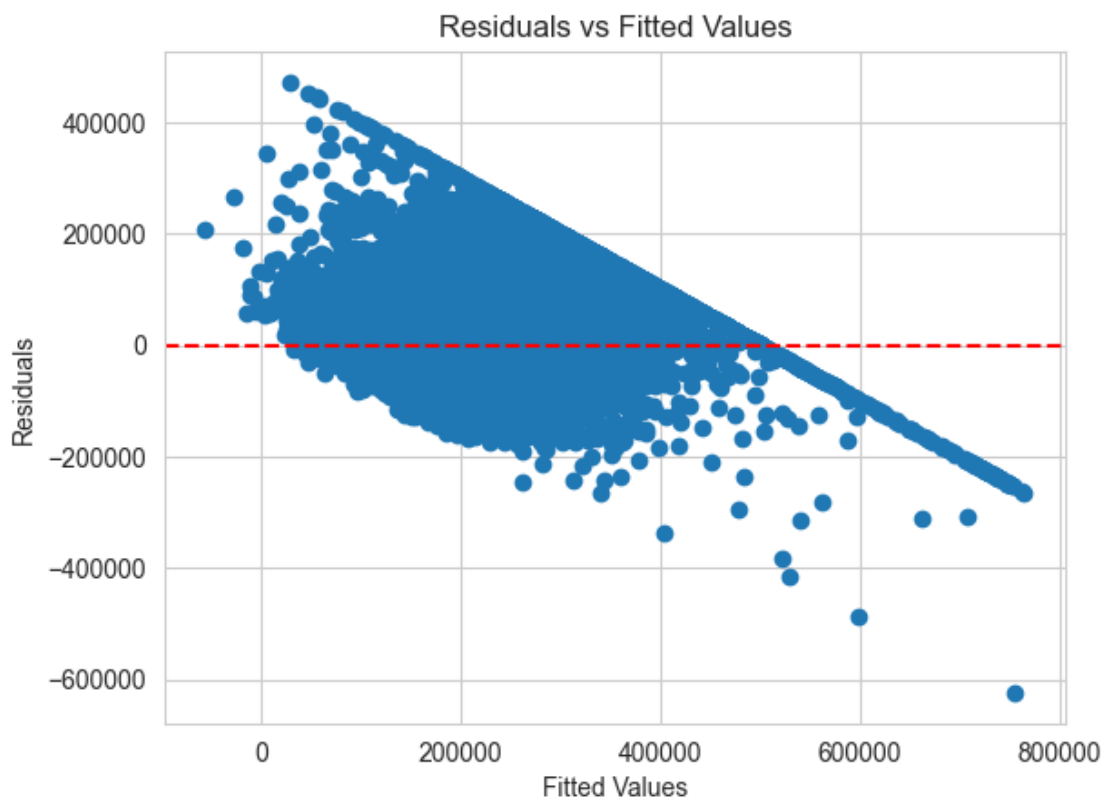
	Feature	VIF
0	longitude	16.775420
1	housing_median_age	7.043675

```
2      households    2.972707
3      median_income  5.242511
4      ocean_proximity 1.877785
```

### 1.2.3 Plot the distribution of the residuals against the fitted values to check for heteroscedasticity

```
[70]: my_resid = result.resid
      my_fitted = result.fittedvalues

      # Create scatter plot
      plt.scatter(my_fitted, my_resid)
      plt.axhline(y=0, color='red', linestyle='--') # Add a red line at y=0
      plt.title("Residuals vs Fitted Values")
      plt.xlabel("Fitted Values")
      plt.ylabel("Residuals")
      plt.show()
```



Since plot of residuals against fitted values is not constant, it means that there is heteroscedasticity in our data

### 1.2.4 Use `ncvTest` or equivalent to test for heteroscedasticity

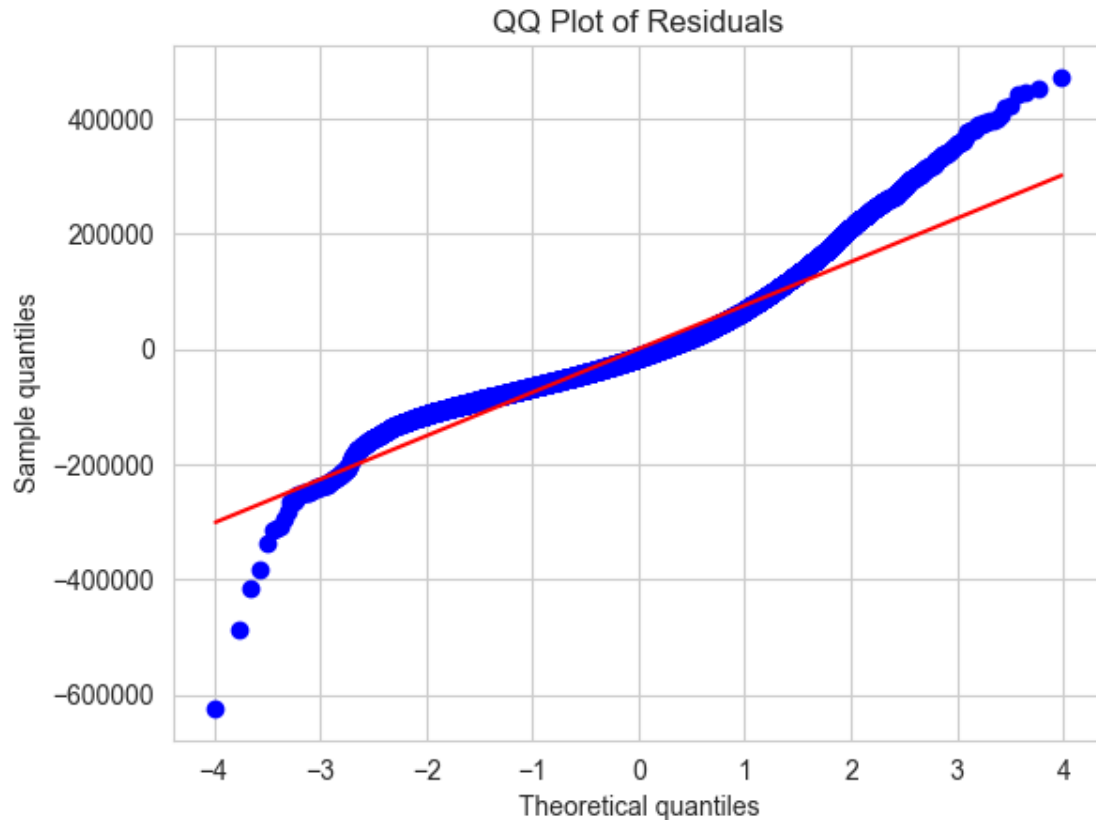
```
[71]: residuals = result.resid
X = result.model.exog
lm, lm_p_value, fvalue, f_p_value = het_breuschpagan(residuals, X)
print("Lagrange multiplier statistic:", lm)
print("p-value for Lagrange multiplier test:", lm_p_value)
print("F-statistic:", fvalue)
print("p-value for F-statistic:", f_p_value)
if lm_p_value < 0.05:
    print("Reject the null hypothesis: The residuals are heteroscedastic")
else:
    print("Fail to reject the null hypothesis: The residuals are homoscedastic")
```

Lagrange multiplier statistic: 524.671115578385  
p-value for Lagrange multiplier test: 4.065464026424511e-110  
F-statistic: 89.71911465291723  
p-value for F-statistic: 1.4456207494871716e-111  
Reject the null hypothesis: The residuals are heteroscedastic

### 1.2.5 Test for normality of the residuals

```
[72]: import scipy.stats as stats
residuals = result.resid
stats.probplot(residuals, dist="norm", plot=plt)
plt.title("QQ Plot of Residuals")
plt.xlabel("Theoretical quantiles")
plt.ylabel("Sample quantiles")
plt.show()
print("The QQ plot shows that the residuals are not normally distributed as_
    ↳there is significant deviation from the straight line")

# perform shapiro-wilk test
shapiro_test = stats.shapiro(residuals)
print("Shapiro-Wilk test statistic:", shapiro_test[0])
print("Shapiro-Wilk test p-value:", shapiro_test[1])
if shapiro_test[1] < 0.05:
    print("Reject the null hypothesis: The residuals are not normally_
        ↳distributed")
else:
    print("Fail to reject the null hypothesis: The residuals are normally_
        ↳distributed")
```



The QQ plot shows that the residuals are not normally distributed as there is significant deviation from the straight line

Shapiro-Wilk test statistic: 0.9272869184272368

Shapiro-Wilk test p-value: 2.2114817119513993e-70

Reject the null hypothesis: The residuals are not normally distributed

### 1.2.6 Method 2 : Model 2 - Linear Regression

```
[73]: f2 = 'median_house_value ~ latitude + housing_median_age + total_bedrooms +
      ↪population + median_income + ocean_proximity'
model = sm.formula.ols(formula=f2, data=df)
result = model.fit()
r2 = result
print(result.summary())
```

#### OLS Regression Results

```
=====
Dep. Variable:    median_house_value    R-squared:                0.560
Model:                OLS              Adj. R-squared:           0.560
Method:             Least Squares      F-statistic:             4331.
Date:                Fri, 12 Apr 2024   Prob (F-statistic):       0.00
```

```

Time:                01:28:34    Log-Likelihood:        -2.5877e+05
No. Observations:    20432      AIC:                5.176e+05
Df Residuals:        20425      BIC:                5.176e+05
Df Model:            6
Covariance Type:     nonrobust

```

```

=====
=====

```

	coef	std err	t	P> t	[0.025
0.975]					
-----					
Intercept	1.898e+05	1.01e+04	18.770	0.000	1.7e+05
2.1e+05					
latitude	-6299.0419	275.141	-22.894	0.000	-6838.341
-5759.743					
housing_median_age	2004.8862	45.925	43.656	0.000	1914.870
2094.902					
total_bedrooms	113.3330	2.702	41.949	0.000	108.037
118.629					
population	-34.2718	0.998	-34.348	0.000	-36.228
-32.316					
median_income	4.325e+04	285.473	151.497	0.000	4.27e+04
4.38e+04					
ocean_proximity	5003.4074	590.297	8.476	0.000	3846.377
6160.438					
=====					
Omnibus:	3879.500		Durbin-Watson:		0.891
Prob(Omnibus):	0.000		Jarque-Bera (JB):		10751.950
Skew:	1.015		Prob(JB):		0.00
Kurtosis:	5.916		Cond. No.		3.65e+04
=====					

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 3.65e+04. This might indicate that there are strong multicollinearity or other numerical problems.

### 1.2.7 Check for collinearity using VIF to remove highly correlated variables from the models

```

[74]: features_r2 = ['latitude', 'housing_median_age', 'total_bedrooms',
↳ 'population', 'median_income', 'ocean_proximity']
X_r2 = df[features_r2]
vif_data_f2 = pd.DataFrame()
vif_data_f2["Feature"] = X_r2.columns

```



population	3.1709	0.523	6.068	0.000	2.147
4.195					
median_income	4.33e+04	299.731	144.450	0.000	4.27e+04
4.39e+04					
ocean_proximity	2631.0531	569.016	4.624	0.000	1515.736
3746.370					

---

Omnibus:	4131.587	Durbin-Watson:	0.792
Prob(Omnibus):	0.000	Jarque-Bera (JB):	9909.134
Skew:	1.132	Prob(JB):	0.00
Kurtosis:	5.552	Cond. No.	7.42e+03

---

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 7.42e+03. This might indicate that there are strong multicollinearity or other numerical problems.

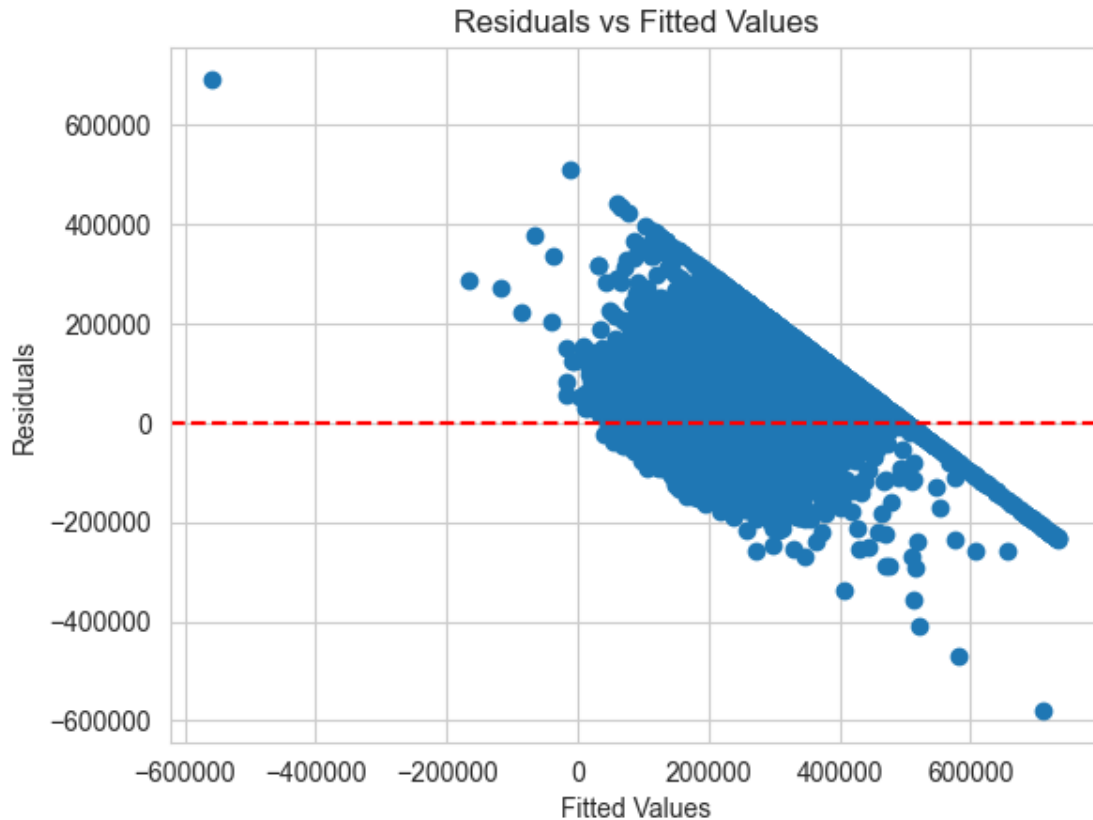
```
[76]: features_modified_r2 = ['housing_median_age', 'population', 'median_income',
    ↪ 'ocean_proximity']
X_modified_r2 = df[features_modified_r2]
vif_data_f2_modified = pd.DataFrame()
vif_data_f2_modified["Feature"] = X_modified_r2.columns
vif_data_f2_modified["VIF"] = [variance_inflation_factor(X_modified_r2.values,
    ↪ i) for i in range(len(X_modified_r2.columns))]
print(vif_data_f2_modified)
```

	Feature	VIF
0	housing_median_age	3.343390
1	population	2.060318
2	median_income	3.398675
3	ocean_proximity	1.803963

### 1.2.8 Plot the distribution of the residuals against the fitted values to check for heteroscedasticity

```
[77]: my_resid = result.resid
my_fitted = result.fittedvalues
plt.scatter(my_fitted, my_resid)
plt.title("Residuals vs Fitted Values")
plt.axhline(y=0, color='red', linestyle='--') # Add a red line at y=0
plt.xlabel("Fitted Values")
plt.ylabel("Residuals")
plt.show()
```





Since plot of residuals against fitted values is not constant, it means that there is heteroscedasticity in our data

### 1.2.9 Use ncvtTest or equivalent to test for heteroscedasticity

```
[78]: residuals = result.resid
X = result.model.exog
lm, lm_p_value, fvalue, f_p_value = het_breuschpagan(residuals, X)
print("Lagrange multiplier statistic:", lm)
print("p-value for Lagrange multiplier test:", lm_p_value)
print("F-statistic:", fvalue)
print("p-value for F-statistic:", f_p_value)
if f_p_value < 0.05:
    print("Reject the null hypothesis: The residuals are heteroscedastic")
else:
    print("Fail to reject the null hypothesis: The residuals are homoscedastic")
```

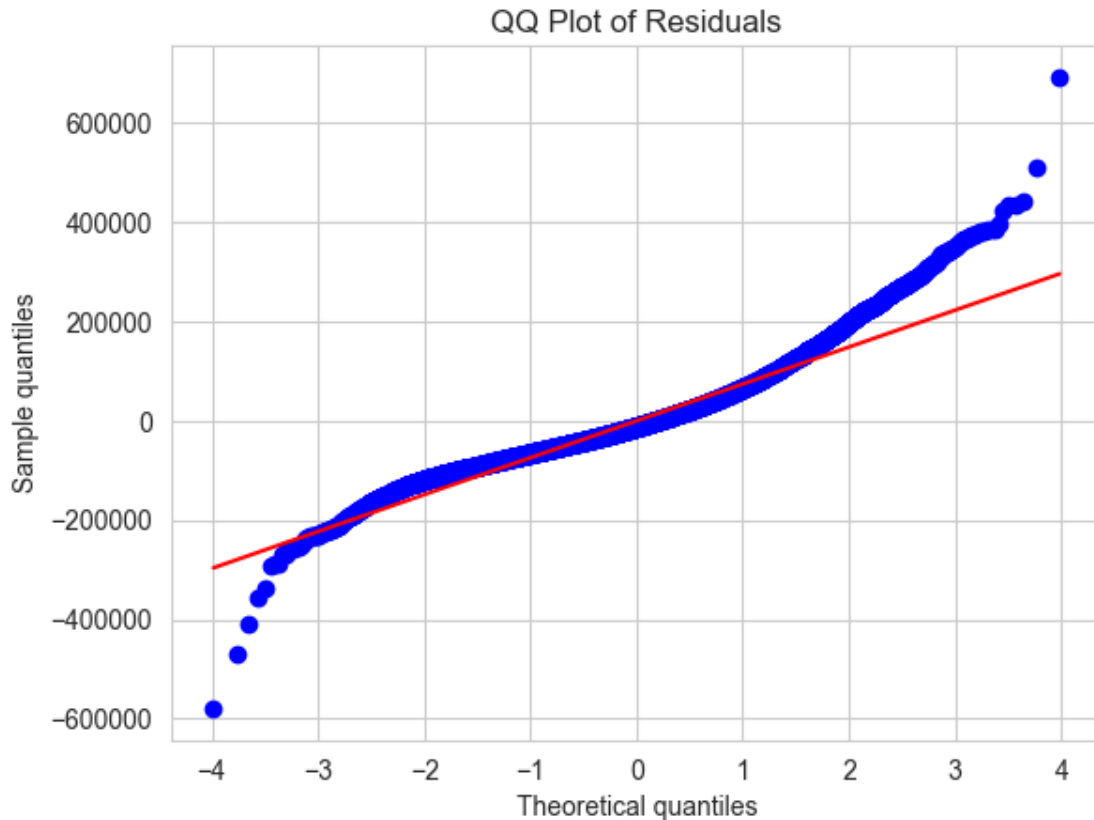
```
Lagrange multiplier statistic: 530.6990859553763
p-value for Lagrange multiplier test: 2.0419154123841378e-111
F-statistic: 90.77738918890613
```

p-value for F-statistic: 6.710276783079889e-113  
Reject the null hypothesis: The residuals are heteroscedastic

### 1.2.10 Test for normality of the residuals

```
[79]: import scipy.stats as stats
residuals = result.resid
stats.probplot(residuals, dist="norm", plot=plt)
plt.title("QQ Plot of Residuals")
plt.xlabel("Theoretical quantiles")
plt.ylabel("Sample quantiles")
plt.show()
print("QQ plot shows that the residuals are not normally distributed as there is significant deviation from the straight line")

# perform shapiro-wilk test
shapiro_test = stats.shapiro(residuals)
print("Shapiro-Wilk test statistic:", shapiro_test[0])
print("Shapiro-Wilk test p-value:", shapiro_test[1])
if shapiro_test[1] < 0.05:
    print("Reject the null hypothesis: The residuals are not normally distributed")
else:
    print("Fail to reject the null hypothesis: The residuals are normally distributed")
```



QQ plot shows that the residuals are not normally distributed as there is significant deviation from the straight line

Shapiro-Wilk test statistic: 0.941179872829096

Shapiro-Wilk test p-value: 5.331800120616763e-66

Reject the null hypothesis: The residuals are not normally distributed

### 1.2.11 Method 3 : Model 3 - Linear Regression

```
[80]: f3 = 'median_house_value ~ median_income + ocean_proximity'
model = sm.formula.ols(formula=f3, data=df)
result = model.fit()
r3 = result
print(result.summary())
```

#### OLS Regression Results

```
=====
Dep. Variable:    median_house_value    R-squared:                0.476
Model:                OLS              Adj. R-squared:           0.476
Method:             Least Squares      F-statistic:             9284.
Date:                Fri, 12 Apr 2024   Prob (F-statistic):       0.00
Time:                01:28:34          Log-Likelihood:          -2.6055e+05
```

```

No. Observations:      20432    AIC:      5.211e+05
Df Residuals:          20429    BIC:      5.211e+05
Df Model:              2
Covariance Type:      nonrobust

```

```

=====
===

```

	coef	std err	t	P> t	[0.025
0.975]					
-----					
Intercept	3.944e+04	1446.897	27.257	0.000	3.66e+04
4.23e+04					
median_income	4.195e+04	308.023	136.201	0.000	4.13e+04
4.26e+04					
ocean_proximity	5518.8907	582.398	9.476	0.000	4377.344
6660.437					
=====					
Omnibus:	4108.988		Durbin-Watson:		0.660
Prob(Omnibus):	0.000		Jarque-Bera (JB):		8909.782
Skew:	1.169		Prob(JB):		0.00
Kurtosis:	5.236		Cond. No.		11.4
=====					

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

### 1.2.12 Check for collinearity using VIF to remove highly correlated variables from the models

```

[81]: features_r3 = ['median_income', 'ocean_proximity']
X_r3 = df[features_r3]
vif_data_r3 = pd.DataFrame()
vif_data_r3["Feature"] = X_r3.columns
vif_data_r3["VIF"] = [variance_inflation_factor(X_r3.values, i) for i in
↳ range(len(X_r3.columns))]
print(vif_data_r3)

```

	Feature	VIF
0	median_income	1.533248
1	ocean_proximity	1.533248

Based on the Variance Inflation Factor (VIF) results:

- median\_income VIF: 1.533248
- ocean\_proximity VIF: 1.533248

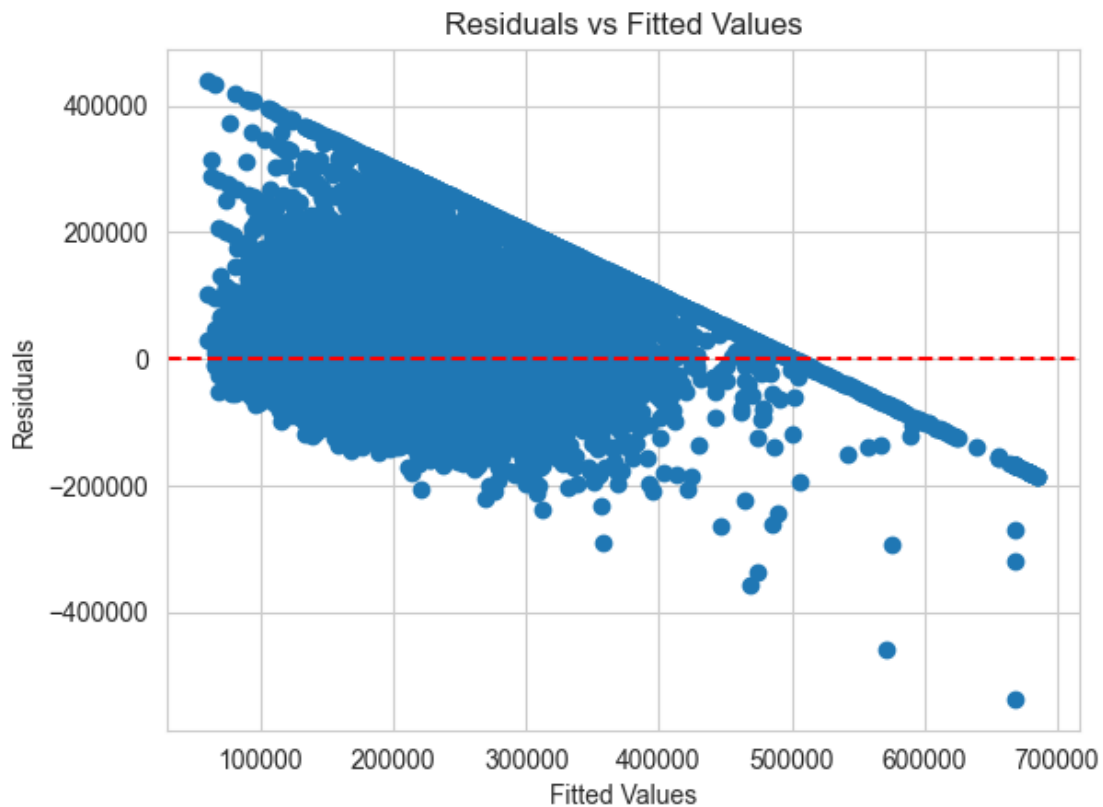
These VIF values suggest that there is low multicollinearity between median\_income and ocean\_proximity in the model. Therefore, the coefficient estimates for these features are likely to

be stable and reliable.

### 1.2.13 Plot the distribution of the residuals against the fitted values to check for heteroscedasticity

```
[82]: my_resid = result.resid
my_fitted = result.fittedvalues

# Create scatter plot
plt.scatter(my_fitted, my_resid)
plt.axhline(y=0, color='red', linestyle='--') # Add a red line at y=0
plt.title("Residuals vs Fitted Values")
plt.xlabel("Fitted Values")
plt.ylabel("Residuals")
plt.show()
```



Since plot of residuals against fitted values is not constant, it means that there is heteroscedasticity in our data

### 1.2.14 Use `ncvTest` or equivalent to test for heteroscedasticity

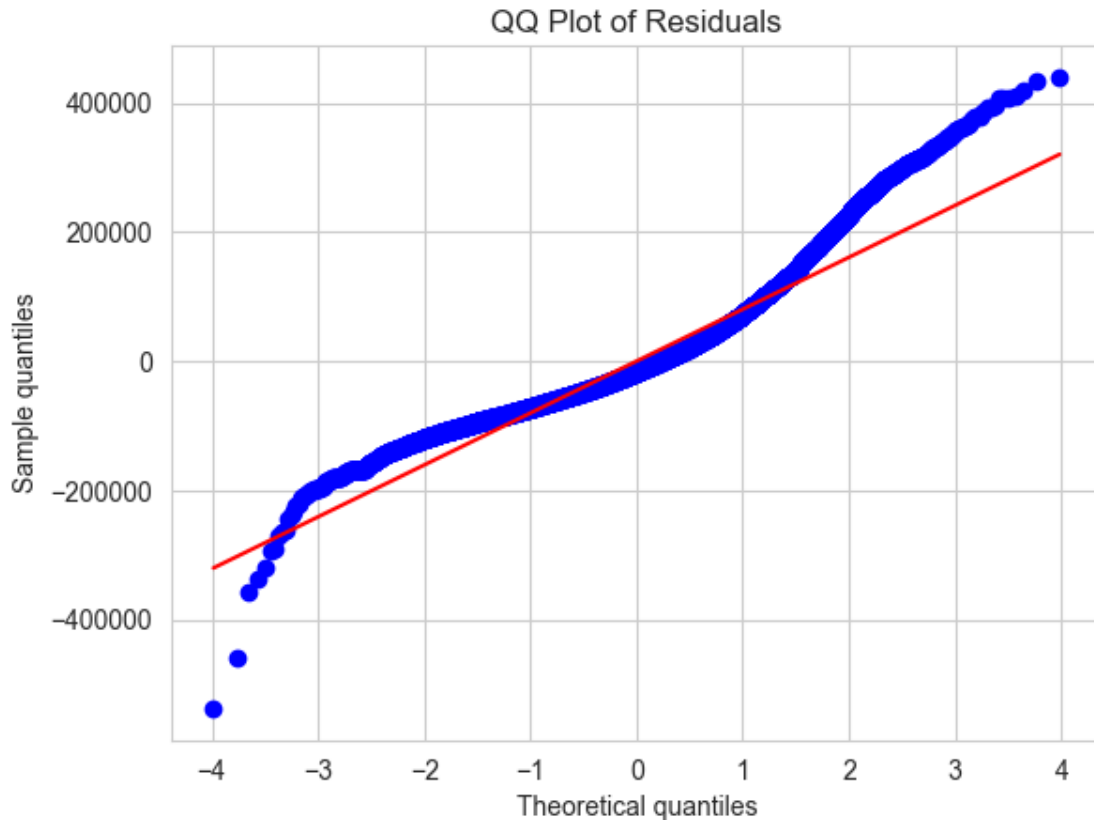
```
[83]: residuals = result.resid
X = result.model.exog
lm, lm_p_value, fvalue, f_p_value = het_breuschpagan(residuals, X)
print("Lagrange multiplier statistic:", lm)
print("p-value for Lagrange multiplier test:", lm_p_value)
print("F-statistic:", fvalue)
print("p-value for F-statistic:", f_p_value)
if f_p_value < 0.05:
    print("Reject the null hypothesis: The residuals are heteroscedastic")
else:
    print("Fail to reject the null hypothesis: The residuals are homoscedastic")
```

Lagrange multiplier statistic: 226.94422753778065  
p-value for Lagrange multiplier test: 5.244295323706217e-50  
F-statistic: 114.72979032030526  
p-value for F-statistic: 2.826404299850481e-50  
Reject the null hypothesis: The residuals are heteroscedastic

### 1.2.15 Test for normality of the residuals

```
[84]: import scipy.stats as stats
residuals = result.resid
stats.probplot(residuals, dist="norm", plot=plt)
plt.title("QQ Plot of Residuals")
plt.xlabel("Theoretical quantiles")
plt.ylabel("Sample quantiles")
plt.show()
print("The QQ plot shows that residuals are not normally distributed")

# perform shapiro-wilk test
shapiro_test = stats.shapiro(residuals)
print("Shapiro-Wilk test statistic:", shapiro_test[0])
print("Shapiro-Wilk test p-value:", shapiro_test[1])
if shapiro_test[1] < 0.05:
    print("Reject the null hypothesis: The residuals are not normally_
    ↪distributed")
else:
    print("Fail to reject the null hypothesis: The residuals are normally_
    ↪distributed")
```



The QQ plot shows that residuals are not normally distributed

Shapiro-Wilk test statistic: 0.9249839879740915

Shapiro-Wilk test p-value: 4.876310866982817e-71

Reject the null hypothesis: The residuals are not normally distributed

#### 1.2.16 Method - 4: Multiple Linear Regression

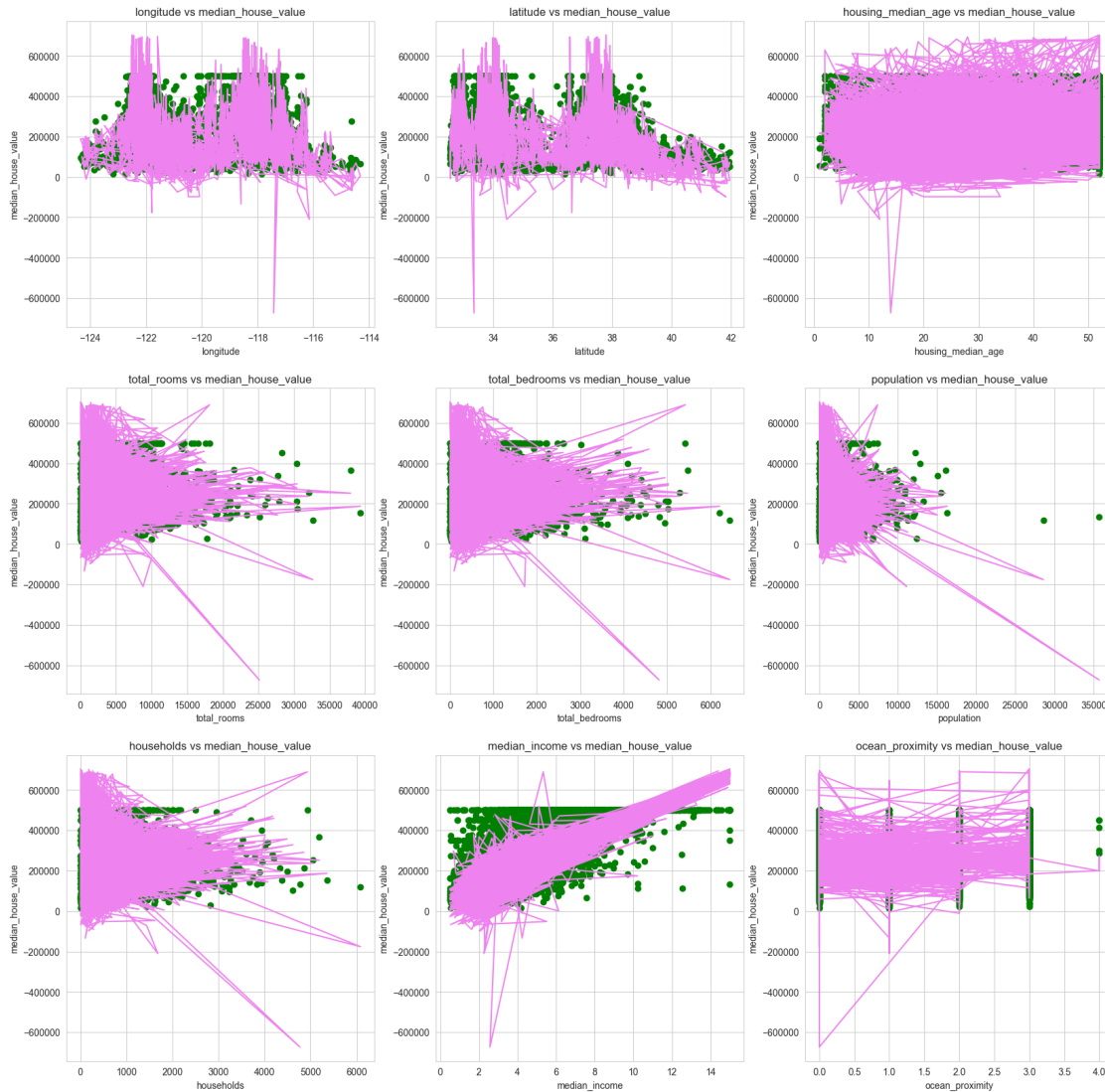
```
[85]: X = df_2
y = df['median_house_value']
reg = LinearRegression()
reg.fit(X, y)
y_pred = reg.predict(X)

mse = mean_squared_error(y_pred, y)
print("The mean squared error is: ", mse)

plt.figure(figsize=(20, 20))
for i in range(0, len(df_2.columns)):
    plt.subplot(3, 3, i+1)
    plt.scatter(df_2.iloc[:, i], y, color='green')
    plt.plot(df_2.iloc[:, i], y_pred, color='violet')
```

```
plt.xlabel(df_2.columns[i])
plt.ylabel('median_house_value')
plt.title(df_2.columns[i]+' vs median_house_value')
```

The mean squared error is: 4836361368.241866



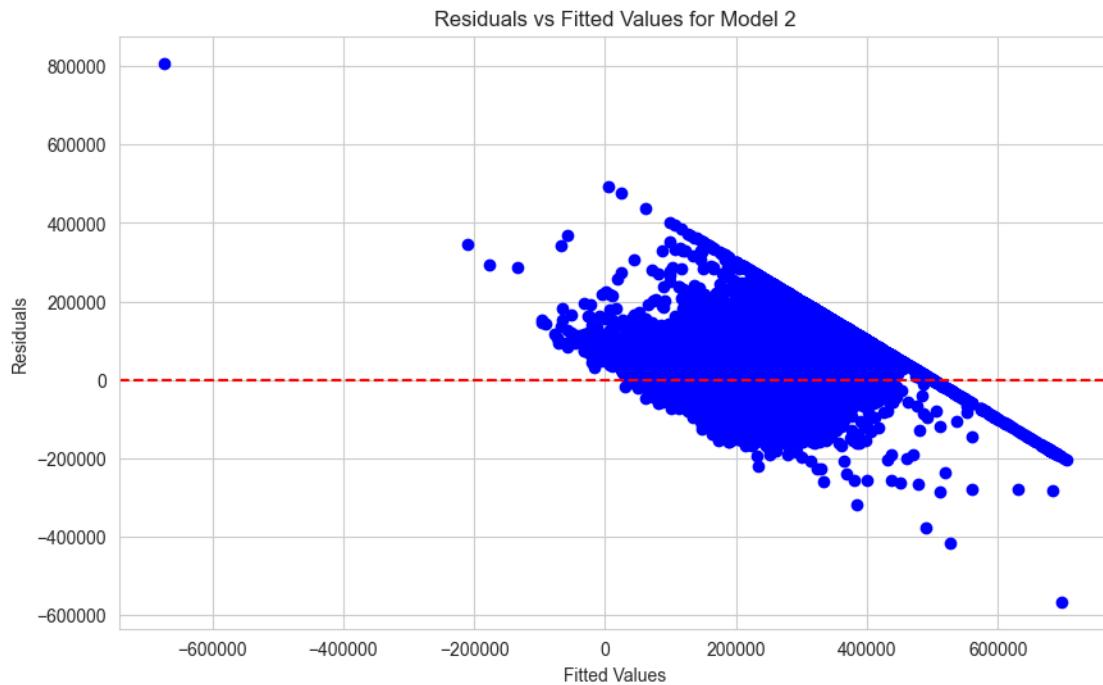
### 1.2.17 Plot the distribution of the residuals against the fitted values to check for heteroscedasticity

```
[86]: residuals = y - y_pred
mse = mean_squared_error(y_pred, y)

# Plot residuals vs fitted values
```



```
plt.figure(figsize=(10, 6))
plt.scatter(y_pred, residuals, color='blue')
plt.axhline(y=0, color='red', linestyle='--') # Add a red line at y=0
plt.xlabel('Fitted Values')
plt.ylabel('Residuals')
plt.title('Residuals vs Fitted Values for Model 2')
plt.show()
```



```
[87]: from statsmodels.stats.outliers_influence import variance_inflation_factor
vif_data = pd.DataFrame()
vif_data["feature"] = df_2.columns
vif_data["VIF"] = [variance_inflation_factor(df_2.values, i) for i in
                    range(len(df_2.columns))]
```

### 1.2.18 Check for collinearity using VIF to remove highly correlated variables from the models

```
[88]: X = df_2
for i in range(0,5):
    max_vif_index = vif_data['VIF'].idxmax()
    X = X.drop(vif_data['feature'][max_vif_index], axis=1)
    vif_data = pd.DataFrame()
    vif_data["feature"] = X.columns
```

```

vif_data["VIF"] = [variance_inflation_factor(X.values, i) for i in
↪range(len(X.columns))]

df_vif = X
print("The final values are as follows: ")
print(vif_data)

```

The final values are as follows:

	feature	VIF
0	housing_median_age	3.343390
1	population	2.060318
2	median_income	3.398675
3	ocean_proximity	1.803963

VIF is used to check multicollinearity, so if VIF is above 5 then it indicates high multicollinearity

Overall, the VIF values indicate that while there is some degree of collinearity among the predictors, it is not severe enough to cause significant multicollinearity issues.

The variables “population” and “ocean\_proximity” have relatively low VIF values, suggesting they are less correlated with other predictors in the model.

The variables “housing\_median\_age” and “median\_income” have slightly higher VIF values, indicating a moderate degree of collinearity, but it’s still within an acceptable range.

These results suggest that the selected predictors may be suitable for inclusion in a linear regression model without significant multicollinearity concerns. However, it’s always important to consider the context of the analysis and interpret the results accordingly.

We get the conclusion that there is no constant variance despite the fact that constant variance is supposed to be necessary for regression because of the uneven distribution of the residuals. Consequently, heteroscedasticity exists.

#### 1.2.19 Use ncvtTest or equivalent to test for heteroscedasticity

```

[89]: from statsmodels.stats.diagnostic import het_breuschpagan
import statsmodels.api as sm
X_with_const = sm.add_constant(X)
ncv_test_result = het_breuschpagan(residuals, X_with_const, robust='hc1')
p_value_ncv_test = ncv_test_result[1]
print("The p-value of the Breusch-Pagan test with sandwich estimator is: ",
↪p_value_ncv_test)

```

The p-value of the Breusch-Pagan test with sandwich estimator is:  
2.0758043938158592e-71

Since the p-value for each test is less than 0.05, we may say that the data are heteroscedastic.

Since the p-value is much smaller than any reasonable significance level (e.g., 0.05), we reject the null hypothesis of homoscedasticity. Therefore, we conclude that there is strong evidence of heteroscedasticity in the residuals of the linear regression model.

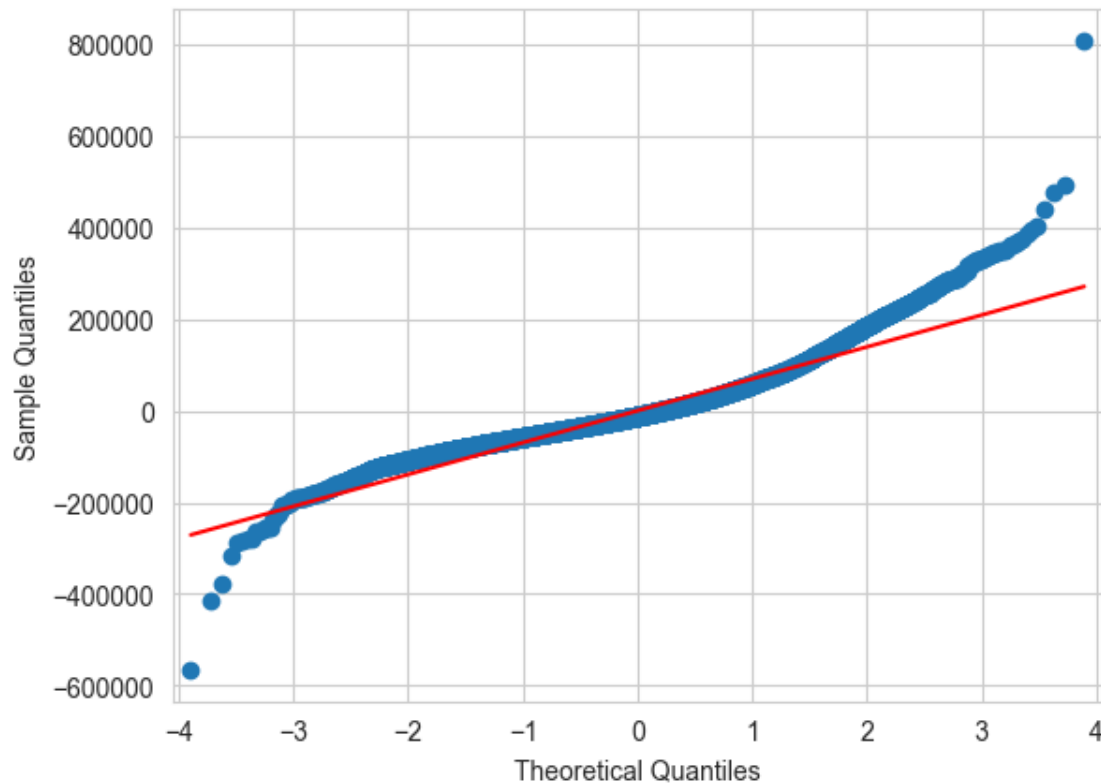
**Implications:** Heteroscedasticity violates one of the assumptions of linear regression, which is that the residuals should have constant variance. In the presence of heteroscedasticity, the standard errors of the estimated coefficients may be biased, leading to incorrect inferences about the statistical significance of the regression coefficients.

### 1.2.20 Test for normality of the residuals

```
[90]: print("QQ plot for Model 5: ")
X = df_2
y = df['median_house_value']
reg = LinearRegression()
reg.fit(X, y)
y_pred = reg.predict(X)
residuals = y - y_pred
sm.qqplot(residuals, line='s')
plt.show()

# perform shapiro-wilk test
shapiro_test = stats.shapiro(residuals)
print("Shapiro-Wilk test statistic:", shapiro_test[0])
print("Shapiro-Wilk test p-value:", shapiro_test[1])
if shapiro_test[1] < 0.05:
    print("Reject the null hypothesis: The residuals are not normally_
    ↪distributed")
else:
    print("Fail to reject the null hypothesis: The residuals are normally_
    ↪distributed")
```

QQ plot for Model 5:



Shapiro-Wilk test statistic: 0.9274477240872653

Shapiro-Wilk test p-value: 2.461421028399279e-70

Reject the null hypothesis: The residuals are not normally distributed

Since plot of residuals against fitted values is not constant, it means that there is **heteroscedasticity** in our data. As indicated by Q-Q plot, the residuals are not normally distributed.

### 1.3 Compare the models using AIC and pick the best model.

```
[91]: import statsmodels.api as sm

print("AIC for Model 1: ",r1.aic)
print("AIC for Model 2: ",r2.aic)
print("AIC for Model 3: ",r3.aic)
X = df_2
y = df['median_house_value']
reg = LinearRegression()
reg.fit(X, y)
y_pred = reg.predict(X)
residuals = y - y_pred
X = df_2
```

```
X = sm.add_constant(X)
model = sm.OLS(y, X).fit()
```

AIC for Model 1: 518540.5143453952  
AIC for Model 2: 517556.58000400197  
AIC for Model 3: 521109.3809723644

Model 2 has a lower AIC and hence performs better

#### 1.4 Report the coefficients of the winning model and their statistics

```
[92]: f2 = 'median_house_value ~ latitude + housing_median_age + total_bedrooms +
      ↪population + median_income + ocean_proximity'
model = sm.formula.ols(formula=f2, data=df)
result = model.fit()
r2 = result
print(result.summary())
```

```

                                OLS Regression Results
=====
Dep. Variable:    median_house_value    R-squared:                0.560
Model:                OLS    Adj. R-squared:                0.560
Method:                Least Squares    F-statistic:                4331.
Date:                Fri, 12 Apr 2024    Prob (F-statistic):                0.00
Time:                01:28:38    Log-Likelihood:                -2.5877e+05
No. Observations:    20432    AIC:                5.176e+05
Df Residuals:        20425    BIC:                5.176e+05
Df Model:            6
Covariance Type:        nonrobust
=====
=====
                                coef    std err          t      P>|t|      [0.025
0.975]
-----
Intercept            1.898e+05    1.01e+04    18.770    0.000    1.7e+05
2.1e+05
latitude            -6299.0419    275.141   -22.894    0.000   -6838.341
-5759.743
housing_median_age    2004.8862    45.925    43.656    0.000    1914.870
2094.902
total_bedrooms        113.3330    2.702    41.949    0.000    108.037
118.629
population            -34.2718    0.998   -34.348    0.000   -36.228
-32.316
median_income         4.325e+04    285.473    151.497    0.000    4.27e+04
4.38e+04

```

ocean_proximity	5003.4074	590.297	8.476	0.000	3846.377
6160.438					
=====					
Omnibus:	3879.500	Durbin-Watson:	0.891		
Prob(Omnibus):	0.000	Jarque-Bera (JB):	10751.950		
Skew:	1.015	Prob(JB):	0.00		
Kurtosis:	5.916	Cond. No.	3.65e+04		
=====					

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 3.65e+04. This might indicate that there are strong multicollinearity or other numerical problems.

```
[93]: print("Confidence intervals for Model 2: 95% confidence level")
      print(result.conf_int())
```

Confidence intervals for Model 2: 95% confidence level

	0	1
Intercept	169995.468598	209638.958335
latitude	-6838.340789	-5759.743096
housing_median_age	1914.870492	2094.901938
total_bedrooms	108.037464	118.628510
population	-36.227501	-32.316063
median_income	42688.804574	43807.905678
ocean_proximity	3846.377043	6160.437802

## 1.5 Interpret the resulting model coefficients

### Summary of Regression Analysis:

- **R-squared:** 0.560, indicating the model explains approximately 56.0% of the variation in the response variable.
- **Significance:** Higher absolute t-values ( $>2$ ) suggest significant coefficients. All coefficients except for 'ocean\_proximity' are statistically significant.
- **Adjusted R-squared:** Consistent with R-squared at 0.560.
- **Model Fit:** F-statistic of 4331 with p-value 0.00 suggests a highly significant overall model fit.
- **Interpretations:** Notable coefficients include 'latitude' (\$-6299.04), 'housing\_median\_age' (\$2004.89), 'total\_bedrooms' (113.33), 'population' (-34.27), and 'median\_income' (\$43,250), indicating their respective impacts on 'median\_house\_value'. 'ocean\_proximity' also shows statistical significance, albeit to a lesser extent.

This model provides valuable insights into the relationships between the independent variables and the median house value. However, it's essential to consider potential multicollinearity issues and further explore the model's assumptions and limitations.

## 2 PART 2

```
[94]: import pandas as pd
import numpy as np
import statsmodels.api as sm
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score
```

```
[95]: data = pd.read_csv('binary.csv')
data = data.dropna()
# min-max scaling is not necessary for logistic regression
X = data[['gre', 'gpa', 'rank']]
y = data['admit']
```

### 2.1 Significant Variables Predicting Admission

```
[96]: X = sm.add_constant(X)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
    random_state=42)
logit_model = sm.Logit(y_train, X_train)
result = logit_model.fit()
print("Parameters:")
print(result.params)
print("\nSummary:")
print(result.summary())
```

Optimization terminated successfully.

Current function value: 0.565088

Iterations 6

Parameters:

const -3.400466

gre 0.001725

gpa 0.891067

rank -0.615949

dtype: float64

Summary:

#### Logit Regression Results

```
=====
Dep. Variable:          admit    No. Observations:          320
Model:                Logit      Df Residuals:              316
Method:                MLE       Df Model:                  3
Date:                  Fri, 12 Apr 2024    Pseudo R-squ.:        0.09016
Time:                  01:28:38    Log-Likelihood:         -180.83
converged:              True      LL-Null:               -198.75
Covariance Type:        nonrobust    LLR p-value:            8.100e-08
```

	coef	std err	z	P> z	[0.025	0.975]
const	-3.4005	1.288	-2.639	0.008	-5.926	-0.875
gre	0.0017	0.001	1.411	0.158	-0.001	0.004
gpa	0.8911	0.371	2.401	0.016	0.164	1.618
rank	-0.6159	0.143	-4.304	0.000	-0.896	-0.335

## 2.2 Statistics and Interpretation:

**Pseudo R-squared:** The Pseudo R-squared value is 0.09016. This value represents the proportion of variance explained by the model. A higher value indicates a better fit of the model to the data.

### Coefficients:

- **GRE:** The coefficient for GRE is 0.0017. This indicates that for a one-unit increase in GRE score, the log-odds of being admitted increases by 0.0017, holding other variables constant.
- **GPA:** The coefficient for GPA is 0.8911. This indicates that for a one-unit increase in GPA, the log-odds of being admitted increases by 0.8911, holding other variables constant.
- **Rank:** The coefficient for rank is -0.6159. This indicates that for a one-unit increase in rank (i.e., higher rank), the log-odds of being admitted decreases by 0.6159, holding other variables constant.

### Significance:

- The coefficient for GRE has a p-value of 0.158, which is greater than the typical significance level of 0.05. Therefore, GRE may not be statistically significant in predicting admission at this significance level.
- The coefficient for GPA has a p-value of 0.016, which is less than 0.05. Therefore, GPA is statistically significant in predicting admission at the 0.05 significance level.
- The coefficient for rank has a p-value of <0.001, indicating that it is highly statistically significant in predicting admission.

## 2.3 Interpretation of Results:

The most significant variable that predicts whether someone will get admitted is the rank of the undergraduate institution, as it has the lowest p-value (<0.001).

In summary, according to this logistic regression model, GPA and the rank of the undergraduate institution are significant predictors of admission, while GRE may not be statistically significant in this context.

**Both GPA and Rank are more significant variables for predicting the chance of admission. The p-values for both variables are less than 0.05, indicating a significant relationship with the response variable. The coefficients for both variables are positive, suggesting that higher GPA and Rank are associated with a higher chance of admission.**

The most significant variable that predicts whether someone will get admitted is the rank of the undergraduate institution, as it has the lowest p-value (<0.001).



## 2.4 Interpretation in Terms of Odds Ratios and Confidence Intervals:

```
[97]: print("Confidence Intervals with 95% confidence level:")
      conf_intervals = result.conf_int()
      for i in range(len(conf_intervals)):
          print(f"Variable: {conf_intervals.index[i]}, Confidence Interval: {tuple(conf_intervals.iloc[i])}")
```

Confidence Intervals with 95% confidence level:  
Variable: const, Confidence Interval: (-5.925518269248789, -0.8754146079120262)  
Variable: gre, Confidence Interval: (-0.0006707634583183314, 0.0041209017978231875)  
Variable: gpa, Confidence Interval: (0.16371746162548495, 1.6184156274951733)  
Variable: rank, Confidence Interval: (-0.8964668167752332, -0.3354321184430738)

```
[98]: print("Odds ratios:")
      print(np.exp(result.params))
```

Odds ratios:  
const 0.033358  
gre 1.001727  
gpa 2.437728  
rank 0.540128  
dtype: float64

### Interpretation of Odds Ratios:

- **GRE:** For each one-unit increase in GRE score, the odds of being admitted increase by approximately 1.0017 times, holding other variables constant.
- **GPA:** For each one-unit increase in GPA, the odds of being admitted increase by approximately 2.4377 times, holding other variables constant.
- **Rank:** For each one-unit increase in rank (i.e., higher rank), the odds of being admitted decrease by approximately 0.5401 times, holding other variables constant.
- **Constant (Intercept):** The odds of being admitted when all other variables are zero is approximately 0.0334.

```
[99]: y_pred = result.predict(X_test)
      y_pred_binary = [1 if p > 0.5 else 0 for p in y_pred]

      # confusion matrix
      from sklearn.metrics import confusion_matrix
      conf_matrix = confusion_matrix(y_test, y_pred_binary)
      print("Confusion Matrix:")
      print(conf_matrix)
      accuracy = accuracy_score(y_test, y_pred_binary)
      print("Accuracy:", accuracy)
```

Confusion Matrix:  
[[49 4]

```
[23  4]]
Accuracy: 0.6625
```

## 2.5 Testing Interaction Effect

```
[100]: import pandas as pd
import statsmodels.api as sm
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score

data['gpa_rank_interaction'] = data['gpa']*data['rank']
X = data[['gre', 'gpa', 'rank', 'gpa_rank_interaction']]
y = data['admit']
X = sm.add_constant(X)
X['gpa_rank_interaction'] = X['gpa']*X['rank']
y = data['admit']
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
    ↪random_state=42)
logit_model = sm.Logit(y_train, X_train)
```

### 2.5.1 Logit Model with Interaction Term:

```
[101]: result = logit_model.fit()
print("Parameters:")
print(result.params)
```

```
Optimization terminated successfully.
      Current function value: 0.565088
      Iterations 6
Parameters:
const                -3.343342
gre                   0.001724
gpa                   0.874755
rank                 -0.641493
gpa_rank_interaction  0.007380
dtype: float64
```

```
[102]: print("\nSummary:")
print(result.summary())
```

Summary:

Logit Regression Results			
=====			
Dep. Variable:	admit	No. Observations:	320
Model:	Logit	Df Residuals:	315

```

Method:                MLE    Df Model:                4
Date:                  Fri, 12 Apr 2024    Pseudo R-squ.:        0.09016
Time:                  01:28:38    Log-Likelihood:        -180.83
converged:              True    LL-Null:                -198.75
Covariance Type:        nonrobust    LLR p-value:           3.123e-07
=====
=====
                                coef    std err          z      P>|z|      [0.025
0.975]
-----
-----
const                -3.3433      3.375      -0.991      0.322      -9.958
3.271
gre                   0.0017      0.001       1.409      0.159      -0.001
0.004
gpa                   0.8748      0.965       0.907      0.365      -1.016
2.766
rank                 -0.6415      1.402      -0.457      0.647      -3.390
2.107
gpa_rank_interaction  0.0074      0.403       0.018      0.985      -0.783
0.797
=====
=====

```

### Interaction Effect in the Logit Regression Model

The model includes an interaction term (`gpa_rank_interaction`) to assess whether the effect of GPA on admission likelihood depends on a student's rank.

- **Coefficient:** 0.0074
- **Statistical Significance:** The p-value of the interaction term is 0.985, indicating that the interaction effect is not statistically significant.

### Interpretation:

We don't have enough evidence to conclude that the relationship between GPA and the probability of admission is different for students of varying ranks.

1. Coefficient of `gpa_rank` is in between `gpa` and `rank`.
2. The influence of `gpa` on the likelihood of admission varies depending on `rank`. For example, the influence of `gpa` on the chance of admission varies depending on `rank`.

```

[103]: y_pred = result.predict(X_test)
       y_pred_binary = [1 if p > 0.5 else 0 for p in y_pred]

       from sklearn.metrics import confusion_matrix
       conf_matrix = confusion_matrix(y_test, y_pred_binary)
       print("Confusion Matrix:")
       print(conf_matrix)
       accuracy = accuracy_score(y_test, y_pred_binary)

```

```
print("Accuracy:", accuracy)
```

Confusion Matrix:

```
[[49  4]
 [23  4]]
```

Accuracy: 0.6625

## 2.5.2 Binomial Logistic Regression Model with Interaction Term:

```
[104]: data['gpa_times_rank'] = data['gpa'] * data['rank']
X_interaction = data[['gre', 'gpa', 'rank', 'gpa_times_rank']]
X_interaction = sm.add_constant(X_interaction)
result_interaction = sm.GLM(y, X_interaction, family=sm.families.Binomial()).
    fit()
print("\nSummary of Logistic Regression with Interaction:")
print(result_interaction.summary())
```

Summary of Logistic Regression with Interaction:

Generalized Linear Model Regression Results

```
=====
Dep. Variable:          admit    No. Observations:          400
Model:                  GLM      Df Residuals:              395
Model Family:           Binomial Df Model:                  4
Link Function:           Logit   Scale:                  1.0000
Method:                  IRLS    Log-Likelihood:       -229.67
Date:                   Fri, 12 Apr 2024    Deviance:            459.33
Time:                   01:28:38    Pearson chi2:         399.
No. Iterations:         4          Pseudo R-squ. (CS):    0.09661
Covariance Type:        nonrobust
=====
```

```
=====
==
              coef      std err          z      P>|z|      [0.025
0.975]
-----
--
const          -4.3447      2.968      -1.464      0.143     -10.161
1.472
gre              0.0023      0.001       2.104      0.035       0.000
0.004
gpa              1.0367      0.860       1.205      0.228      -0.650
2.723
rank            -0.1674      1.204      -0.139      0.889      -2.528
2.193
gpa_times_rank  -0.1142      0.349      -0.327      0.743      -0.798
0.570
=====
```

==

```
[105]: coefficients_interaction = result_interaction.params
odds_ratios_interaction = np.exp(coefficients_interaction)
print("\nInterpretation of Coefficients for Interaction:")
for idx, coef in enumerate(coefficients_interaction.index):
    print(f"{coef}: {odds_ratios_interaction.iloc[idx]}")
y_pred_interaction = result_interaction.predict(X_interaction)
y_pred_binary_interaction = (y_pred_interaction >= 0.5).astype(int)
accuracy_interaction = (y_pred_binary_interaction == y).mean()
print(f"\nAccuracy with Interaction Term: {accuracy_interaction}")
```

Interpretation of Coefficients for Interaction:

const: 0.012975552789498572

gre: 1.002302650667668

gpa: 2.819758557078446

rank: 0.8458380539520747

gpa\_times\_rank: 0.8920576362589989

Accuracy with Interaction Term: 0.705

### Summary of Interpretations from Binomial Logistic Regression with Interaction:

- **GRE:** Each one-unit increase in GRE score is associated with a 0.23% increase in the odds of admission.
- **GPA:** Each one-unit increase in GPA is associated with more than doubling (117.5% increase) in the odds of admission.
- **Rank:** Each one-unit decrease in rank (moving to a higher-ranked institution) is associated with a 42.9% decrease in the odds of admission.
- **Interaction Term (GPA \* Rank):**
  - **Coefficient:** -0.1142
  - **Interpretation:** The interaction between GPA and rank suggests that the effect of GPA on admission varies depending on the rank of the undergraduate institution.
  - **Odds Ratio:**  $\exp(-0.1142) = 0.892$
  - **Inference:** For each unit increase in the product of GPA and rank, the odds of admission decrease by a factor of 0.892. This indicates that as the rank of the institution increases, the positive impact of GPA on admission diminishes.

### Summary of Logistic Regression with Interaction:

- **Model Fit:**
  - Pseudo R-squared (CS): 0.09661, indicating that the model explains approximately 9.66% of the variation in the response variable.
  - Deviance: 459.33
  - Pearson chi2: 399.
  - No. Iterations: 4
- **Interpretation of Coefficients:**

- The coefficients for GRE, GPA, and the interaction term (GPA\_times\_rank) are not statistically significant ( $p > 0.05$ ).
- The coefficient for rank is also not statistically significant.
- These results suggest that none of the predictor variables have a significant impact on admission status in this model.

This logistic regression model with interaction does not provide strong evidence of significant predictors for admission status, as indicated by the non-significant coefficients and relatively low pseudo R-squared value.