

# Data and Applications

## Homework 4

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### 1. Part One

#### 1. Q1:

A minimal cover of a set of functional dependencies (FD)  $E$  is a minimal set of dependencies  $F$  that is equivalent to  $E$ .

A set FD is minimal if it satisfies the following conditions:

1. Every dependency in  $F$  has a single attribute for its right hand-side
2. We cannot replace any dependency  $X \rightarrow A$  in  $F$  with a dependency  $Y \rightarrow A$ , where  $Y$  is a proper subset of  $X$ , and still have a set of dependencies that is equivalent to  $F$
3. We cannot remove any dependency from  $F$  and still have a set of dependencies that are equivalent to  $F$

A relation is said to be in BCNF in DBMS if the relation is already in 3NF and also , for every functional dependency (say  $X \rightarrow Y$ ),  $X$  is either the super key or candidate key.

#### (a) ABC:

##### i. The FD's are:

- $AB \rightarrow C$
- $AC \rightarrow B$
- $BC \rightarrow A$

$S = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow A\}$  is the minimal cover and is also the set of dependencies that hold over the set

##### ii. The strongest normal form that is not violated by these set of attributes:

#### **BCNF**

as  $AB$ ,  $AC$  and  $BC$  are candidate keys for  $R1$  where  $R1 = ABC$

##### iii. $R1$ is in BCNF where $R1 = ABC$

(b) ABCD:

i. The FD's are:

- $AB \rightarrow C$
- $AC \rightarrow B$
- $B \rightarrow D$
- $BC \rightarrow A$

$S = \{AB \rightarrow C, AC \rightarrow B, B \rightarrow D, BC \rightarrow A\}$  is the minimal cover and is also the set of dependencies that hold over the set

ii. The strongest normal form that is not violated by the relation containing these attributes: **1NF**.

Here AB, BC, AC are keys, therefore....

the Functional dependency  $B \rightarrow D$  violates 2NF and thereby **violates BCNF** as B is a proper subset of AB.

iii. Decomposing to: ABC, BD

This is a BCNF decomposition

ABC is in BCNF as stated in (a)

BD is in BCNF as the Functional Dependency  $B \rightarrow D$  is in accordance with regulations of BCNF in the relation BD

(c) ABCEG:

i. The FD's are:

- $AB \rightarrow C$
- $AC \rightarrow B$
- $BC \rightarrow A$
- $E \rightarrow G$

$S = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow A, E \rightarrow G\}$  is the minimal cover and is also the set of dependencies that hold over the set.

ii. The strongest normal form that is not violated by the relation containing these attributes: **1NF**.

Here ABE, BCE, ACE are keys, therefore....

the Functional dependency  $E \rightarrow G$  violates 2NF and thereby **violates BCNF** as E is a proper subset of ABE.

iii. Decompose to: ABE, ABC, EG

This is a BCNF decomposition.

ABC is in BCNF as stated in (a)

ABE is in BCNF as it doesn't violate regulations of BCNF

EG is in BCNF as the Functional Dependency  $E \rightarrow G$  is in accordance with the regulations of BCNF

(d) **DCEGH:**

- i. The FD's are:

➤  $E \rightarrow G$

$S = \{E \rightarrow G\}$  is the minimal cover and is also the set of dependencies that hold over the set.

- ii. The strongest normal form that is not violated by the relation containing these attributes: **1NF**

Here DCEH is key, therefore...

the Functional dependency  $E \rightarrow G$  violates 2NF and thereby **violates BCNF** as E is a proper subset of DCEH.

- iii. Decompose to: DCEH, EG

This is a BCNF decomposition.

DCEH is in BCNF as it doesn't violate any regulations of BCNF

EG is in BCNF as the Functional Dependency  $E \rightarrow G$  is in accordance with the regulations of BCNF

(e) **ACEH:**

- i. No Functional Dependencies exist.

$S = \{\}$  is the minimal cover and is also the set of dependencies that hold over the set.

- ii. The strongest normal form that is not violated by the relation containing these attributes: **BCNF**

Here ACEH is the key

- iii. Already in BCNF

## 2. Q2:

The Closure of Functional Dependency means the complete set of all possible attributes that can be functionally derived from given functional dependency.

If "F" is a functional dependency, then closure of functional dependency can be denoted using " $\{F\}^+$ "

A Decomposition  $D = \{R_1, R_2, R_3, \dots, R_n\}$  of R is dependency preserving with respect to a set F of Functional dependency if

$$(F_1 \cup F_2 \cup \dots \cup F_m) = \{F\}^+$$

A decomposition  $D = \{R_1, R_2, R_3, \dots, R_n\}$  of a set R of functional dependencies is said to be lossless join if:

$$\{R_1 \cup R_2 \cup \dots \cup R_n\} = F$$

$$\{R_i \cap R_j\} \neq \emptyset \text{ for all } 1 \leq i, j \leq n$$

$\{R_i \cap R_j\}$  must be key of at least one of  $R_i, R_j$

a) The decomposition  $D = \{AB, ABDE, EG, BC\}$  is not dependency preserving as the FD  $AC \rightarrow B$  and  $AB \rightarrow C$  are not preserved

The decomposition  $D = \{AB, ABDE, EG, BC\}$  is not a lossless decomposition of R with set of dependencies given by F.

Now consider the following instances of R

$\{(a_2, b, c_2, d_2, e_2, g_2), (a_1, b, c_1, d_1, e_1, g_1)\}$

Due to the functional dependencies in F given by  $AB \rightarrow C$  and  $BC \rightarrow A$ ,  $c_1 \neq c_2$  iff  $a_1 \neq a_2$ .

Hence the join  $AB \bowtie BC$  contain the tuples

$\{(a_1, b, c_1), (a_2, b, c_1), (a_2, b, c_2), (a_1, b, c_2)\}$

Hence the join of AB, EG, BC, ABDE contains  $\geq 4$  tuples (8 in reality).

Hence D is a lossy decomposition.

b) The decomposition  $D = \{ABC, ACDE, ADG\}$  is lossless Decomposition.

Proof: The join of ADG, ACDE and ABC can be constructed in 2 steps

Step-1: We construct the join of ABC and ACDE.

This is lossless decomposition because intersection of their attributes is given by AC which is a key for ABCDE and ABCDEG. Hence this decomposition is lossless decomposition.

Step-2: Now we join this intermediate join with ADG.

This is lossless decomposition because the intersection of their attributes is given by AD and  $AD \rightarrow ADG$ .

Hence from Step-1 and Step 2, D is a lossless decomposition

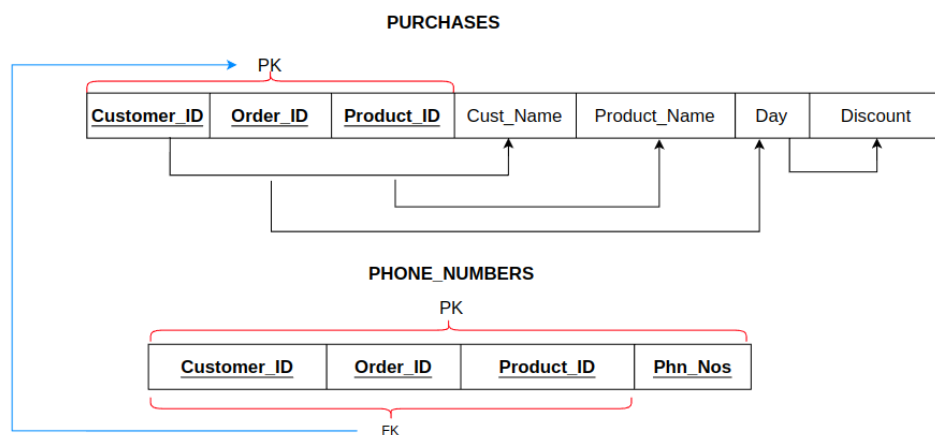
The decomposition  $D = \{ABC, ACDE, ADG\}$  is not dependency preserving as the FD  $B \rightarrow D$  and  $E \rightarrow G$  are not preserved.

## 2. Part Two

**Note:** Blue arrows in the Relational diagrams denote foreign key constraints and Black arrows show functional dependencies

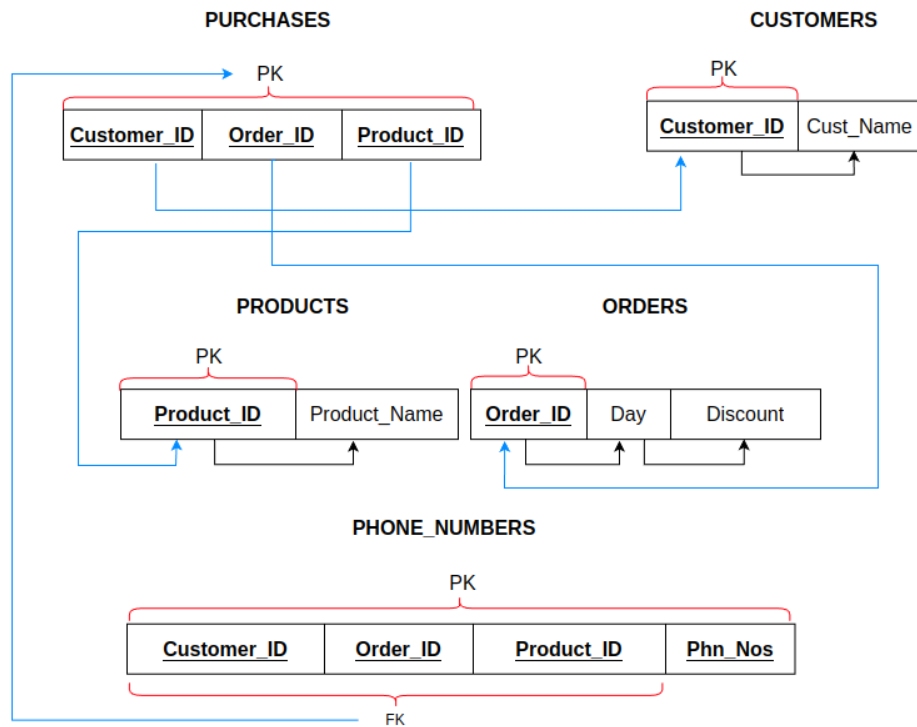
(a) 1NF:

- i. The composite key attribute Customer\_ID + Order\_ID + Product\_ID has been broken down into the corresponding simple attributes – Customer\_ID, Order\_ID and Product\_ID
- ii. A new relation “PHONE\_NUMBERS” was created with {Customer\_ID, Order\_ID, Product\_ID} as foreign key referencing {Customer\_ID, Order\_ID, Product\_ID} of “PURCHASES” and Phn\_Nos was added as a simple attribute to remove the multivalued attribute Phn\_Nos of the original “PURCHASES” and {Customer\_ID, Order\_ID, Product\_ID, Phn\_Nos} was set as the primary key attribute of the relation “PHONE\_NUMBERS”



(b) 2NF:

- i. Since there are partial functional dependencies involving Customer\_ID, Order\_ID and Product\_ID individually as shown in the above 1NF representation where {Customer\_ID, Order\_ID, Product\_ID} was the primary key, they violated the guidelines of 2NF.
- ii. Thus, new tables “CUSTOMERS”, “PRODUCTS” and “ORDERS” were added as follows:
  - “CUSTOMERS” table was created with Customer\_ID as primary key and Cust\_Name as a simple attribute. The Customer\_ID attribute of “PURCHASES” is set to be a foreign key that references Customer\_ID of “CUSTOMERS”.
  - “PRODUCTS” table was created with Product\_ID as primary key and Product\_Name as a simple attribute. The Product\_ID attribute of “PURCHASES” is set to be a foreign key that references Product\_ID of “PRODUCTS”
  - “ORDERS” table was created with Order\_ID as primary key and Day and Discount as simple attributes. The Order\_ID attribute of “PURCHASES” is set to be a foreign key that references ORDER\_ID of “ORDERS”



(c) 3NF:

- Since there is a transitive functional dependency from Order\_ID to Day and from Day to Discount but Day is a non-prime attribute. This violates the guidelines of 3NF.
- Thus, another table "DISCOUNTS" was created with Day as key attribute and functional dependency from Day to Discount in the "DISCOUNTS" table. Also, the "ORDERS" table was updated by removing the Discount column and setting the Day attribute as foreign key referencing Day of "DISCOUNTS"

