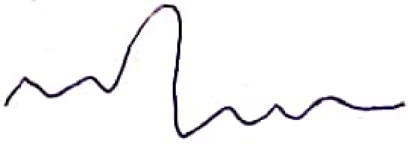
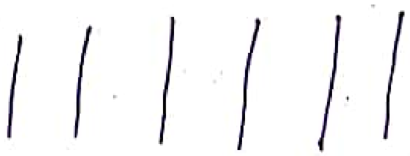
* Sampling Theorem

Nyquest : $f_s \geq 2 f_M$ → max- frequency of signal

Signal → 

Impulse train → 

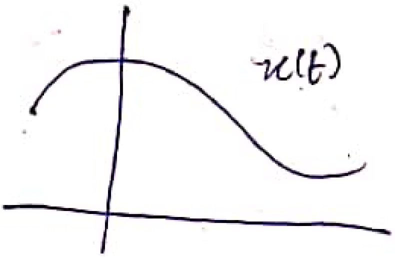
$f_s = \frac{1}{T_s}$

→ (0 to f_{max})

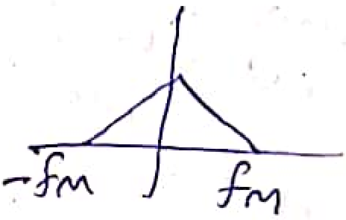
Low-pass sampling theorem:

Band pass
↳ f_{min} to f_{max}

can do much lesser than $2 f_{max}$



atleast ⟹



*

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\delta'(t) = \sum_{n=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T}$$

$$\delta'(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t) + (b_n \sin(n\omega_0 t))$$

⇓

$$c_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta'(t) \cdot e^{-jk\omega_0 t} \, dt$$

$$c_k = 1/T$$

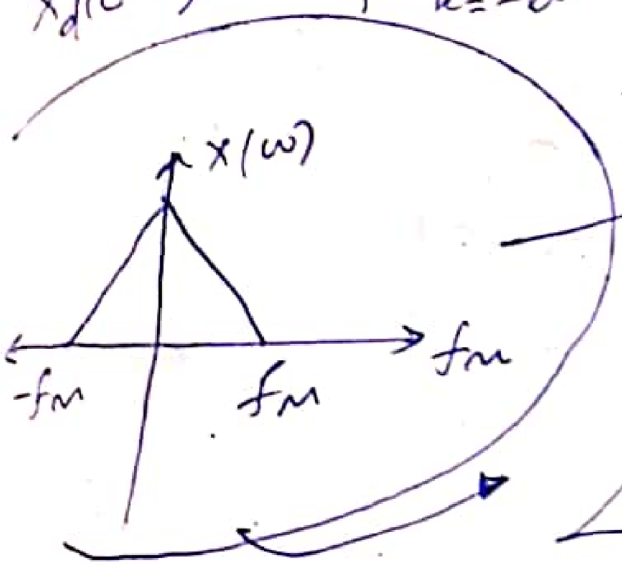
$$\delta'(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} \cdot e^{kj\omega_0 t}$$

$$x_d(n) = x(t)\cdot s'(t)$$

$$= x(t)\cdot \sum_{n=-\infty}^{\infty} \tfrac{1}{T}\cdot e^{jk\omega_0 t}$$

$$X_d(e^{j\omega}) = \frac{1}{T}\left[FT\left\{x(t)\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}\right\}\right]$$

$$X_d(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(e^{j(\omega-k\omega_0)})$$

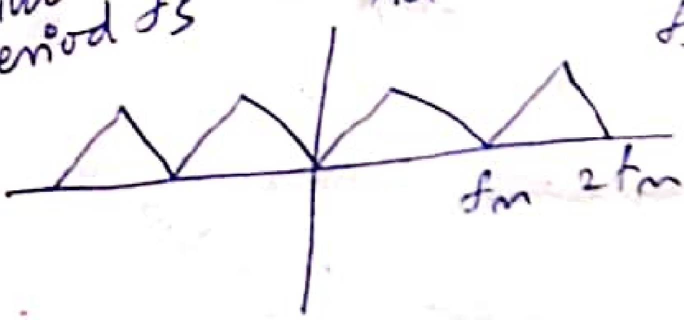


Assumed spectrum

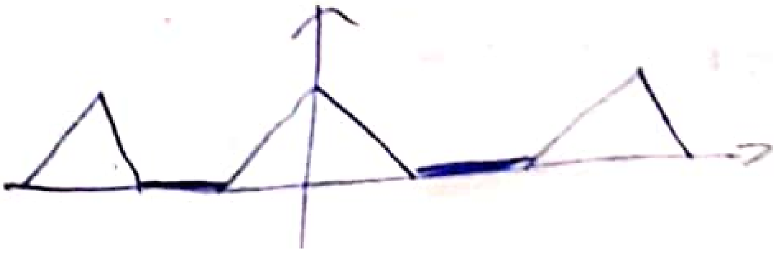
Discrete signal became periodic version of continuous signal but by a factor of $1/T$

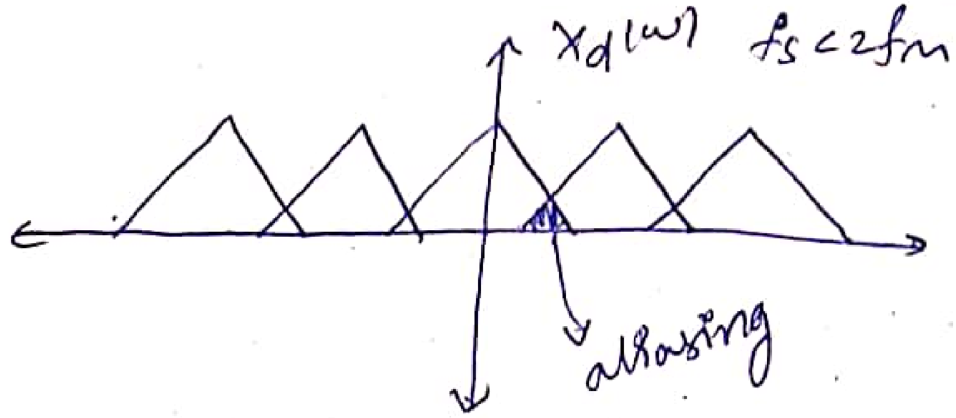
with period f_s

$X_d(\omega)$ $\quad f_s = 2f_m$



$X_d(\omega)$ $\quad f_s > 2f_m$





0

$\longrightarrow$ 0.3125 $\Rightarrow$ 000

5/8

$\longrightarrow$ 0.3125 + d $\Rightarrow$ 001

5/4

15/8

5/2

25/8

30/8

35/8

5

0 – 5V
↓
3-bits

step-size : $\frac{V_{max} - V_{min}}{2^N}$

Max-quantization error: $\frac{V_{max} - V_{min}}{2^N + 1}$

3.75 ——— $\underline{0.3125}$ → error
↳ encoding

3.2 $\underline{101}$ 0.2375

⇒ Music – 128 kbps — $f_s \times N$

* Different kinds of signals & system

i) periodic/aperiodic

ii) CT/DT

iii) even & odd

iv) 1D, 2D, 3D

v) Energy signal / Power signal

$$\sum_{n=-\infty}^{\infty} [x(n)]^2$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Energy is finite & power = 0

power → finite
Energy → ∞

vi) Deterministic / Random

$x(t)$

* Discrete-systems

$x(n)$ → discrete

$x(t)$ → continuous

$= \{1, 2, 3, 4\}$

$= x(0) \cdot \delta(n) + x(1) \cdot \delta(n-1) + x(2) \cdot \delta(n-2) + \cdots$

$$y(n) = x(n) + x(n-1) + y(n-2)\cdot 2 + y(n-1)$$

$$x(n) \longrightarrow \boxed{} \longrightarrow y(n)$$

1) Delay
2) Adder
3) Multiplier

* Linear

$$a_1 x_1(t) + b x_2(t) \rightarrow a_1 y_1(t) + b \cdot y_2(t)$$

$$y(n) = x(n-1)$$

* Time invariance

$$x(n) \rightarrow y(n)$$

$$x(n-d) \rightarrow y(n-d)$$

* stable-systems

$$x(n) \le |M_x|$$

$$y(n) \le |M_y|$$

* Invariability

$$x(n) \longrightarrow \boxed{T} \xrightarrow{y(n)} \boxed{T'} \longrightarrow x(n)$$

* Casual systems

$y(n)$ ← $x(n)$ — $x(n), x(n-1),$ $y(n-2), ---$

output depends on present inputs, past inputs, past outputs

* Memory → memoryless

* $\frac{Max - Min}{2^{N+1}}$ → quantisation error

Bit-rate → $f_s \times N$

* Signals → Discrete / Digital Analysis
  → periodic / aperiodic
  → odd, even
  → Energy ⇒ $E \to$ finite, $P = 0$
  Power ⇒ $E \to \infty$, $P =$ finite
  → 1D, 2D, 3D

* System
  → Linear-system → Homogenity
  → Superposition

$a x_1(n) + b x_2(n)$

→ $a y_1(n) + b y_2(n)$

→ Time-invariant, Time-variant

→ $x(n) \to y(n)$

$x(n-k) \to y(n-k)$

* → Memoryless, Memory

↓

depends only on $x(n)$

→ Causal system → practical

→ stability → Bounded

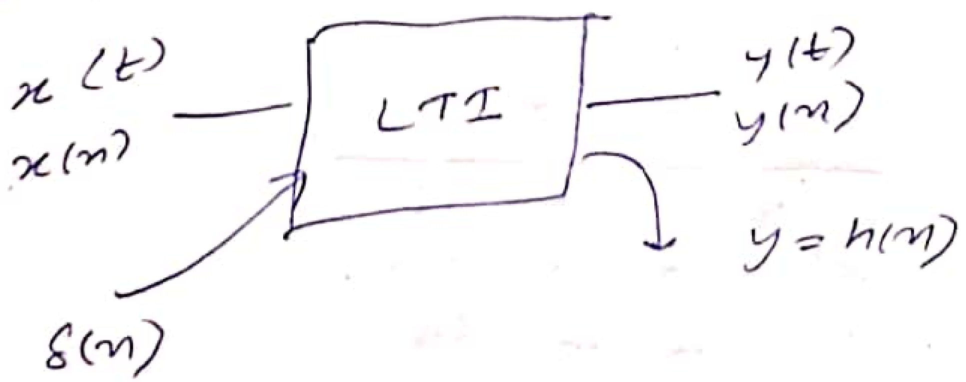
$\begin{cases} \text{casual} \\ \text{linear} \\ \text{Time-inv} \\ \text{Stable} \end{cases}$ LTI

→ convolution

$$x_1(n) * x_2(n) = \sum_{n=-\infty}^{\infty} x_1(m) \cdot x_2(n-M)$$

$$x_1(t) * x_2(t) = \int_{\tau=-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau)\, d\tau$$

→



$$y(n) = x(n) * h(n)$$

→ $x(0) = \{ ---, x(-2), x(-1), x(0), ---- \}$

$$x(n) = --- + x(-2) \cdot \delta(n+2) + x(-1) \cdot \delta(n+1)$$
$$+ x(0) \cdot \delta(0) + ---$$
$$+ x(1) \cdot \delta(n-1) + ---$$
$$+ x(2) \cdot \delta(n-2) + ---$$

$$y(n) = \cdots + x(-2)\cdot h(n+2) + \cdots$$
$$+ \cdots + x(1)\cdot h(n-1)$$
$$= \sum_{m=-\infty}^{\infty} x(m)\cdot h(n-m)$$

$\Rightarrow$ $y(n) = x(n) + x(n-1)$

$h(n) = \delta(n) + \delta(n-1)$ } impulse response

---

* Linear convolution

$x(n) = \{1,2,3,4\}$, $h(n) = \{4,5,6\}$

$y(n) = x(n) * h(n) = \{4, 13, 28, 43, 38, 24\}$

	1	2	3	4
4	4	8	12	16
5	5	10	15	20
6	6	12	18	24

$\|y(n)\| = \|x(n)\| + \|h(n)\| - 1$

* Circular convolution

$\Rightarrow$ $x(t) * h(t) \rightarrow X(\omega)\cdot H(\omega)$

$DFT\{x(n) * h(n)\} \rightarrow X_1(k)\cdot X_2(k)$

$\downarrow N = \text{size}$

$\Rightarrow \{1,2,3,4\} \quad \{1,2,3,4\}$

$\{4,5,6\} \longrightarrow \{4,5,6,0\}$

$$y(n)_N = \sum x(m)_N \cdot h(n-m)_N$$

$\{1,2,3,4\}, \{4,5,6\}$

$$\begin{vmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{vmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 42 \\ 37 \\ 28 \\ 43 \end{bmatrix}$$

→ Circular convolution

↳ Linear convolution

$\cancel{\begin{bmatrix} 4 \\ 5 \end{bmatrix}}$

$\{1,2,3,4,0,0\}$

$\{4,5,6,0,0,0\}$