2. SYSTEMS

② Systems

① If $x(n)$ is input & corresponding output is $y(n)$, then if $y(n-k)$ is same as the output produced when $x(n-k)$ is given as input, the system is time-invariant

ⓐ $y(t) = t^2 x(t-1)$

Consider an arbitrary input $x_1(t)$.

Let $y_1(t) = t^2 x_1(t-1)$ be the corresponding output

→ Consider $x_2(t)$ by shifting $x_1(t)$ in time

$x_2(t) = x_1(t-t_0)$

The output corresponding to this input is

$y_2(t) = t^2 x_2(t-1)$

$= t^2 x_1(t-t_0-1)$

$y_1(t-t_0) = (t-t_0)^2 x_1(t-t_0-1) \neq y_2(t)$

Therefore, the system is not time-invariant

(b) $y[n] = x[n-1] + x[n+1]$

Now, if input is $x(n-k)$, let the output be y'

$$y'(n) = 3\cdot x(n-k-1) + x(n-k+1)$$

$$= x((n-k)-1) + x((n-k)+1)$$

$$= y(n-k)$$

→ Same delay is produced in output

↳ Hence the above system is time-invariant

(c) $y[n] = 1/x[n]$

$$y(n-k) = 1/x(n-k)$$

Now, if input is $x(n-k)$, let the output be y'

$$y'(n) = \frac{1}{x(n-k)} = y(n-k)$$

→ same delay is produced in output

↳ Hence the system is time-invariant

(d)

Given the system S with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n] \cdot (g(n) + g(n-1))$$

(i) Given that $g(n) = 1 \ \forall n$

Then,

$$y[n] = x[n] \cdot (1+1)$$
$$= 2x[n]$$

→ Consider a shift in $y[n]$ by n_0

$$y[n-n_0] = 2 \cdot x(n-n_0)$$

⇒ If the input is shifted by n_0, then the $x[n-n_0]$ output corresponding to this input is given by,

$$y'(n) = 2 \cdot x(n-n_0) = y(n-n_0)$$ (same delay is produced in output)

→ Hence, the above system is $\boxed{\text{time-invariant}}$

(ii) Given that $g(n) = n$

Then,

$$y(n) = x[n] \cdot (n+n-1)$$
$$= (2n-1) \cdot x[n]$$

Now if input is shifted by n_0, $x(n-n_0)$ let the output be y'

$$y'(n) = (2n-1) \cdot x(n-n_0) \neq y(n-n_0)$$

$$(2n-2n_0-1) \cdot x(n-n_0)$$

→ Consider a shift in $y[n]$ by n_0

$$y[n-n_0] = (2(n-n_0)-1)\, x(n-n_0)$$
$$= (2n+1-2n_0)\, x(n-n_0)$$

shift by n_0 in the input doesn't have a corresponding shift in the output

→ Hence the above system is time-variant

(c) Given that

$$g(n) = 1+(-1)^n$$

Then,

$$y(n) = x(n)\left[1+(-1)^n + 1 + (-1)^{n-1}\right]$$
$$= x(n)\left[2+(-1)^{n-1}[1+(-1)]\right]$$
$$= 2x(n)$$

Let the input be shifted by n_0, let the output be y'

↳ $x[n-n_0]$

$$y'(n) = 2x(n-n_0) = y(n-n_0)$$

↑ a shift of n_0 in the output

→ A shift of n_0 in the input have a corresponding shift in the output with same delay

Hence the system S is time-invariant

(2.) Let $x_1(n) \rightarrow y_1(n)$, $x_2 \rightarrow y_2(n)$

The system is linear when

$$a_1 x_1(n) + a_2 x_2(n) \longrightarrow a_1 y_1(n) + a_2 y_2(n)$$

(a) $y(t) = x(\sin t)$.

Consider 2 arbitrary inputs $x_1(t)$ and $x_2(t)$

$x_1(t) \rightarrow y_1(t) = x_1(\sin t)$

input

$x_2(t) \rightarrow y_2(t) = x_2(\sin t)$

(a & b are arbitrary)

Let $x_3(t) = a x_1(t) + b x_2(t)$ ⇒ linear combination of $x_1(t)$ & $x_2(t)$

If $x_3(t)$ is the input to given system, then corresponding output $y_3(t)$ is

$$y_3(t) = x_3(\sin t)$$

$$= a x_1(\sin t) + b x_2(\sin t)$$

$$= a y_1(t) + b y_2(t)$$

⇒ Therefore, the system is linear

(b) $y(t) = \begin{cases} 0 & , t<0 \\ x(t)+x(t-2) & , t\geq 0 \end{cases}$

consider 2 arbitrary inputs $x_1(t)$ & $x_2(t)$

$x_1(t) \rightarrow y_1(t) = \begin{cases} 0 & , t<0 \\ x_1(t)+x_1(t-2) & ; t\geq 0 \end{cases}$

$x_2(t) \rightarrow y_2(t) = \begin{cases} 0 & , t<0 \\ x_2(t)+x_2(t-2) & ; t\geq 0 \end{cases}$

Let $x_3(t) = ax_1(t) + bx_2(t)$ → a & b are arbitrary

→ Linear combination of $x_1(t)$ & $x_2(t)$

→ If $x_3(t)$ is input to given system, then corresponding

$y_3(t) = \begin{cases} 0 & , t<0 \\ x_3(t)+x_3(t-2) & , t\geq 0 \end{cases}$

$= \begin{cases} 0 & , t<0 \\ ax_1(t)+ax_1(t-2)+ bx_2(t)+bx_2(t-2) & , t\geq 0 \end{cases}$

$= \begin{cases} 0 & , t<0 \\ ay_1(t)+by_2(t) & , t\geq 0 \end{cases}$

→ Therefore, the system is linear

satisfies both additivity & Homogeneity

(c) $y(t) = \dfrac{d(x(t))}{dt}$

$\Rightarrow$ Consider 2 arbitrary inputs $x_1(t)$ & $x_2(t)$

$x_1(t) \rightarrow y_1(t) = \dfrac{dx_1(t)}{dt}$

$x_2(t) \rightarrow y_2(t) = \dfrac{dx_2(t)}{dt}$

$\rightarrow$ a & b are arbitrary scalars

Let $x_3(t) = ax_1(t) + bx_2(t)$

$\hookrightarrow$ linear combination of $x_1(t)$ & $x_2(t)$

$\rightarrow$ If $x_3(t)$ is input to given system, then

$$y_3(t) = \frac{dx_3(t)}{dt}$$

$$= \frac{d}{dt}(ax_1(t) + bx_2(t))$$

$$= a \cdot \frac{d}{dt}(x_1(t)) + b \cdot \frac{d}{dt}(x_2(t))$$

$$\boxed{y_3(t) = ay_1(t) + by_2(t)}$$

$\downarrow$ satisfies both additivity & homogeneity properties

$\rightarrow$ Therefore, the system is linear

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(d) $y[n] = \sum_{m=0}^{M} a \cdot x[n-m] + \sum_{m=1}^{N} b \cdot x[n-m]$

$$y_1(n) = \sum_{m=0}^{M} a \cdot x_1(n-m) + \sum_{m=1}^{N} b \cdot x_1(n-m)$$

$$y_2(n) = \sum_{m=0}^{M} a \cdot x_2(n-m) + \sum_{m=1}^{N} b \cdot x_2(n-m)$$

Let $y_3(n)$ be the output when $a_1x_1(n) + a_2x_2(n)$ is given as input

$$y_3(n) = \sum_{m=0}^{M} a \cdot [a_1x_1(n-m) + a_2x_2(n-m)] + \sum_{m=1}^{N} b \cdot [a_1x_1(n-m) + a_2x_2(n-m)]$$

$$= a_1 \cdot \sum_{m=0}^{M} a x_1(n-m) + a_2 \sum_{m=0}^{M} a x_2(n-m) + a_1 \sum_{m=1}^{N} b \cdot x_1(n-m) + a_2 \cdot \sum_{m=1}^{N} b x_2(n-m)$$

$$= a_1 \left[\sum_{m=0}^{M} a \cdot x(n-m) + \sum_{m=1}^{N} b \cdot x(n-m) \right] + a_2 \left[\sum_{m=0}^{M} a \cdot x(n-m) + \sum_{m=1}^{N} b \cdot x(n-m) \right]$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

→ Therefore, the system is Linear

ⓔ $y[n] = a.x[n] + \frac{b}{x[n-1]}$

Let $y_1(n) = a x_1(n) + \frac{b}{x_1(n-1)}$.

$y_2(n) = a x_2(n) + \frac{b}{x_2(n-1)}$

Let $y_3(n)$ be the output when $a_1 x_1(n) + a_2 x_2(n)$ is input

$$y_3(n) = a\left(a_1 x_1(n) + a_2 x_2(n)\right) + \frac{b}{a_1 x_1(n-1) + a_2 x_2(n-1)}$$

Now,

$$a_1 y_1(n) + a_2 y_2(n) = a_1\left(a x_1(n) + \frac{b}{x_1(n-1)}\right) + a_2\left(a x_2(n) + \frac{b}{x_2(n-1)}\right)$$

$$\neq y_3(n)$$

$$\frac{a_1 b}{x_1(n-1)} + \frac{a_2 b}{x_2(n-1)} \neq \frac{b}{a_1 x_1(n-1) + a_2 x_2(n-1)}$$

⇒ Therefore, the system is non-linear

(3) A system is causal if it doesn't depend on future-inputs

(a) $y(t) = x(t-2) + x(2-t)$

Consider the output at $t=0$; i.e,

$$y(0) = x(0-2) + x(2-0)$$
$$= x(-2) + x(2)$$

Output $y(0)$, at $t=0$, depends upon the past value $x(-2)$ and future value $x(2)$

→ Therefore, the system is not causal

(b) $y(t) = x(t) \cdot \cos 3t$

Consider the output at $t=t'$, i.e

$$y(t') = x(t') \cdot \cos(3t')$$

The output at $t=t'$, depends on the present value $x(t')$

→ Therefore, the system is causal

(c) $y(t) = \int_{-\infty}^{2t} x(k)\,dk$

Consider the output at $t=t'$, i.e

$$y(t') = \int_{-\infty}^{2t'} x(k)\,dk$$

The output $y(t')$ at $t=t'$ depends on the past-inputs and the future-inputs i.e,

i.e, $-\infty < t \le t'-1$ and

$t'+1 \le t < 2t'$

→ Therefore, the system is not causal

(d) $y[n] = \sum_{k=0}^{\infty} x[n+k]$

as $k \geq 0$

$n+k \geq n$

→ $y(n)$ depends on future inputs

Therefore, the system is not causal system

(e) $y[n] = \sum_{k=0}^{\infty} x[n-k]$

as $k \geq 0 \Rightarrow -k \leq 0$

$n-k \leq n$

→ $y(n)$ depends only on present & past inputs

Therefore, the system is causal