DSA - Assignment - 3

Name : Gowlapalli Rohit

Roll No : 2021101113

① Given the sequences $x[n]$ and $h[n]$ where

$$x[n] = \{-1, 1, 0, 1\} \Rightarrow l_1 = 4$$

$$h[n] = \{1, 2, 3, 4, 5\} \Rightarrow l_2 = 5$$

Length of linear convolution $= l_1 + l_2 - 1$

$= 4 + 5 - 1$

$= 8$

$$\{1, 2, 3, 4, 5\} * \{-1, 1, 0, 1\}$$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| -1 | -1 | -2 | -3 | -4 | -5 |
| 1 | 1 | 2 | 3 | 4 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 2 | 3 | 4 | 5 |

⇒ linear convolution

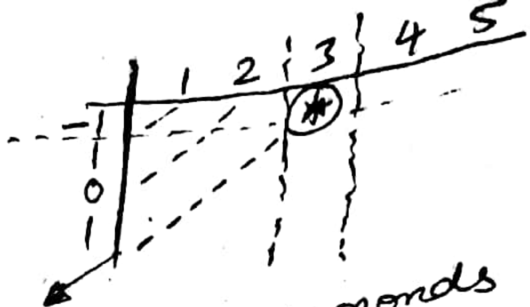
$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) = y[n]$$

Matrix method to calculate linear convolution

$x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$= y[n]$$




This sum corresponds to the arrow in final convoluted output

$$y[-2] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-2-k)$$

$$= -1 \cdot 1 = -1$$

$$y[-1] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-1-k) = 1(1) + (-1)(2) = -1$$

$$y[0] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(-k) = 0(1) + 1(2) + (-1)(3) = -1$$

$$y[1] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(1-k) = 1(1) + 0(2) + 1(3) + (-1)(4)$$

$$= 0$$

$$y[2] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(2-k) = 1(2) + 0(3) + 1(4) + (-1)(5)$$

$$= 1$$

$$y[3] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(3-k) = 1(3) + 0(4) + 1(5)$$

$$= 8$$

$$y[4] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(4-k) = 1(4) + 0(5) = 4$$

$$y[5] = \sum_{k=-\infty}^{\infty} x(k) \cdot h(5-k) = 1(5) = 5$$

$$x(n) * h(n) = y(n) = \{-1, -1, \underset{\uparrow}{-1}, 0, 1, 8, 4, 5\}$$

$\boxed{\text{Linear convolution of } x(n) \text{ \& } h(n) \text{ is given by } \{-1, -1, \underset{\uparrow}{-1}, 0, 1, 8, 4, 5\}}$

Length of circular convolution $= \max(l_1, l_2)$

$= \max(4,5)$

$= 5$

$\Rightarrow$ Inorder to calculate circular convolution we pad $x[n]$ with $\max(l_1, l_2) - l_1$ zeroes

$= 5-4$ zeroes

$= 1$ zeroes

circular convolution

$\{-1,1,0,1\} \circledast \{1,2,3,4,5\}$

$\downarrow$ padded with 1 zero

$\{-1,1,0,1,0\}$

multiplied using matrix method of circular convolution

$$(h[n])_N = \{1, 2, \underset{\uparrow}{3}, 4, 5\} = \{\underset{\uparrow}{3}, 4, 5, 1, 2\}$$

Circular convolution array

$$y(n)_N = \sum_{m=-\infty}^{\infty} x(m)_N \, h(n-m)_N$$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3+5+2 \\ 3-4+1 \\ 4-5+2 \\ 3+5-1 \\ 4+1-2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 0 \\ 1 \\ 7 \\ 3 \end{bmatrix}$$

$$\boxed{\{\underset{\uparrow}{-1}, 1, 0, 1\} \circledast \{1, 2, \underset{\uparrow}{3}, 4, 5\} = \{4, 0, 1, 7, 3\}}$$

Circular convolution of $x(n)$ and $h(n)$ is given by $\{\underset{\uparrow}{4}, 0, 1, 7, 3\}$

(2) Given a relaxed LTI system with impulse response $h[n] = a^n u[n]$, $|a|<1$

when input sequence is a unit step-sequence i.e $x[n] = u[n]$

$$x(n) \longrightarrow \boxed{\begin{matrix}\text{LTI}\\ h(n)\end{matrix}} \longrightarrow y(n)$$

$$\boxed{\begin{array}{c} y(n) = x(n) * h(n) \\ \text{ZT}\downarrow\ \uparrow\text{IZT} \qquad \text{ZT}\downarrow\ \uparrow\text{IZT} \\ Y(z) = X(z)\cdot H(z) \end{array}}$$

$$x(n) = u(n) \xrightarrow{\text{ZT}} X(z) = \frac{1}{1-z^{-1}}$$

$$h(n) = a^n u(n) \xrightarrow{\text{ZT}} H(z) = \frac{1}{1-az^{-1}}$$

$$X(z)\cdot H(z) = \frac{1}{(1-z^{-1})(1-az^{-1})}$$

$$X(z)\cdot H(z) = \left(\frac{1}{1-a}\right)\left(\frac{1}{1-z^{-1}}\right) + \left(\frac{a}{a-1}\right)\left(\frac{1}{1-az^{-1}}\right)$$

$$\boxed{Y(z) = \frac{\left(\frac{1}{1-a}\right)}{1-z^{-1}} + \frac{\left(\frac{a}{a-1}\right)}{1-az^{-1}}}$$

$$y(n) = \text{IZT}\left(Y(z)\right) = \text{IZT}\left[\frac{\left(\frac{1}{1-a}\right)}{1-z^{-1}} + \frac{\left(\frac{a}{a-1}\right)}{1-az^{-1}}\right]$$

$$= \left(\frac{1}{1-a}\right)u[n] + \left(\frac{a}{a-1}\right)a^n \cdot u[n]$$

Output

$$\boxed{y(n) = u[n]\cdot\left[\frac{1-a^{n+1}}{1-a}\right]}$$

$$\boxed{y[n] = \begin{cases} \frac{(1-a^{n+1})}{1-a}, & n \geq 0 \\ 0, & n < 0 \end{cases}}$$

(3) Given the shift-invariant system

$$y[n] = 0.1x[n] + 0.2x[n-1] + 0.3x[n-2] + 0.4x[n-4]$$

$$x(n) \longrightarrow \boxed{\text{LTI} \; h(n)} \longrightarrow y(n)$$

$$y(n) = x(n) * h(n)$$

$\downarrow$ F.T F.T $\downarrow$

$$\boxed{Y(z) = X(z) \cdot H(z)}$$

Time-shifting property: $x(n) \xrightarrow{Z.T} X(z)$

$x(n-k) \xrightarrow{Z.T} z^{-k} \cdot X(z)$

Applying z-transform on the equation

$$Y(z) = 0.1X(z) + 0.2(z^{-1})X(z) + 0.3(z^{-2})X(z) + 0.4(z^{-4})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.1 + 0.2z^{-1} + 0.3z^{-2} + 0.4z^{-4}$$

$$H(z) = 0.1 + 0.2z^{-1} + 0.3z^{-2} + 0.4z^{-4}$$

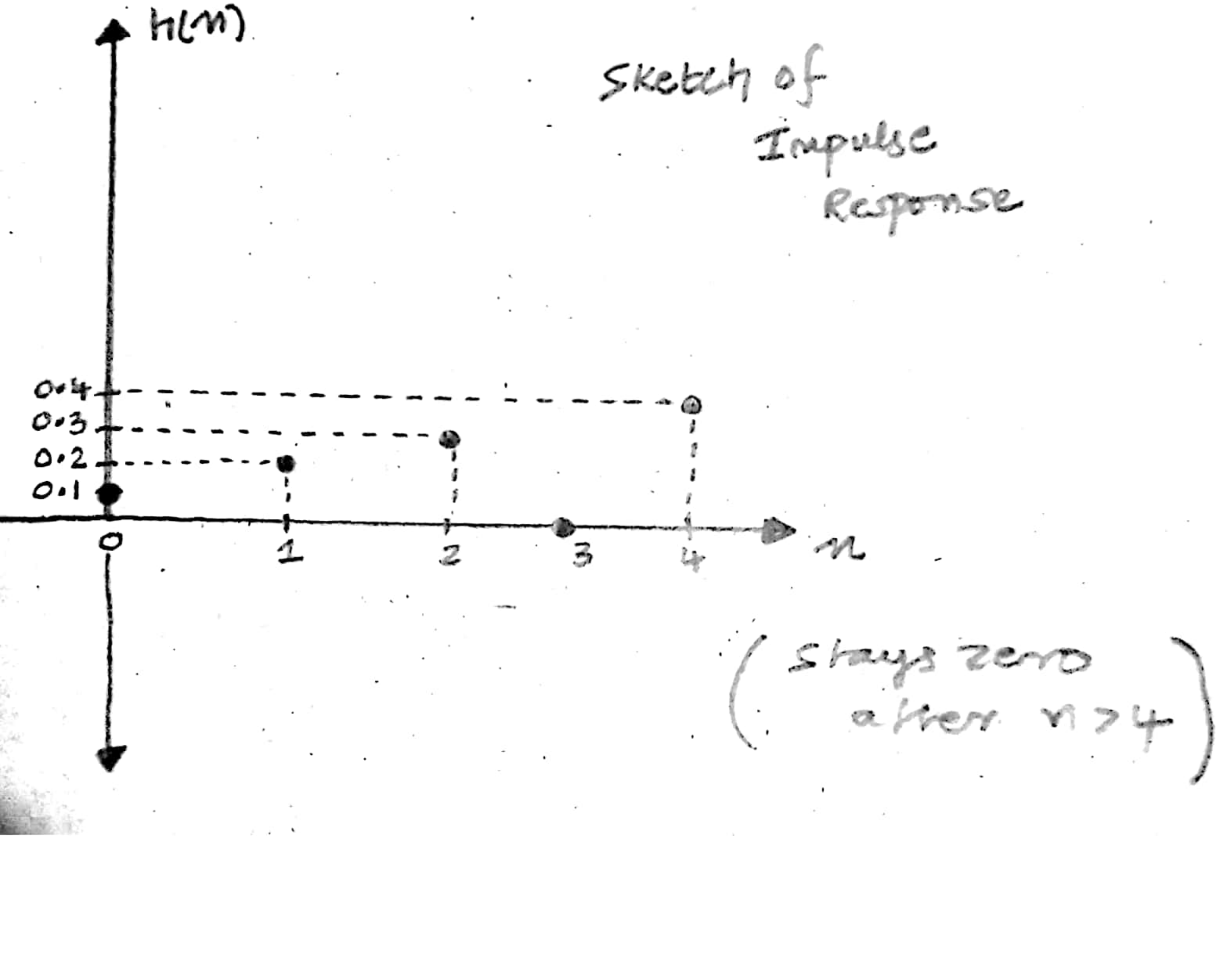
$\downarrow$ Applying inverse z-transform

$\left[ \begin{array}{l} \delta(n) \xrightarrow{ZT} 1 \\ \delta(n-k) \xrightarrow{ZT} z^{-k} \end{array} \right]$

$$\boxed{\begin{aligned} h(n) = 0.1\,\delta(n) &+ 0.2\,\delta(n-1) \\ &+ 0.3\,\delta(n-2) \\ &+ 0.4\,\delta(n-4) \end{aligned}}$$

where $\delta$ is kronecker delta function

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{else} \end{cases}$$



h(n)
Sketch of Impulse Response
0.4
0.3
0.2
0.1
0
1
2
3
4
n
(stays zero after n > 4)

(4) $y[n] = x[n] - 0.5x[n-1] + 0.36x[n-2]$

Applying Time-shifting property: $x(n) \xrightarrow{ZT} X(z)$

$x(n-k) \xrightarrow{ZT} z^{-k}X(z)$

Applying z-transform on both sides,

$$Y(z) = X(z) - 0.5(z^{-1})X(z) + 0.36z^{-2}X(z)$$

Therefore, the transfer function, i.e the ratio of $Y(z)$ to $X(z)$ can be found as

$$H(z) = Y(z)/X(z)$$

$$= 1 - 0.5z^{-1} + 0.36z^{-2}$$

$$= \frac{B(z)}{A(z)}$$

$B(z)$ → numerator polynomial $B(z)$

$A(z)$ → denominator polynomial $A(z)$

From the derived transfer function,

$$\boxed{A(z) = 1}$$

$$\boxed{B(z) = 1 - 0.5z^{-1} + 0.36z^{-2}}$$

5

(a) $x(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

z-transform

$z = re^{j\omega}$

ROC → Region of convergence

$$X(z) = \underbrace{\sum_{n=-\infty}^{n=-3} 0 \cdot z^{-n}}_{0} + \sum_{n=-2}^{n=3} x(n) \cdot z^{-n} + \underbrace{\sum_{n=4}^{n=\infty} 0 \cdot z^{-n}}_{=0}$$

$$= \sum_{n=-2}^{n=3} x(n) \cdot z^{-n} = 2z^2 + 4z + 5 + 7/z + 1/z^3 + 0/z^2$$

$$= 2z^2 + 4z + 5 + 7/z + 1/z^3$$

$$\boxed{ROC : \mathbb{C} - \{0\} - \{\infty\}}$$

z-plane

X(z) is finite for all possible values of z except 0 and $\infty$

(b) Given that

$$x(n) = a^n u(n) + b^n u(-n-1)$$

$z = re^{j\omega}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

ROC → Region of convergence

z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} [a^n u(n) + b^n u(-n-1)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) z^{-n} + \sum_{n=-\infty}^{\infty} b^n u(-n-1) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{n=-1} a^n \cdot 0 + \sum_{n=0}^{\infty} a^n \cdot 1 \cdot z^{-n} + \sum_{n=-\infty}^{n=-1} b^n \cdot 1 \cdot z^{-n}$$

$$+ \sum_{n=0}^{\infty} b^n \cdot 0 \cdot z^{-n}$$

$$= 0 + \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=-\infty}^{n=-1} b^n z^{-n} + 0$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{n=-\infty}^{n=-1} b^n z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} + \sum_{t=1}^{\infty} z^t / b^t \quad \text{where } t = -n$$

Now, applying the formula for Infinite G.P's

$$\left( a + ar + ar^2 + \cdots = \frac{a}{1-r}, \; |r| < 1 \right)$$

$$= \frac{1}{1 - a/z} + \left(\frac{z}{b}\right)\left[\frac{1}{1 - z/b}\right]$$

$$= \frac{z}{z-a} + \frac{z}{b-z}$$

Here for $X(z)$ to be finite,

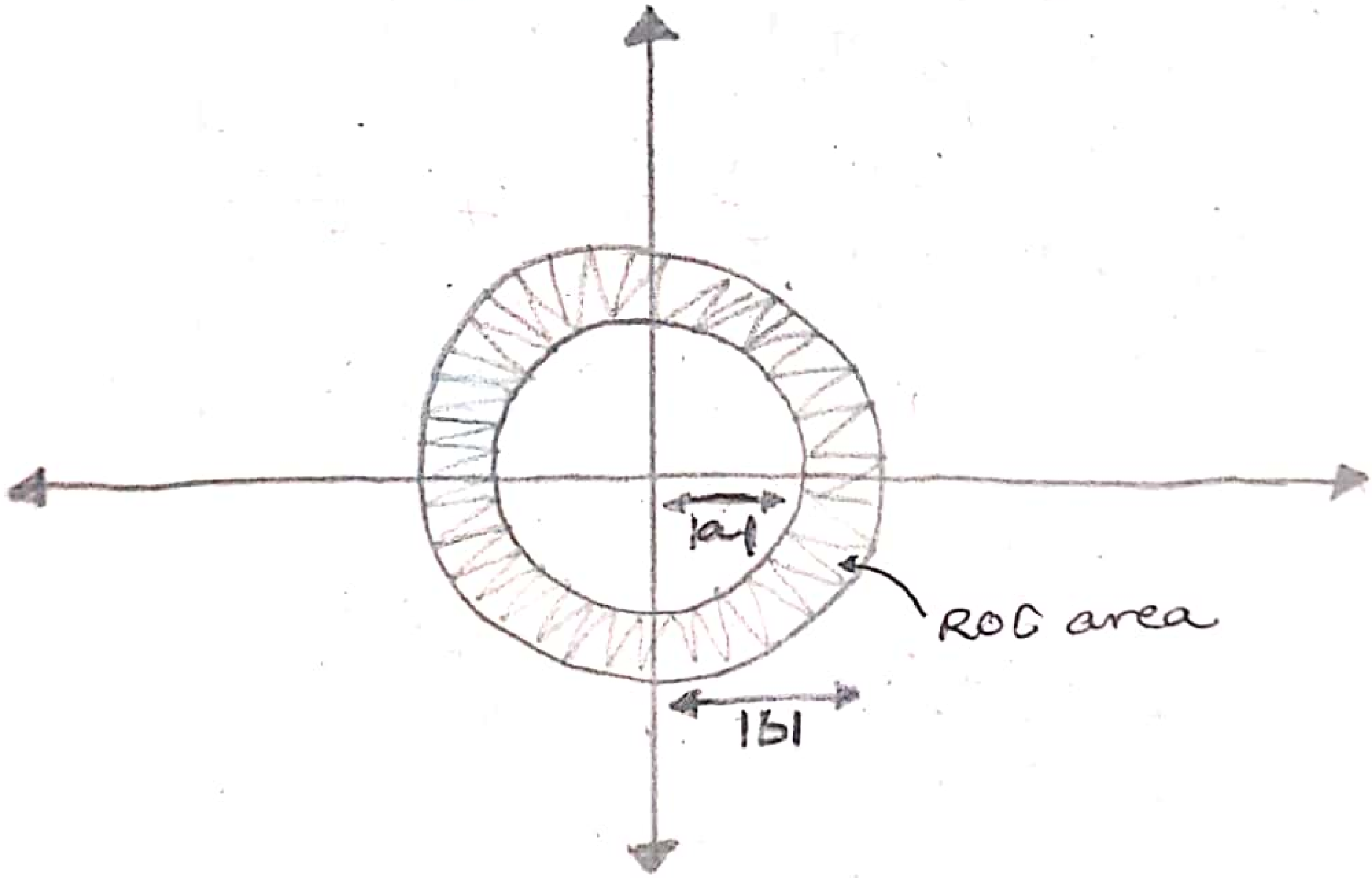
$$|a/z| < 1 \text{ and } |z/b| < 1$$

$\Rightarrow |z| > |a|$ and $|z| < |b| \Rightarrow ROC: (|z| < |b|) \cap (|z| > |a|)$

$\Rightarrow$ $\boxed{ROC: |a| < |z| < |b|}$ [assuming $|b| > |a|$]

$\downarrow$ else no ROC

Scanned with CamScanner


|a|
ROC area
|b|

(6) Given that $x_1(n) = 3\delta(n) + 2\delta(n-1)$

$x_2(n) = 2\delta(n) - \delta(n-1)$

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}}$$

(a) $X(z) = Z(x_1(n) * x_2(n))$

From the properties of z-transform

if $x(n) \rightarrow x_1(n) * x_2(n)$

$$\begin{array}{ccc} x(n) & \rightarrow & x_1(n) * x_2(n) \\ \downarrow ZT & & \downarrow ZT \\ X(z) & \rightarrow & X_1(z) \cdot X_2(z) \end{array}$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [3\delta(n) + 2\delta(n-1)] z^{-n}$$

$$= 3\sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n} + 2\sum_{n=-\infty}^{\infty} \delta(n-1) \cdot z^{-n}$$

$$= 3 \cdot \left[\underbrace{\sum_{n=-\infty}^{n=-1} 0 \cdot z^{-n}}_{=0} + z^{-0}(1) + \underbrace{\sum_{n=1}^{\infty} 0 \cdot z^{-n}}_{=0}\right]$$

$$+ 2\left[\underbrace{\sum_{n=-\infty}^{n=0} 0 \cdot z^{-n}}_{=0} + z^{-1} \cdot (1) + \underbrace{\sum_{n=2}^{\infty} 0 \cdot z^{-n}}_{=0}\right]$$

$$\boxed{X_1(z) = 3 + 2z^{-1}}$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} x_2(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [2\delta(n) - \delta(n-1)]\, z^{-n}$$

$$= \left(2 \cdot \sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n}\right) - \left(\sum_{n=-\infty}^{\infty} \delta(n-1) \cdot z^{-n}\right)$$

$$= 2\left[\sum_{n=-\infty}^{n=-1} 0 \cdot z^{-n} + z^{0}(1) + \sum_{n=1}^{\infty} 0 \cdot z^{-n}\right] \quad (=0) \quad (=0)$$

$$- \left[\sum_{n=-\infty}^{n=0} 0 \cdot z^{-n} + 1 \cdot z^{-1} + \sum_{n=2}^{\infty} 0 \cdot z^{-n}\right] \quad (=0) \quad (=0)$$

$$\Rightarrow 2 - z^{-1} \quad \Rightarrow \boxed{X_2(z) = 2 - z^{-1}}$$

$$X_1(z) \cdot X_2(z) = (2 - z^{-1}) \cdot (3 + 2z^{-1})$$

$$= 6 + 4z^{-1} - 3z^{-1} - 2z^{-2}$$

$$= 6 + z^{-1} - 2z^{-2}$$

$$\boxed{X_1(z) \cdot X_2(z) = 6 + 1/z - 2/z^2}$$

$$\boxed{X(z) = 6 + 1/z - 2/z^2}$$

ROC $= \mathbb{C} - \{0\}$

Region of convergence

Complex plane

(b) $x(n) = x_1(n) * x_2(n)$

$$= \sum_{k=0}^{\infty} x_1(k) \cdot x_2(n-k)$$

$$x_1(n), x_2(n) \xrightarrow{\text{z-Transform}} X_1(z), X_2(z)$$

$\downarrow$ convolution of $x_1(n), x_2(n)$

$$x_1(n) * x_2(n) \xleftarrow[\text{z-transform}]{\text{Inverse}} X_1(z) \cdot X_2(z)$$

$$= x(n) \qquad\qquad = X(z)$$

$$x(n) = \text{Inverse-z-transform}([X_1(z) \cdot X_2(z)])$$

$$= \text{Inverse-z-transform}(X(z))$$

$$= IZT[6 + z^{-1} - 2z^{-2}]$$

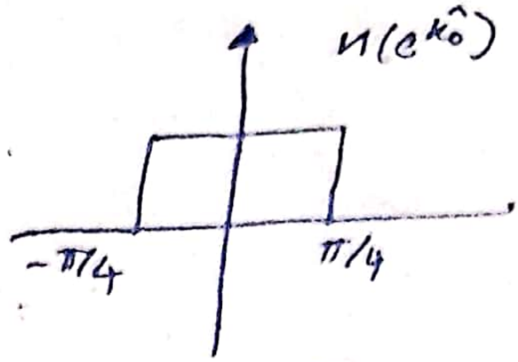
From part-(a)

$$\boxed{x(n) = 6\delta(n) + \delta(n-1) - 2\delta(n-2)}$$

$$\left[\begin{array}{c} x(n) = \delta(n) \\ \downarrow ZT \\ X(z) = 1 \end{array}\right]$$

$$\left[\begin{array}{l} \text{property of Time-shifting:} \\ x(n) \xrightarrow{ZT} X(z) \\ x(n-k) \xrightarrow{ZT} X(z) \cdot z^{-k} \end{array}\right]$$

⑦




ⓐ DTFT $\rightarrow h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{jn\omega}\, d\omega$$

$$= \frac{1}{2\pi j n} \left[ e^{j\omega n} \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{\sin(\pi/4\, n)}{\pi n}$$

Take $N = 11$ $\rightarrow (h(-5), h(-4), \ldots h(5)) \times z^{-5}$

$\downarrow$

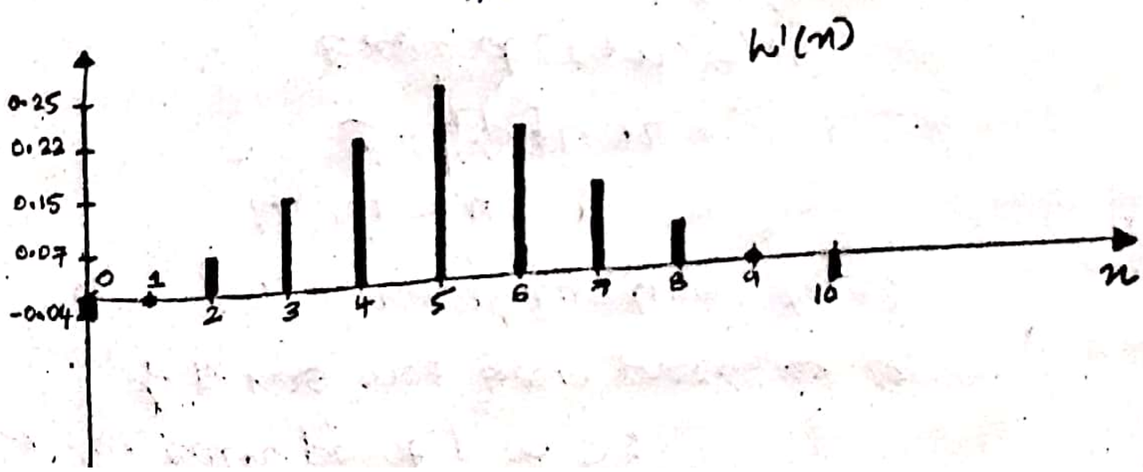
$-5 \le n \le 5$

$\downarrow$

$(h(0), \ldots, h(10))$

$$H'(e^{j\omega}) = H(e^{j\omega}) \cdot e^{-5j\omega}$$

$$h'[n] = \frac{\sin\left(\frac{\pi(n-5)}{4}\right)}{\pi(n-5)}$$



(b) Hamming window:

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{M-1}\right), \quad 0 \le n \le M-1$$

$M = N = 8+1 = 9$ ← Let $N = 9$

$$h(n) = h_d(n) \cdot w(n)$$

when $N = 9$,

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{8}\right)$$

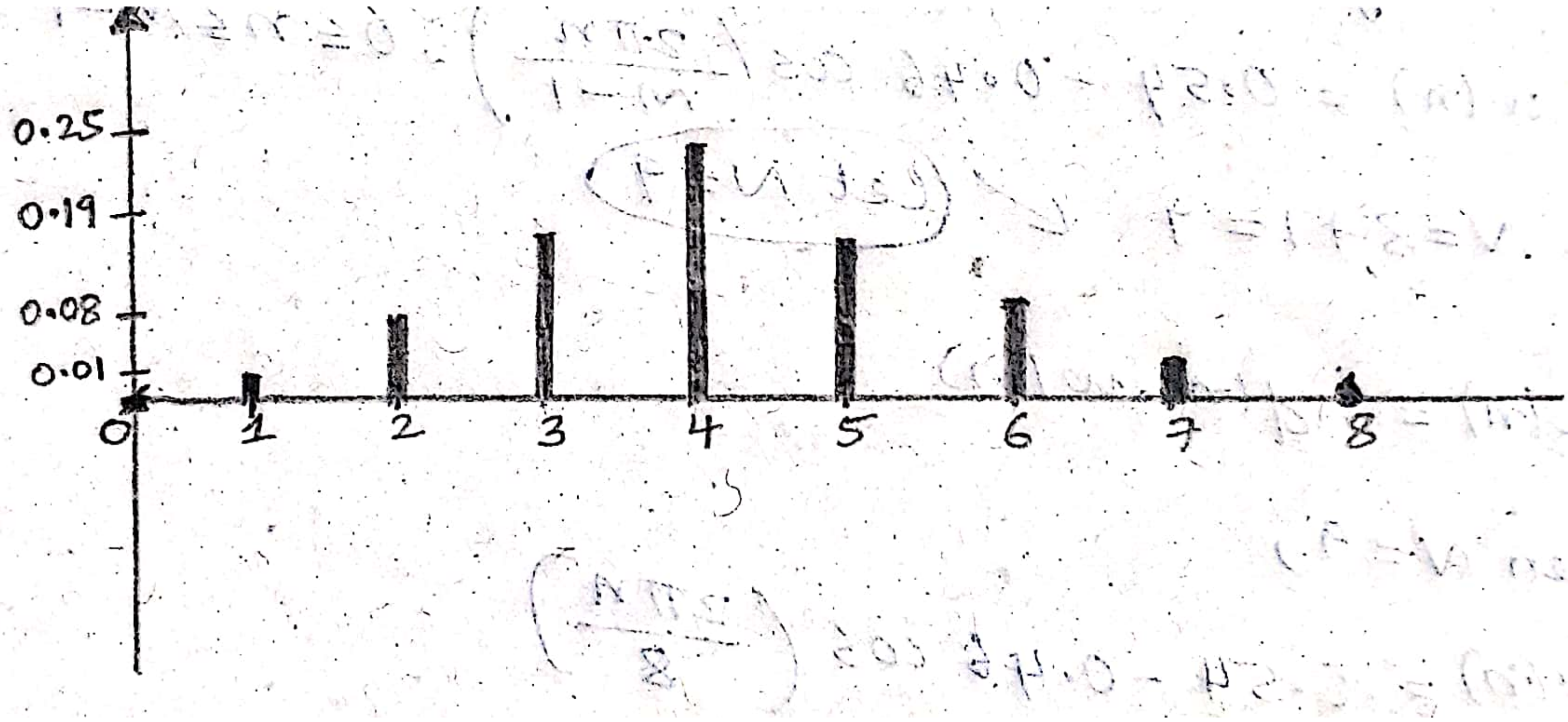
$$= 0.54 - 0.46 \cos(\pi n/4)$$

$$h(n) = \frac{\sin(\pi n/4)}{\pi n} \times (0.54 - 0.46 \cos(\pi n/4))$$

$$h'(n) \longrightarrow \int H(z) \cdot e^{-4j\omega} = H(z) \cdot z^{-4}$$

$$h'(n) = \frac{\sin\left(\frac{\pi(n-4)}{4}\right) \times (0.54 - 0.46 \cos\left(\frac{\pi n}{4}\right)}{\pi(n-4)}$$

$\alpha = \frac{N-1}{2}$



0.25
0.19
0.08
0.01
0
1
2
3
4
5
6
7
8

**Q8]**

## <u>Name in the Audio:</u> Gowlapalli Rohit

<u>Coarticulation: ▪</u>

- This refers to the change in speech articulation of the current speech segment due to neighbouring speech or noise.
- This phenomenon arises in speech articulation because the movements of articulators are affected by the neighbouring phoneme.
- <u>Example:</u> If the word ‘tulip’ is to be pronounced, then the entire word pronunciation planning completely depends on the next character. If “tulip” were produced in a piecemeal fashion, with each sound planned only after the preceding sound was produced, the rounding of the lips required for “u” would only occur after “t” was uttered.
- From the above example, we understand that articulation of the current speech is being affected because of the preceding sound. So, this stands as a good example for coarticulation.

<u>Formant</u>

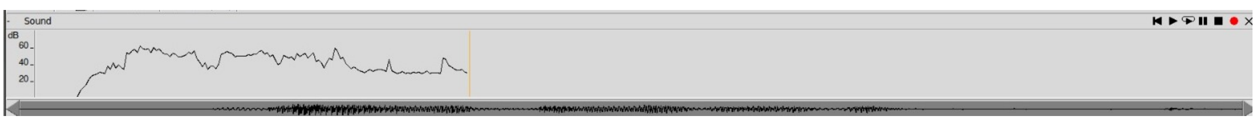
- The spectral peaks of the spectrum are referred to as formants.
- The peaks which are determined within the spectrum envelope are termed to be formant.
- Formants are basically frequency peaks with a high degree of energy. They are especially prominent in vowels.

- Below is the formant plot generated using the wave surfer for the .wav file with an audio


## <u>Spectrogram</u>

- Spectrogram basically represents the signal strength or loudness of a signal over a time at various frequencies present in a waveform.
- In simple words, Spectrogram is a picture of sound.
- Below is the spectrogram of my audio file


## <u>Power-Plot</u>




## <u>Wave-form</u>

## Q8]

## Name in the Audio: Gowlapalli Rohit

Coarticulation: ▪

- This refers to the change in speech articulation of the current speech segment due to neighbouring speech or noise.
- This phenomenon arises in speech articulation because the movements of articulators are affected by the neighbouring phoneme.
- Example: If the word ‘tulip’ is to be pronounced, then the entire word pronunciation planning completely depends on the next character. If “tulip” were produced in a piecemeal fashion, with each sound planned only after the preceding sound was produced, the rounding of the lips required for “u” would only occur after “t” was uttered.
- From the above example, we understand that articulation of the current speech is being affected because of the preceding sound. So, this stands as a good example for coarticulation.

Formant

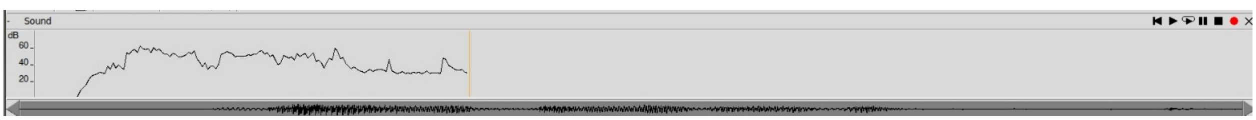
- The spectral peaks of the spectrum are referred to as formants.
- The peaks which are determined within the spectrum envelope are termed to be formant.
- Formants are basically frequency peaks with a high degree of energy. They are especially prominent in vowels.

- Below is the formant plot generated using the wave surfer for the .wav file with an audio


## Spectrogram

- Spectrogram basically represents the signal strength or loudness of a signal over a time at various frequencies present in a waveform.
- In simple words, Spectrogram is a picture of sound.
- Below is the spectrogram of my audio file


## Power-Plot




## Wave-form