

DSA-Assignment-1

Deadline: 20th March 2024

-
1. Solve all the question and submit a handwritten document
 2. Plagiarism will be penalised
 3. Submit a pdf of the form <roll_no>_dsa1.pdf
-

1 Even Odd Functions - 4 marks

1. A signal $x[n]$ is defined to be odd if $x[-n] = -x[n]$ for all n . The signal $x[n]$ is defined to be even if $x[-n] = x[n]$ for all n .
 - (a) **(2 mark)** Any signal $g[n]$ can be written as the sum of even and odd parts $g_e[n] + g_o[n]$. Find these parts in terms of $g[n]$.
 - (b) **(2 mark)** If $x[n]$ is an even signal and $y[n]$ is an odd signal, then show that $x[n]y[n]$ is an odd signal.

2 Fourier Series - 10 marks

1. **(1 mark)** What are Dirichlet Conditions? State them:
2. **(1 mark)** Give Formulas for Trigonometric Fourier Series and Exponential Fourier Series:
3. Find the Trigonometric Fourier Series of the following functions:
 - (a) **(3 mark)** Square wave function with period 2π : $f(x + 2\pi) = f(x)$

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x < \pi \\ -1 & \text{if } \pi \leq x < 2\pi \end{cases}$$

- (b) **(3 mark)** $y = |t|$, where $-\pi \leq t < \pi$ and $y(t) = y(t + 2\pi)$.
- (c) **(2 mark)** Give your observations about a_n and b_n in the above two questions

3 Fourier Transform - (CTFT,DTFT) - 20 marks

1. **(1 mark)** What are the formulas for DTFT and CTFT, and how do they differ in signal processing?
2. **(2 marks)** Consider a discrete-time signal $x[n]$ of length N . The DTFT of the signal is given by $X(e^{j\omega})$. Show that DTFT is periodic with period 2π .

3. **(4 marks)** What are symmetry properties of Fourier Transform ? Prove that Fourier transform of a real and even signal is real and even.

4. Prove below properties of CTFT **(9 marks)**

- (a) **(1 mark)** Linearity
- (b) **(1 mark)** Frequency Shifting
- (c) **(1 mark)** Time Reversal
- (d) **(2 mark)** Find Fourier Transform of the given signal (Hint: use duality property)

$$x(t) = \frac{\sin^2 kt}{t^2}$$

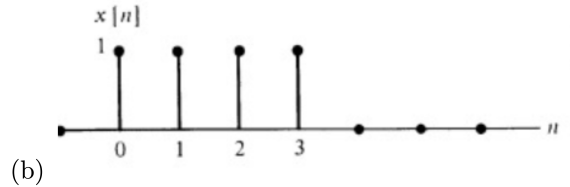
(e) **(2 mark)** Use property of differentiation in time domain and frequency domain to find

$$\mathcal{CTFT} \left\{ \frac{d^2}{dt^2} x(t-1) \right\}$$

- (f) **(1 mark)** Convolution Property
- (g) **(1 mark)** Parseval's Relation

5. Compute the DTFT for the following signals: **(2 x 2 = 4 marks)**

(a) $x[n] = \left(\frac{1}{4}\right)^n u[n+2]$



4 Fourier Transform - (DFT) - 32 marks

1. **(2 marks)** Compute 8-point DFT for the signal $x(n) = \frac{1}{4}u(4-n)$, where $u(\cdot)$ represents the unit step function.

2. **(4 marks)** Let $x[n]$ values be non-zero for $0 \leq n \leq N-1$, else zero. Let $y[n] = x[n] + x[n + \frac{N}{2}]$, $0 \leq n \leq N-1$ else zero and $Y(k)$ is the $\frac{N}{2}$ -point DFT of $y[n]$. Then, what is the relation between $Y(k)$ and $X(k)$?

3. **(2 marks)** Given $X[k] = k^2$, $0 \leq k \leq 7$ be 8-point DFT of a sequence $x[n]$, find the value of $\sum_{n=0}^3 x[2n+1]$

4. **(4 x 3 = 12 marks)** Determine the Inverse Fourier transform of the following:

(a) **(4 marks)**

$$X(e^{jw}) = \cos^3 w + \cos^2 w$$

(b) **(4 marks)**

$$X(e^{jw}) = \frac{e^{-4jw} + e^{-3jw} - e^{-jw} - 1}{e^{-jw} + 1}$$

(c) **(4 marks)**

$$X(e^{jw}) = \frac{3e^{-jw} - 1}{3 - e^{-jw}}$$

5. Let $x[n] = \{1, 2, 3, 6\}$ then

(a) **(2 marks)** Compute 6-point DFT of $x[n]$ and is represented as $X(k)$. Comment on the relation between:

- $X(1)$ and $X(5)$
- $X(2)$ and $X(4)$

(b) **(4 marks)** Compute 6-point DFT of $x[n - 10]$ and is represented as $Y(k)$; What is relation between $Y(k)$ and $X(k)$;

(c) **(2 marks)** Obtain $y[n]$ by computing IDFT of $Y(k)$; What is the relation between $y[n]$ and $x[n]$?

(d) **(4 marks)** Find DFT of $x[n]\cos(\frac{2\pi}{N}k_0n)$ in terms of $X(k)$; here k_0 is an integer constant.

5 Convolution - 12 marks

1. Find the Convolution of the following functions:-

(a) **(6 marks)**

$$f[n] = 2\delta[n + 10] + 2\delta[n - 10], \quad g[n] = 3\delta[n + 5] + 2\delta[n - 5]$$

(b) **(6 marks)**

$$f[n] = (-1)^n, \quad g[n] = \delta[n] + \delta[n - 1]$$

6 Sampling - 24 marks

1. **(6 marks)** What is aliasing? What can be done to reduce aliasing?

Let $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$ be a continuous-time signal. Find the Nyquist rate and Nyquist interval for this signal.

2. **(6 marks)** A waveform, $x(t) = 10\cos(1000t + \pi/3) + 20\cos(2000t + \pi/6)$ is to be uniformly sampled for digital transmission. What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction? If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?

3. Consider three signals $x_1(t)$ and $x_2(t)$ and $x_3(t)$ with Fourier transforms satisfying:

$$X_1(\Omega) = 0, |\Omega| \geq 120$$

$$X_2(\Omega) = 0, |\Omega| \leq 60, |\Omega| \geq 100$$

Determine the minimum frequency f_s , at which we must sample the following signals to prevent aliasing.

- (a) **(3 marks)** $x(t) = x_1(t) + x_2(t)$
- (b) **(6 marks)** $x(t) = x_1(t)x_2(t)$
- (c) **(3 marks)** $x(t) = \cos(3.6\pi t + 9.23)$

7 Quantization - 48 marks

1. Consider the analog waveform $x(t)$ and answer the following questions.

$$x(t) = \begin{cases} -2 \sin(\pi x/4) & 0 \leq x < 4 \\ x - 4 & 4 \leq x < 5 \\ 1 & 5 < x < 7 \\ 8 - x & 7 \leq x \leq 10 \end{cases}$$

It is sampled at 1000 Hz and quantized with a 2-bit quantizer with input range -2V to 2V.

- (a) **(1 marks)** Indicate the sample points.
- (b) **(5 marks)** State the quantization intervals and the corresponding digital words.
- (c) **(5 marks)** Sketch the digital word assigned to each sample point.
- (d) **(5 marks)** Indicate the stream of bits generated after the quantization is complete.
- (e) **(2 marks)** What is the resulting bit rate?
- (f) **(2 marks)** What is the quantization error?

Answer all of the above questions for a 3-bit quantizer as well **(20 marks)**

- 2. **(8 marks)** Mention advantages/disadvantages of increasing quantization bits.