DSA-Assignment-1

Deadline: 20th January 2025

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1. Solve all the question and submit a handwritten document
2. Plagiarism will be penalised
3. Submit a pdf of the form <**roll_no**>**_dsa1.pdf**

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1 Odd and Even Functions

1. Classify each of the following functions as **odd**, **even**, or **neither**

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = |x| - 2, \quad -2 \leq x < 2, \quad f(x+4) = f(x)$

(c)

$$f(x) = \begin{cases} x - 1 & 0 \leq x \leq 2, \\ -1 - x & -2 < x < 0 \end{cases}, \quad f(x+4) = f(x)$$

2 Fourier Series

1. What are the Dirichlet conditions for a signal? Provide a clear statement of these conditions.
2. Plot the following functions and find their Fourier series representations:

(a) Square Wave Function:

$$f(x) = \begin{cases} 1 & 0 \leq x < \pi, \\ -1 & \pi \leq x < 2\pi \end{cases} \quad period = 2\pi$$

(b) Sawtooth Wave Function:

$$f(x) = x, \quad -\pi \leq x < \pi, \quad period = 2\pi$$

(c) Exponential Function:

$$f(x) = e^x, \quad -\pi \leq x < \pi, \quad period = 2\pi$$

(d) Piecewise Linear Function:

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0, \\ x & 0 \leq x < \pi \end{cases} \quad period = 2\pi$$

3. Show that:

(a)
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

(b)
$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

[Hint: Use the Fourier series representations from the previous question]

3 Fourier Transform

1. Find fourier transform of the following function $x(t)$:

$$x(t) = \begin{cases} 1 & 1 \leq |t| \leq 3, \\ -1 & |t| < 1, \\ 0 & \text{otherwise} \end{cases}$$

2. Find and sketch the magnitude and phase spectrum of Fourier transform of the following:

   (a)
$$x(t) = \begin{cases} a & -T \leq t \leq T, \\ 0 & \text{otherwise} \end{cases}$$

   (b) $x(t) = \delta(t - a)$, where a is real

3. (a) Determine the transform of the following signal:

$$x(t) = t \left(\frac{\sin(t)}{\pi t} \right)^2$$

   (b) Use Parseval's Law and the result from the previous part to determine the value of

$$A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin(t)}{\pi t} \right)^4 dt$$

4. For the following signals:

   (a) $x(t) = e^{-|a|t} \cdot u(t)$

   (b) $x(t) = e^{(-1+2j)t} \cdot u(t)$

   Compute the following for each signal:

   (a) $|X(\omega)|$

   (b) $\angle X(\omega)$

   (c) $\Re\{X(\omega)\}$

   (d) $\Im\{X(\omega)\}$

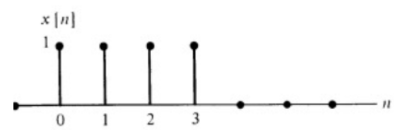
4 DTFT

1. Consider a discrete-time signal $x[n]$ of length N. The DTFT of the signal is given by $X(e^{j\omega})$. Show that the DTFT is periodic with period 2π.

2. For a DTFT, Prove the following properties:
   (a) Linearity
   (b) Symmetry
   (c) Duality
   (d) Time shifting
   (e) Time scaling
   (f) Frequency Shifting
   (g) Time Reversal
   (h) Convolution Property
   (i) Parseval's Relation

3. Compute the DTFT for the following signals:

$$(a) \quad x[n] = \left(\frac{1}{5}\right)^n u[n+1]$$



(b)

5 DFT

1. Compute the 8-point DFT for the following:
   (a) $x(n) = u(4-n)/4$, where $u(n)$ is the unit step function.
   (b) $x(n) = \sin(\pi n/4) + \cos(\pi n/4)$
   (c) $x[n] = \{1, -1-j, -1, -1+j\}$
   (d) $x[n] = \{1, 1, 1, 1, 1, 1, 1, 1\}$

2. Determine the Inverse Fourier transform of the following:
   (a) $X(e^{j\omega}) = \cos^3(\omega) + \cos^2(\omega)$
   (b) $X(e^{j\omega}) = \frac{e^{-4j\omega}+e^{-3j\omega}-e^{-j\omega}-1}{e^{-j\omega}+1}$
   (c) $X(e^{j\omega}) = \frac{3e^{-j\omega}-1}{3-e^{-j\omega}}$

3. Given $X[k] = k^2$, $0 \le k \le 7$ be the 8-point DFT of a sequence $x[n]$, find the value of:

$$P = \sum_{n=0}^{7} x[2n+1]$$

6 Sampling

1. What is aliasing? What can be done to reduce aliasing? Let $x(t) = \frac{1}{2\pi} cos(4000\pi t) cos(1000\pi t)$ be a continuous-time signal. Find the Nyquist rate and Nyquist interval for this signal.
2. A waveform, $x(t) = 10cos(1000t + \pi/3) + 20cos(2000t + \pi/6)$ is to be uniformly sampled for digital transmission. What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction? If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?
3. Consider two signals $x_1(t)$ and $x_2(t)$ with Fourier transforms satisfying:

$$X_1(\Omega) = 0, |\Omega| \geq 120$$
$$X_2(\Omega) = 0, |\Omega| \leq 60, |\Omega| \geq 100$$

   Determine the minimum frequency f_s, at which we must sample the following signals to prevent aliasing.

   (a) $x(t) = x_1(t) + x_2(t)$

   (b) $x(t) = x_1(t)x_2(t)$

   (c) $x(t) = \cos(3.6\pi t + 9.23)$

7 Quantization

1. Consider the analog waveform $x(t)$ and answer the following questions.

$$x(t) = \begin{cases} -2\sin(\pi x/4) & 0 \leq x < 4 \\ x - 4 & 4 \leq x < 5 \\ 1 & 5 < x < 7 \\ 8 - x & 7 \leq x \leq 10 \end{cases}$$

It is sampled at 1000 Hz and quantized with a 2-bit quantizer with input range -2V to 2V.

(a) Indicate the sample points.

(b) State the quantization intervals and the corresponding digital words.

(c) Sketch the digital word assigned to each sample point.

(d) Indicate the stream of bits generated after the quantization is complete.

(e) What is the resulting bit rate?

(f) What is the quantization error?

Answer all of the above questions for a 3-bit quantizer as well.

2. Mention advantages/disadvantages of increasing quantization bits.

8 Convolution

1. Find the Convolution of the following functions:-

(a)

$$f[n] = 2\delta[n+10] + 2\delta[n-10], \quad g[n] = 3\delta[n+5] + 2\delta[n-5]$$

(b)

$$f[n] = (-1)^n, \quad g[n] = \delta[n] + \delta[n-1]$$