Digital Signal Analysis

Assignment - 2

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Q1: a, $x(n) = \cos(0.03\pi n)$

we know that, If $\cancel{N}$ is the period of the function $f(x)$

then $f(x+N) = f(x)$

Now, if N is the period of $x(n)$ then

$$x(n+N) = x(n)$$

$$\Rightarrow \cos(0.03\pi(n+N)) = \cos(0.03\pi n)$$

we know that,

$$\cos\theta = \cos x$$

$$\Rightarrow \theta = x \pm 2k\pi \text{ where } k \in \mathbb{Z}$$

$$\Rightarrow 0.03\pi(n+N) = 0.03\pi n \pm 2k\pi$$

$$\Rightarrow 0.03\cancel{\pi}N = \pm 2k\cancel{\pi}$$

$$\Rightarrow N = \frac{\pm 2k}{0.03} = \frac{\pm 200k}{3}$$

as $x(n)$ is discrete and n cannot take fractional values k is a multiple of 3. And as N is fundamental period and it can't take negative values $k=3$

$$\Rightarrow \boxed{N=200}$$

$\therefore$ fundamental period of $\cos(0.03\pi n)$ is 200.

b) $x(n) = \cos\left(\frac{\pi n^2}{4}\right)$

let N be the fundamental period, then

$x(N+n) = x(n)$

$\Rightarrow \cos\left(\frac{\pi}{4}(n+N)^2\right) = \cos\left(\frac{\pi}{4}n^2\right)$

$\Rightarrow \frac{\pi}{4}(n+N)^2 = \pi/_4\, n^2 \pm 2k\pi$ $\left(\because \cos\theta = \cos\alpha \Rightarrow \theta = \alpha \pm 2k\pi \text{ where } k \in W\right)$

$\Rightarrow \cancel{\pi/_4 n^2} + \frac{\pi}{4}N^2 + \frac{\pi}{2}nN = \cancel{\frac{\pi}{4}n^2} \pm 2k\pi$

$\Rightarrow \frac{N^2}{4} + \frac{nN}{2} = \pm 2k$

$\Rightarrow N^2 + 2nN = \pm 8k$

$\Rightarrow N^2 + 2nN = 8k \quad (k \in I)$

$\Rightarrow N(N+2n) = 8k$

$\Rightarrow \cancel{2}N\left(\frac{N}{2}+n\right) = \overset{4}{\cancel{8}}k$

i.e for ~~every~~ any n $N\left(\frac{N}{2}+n\right)$ should be a multiple of 4

$\Rightarrow$ N should be a multiple of 4

and N is fundamental period (ie smallest positive number satisfying)

so, $\boxed{N=4}$

$\therefore$ fundamental period of $\cos\left(\frac{\pi n^2}{4}\right)$ is 4.

c) $x(n) = 5$

here $x(n)$ is a constant function

$\Rightarrow x(n+N) = x(n) \quad \forall N \in R$

but as $x(n)$ is a discrete function and N ~~can~~ is smallest ~~deleted~~ positive number $\boxed{N=1}$

$\therefore$ fundamental period of $x(n)$ is 1.

d. Given,

$$x(n) = \cos(5\pi n) + \cos\left(\frac{4}{5}\pi n\right)$$

this is of the form $x_1(n) + x_2(n)$. So, period of $x(n)$ is L.C.M of period of $x_1(n)$ and $x_2(n)$

Let N_1, N_2 be the periods of $x_1(n)$ & $x_2(n)$ respectively

$\Rightarrow x_1(n+N_1) = x_1(n)$ and $x_2(n+N_2) = x_2(n)$

$\Rightarrow \cos(5\pi(n+N_1)) = \cos(5\pi n)$ and $\cos\left(\frac{4}{5}\pi(n+N_2)\right) = \cos\left(\frac{4}{5}\pi n\right)$

$\Rightarrow 5\pi(n+N_1) = 5\pi n + 2k_1\pi$ and $\frac{4}{5}\pi(n+N_2) = \frac{4}{5}\pi n + 2k_2\pi$

where $k_1, k_2 \in I$

$\Rightarrow 5N_1 = 2k_1$ and $\frac{\overset{2}{\cancel{4}}}{5}N_2 = \cancel{2}k_2$

$\Rightarrow \boxed{N_1 = 2}$ and $\boxed{N_2 = 5}$ ($\because N_1, N_2$ are fundamental frequencies, they are smallest positive possible numbers)

$\Rightarrow$ fundamental period of $x(n)$, $N = \text{lcm}(2, 5)$

$= 10$

$\therefore$ fundamental period of $\cos(5\pi n) + \cos\left(\frac{4}{5}\pi n\right) = 10$.

e. Given,

$$x(n) = \sin(5\pi n + 2)$$

Let N be the fundamental period of $x(n)$

$\Rightarrow x(n+N) = x(n)$

$\Rightarrow \sin(5\pi(N+n)+2) = \sin(5\pi n + 2)$

$\Rightarrow 5\pi(N+n) + 2 = (-1)^k(5\pi n + 2) + k\pi$ where $k \in I$

($\because \sin\theta = \sin x \Rightarrow \theta = (-1)^n x + n\pi$ $n \in I$)

$\Rightarrow \quad 5\pi(N+n)+2 = (-1)^k(5\pi n+2)+k\pi$

when k is odd $\quad 5\pi(N+n)+2 = -5\pi n-2+k\pi$

$\pi(5N+10n-k) = -4$

$\Rightarrow \quad 5N = -\frac{4}{\pi}+k-10n$

$\Rightarrow$ N is irrational which is a contradiction

So, k is even, let $k=2m$

$\Rightarrow \quad 5\pi(N+n)+\cancel{2} = 5\pi n+\cancel{2}+2m\pi$

$\Rightarrow \quad 5\cancel{\pi}N = 2m\cancel{\pi}$

$\Rightarrow \quad N = \frac{2m}{5}$

$\Rightarrow \boxed{N=2}$ ($\because$ N is smallest positive integer possible)

$\therefore$ fundamental period of $\sin(5\pi n+2)$ is 2

d Given,

$x(n) = \cos(n+\pi)$

Let N be the fundamental period of $x(n)$

$\Rightarrow \quad x(n+N) = x(n)$

$\Rightarrow \cos(n+N+\pi) = \cos(n+\pi)$

$\Rightarrow \cancel{n}+N+\cancel{\pi} = \cancel{n}+\cancel{\pi}+2k\pi$ where $k\in I$ ($\because \cos x=\cos\theta \Rightarrow x=\theta\pm 2k\pi \; k\in W$)

$\Rightarrow \quad N=(2k)\pi$

$\because k\in I,\ 2k\in I$

$\Rightarrow$ N is irrational value, which is not possible as it can take only positive integral values

$\therefore x(n)$ is not a periodic function.

2. we know that,

A function $f(x)$ can be written as a sum of even and odd function as $f(x) = \left(\frac{f(x)+f(-x)}{2}\right) + \left(\frac{f(x)-f(-x)}{2}\right)$

here $\frac{f(x)+f(-x)}{2}$ is the even part ~~as~~

let $E(x) = \frac{f(x)+f(-x)}{2}$ then ~~f~~ $E(-x) = \frac{f(-x)+f(x)}{2} = E(x)$

$\Rightarrow \frac{f(x)+f(-x)}{2}$ is the even part

$\frac{f(x)-f(-x)}{2}$ is the odd part of $f(x)$

let $O(x) = \frac{f(x)-f(-x)}{2}$ then $O(-x) = \frac{f(-x)-f(x)}{2} = -O(x)$

$\Rightarrow \frac{f(x)-f(-x)}{2}$ is the odd part.

a. $e^{(a\pi n/b)j}$

let $x(n) = e^{(a\pi n/b)j}$

$\Rightarrow$ odd ~~even~~ part of $x(n) = \frac{x(n)-x(-n)}{2}$

$$= \frac{e^{\left(\frac{a\pi n}{b}\right)j} - e^{-\frac{a\pi n}{b}(j)}}{2}$$

$$= \frac{\cos\left(\frac{a\pi n}{b}\right) + j\sin\left(\frac{a\pi n}{b}\right) - \left(\cos\left(-\frac{a\pi n}{b}\right) + j\sin\left(-\frac{a\pi n}{b}\right)\right)}{2}$$

$$= \frac{\cancel{\cos\left(\frac{a\pi n}{b}\right)} + j\sin\left(\frac{a\pi n}{b}\right) - \cancel{\cos\left(\frac{a\pi n}{b}\right)} + j\sin\left(\frac{a\pi n}{b}\right)}{2}$$

$$\boxed{\text{odd part of } x(n) = j\sin\left(\frac{a\pi n}{b}\right)}$$

even part of $x(n) = \dfrac{x(n)+x(-n)}{2}$

$= \dfrac{e^{\left(\frac{a\pi n}{b}\right)j} + e^{-\frac{a\pi n}{b}j}}{2}$

$= \dfrac{\cos\left(\frac{a\pi n}{b}\right) + \cancel{j\sin\left(\frac{a\pi n}{b}\right)} + \cos\left(-\frac{a\pi n}{b}\right) + \cancel{j\sin\left(-\frac{a\pi n}{b}\right)}}{2}$

even part of $x(n) = \cos\left(\frac{a\pi n}{b}\right)$

$\Rightarrow \boxed{\text{even part of } x(n) = \cos\left(\frac{a\pi n}{b}\right)}$

b) $a\cos(b\pi n + 1)$

let $x(n) = a\cos(b\pi n + 1)$

$\Rightarrow$ odd part of $x(n) = \dfrac{x(n) - x(-n)}{2}$

$= \dfrac{a\cos(b\pi n + 1) - a\cos(-b\pi n + 1)}{2}$

$= \dfrac{a}{2}\left(\cos(b\pi n + 1) - \cos(-b\pi n + 1)\right)$

$= \dfrac{a}{\cancel{2}} \times \cancel{2} \sin\left(\dfrac{-b\pi n + \cancel{1} - b\pi n - \cancel{1}}{2}\right) \sin\left(\dfrac{\cancel{b\pi n} + 1 + (\cancel{-b\pi n}) + 1}{2}\right)$

$\left(\because \cos(A) - \cos B = 2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{A+B}{2}\right)\right)$

$\boxed{\text{odd part of } x(n) = a\sin(-b\pi n)\sin 1}$

even part of $x(n) = \dfrac{x(n) + x(-n)}{2}$

$= \dfrac{a\cos(b\pi n + 1) + a\cos(-b\pi n + 1)}{2}$

$= \dfrac{a}{2}\left(\cos(b\pi n + 1) + \cos(-b\pi n + 1)\right)$

$= \dfrac{a}{\cancel{2}} \times \cancel{2} \cos\left(\dfrac{\cancel{b\pi n} + 1 + (\cancel{-b\pi n}) + 1}{2}\right) \cos\left(\dfrac{b\pi n + \cancel{1} + b\pi n - \cancel{1}}{2}\right)$

$= a\cos(1)\cos(bn\pi)$ $\left(\because \cos A+\cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right)$

$\Rightarrow \boxed{\text{even part of } x(n) = a\cos 1 \cos(bn\pi)}$

3 a, $x(n) = \left(\frac{1}{4}\right)^n u(n)$

we know that,

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$\Rightarrow E = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{4}\right)^n u(n)\right|^2$$

$$= \sum_{-\infty}^{-1} \left|\left(\frac{1}{4}\right)^n u(n)\right|^2 + \sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^n u(n)\right|^2$$

$$= \sum_{-\infty}^{-1} \left|\left(\frac{1}{4}\right)^n \cdot 0\right|^2 + \sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^n \times 1\right|^2$$

$$= 0 + \underbrace{1 + \frac{1}{16} + \frac{1}{4^4} + \frac{1}{4^6} + \dots \infty}_{\text{sum of infinite G.P.}}$$

$$= \frac{1}{1 - 1/16} = \frac{16}{15}$$

$$\Rightarrow E = \frac{16}{15} \quad (<\infty)$$

Power of the signal $= P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$\Rightarrow P = \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{N} \left|\left(\frac{1}{4}\right)^n u(n)\right|^2 \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=0}^{N} \left|\left(\frac{1}{4}\right)^n \times 1\right|^2 + \sum_{n=-N}^{-1} \left|\left(\frac{1}{4}\right)^n \times 0\right| \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(1 + \frac{1}{4^2} + \frac{1}{4^4} + \dots + \frac{1}{4^{2N}} \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{1 - \left(\frac{1}{4^2}\right)^{N+1}}{1 - 1/4^2} \right)$$

$\left(\because \text{sum of } N \text{ terms of G.P} = \frac{a(1-r^n)}{1-r}\right)$

$$\Rightarrow P = \lim_{N\to\infty} \frac{1}{(2N+1)} \times \frac{16}{15}\left(1-\left(\frac{1}{4^2}\right)^{N+1}\right)$$

$$= \frac{16}{15} \times \lim_{N\to\infty} \frac{1-\left(\frac{1}{4^2}\right)^{N+1}}{2N+1} \qquad \left(\because \text{ as } N\to\infty \ \frac{1}{4^{2(n+1)}}\to 0 \text{ and } 2N+1\to\infty\right)$$

$$= \frac{16}{15} \lim_{n\to\infty} \frac{1-0}{\infty} = 0$$

i.e Energy of the signal is finite and power of the signal is zero.

therefore Given $x(n)$ is energy signal.

b) $x(n) = a^n u(n)$, $a \in R$

we know that,

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$\Rightarrow E = \sum_{-\infty}^{\infty} |a^n u(n)|^2$$

$$= \sum_{-\infty}^{-1} |a^n u(n)|^2 + \sum_{0}^{\infty} |a^n u(n)|^2$$

$$= \sum_{-\infty}^{-1} |a^n (0)|^2 + \sum_{0}^{\infty} |a^n \times 1|^2$$

$$= 0 + 1 + a^2 + a^4 + a^6 + \dots$$

$$E = \begin{cases} \frac{1}{1-a^2} & \text{if } |a|<1 \\ \infty & \text{if } |a|\geq 1 \end{cases}$$

Power of the signal $= P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} |a^n (0)|^2 + \sum_{n=0}^{N} |a^n \cdot 1|^2 \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\left(0 + 1 + a^2 + \dots + a^{2N}\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\left(\frac{1-a^{2(N+1)}}{1-a^2}\right)$$

$$= \frac{1}{1-a^2} \lim_{N\to\infty} \frac{1}{2N+1}\left(1-a^{2(N+1)}\right)$$

$$P = \begin{cases} 0 & \text{if } |a|<1 \\ 1/2 & |a|=1 \\ \infty & |a|>1 \end{cases}$$

Therefore,
when $|a|<1$ $P=0$, and $E =$ finite $= \frac{1}{1-a^2}$

$\Rightarrow$ signal is energy signal

when $|a|=1$, $P = 1/2$ (finite), $E = \infty$

$\Rightarrow$ signal is power signal

when $|a|>1$, $P=\infty$, $E=\infty$

$\Rightarrow$ signal is neither power signal nor energy signal.

(5) Given,

$x(n) = a^n \delta(n)$, $a \in R$.

we know that,

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} |a^n \delta(n)|^2$$

$$= \sum_{n=-\infty}^{-1} |a^n \times 0|^2 + a^0 \times 1 + \sum_{n=1}^{\infty} |a^n \times 0|^2$$

$\boxed{E = 1}$ finite.

Power of the signal $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |a^n \delta(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} |a^n \times 0|^2 + (a^0 \cdot 1)^2 + \sum_{n=1}^{N} |a^n \times 0|^2 \right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (0 + 1 + 0)$$

$$\boxed{P = 0}$$

$\therefore$ Energy of the signal is finite and power of the signal is zero

$\Rightarrow$ The given signal is energy signal.

d) Given, $x(n) = \sin\left(\frac{n\pi}{4}\right)$

we know that,

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} \left|\sin\frac{n\pi}{4}\right|^2$$

we know that

$$\left(\sin\frac{n\pi}{4}\right)^2 = \begin{cases} 1/2 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n = 0. \end{cases}$$

$$\Rightarrow E = (\infty \cdots + \tfrac{1}{2} + 1 + \tfrac{1}{2} + \cdots 0 + \tfrac{1}{2} + 1 + \cdots \infty)$$

$$= \infty$$

$\Rightarrow$ Energy is infinite

power of the signal, $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin\frac{n\pi}{4}\right|^2$

$$\Rightarrow \quad 0 < P < \lim_{N\to\infty} \frac{2N}{2N+1} \qquad \left(\because \sin(0) = 0 \text{ and } \sin x \leq 1 \right)$$

$$\Rightarrow \quad 0 < P < 1$$

$\Rightarrow$ power is finite

$\therefore$ power is finite and energy is infinite

$\Rightarrow$ the given signal is power signal.

4) Given,

Analog signal $x_a(t) = 5 \sin 200\pi t$

$$= 5 \sin(100 \times 2\pi t)$$

$\Rightarrow$ frequency of the signal $= \dfrac{100 \times 2\pi}{2\pi}$

$$= 100$$

$$\Rightarrow \boxed{f_m = 100}$$

According to Nyquist theorem,

minimum sampling frequency is $2 * f_m$

$\Rightarrow f_s = 2 \times 100$

$$\boxed{f_s = 200}$$

Now, if $f_s = 250$, required to find the discrete signal

we know that,

discrete signal after sampling $= x(nT_s)$

$$= x\left(\frac{n}{f_s}\right)$$

$$= x\left(\frac{n}{250}\right)$$

$$= 5\sin\left(\frac{\cancel{200}4\pi n}{\cancel{250}\,5}\right)$$

$$= 5\sin\left(\frac{4\pi n}{5}\right)$$

$\therefore$ discrete-time signal after sampling is $5\sin\left(\frac{4\pi n}{5}\right)$.

5) Given,

Analog signal, $x_a(t) = 3\cos 80\pi t + 5\sin 40\pi t - 10\cos 160\pi t$

$$= 3\cos 2\pi(40)t + 5\sin 2\pi(20)t - 10\cos 2\pi(80)t$$

$\Rightarrow$ frequencies in the signal are 40, 20, 80

$\Rightarrow$ maximum frequency of the signal $f_m = 80$

we know that,

Nyquist rate $= 2 \times f_m$

$$= 2 \times 80$$

$$\boxed{f_s = 160\,Hz}$$

sampled version of the signal $x_a(n) = x_a(nT_s)$

$$= x_a(n/f_s)$$

$$= x_a\left(\frac{n}{160}\right)$$

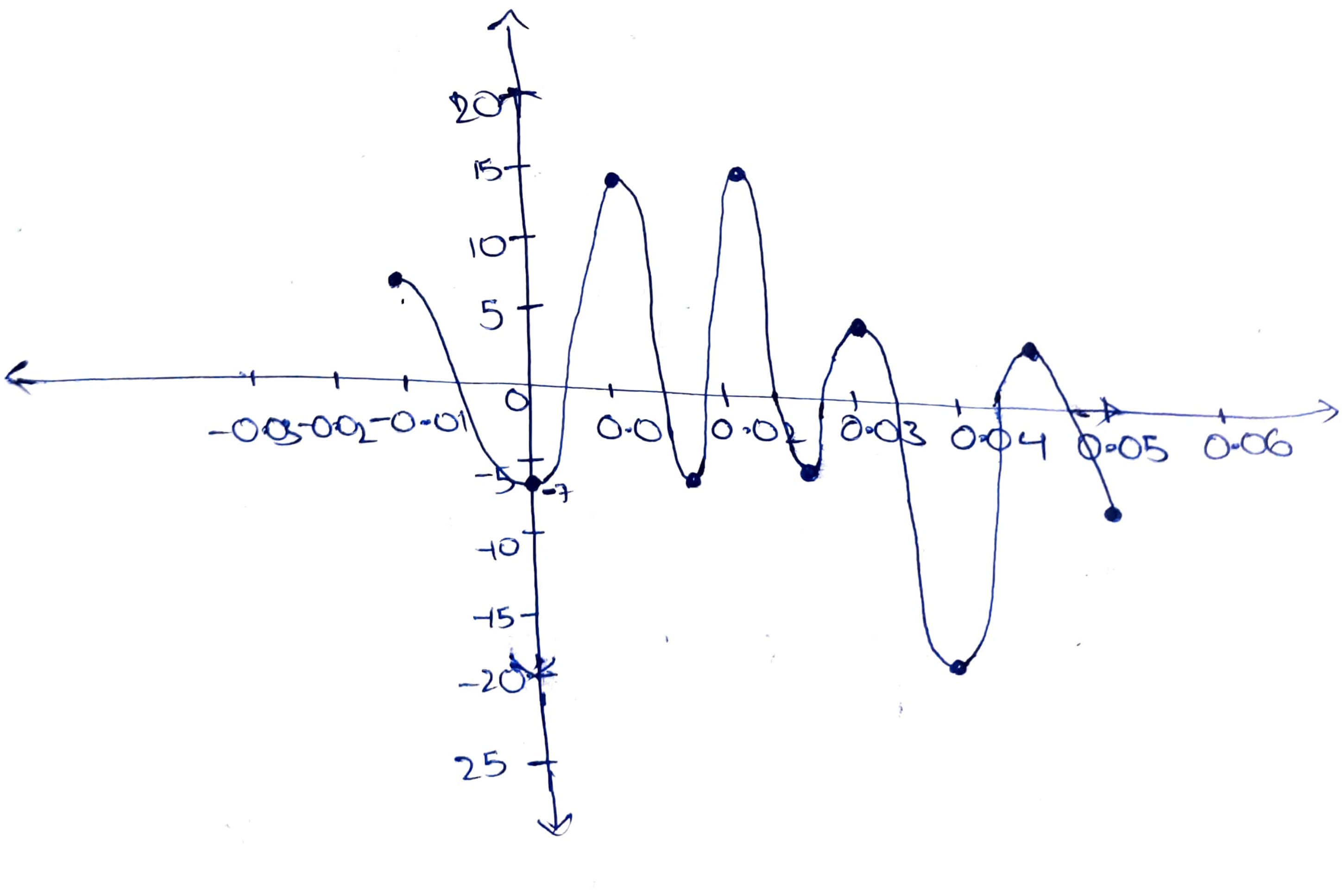
$$= 3\cos\left(\frac{80\pi n}{160}\right) + 5\sin\left(\frac{40\pi n}{160}\right) - 10\cos\left(\frac{160\pi n}{160}\right)$$

$$= 3\cos\frac{\pi n}{2} + 5\sin\frac{\pi n}{4} - 10\cos\pi n$$

$\therefore$ sampled version of the signal is

$$3\cos\frac{\pi n}{2} + 5\sin\frac{n\pi}{4} - 10\cos n\pi$$

<u>wave form</u>:



6. Given,

Analog signal $x_a(t) = 5\cos 2000\pi t + 5\sin(6000\pi t) - 7\cos 12000\pi t$

$= 5\cos 1000\cdot 2\pi t + 5\sin 3000 \times 2\pi t - 7\cos 6000 \times 2\pi t$

$\Rightarrow$ frequencies in the signal are 1000, 3000, 6000

maximum frequency in the signal is 6000

$\Rightarrow f_m = 6000$

$\Rightarrow$ According to Nyquist theorem, minimum sampling frequency is $2 \times 6000 = 12,000$.

$\Rightarrow \boxed{f_s = 12,000}$

Sampled version of the signal $x_a(n) = x_a(nT_s)$

$= x_a(n/F_s)$ $\left(\because \text{given, } F_s = 5000\right)$

$= x_a\left(\frac{n}{5000}\right)$

$= 5\cos \frac{2000\pi n}{5000} + 5\sin\left(\frac{6000\pi n}{5000}\right) - 7\cos\left(\frac{12000\pi n}{5000}\right)$

$= 5\cos \frac{2n\pi}{5} + 5\sin \frac{6n\pi}{5} - 7\cos \frac{12n\pi}{5}$

$= 5\cos\frac{2n\pi}{5} + 5\sin\frac{6n\pi}{5} - 7\cos\left(\frac{12n\pi}{5}\right)$

$= 5\cos\frac{2n\pi}{5} + 5\sin\frac{6n\pi}{5} - 7\cos\left(2n\pi + \frac{2n\pi}{5}\right)$

$= 5\sin\frac{6n\pi}{5} - 2\cos\frac{2n\pi}{5}$

$\therefore$ Sampled version of the signal is

$5\sin\frac{6n\pi}{5} - 2\cos\frac{2n\pi}{5}$

7) Given,

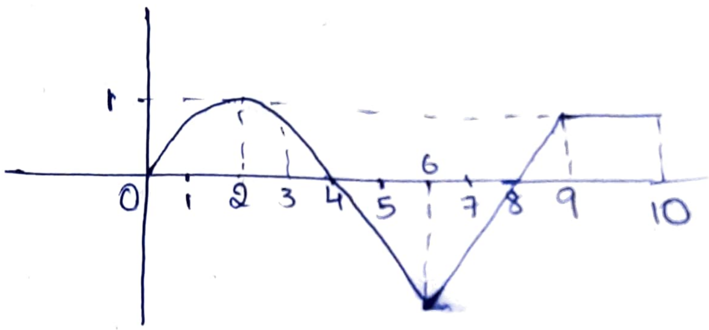
~~Analog~~ waveform $x(t) = \begin{cases} \sin\left(\frac{\pi t}{4}\right), & 0 \leq t \leq 4 \\ 4-t & 4 \leq t \leq 6 \\ t-8 & 6 < t < 9 \\ 1 & 9 \leq t \leq 10 \end{cases}$

a) ~~we know~~ Given that,

sampling frequency $f_s = 1000$

$\Rightarrow$ sampling points are $\frac{1}{f_s}, \frac{2}{f_s}, \frac{3}{f_s}, \ldots$

$= \frac{1}{1000}, \frac{2}{1000}, \frac{3}{1000}, \ldots$



b) Given, input range is $-2v$ to $2v$. and 2-bit quantizer $\Rightarrow$ $N=2$.

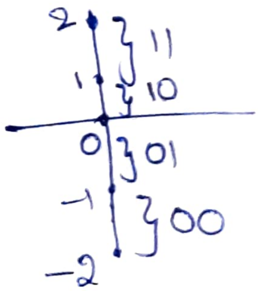
$\Rightarrow$ Quantisation step size $= \frac{2-(-2)}{2^2} = \frac{4}{4} = 1$

-2 to -1 $\rightarrow$ 00

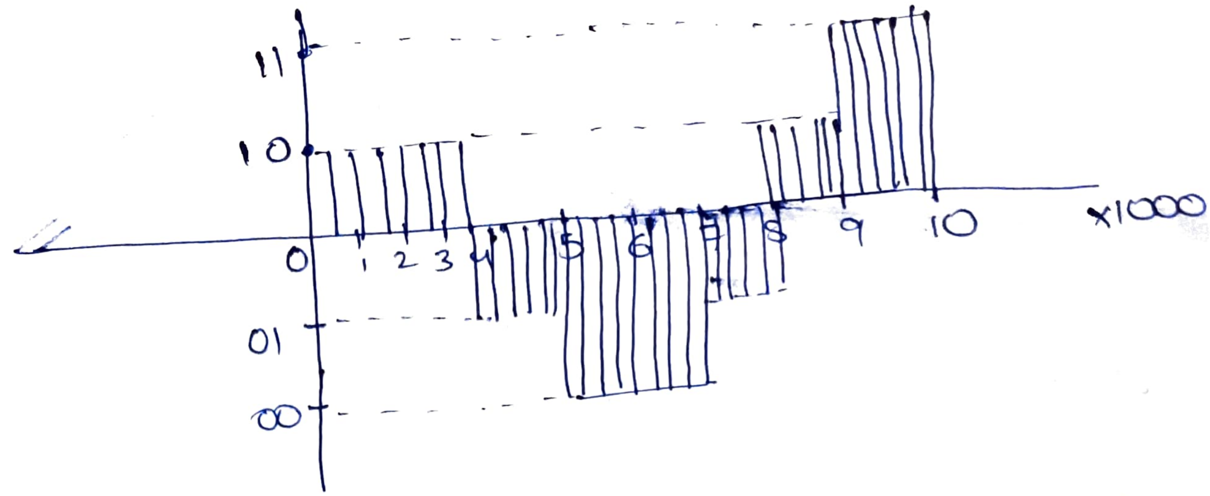
-1 to 0 $\rightarrow$ 01

0 to 1 $\rightarrow$ 10

1 to 2 $\rightarrow$ 11



c)



d) stream of bits generated after the quantisation is complete are

$\underbrace{\overline{10}\,1010101010\;\cdots\;10}_{4000\text{times}}\;\underbrace{\overline{01}\,0101\cdots01}_{1000\text{times}}\;\underbrace{\overline{00}\,00\cdots00}_{2000\text{ times}}\;\underbrace{\overline{01}\cdots}_{1000\text{ times}}$

$\cdots01\;\underbrace{\overline{10}\,10\cdots1010}_{1000\text{times}}\;\underbrace{\overline{11}\,111\cdots11}_{1000\text{times}}$

e) we know that,

bit rate = (no. of bits in quatizer) × sampling frequency

= 2×1000

= 2000 bits per second.

f) Quatization error = $\dfrac{\text{Quantization step size}}{2}$

$= \dfrac{1}{2} = 0.5$.

g 3 bit quantizer

=> quantization step size = $\frac{4}{2^3}$

$= 1/2$

=> maximum error = $1/2 \times 2$

$= 1/4 = 0.25$.

=> bit rate = 3×1000

$= 3000$.

-2	→ 000	Bitstream	
-1.5	→ 001	0-666 → 100	
-1	→ 010	667-3333 → 101	
-0.5	→ 011	3333 → 4000 → 100	
0		4000 → 4500 → 011	
0.5	→ 100	4500 → 5000 → 010	
1	→ 101	5000 → 5500 → 001	
1.5	→ 110	5500 → 6500 → 000	
2	→ 111	6500 → 7000 → 001	

7000 → 7500 → 010

7500 → 8000 → 011

8500 → 9000 → 101

9000 → 1000 → 110

b) 3 bit quantization is better than 2 bit quantizer because. It reduces the quantization error, and we can get more accurate values;

8) if $x(n)$ is input and corresponding output is $y(n)$ then if $y(n-k)$ is same as the output produced when $x(n-k)$ is given as input, the system is time invariant.

a) $y(n) = \dfrac{a}{x(n)}$

$y(n-k) = \dfrac{a}{x(n-k)}$

now, if input is $x(n-k)$ let the output be y'

$y'(n) = \dfrac{a}{x(n-k)} = y(n-k)$

$\Rightarrow$ the system is time invariant

b) $y(n) = 3x(n) + 5x(n-2)$

now, if input is $x(n-k)$ let the output be y'

$y'(n) = 3(x(n-k)) + 5(x(n-k-2))$

$= 3(x(n-k)) + 5x((n-k)-2)$

$= y(n-k)$

$\Rightarrow$ same delay is produced in output

$\therefore$ time invariant

c) $y(n) = x(-n)$

If $x(n-k)$ is given as input to the system, output

$y'(n) = x(-n-k)$

Now, $y(n-k) = x(-(n-k))$

$= x(-n+k) \neq y'(n)$

$\Rightarrow$ the system is time variant.

d) Given, $y(n) = n x(n)$

If $x(n-k)$ is given as input to the system,

Output $y'(n) = n \cdot x(n-k)$

Now, $y(n-k) = (n-k)\, x(n-k)$

$\neq y'(n)$

$\Rightarrow$ the system is time variant

e) Given,

$y(n) = x(5n)$

If $x(n-k)$ is given as input to the system,

output $y'(n) = x(5n-k)$

Now, $y(n-k) = x(5(n-k))$

$= x(5n-5k)$

$\neq x(5n-k) \neq y'(n)$

$\Rightarrow$ the system is time variant.

9) ~~Given~~
Let $x_1(n) \to y_1(n)$, $x_2 \to y_2(n)$.
the system is linear when $a_1x_1(n)+a_2x_2(n) \to a_1y_1(n)+a_2y_2(n)$

a) Given,
$y(n) = 5x(n) + 7x(n-1)$.

$\Rightarrow y_1(n) = 5x_1(n) + 7x_1(n-1)$ and

$y_2(n) = 5x_2(n) + 7x_2(n-1)$

Let $y_3(n)$ be the output when $a_1x_1(n)+a_2x_2(n)$ is input

$$\Rightarrow y_3(n) = 5(a_1x_1(n) + a_2x_2(n)) + 7(a_1x_1(n-1) + a_2x_2(n-1))$$

$$= 5a_1x_1(n) + 7a_1x_1(n-1) + 5a_2x_2(n) + 7a_2x_2(n-1)$$

$$= a_1(5x_1(n) + 7x_1(n-1)) + a_2(5x_2(n) + 7x_2(n-1))$$

$$= a_1y_1(n) + a_2y_2(n)$$

Therefore, the system is linear.

b) Given,
$$y(n) = 4x(n) - \frac{9}{x(n-1)}$$

$$\Rightarrow y_1(n) = 4x_1(n) - \frac{9}{x_1(n-1)} \text{ and}$$

$$y_2(n) = 4x_2(n) - \frac{9}{x_2(n-1)}$$

Let $y_3(n)$ be the output when $a_1x_1(n)+a_2x_2(n)$ is input

$$\Rightarrow y_3(n) = 4(a_1x_1(n) + a_2x_2(n)) - \frac{9}{a_1x_1(n-1)+a_2x_2(n-1)} \quad \text{-- (1)}$$

Now, $$a_1y_1(n) + a_2y_2(n) = a_1\left(4x_1(n) - \frac{9}{x_1(n-1)}\right) + a_2\left(4x_2(n) - \frac{9}{x_2(n-1)}\right)$$

$$= \cancel{b_1}\left(a_1x_1(n) + a_2x_2(n)\right) - q\left(\frac{a_1}{x_1(n-1)} + \frac{a_2}{x_2(n-1)}\right)$$

$$\neq y_3(n)$$

$$\left(\because \frac{1}{a_1x_1(n-1) + a_2x_2(n-1)} \neq \frac{a_1}{x_1(n-1)} + \frac{a_2}{x_2(n-1)}\right)$$

Therefore, the system is non linear.

5) Given,

$$y(n) = \sum_{m=0}^{N} b_m x(n-m) - \sum_{m=1}^{N} d_m x(n-m)$$

$$\Rightarrow y_1(n) = \sum_{m=0}^{N} b_m x_1(n-m) - \sum_{m=1}^{N} d_m x_1(n-m) \text{ and}$$

$$y_2(n) = \sum_{m=0}^{N} b_m x_2(n-m) - \sum_{m=1}^{N} d_m x_2(n-m)$$

Let $y_3(n)$ be the output when $a_1x_1(n) + a_2x_2(n)$ is given as input

$$y_3(n) = \sum_{m=0}^{N} b_m\left(a_1x_1(n-m) + a_2x_2(n-m)\right) - \sum_{m=1}^{N} d_m\left(a_1x_1(n-m) + a_2x_2(n-m)\right)$$

$$= a_1\sum_{m=0}^{N} b_m x_1(n-m) + a_2\sum_{m=0}^{N} b_m x_2(n-m) - a_1\sum_{m=1}^{N} d_m x_1(n-m) - a_2\sum_{m=1}^{N} d_m x_2(n-m)$$

$$= a_1\left(\sum_{m=0}^{N} b_m x_1(n-m) - \sum_{m=1}^{N} d_m x_1(n-m)\right) + a_2\left(\sum_{m=0}^{N} b_m x_2(n-m) - \sum_{m=1}^{N} d_m x_2(n-m)\right)$$

$$= a_1 y_1(n) + a_2 y_2(n)$$

Therefore, the system is linear.

10) we know that, the system is causal when it does not depend on future inputs.

a) Given,

$y(n) = ax(n) + bx(n-1)$

As the value of $y(n)$ depends only on present an past inputs, the system is causal system.

b) Given,

$y(n) = ax(n-1) + bx(n+1)$

As the present value of $y(n)$ depends ~~only~~ on past input and future input, the system is non causal system.

c) Given,

$$y(n) = \sum_{K=0}^{\infty} x(n-K)$$

as $K \geq 0$, $n-K \leq n$

$\Rightarrow y(n)$ depends only on present & past inputs

therfore, the system is causal.

d) Given,

$$y(n) = \sum_{K=0}^{\infty} x(n+K)$$

as $K \geq 0$, $n+K \geq n$

$\Rightarrow y(n)$ depends on future inputs

Therefore, the system is not causal system.