DSA-Assignment-1

Deadline: 20th January 2025

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1. Solve all the question and submit a handwritten document
2. Plagiarism will be penalised
3. Submit a pdf of the form <**roll_no**>**_dsa1.pdf**

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1 Odd and Even Functions - 6M

1. Classify each of the following functions as **odd**, **even**, or **neither**
   (a) $f(x) = \sin x + \cos x$ **[2M]**
   (b) $f(x) = |x| - 2, \quad -2 \leq x < 2, \quad f(x+4) = f(x)$ **[2M]**
   (c) **[2M]**
   $$f(x) = \begin{cases} x - 1 & 0 \leq x \leq 2, \\ -1 - x & -2 < x < 0 \end{cases}, \quad f(x+4) = f(x)$$

2 Fourier Series - 15M

1. **[1M]** What are the Dirichlet conditions for a signal? Provide a clear statement of these conditions.
2. Plot the following functions and find their Fourier series representations:
   (a) **[3M]** Square Wave Function:
   $$f(x) = \begin{cases} 1 & 0 \leq x < \pi, \\ -1 & \pi \leq x < 2\pi \end{cases} \quad period = 2\pi$$
   (b) **[3M]** Sawtooth Wave Function:
   $$f(x) = x, \quad -\pi \leq x < \pi, \quad period = 2\pi$$
   (c) **[3M]** Exponential Function:
   $$f(x) = e^x, \quad -\pi \leq x < \pi, \quad period = 2\pi$$
   (d) **[3M]** Piecewise Linear Function:
   $$f(x) = \begin{cases} 0 & -\pi \leq x < 0, \\ x & 0 \leq x < \pi \end{cases} \quad period = 2\pi$$

3. Show that:

   (a) **[1M]**

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

   (b) **[1M]**

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

[Hint: Use the Fourier series representations from the previous question]

3 Fourier Transform - 24M

1. **[3M]** Find fourier transform of the following function $x(t)$:

$$x(t) = \begin{cases} 1 & 1 \leq |t| \leq 3, \\ -1 & |t| < 1, \\ 0 & \text{otherwise} \end{cases}$$

2. Find and sketch the magnitude and phase spectrum of Fourier transform of the following:

   (a) **[3M]**

$$x(t) = \begin{cases} a & -T \leq t \leq T, \\ 0 & \text{otherwise} \end{cases}$$

   (b) **[3M]** $x(t) = \delta(t - a)$, where a is real

3. (a) **[4M]** Determine the transform of the following signal:

$$x(t) = t\left(\frac{\sin(t)}{\pi t}\right)^2$$

   (b) **[3M]** Use Parseval's Law and the result from the previous part to determine the value of

$$A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin(t)}{\pi t}\right)^4 dt$$

4. For the following signals:

   (a) $x(t) = e^{-|a|t} \cdot u(t)$

   (b) $x(t) = e^{(-1+2j)t} \cdot u(t)$

   Compute the following for each signal:

   (a) **[1M]** $|X(\omega)|$

   (b) **[1M]** $\angle X(\omega)$

   (c) **[1M]** $\Re\{X(\omega)\}$

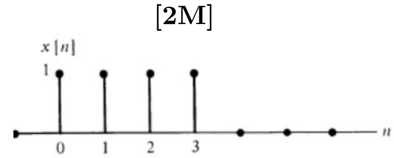
   (d) **[1M]** $\Im\{X(\omega)\}$

4 DTFT - 15M

1. **[2M]** Consider a discrete-time signal $x[n]$ of length N. The DTFT of the signal is given by $X(e^{j\omega})$. Show that the DTFT is periodic with period 2π.
2. For a DTFT, Prove the following properties:
   (a) **[1M]** Linearity
   (b) **[1M]** Symmetry
   (c) **[1M]** Duality
   (d) **[1M]** Time shifting
   (e) **[1M]** Time scaling
   (f) **[1M]** Frequency Shifting
   (g) **[1M]** Time Reversal
   (h) **[1M]** Convolution Property
   (i) **[1M]** Parseval's Relation
3. Compute the DTFT for the following signals:

$$(a)\mathbf{[2M]} \quad x[n] = \left(\frac{1}{5}\right)^n u[n+1]$$

[2M]

(b) 

5 DFT - 16M

1. Compute the 8-point DFT for the following:
   (a) **[2M]** $x(n) = u(4-n)/4$, where $u(n)$ is the unit step function.
   (b) **[2M]** $x(n) = \sin(\pi n/4) + \cos(\pi n/4)$
   (c) **[2M]** $x[n] = \{1, -1-j, -1, -1+j\}$
   (d) **[2M]** $x[n] = \{1, 1, 1, 1, 1, 1, 1, 1\}$
2. Determine the Inverse Fourier transform of the following:
   (a) **[2M]** $X(e^{j\omega}) = \cos^3(\omega) + \cos^2(\omega)$
   (b) **[2M]** $X(e^{j\omega}) = \frac{e^{-4j\omega}+e^{-3j\omega}-e^{-j\omega}-1}{e^{-j\omega}+1}$
   (c) **[2M]** $X(e^{j\omega}) = \frac{3e^{-j\omega}-1}{3-e^{-j\omega}}$
3. **[2M]** Given $X[k] = k^2$, $0 \le k \le 7$ be the 8-point DFT of a sequence $x[n]$, find the value of:

$$P = \sum_{n=0}^{7} x[2n+1]$$

6 Sampling - 20M

1. **[4M]** What is aliasing? What can be done to reduce aliasing? Let $x(t) = \frac{1}{2\pi} cos(4000\pi t) cos(1000\pi t)$ be a continuous-time signal. Find the Nyquist rate and Nyquist interval for this signal.
2. **[5M]** A waveform, $x(t) = 10cos(1000t + \pi/3) + 20cos(2000t + \pi/6)$ is to be uniformly sampled for digital transmission. What is the maximum allowable time interval between sample values that will ensure perfect signal reproduction? If we want to reproduce 1 hour of this waveform, how many sample values need to be stored?
3. Consider two signals $x_1(t)$ and $x_2(t)$ with Fourier transforms satisfying:

$$X_1(\Omega) = 0, |\Omega| \geq 120$$
$$X_2(\Omega) = 0, |\Omega| \leq 60, |\Omega| \geq 100$$

   Determine the minimum frequency f_s, at which we must sample the following signals to prevent aliasing.

   (a) **[3M]** $x(t) = x_1(t) + x_2(t)$

   (b) **[5M]** $x(t) = x_1(t)x_2(t)$

   (c) **[3M]** $x(t) = \cos(3.6\pi t + 9.23)$

7 Quantization - 40M

1. Consider the analog waveform $x(t)$ and answer the following questions.

$$x(t) = \begin{cases} -2\sin(\pi x/4) & 0 \leq x < 4 \\ x - 4 & 4 \leq x < 5 \\ 1 & 5 < x < 7 \\ 8 - x & 7 \leq x \leq 10 \end{cases}$$

It is sampled at 1000 Hz and quantized with a 2-bit quantizer with input range -2V to 2V.

(a) **[1M]** Indicate the sample points.

(b) **[4M]** State the quantization intervals and the corresponding digital words.

(c) **[4M]** Sketch the digital word assigned to each sample point.

(d) **[4M]** Indicate the stream of bits generated after the quantization is complete.

(e) **[2M]** What is the resulting bit rate?

(f) **[2M]** What is the quantization error?

Answer all of the above questions for a 3-bit quantizer as well.

2. **[6M]** Mention advantages/disadvantages of increasing quantization bits.

8 Convolution - 8M

1. Find the Convolution of the following functions:-

(a) **[4M]**

$$f[n] = 2\delta[n + 10] + 2\delta[n - 10], \quad g[n] = 3\delta[n + 5] + 2\delta[n - 5]$$

(b) **[4M]**

$$f[n] = (-1)^n, \quad g[n] = \delta[n] + \delta[n - 1]$$