MA 6.101
Probability and Statistics

Tejas Bodas

Assistant Professor, IIIT Hyderabad

Logistics

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- Second half of the course by Prof. Pawan Kumar.

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- Phones in pocket, laptops in bag!

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  1. Intro. to Probability and Statistics for Engineers and Scientists by Sheldon Ross (Elsevier)
  2. Probability and Statistics by DeGroot and Schervish (Addison-Wesley)
  3. Introduction to probability by Bertsekas and Tsisiklis (Athena Scientific)
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- Some urls
  1. https://www.statlect.com/
  2. https://www.randomservices.org/
  3. https://www.probabilitycourse.com/

Evaluation scheme

- Quiz 1 : 15%.
- Midsem exam: 30%.
- Quiz 2: 15%
- Endsem 40 %.

Course Outline

- Module 1 (4 Lectures)
  Motivation & Probability basics

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- Module 4 (10 lectures)
  Probability inequalities and Statistics

Where is probability & statistics useful?

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- Machine learning

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- Biostatistics (clinical trials)

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- Draw inferences from the underlying data
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- Establish proof of concept for robustness of algorithms under randomness.

Laws

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This course is about laws of uncertainty/randomness.

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For example in a coin toss, if you could mathematically capture all possible influences on the coin exactly (force of tossing, metal density, temperature, wind speed etc) you can predict the outcome of each coin toss experiment with certainty.

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Probability theory is all about finding regularity and patterns is seemingly random experiments (experiments lacking a deterministic understanding of it) and expressing them mathematically.

Random experiments and Sample space

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- Any guesses for $\mathbb{P}(C_1), \mathbb{P}(D_1), \mathbb{P}(D_2), \mathbb{P}(U_1)$ and $\mathbb{P}(U_2)$?

Probability theory

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Probability measure $\mathbb{P}$ is a **set function**, i.e. it acts on sets and measures the probability of such sets.

Set theory 101

Visualizing operations on events using Venn diagram!

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- Uncountable sets: These are sets which are not countable.

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- Examples from $U[0, 1]$:
  - $I_n = [0, 1 - \frac{1}{n}]$
  - $D_n = [0, \frac{1}{n}]$

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- What is $[0, 1] \times [0, 1]$? unit square!

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- For discrete sets Ω, often the power set is denoted by 2^Ω.

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- $l : \mathcal{D} \to \mathbb{R}_+$ where $\mathcal{D} = \{[a, b] : a \leq b, a, b \in \mathbb{R}\}$ and where $l([c, d]) = d - c$.

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- So given $\mathbb{P}(A)$ and $\mathbb{P}(B)$, can we deduce $\mathbb{P}(A \cup B)$ or $\mathbb{P}(A/B)$?
- We want to understand how the probability measure $\mathbb{P}$ acts on sets such as $A \cup B,\ A \backslash B,\ A \times B$.