Assignment-1

2021101113

Science - II

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> **All these commands are tested on Ubuntu Version 20.04.3 LTS (Focal Fossa)**

```
Q3
├── Q3-Bonus.py
├── Q3-c.py
├── Q3.pdf
└── Q3.py
```

> - Q3

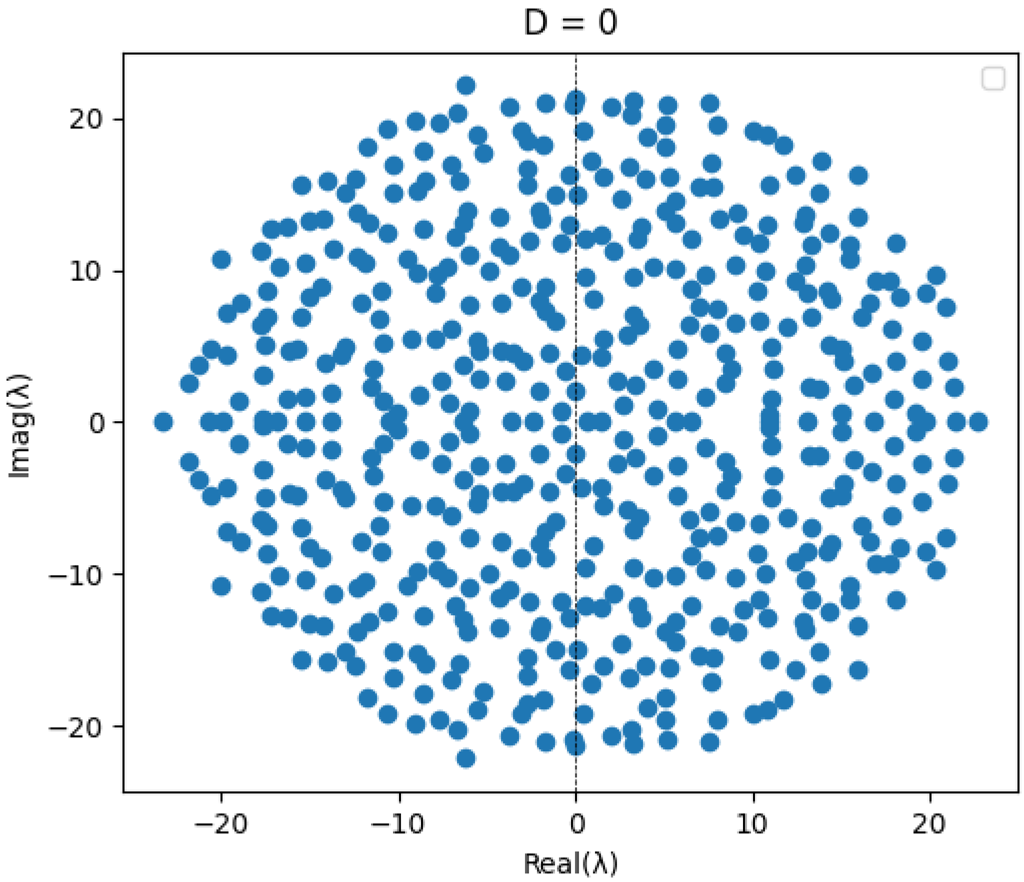
`$ python3 Q3.py`

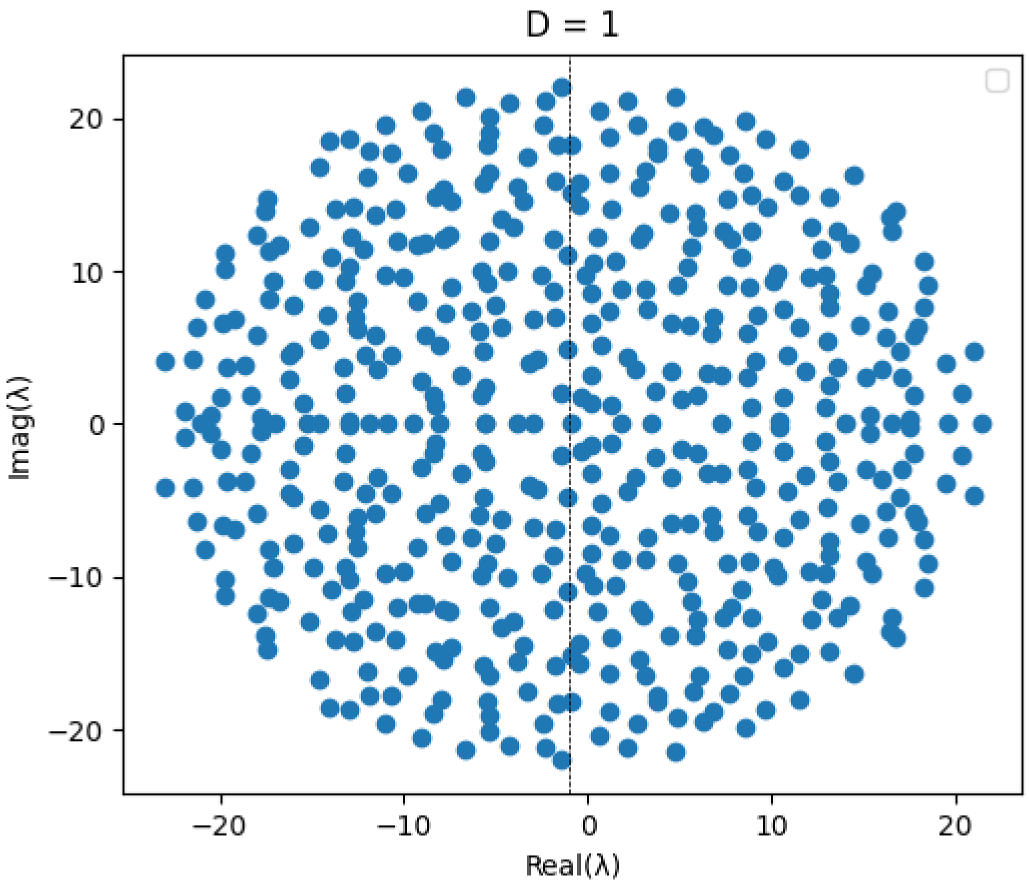
```
// code for eigen value plot for D = 0,1,5,10
import numpy as np
import matplotlib.pyplot as plt
M = np.random.normal(loc=0, scale=1, size=(500, 500))
for D in [0,1,5,10]:
    # Add diagonal elements to matrix
    np.fill_diagonal(M, -D)
    eigenvalues = np.linalg.eigvals(M)
    plt.scatter(np.real(eigenvalues), np.imag(eigenvalues))

plt.xlabel("Real(λ)")
plt.ylabel("Imag(λ)")
plt.show()
```

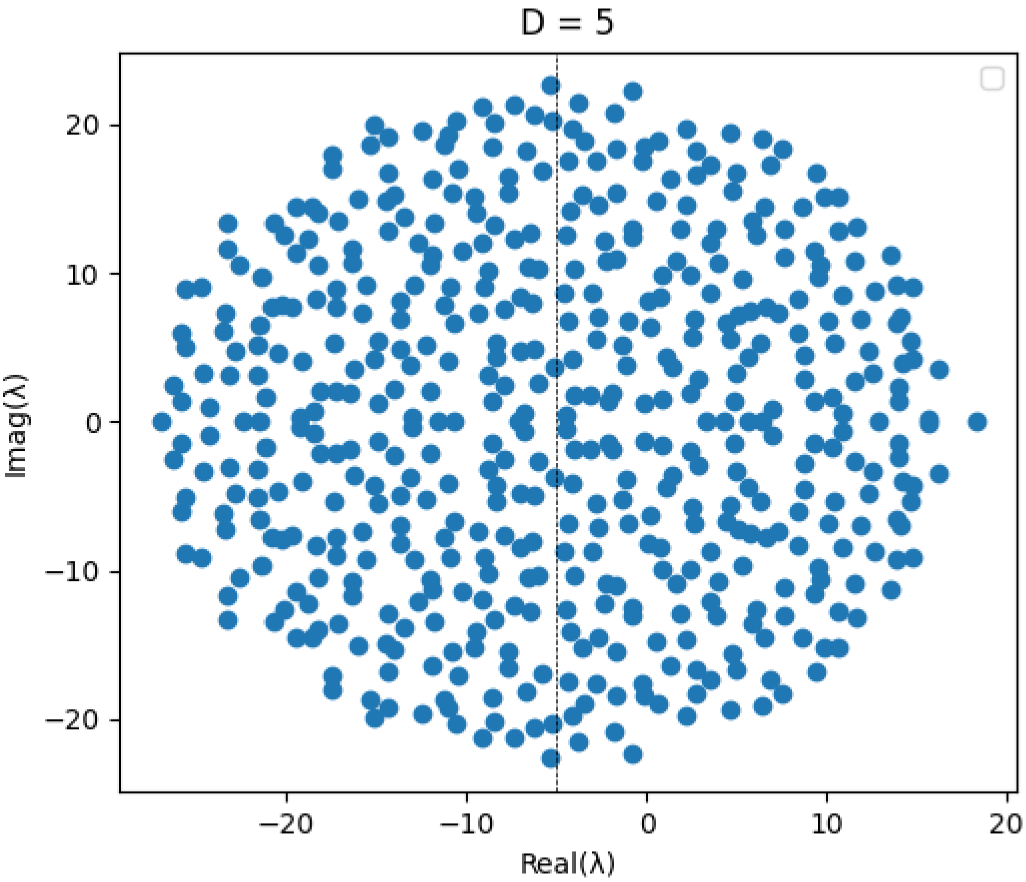
```
// code for part c
import numpy as np
import matplotlib.pyplot as plt
N = 500
M = np.random.normal(loc=0, scale=1, size=(500, 500))

# Plot eigenvalues for different values of D
# for D in [1]:
```

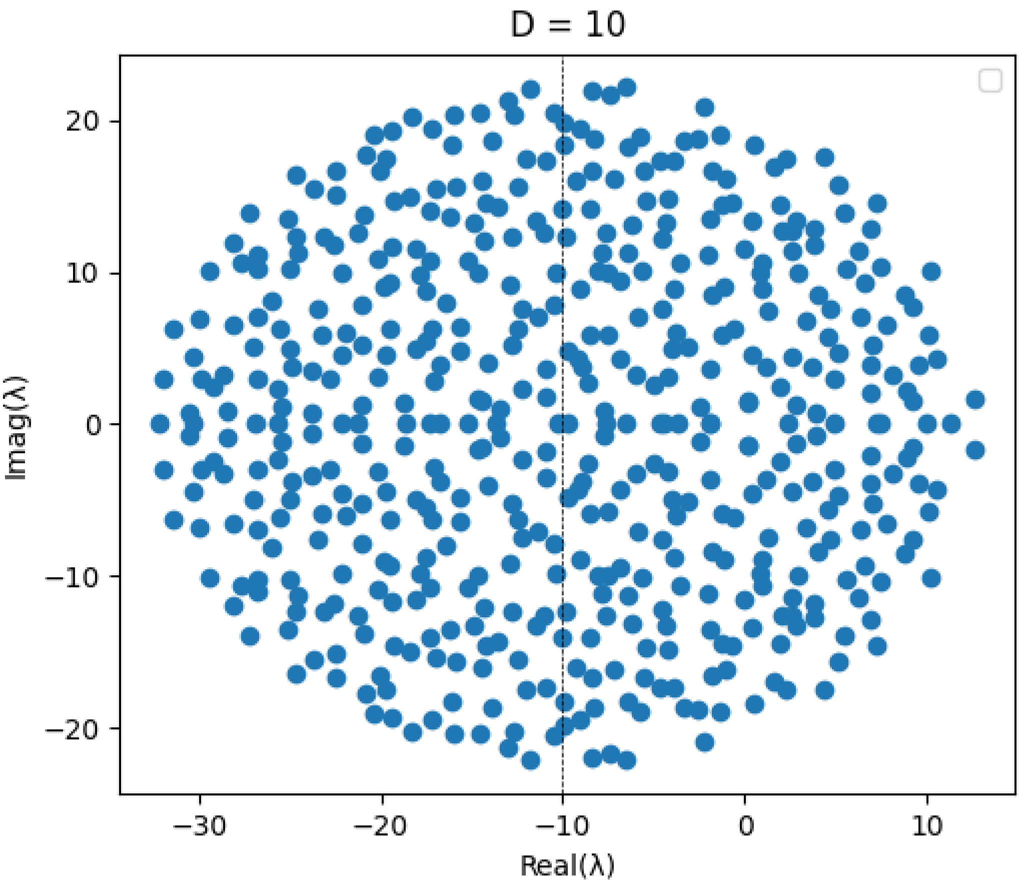
```
    # Add diagonal elements to matrix
D = 1
np.fill_diagonal(M, -D)
M = (M + M.T)/2
eigenvalues = np.linalg.eigvals(M)
plt.scatter(np.real(eigenvalues), np.imag(eigenvalues))
plt.xlabel("Real(λ)")
plt.ylabel("Imag(λ)")
plt.show()
```

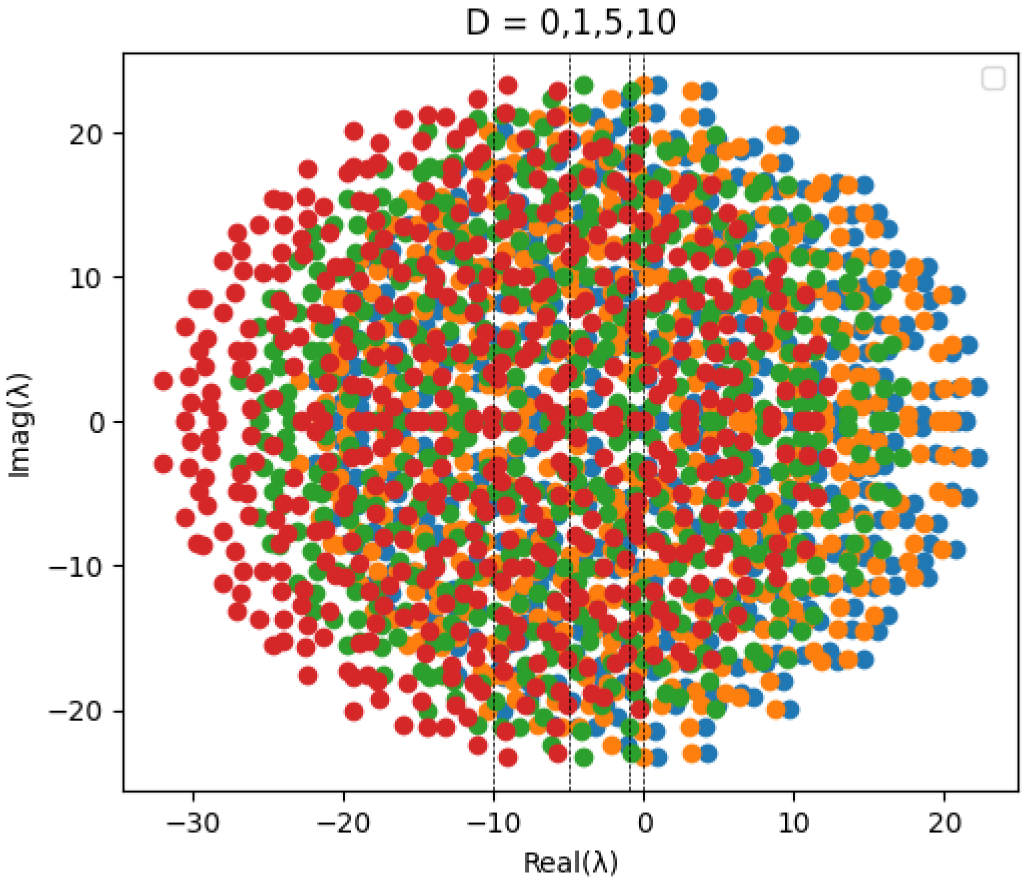
```
// code for bonus part
import numpy as np
import matplotlib.pyplot as plt
N = 500
M = np.random.normal(loc=0, scale=1, size=(500, 500))
for D in [0,1,5,10]:
    N = (M - M.T)/2
    np.fill_diagonal(N, -D)
    eigenvalues = np.linalg.eigvals(N)
    plt.scatter(np.real(eigenvalues), np.imag(eigenvalues))
plt.xlabel("Real(λ)")
plt.ylabel("Imag(λ)")
plt.show()
```


D = 0
Imag(λ)
Real(λ)
20
10
0
−10
−20
−20
−10
0
10
20


D = 1
Imag(λ)
Real(λ)
20
10
0
−10
−20
−20
−10
0
10
20

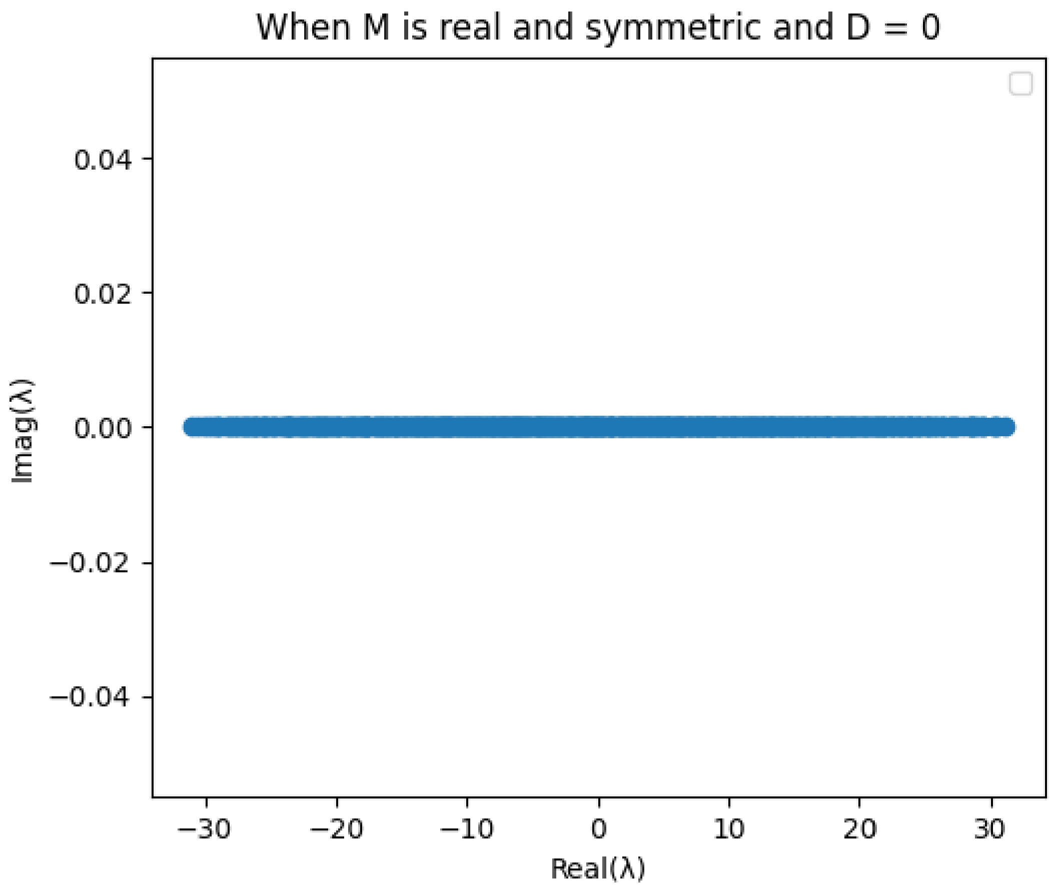

D = 5
Imag(λ)
Real(λ)
20
10
0
−10
−20
−20
−10
0
10
20


D = 10
Imag(λ)
Real(λ)
20
10
0
−10
−20
−30
−20
−10
0
10



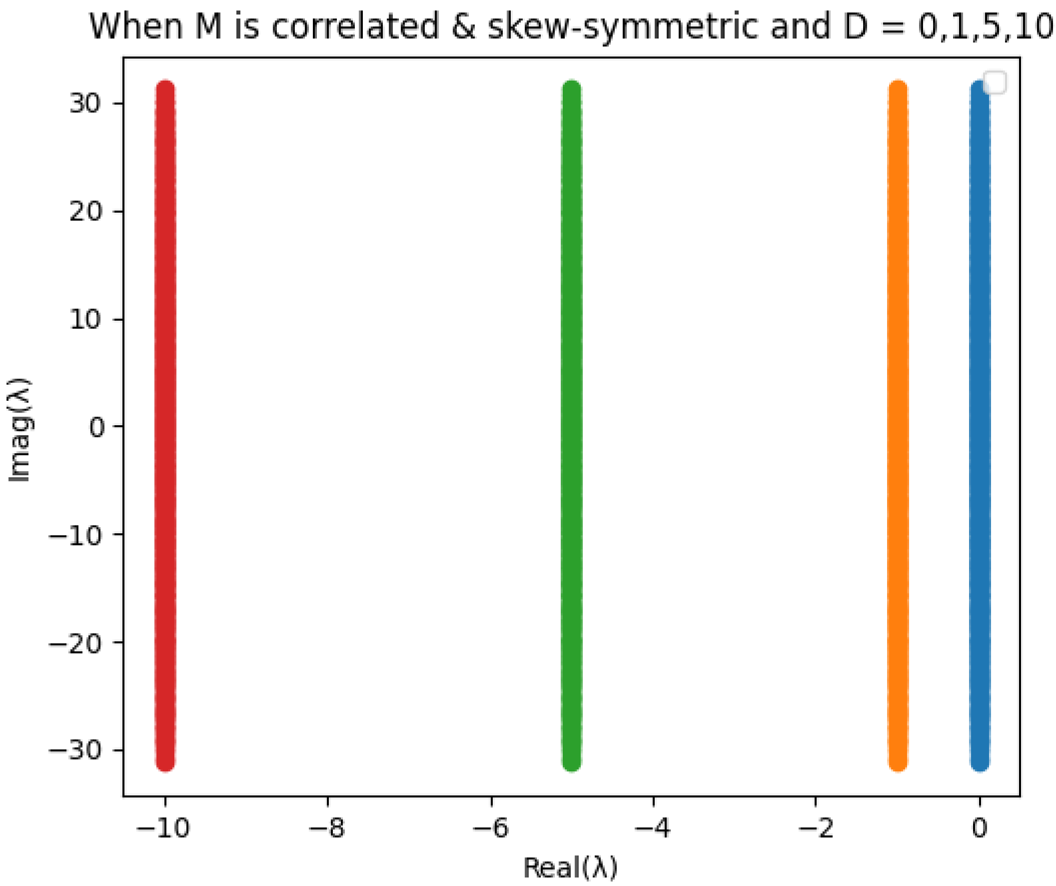

```
a) The shape in the plot is likely to be a cloud of scattered points ,
eccentricity is close to that of a circle , as the eigen values are complex
numbers
b) As the value of D increases , the eigen values will move away from the
origin of complex plane (-D is used in the matrix) , the cloud is
approximately symmetric about the point (-D,0)
```

```
c) If the matrix is real and symmetric , then all eigen values will be real
numbers and the plot will show a scatter of points along the real axis only
```

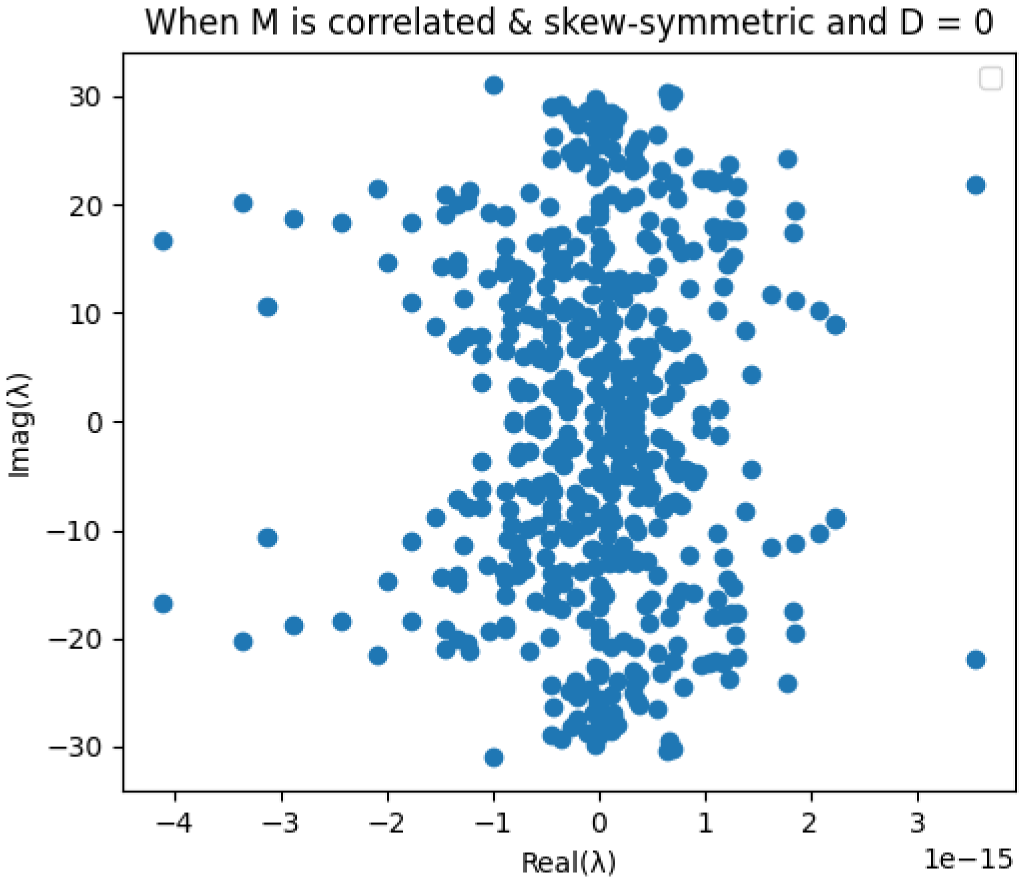



```
What will happen if the random elements in the matrix are correlated? E.g
if Mij >0 then Mji <0.

1. Large complex systems which are assembled (connected) at random may be
expected to be stable up to a certain critical level of connectance, and
then, as this increases, to suddenly become unstable

2. As you can observe the previous plots where all the elements were
randomly chosen from the normal distribution compared to  the plot for a
skewsymmetric matrix , the graph has drastically changed its shape from a
cloud of scattered points to a straight line due to the level of
correlation

3. If the random elements in the matrix are correlated (Consider the
example where if Mij > 0 and Mji<0) , then the matrix will be skew-
symmetric . A skew symmetric matrix is a square matrix whose transpose is
equal to its negative . If A is skew-symmetric , then A^T = -A

4. Skew symmetric matrices have some special properties , one of them is
that their eigen values will have zero real part and the plot will show a
scatter of points along the imaginary axis only and they are always
orthogonal matrices and hence their eigenvectors form an orthogonal basis
```

```
// ! Here the matrices are obtained after fetching a skew-symmetric matrix
and then replacing the Diagonal matrix with -D (centre of origin shifted)
```




```
You can observe slight deviations due to floating point errors and moreover
observe the scale of the below graph
```


When M is correlated & skew-symmetric and D = 0
30
20
10
0
−10
−20
−30
Imag(λ)
−4
−3
−2
−1
0
1
2
3
Real(λ)
1e−15