Training Day-113 Report:

Central Limit Theorem

What is the Central Limit Theorem?

The **Central Limit Theorem (CLT)** states that the sampling distribution of the sample mean approaches a normal distribution as the sample size increases, regardless of the original distribution of the population.

This theorem is a cornerstone of statistics because it justifies the use of normal distribution in hypothesis testing and confidence interval estimation, even when the underlying population distribution is unknown.

Key Points of the Central Limit Theorem

1. **Normality of Sampling Distribution:**
   - For large sample sizes ($n \geq 30$n \geq 30), the distribution of sample means will be approximately normal.
2. **Mean and Standard Deviation:**
   - The mean of the sampling distribution ($\mu_{\bar{X}}$\mu_{\bar{X}}) equals the population mean (μ\mu).
   - The standard deviation of the sampling distribution ($\sigma_{\bar{X}}$\sigma_{\bar{X}}), also called the **Standard Error (SE)**, is given by: $\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}
3. **Independence of Sample Size:**
   - The CLT holds true regardless of the population's original shape, as long as the samples are independent and identically distributed.

Applications of Central Limit Theorem

1. **Hypothesis Testing:**
   - Enables the use of Z-tests and T-tests, which assume normality of the sampling distribution.
2. **Confidence Intervals:**
   - Helps calculate intervals to estimate population parameters.
3. **Quality Control:**
   - Used in evaluating sample statistics to infer about population metrics.

Example of CLT

Suppose a population has a mean (μ\mu) of 100 and a standard deviation (σ\sigma) of 15. If we take random samples of size n=50n = 50:

- The mean of the sampling distribution remains 100100.
- The standard error is: σX¯=1550=2.12\sigma_{\bar{X}} = \frac{15}{\sqrt{50}} = 2.12

If the sample means are plotted, their distribution will approximate a normal curve, even if the population distribution is not normal.

Hypothesis Testing

What is Hypothesis Testing?

Hypothesis Testing is a statistical method used to evaluate assumptions (hypotheses) about a population parameter based on sample data.

Key Steps in Hypothesis Testing

1. **State the Hypotheses:**
   - Null Hypothesis (H0H_0): Assumes no effect or no difference.
   - Alternative Hypothesis (HaH_a): Contradicts H0H_0; claims an effect or difference exists.
2. **Set the Significance Level (α\alpha):**
   - Commonly used values are 0.05 (5%) or 0.01 (1%).
3. **Choose the Appropriate Test:**
   - Use Z-tests, T-tests, Chi-square tests, or others depending on data type and sample size.
4. **Calculate the Test Statistic:**
   - Example: For Z-test, compute: Z=X¯−μσnZ = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}
5. **Determine the p-value:**
   - Compare the p-value to α\alpha to decide whether to reject H0H_0.
6. **Make a Decision:**
   - If p≤αp \leq \alpha, reject H0H_0.
   - If p>αp > \alpha, fail to reject H0H_0.

Types of Errors

1. **Type I Error (α\alpha):**
   - Rejecting H0H_0 when it is true.

2. **Type II Error (β):**
    - Failing to reject H_0 when it is false.

Example of Hypothesis Testing

Example 1: Mean Testing

Suppose the average height of students is claimed to be 170 cm ($H_0: \mu = 170$). A sample of 30 students has a mean height of 172 cm, with a standard deviation of 5 cm. Use a significance level of 0.05.

1. **State Hypotheses:**
    - $H_0: \mu = 170$, $H_a: \mu \neq 170$.
2. **Calculate Test Statistic:**

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{172 - 170}{\frac{5}{\sqrt{30}}} = 2.19$$

3. **Find p-value:**
    - From Z-tables, $p = 0.0282$.
4. **Decision:**
    - Since $p = 0.0282 < \alpha = 0.05$, reject H_0.
    - Conclusion: The mean height is significantly different from 170 cm.

Applications of Hypothesis Testing

1. **Medicine:**
    - Test the efficacy of new drugs.
2. **Manufacturing:**
    - Evaluate quality differences in production.
3. **Market Research:**
    - Determine customer preferences or behavior changes.