Training Day-110 Report:

Binomial Distribution

What is Binomial Distribution?

The **Binomial Distribution** is a discrete probability distribution that models the number of successes in a fixed number of independent trials of a binary experiment. Each trial has only two possible outcomes: **success** or **failure**.

It is widely used in probability and statistics for modeling events with fixed probabilities.

Characteristics of Binomial Distribution

1. **Fixed Number of Trials (nn):**
   The experiment consists of a predetermined number of trials.
2. **Binary Outcomes:**
   Each trial results in one of two outcomes: **success** (with probability pp) or **failure** (with probability 1−p1-p).
3. **Independent Trials:**
   The outcome of one trial does not influence the outcomes of other trials.
4. **Constant Probability (pp):**
   The probability of success remains constant across all trials.

Formula for Binomial Distribution

The probability of exactly kk successes in nn trials is given by:

P(X=k)=(nk)pk(1−p)n−kP(X = k) = \binom{n}{k} p^k (1-p)^{n-k}

Where:

- P(X=k)P(X = k) is the probability of kk successes.
- (nk)=n!k!(n−k)!\binom{n}{k} = \frac{n!}{k!(n-k)!} is the binomial coefficient.
- pp is the probability of success.
- 1−p1-p is the probability of failure.
- nn is the number of trials.
- kk is the number of successes.

Mean and Variance

- **Mean (μ\mu):**

μ=n·p\mu = n \cdot p

- **Variance (σ2\sigma^2):**

σ2=n·p·(1−p)\sigma^2 = n \cdot p \cdot (1-p)

- **Standard Deviation (σ\sigma):**

σ=n·p·(1−p)\sigma = \sqrt{n \cdot p \cdot (1-p)}

Examples

1. Tossing a Coin:

Suppose you flip a coin 10 times (n=10n = 10), and the probability of heads (pp) is 0.5. The probability of getting exactly 6 heads (k=6k = 6) is:

P(X=6)=(106)(0.5)6(0.5)4=210·(0.5)10=0.205P(X = 6) = \binom{10}{6} (0.5)^6 (0.5)^4 = 210 \cdot (0.5)^{10} = 0.205

2. Defective Items in a Batch:

A factory produces items with a 95% success rate (p=0.95p = 0.95). If 20 items (n=20n = 20) are randomly selected, the probability of exactly 18 defect-free items (k=18k = 18) is:

P(X=18)=(2018)(0.95)18(0.05)2P(X = 18) = \binom{20}{18} (0.95)^{18} (0.05)^2

Calculate this using the binomial coefficient and powers of pp and 1−p1-p.

Applications of Binomial Distribution

1. **Quality Control:**
   - Used to determine the probability of defective items in a production line.
2. **Epidemiology:**
   - Models the spread of diseases or the effectiveness of vaccines.
3. **Finance:**
   - Evaluates probabilities in risk analysis and decision-making.
4. **Machine Learning:**
   - Used in probabilistic models such as Naive Bayes classifiers.