Training Day-109 Report:

What is Expected Value?

The **Expected Value (EV)** is a fundamental concept in probability and statistics that represents the long-term average or mean value of random variable outcomes if an experiment is repeated infinitely. It provides a measure of the central tendency of a probability distribution.

Formula for Expected Value

1. **For Discrete Random Variables:**
   If XX is a discrete random variable with possible values x1,x2,...,xnx_1, x_2, ..., x_n and corresponding probabilities P(x1),P(x2),...,P(xn)P(x_1), P(x_2), ..., P(x_n), then:

E(X)=∑i=1nxi·P(xi)E(X) = \sum_{i=1}^n x_i \cdot P(x_i)

2. **For Continuous Random Variables:**
   If XX is a continuous random variable with probability density function f(x)f(x), then:

E(X)=∫−∞∞x·f(x) dxE(X) = \int_{-\infty}^{\infty} x \cdot f(x) \, dx

Properties of Expected Value

1. **Linearity:**
   - For random variables XX and YY, and constants aa and bb:
     E(aX+bY)=aE(X)+bE(Y)E(aX + bY) = aE(X) + bE(Y)
2. **Non-Negativity:**
   - For a non-negative random variable XX, E(X)≥0E(X) \geq 0.
3. **Expectation of a Constant:**
   - If cc is a constant, E(c)=cE(c) = c.

Examples

1. Discrete Random Variable (Dice Roll):

Suppose you roll a fair six-sided die. The possible outcomes are {1,2,3,4,5,6}\{1, 2, 3, 4, 5, 6\}, each with a probability of 16\frac{1}{6}.

E(X)=∑i=16xi·P(xi)=1·16+2·16+...+6·16=3.5E(X) = \sum_{i=1}^6 x_i \cdot P(x_i) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + ... + 6 \cdot \frac{1}{6} = 3.5

The expected value is 3.5, which is the average result of rolling the die many times.

2. Continuous Random Variable (Uniform Distribution):

If XX is uniformly distributed between 0 and 10, the PDF is:

f(x)={110,0≤x≤100,otherwisef(x) = \begin{cases} \frac{1}{10}, & 0 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}

The expected value is:

E(X)=∫010x·110dx=110∫010xdx=110·x22|010=5E(X) = \int_0^{10} x \cdot \frac{1}{10} dx = \frac{1}{10} \int_0^{10} x dx = \frac{1}{10} \cdot \frac{x^2}{2} \Big|_0^{10} = 5

Applications of Expected Value

1. **Economics and Finance:**
   - Used in decision-making under uncertainty, such as calculating expected returns in investments.
2. **Insurance:**
   - Helps calculate premiums based on expected claims.
3. **Game Theory:**
   - Used to evaluate strategies and outcomes in competitive scenarios.
4. **Machine Learning:**
   - Forms the basis of loss functions in supervised learning models.