Training Day-112 Report:

Z-Score:

What is Z-Score?

The **Z-Score** (also called the **Standard Score**) measures how many standard deviations a data point is from the mean of a distribution. It allows comparison of data points from different distributions by standardizing them.

A Z-score helps to determine whether a value is typical or unusual in a given dataset.

Formula for Z-Score

Z=X−μσZ = \frac{X - \mu}{\sigma}

Where:

- ZZ = Z-score
- XX = Individual data point
- μ\mu = Mean of the distribution
- σ\sigma = Standard deviation of the distribution

Interpretation of Z-Score

1. **Positive Z-Score (Z>0Z > 0):**
   - Indicates that the data point is above the mean.
2. **Negative Z-Score (Z<0Z < 0):**
   - Indicates that the data point is below the mean.
3. **Z-Score of 0:**
   - Indicates that the data point is exactly at the mean.
4. **Magnitude of Z-Score:**
   - The further the Z-score is from 0, the more unusual the data point is.

Example of Z-Score Calculation

Example 1: Exam Scores

Suppose exam scores are normally distributed with a mean (μ\mu) of 75 and a standard deviation (σ\sigma) of 10. A student scores 85 on the exam.

To calculate the Z-score:

Z=85−7510=1010=1Z = \frac{85 - 75}{10} = \frac{10}{10} = 1

The Z-score is **1**, meaning the student scored 1 standard deviation above the mean.

Applications of Z-Score

1. **Comparing Data Points:**
   - Standardizes values from different distributions for comparison.
   - Example: Comparing test scores from two tests with different scales.
2. **Outlier Detection:**
   - Data points with Z-scores greater than 3 or less than -3 are often considered outliers.
3. **Probability Calculations:**
   - Z-scores are used with standard normal tables to calculate probabilities for normal distributions.
4. **Standardizing Data in Machine Learning:**
   - Z-scores are used to normalize features to have a mean of 0 and standard deviation of 1.

Z-Score and the Standard Normal Table

The **Standard Normal Table** (Z-Table) provides the cumulative probability for a given Z-score.

- Example: A Z-score of **1.25** corresponds to a cumulative probability of approximately **0.8944**.
  This means that 89.44% of the data lies below this Z-score.

Examples with Probability

Example 1: Finding Probabilities

A dataset of weights is normally distributed with a mean of 70 kg and a standard deviation of 5 kg. What is the probability that a randomly chosen person weighs less than 75 kg?

1. Calculate the Z-score:

Z=75−705=1Z = \frac{75 - 70}{5} = 1

2. Use the Z-Table:
   - A Z-score of **1** corresponds to a cumulative probability of **0.8413**.
   - Therefore, there is an **84.13%** chance that a person weighs less than 75 kg.

Example 2: Finding Intervals

What is the probability of a person weighing between 65 kg and 75 kg?

1. Calculate the Z-scores:
   - For 65 kg: Z=65−705=−1Z = \frac{65 - 70}{5} = -1.
   - For 75 kg: Z=75−705=1Z = \frac{75 - 70}{5} = 1.

2. Use the Z-Table:
    - Cumulative probability for $Z = -1$: **0.1587**.
    - Cumulative probability for $Z = 1$: **0.8413**.
3. Subtract probabilities:

$$P(65 \leq X \leq 75) = 0.8413 - 0.1587 = 0.6826$$

- Therefore, there is a **68.26%** chance that a person weighs between 65 kg and 75 kg.