Training Day-111 Report:

Normal Distributions

What is a Normal Distribution?

A **Normal Distribution**, also known as a **Gaussian Distribution**, is a continuous probability distribution characterized by its symmetric, bell-shaped curve. It is one of the most important distributions in statistics and is widely used in various fields due to its natural occurrence in many real-world phenomena.

The Normal Distribution is defined by two parameters:

1. **Mean (μ\mu):** Determines the center of the distribution.
2. **Standard Deviation (σ\sigma):** Determines the spread or width of the distribution.

Properties of Normal Distribution

1. **Symmetry:**
   - The curve is symmetric around the mean (μ\mu).
2. **Bell Shape:**
   - The curve has a peak at the mean and tails off equally on both sides.
3. **Empirical Rule (68-95-99.7 Rule):**
   - About **68%** of the data lies within one standard deviation of the mean (μ±σ\mu \pm \sigma).
   - About **95%** of the data lies within two standard deviations (μ±2σ\mu \pm 2\sigma).
   - About **99.7%** of the data lies within three standard deviations (μ±3σ\mu \pm 3\sigma).
4. **Total Area Under the Curve:**
   - The total area under the curve is equal to 1.

Probability Density Function (PDF)

The PDF of a normal distribution is given by:

f(x)=12πσ2e−(x−μ)22σ2f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}

Where:

- xx is the random variable.
- μ\mu is the mean.

- σ2\sigma^2 is the variance.
- ee is the base of the natural logarithm (e≈2.718e \approx 2.718).

Standard Normal Distribution

A **Standard Normal Distribution** is a special case of the normal distribution with:

- Mean (μ\mu) = 0
- Standard Deviation (σ\sigma) = 1

To standardize a normal random variable XX, use the **Z-score**:

Z=X−μσZ = \frac{X - \mu}{\sigma}

Applications of Normal Distribution

1. **Statistics:**
   - Used in hypothesis testing, confidence intervals, and regression analysis.
2. **Natural Sciences:**
   - Models phenomena like heights, weights, and measurement errors.
3. **Finance:**
   - Analyzes stock market returns and risk assessments.
4. **Machine Learning:**
   - Assumes normality in algorithms like Gaussian Naive Bayes.
5. **Quality Control:**
   - Evaluates processes under the assumption of normality in production lines.

Examples

1. Height of Individuals:

Suppose the heights of a population are normally distributed with a mean of 170 cm and a standard deviation of 10 cm. The probability of a randomly selected individual being between 160 and 180 cm is:

P(160≤X≤180)=∫160180f(x)dxP(160 \leq X \leq 180) = \int_{160}^{180} f(x) dx

Use the Z-score and standard normal tables for calculation.

2. Exam Scores:

If exam scores are normally distributed with a mean of 75 and a standard deviation of 8, the probability of scoring above 85 is:

Z=85−758=1.25Z = \frac{85 - 75}{8} = 1.25

Look up Z=1.25Z = 1.25 in standard normal tables to find the probability.