Training Day-108 Report:

Probability Distribution Function:

What is a Probability Distribution Function?

A **Probability Distribution Function (PDF)** is a mathematical function that describes the likelihood of a random variable taking on a particular value. It provides a relationship between the possible values of a random variable and their associated probabilities.

- **Discrete Random Variables:** A PDF assigns probabilities to specific values.
- **Continuous Random Variables:** A PDF defines a density function, where the probability is the area under the curve within a specified interval.

Key Terminologies Related to PDF

- **Random Variable:** A variable that can take on different values determined by chance.
    - **Discrete Random Variable:** Takes on specific, countable values (e.g., number of heads in coin flips).
    - **Continuous Random Variable:** Takes on any value within a range (e.g., height or weight).
- **Probability Mass Function (PMF):** Used for discrete random variables to assign probabilities to each value.
- **Cumulative Distribution Function (CDF):** The probability that a random variable is less than or equal to a given value.
- **Expected Value (Mean):** The weighted average of all possible values of a random variable.
- **Variance:** A measure of the spread or dispersion of the random variable around its mean.

Types of Probability Distributions

1. Discrete Probability Distributions

- **Binomial Distribution:** Deals with experiments consisting of independent trials with two outcomes (success or failure).
- **Poisson Distribution:** Models the probability of a given number of events occurring

in a fixed interval.

2. Continuous Probability Distributions

- **Normal Distribution (Gaussian):** A symmetric bell-shaped curve widely used in statistics and real-world modeling.
- **Exponential Distribution:** Describes the time between events in a Poisson process.

Properties of PDF

1. **Non-negativity:** The probability value is always non-negative, i.e., $f(x) \geq 0$ for all values of x.
2. **Normalization:** The total probability over all possible values is 1, i.e., $\int_{-\infty}^{\infty} f(x)\,dx = 1$
3. **Probability Calculation:** For a continuous variable, the probability for an interval [a, b] is given by: $P(a \leq X \leq b) = \int_a^b f(x)\,dx$

Applications of PDF

1. **Statistical Analysis:** Used to describe the probability distributions of data in fields like economics, biology, and engineering.
2. **Machine Learning:** PDFs are integral in probabilistic models like Naive Bayes and Hidden Markov Models.
3. **Risk Analysis:** PDFs help estimate the likelihood of events in fields like insurance and finance.
4. **Simulation:** Generate random samples following specific distributions for testing and modeling.

Examples

1. Discrete PDF (Binomial Distribution):

A factory produces items with a 95% success rate. What is the probability of 3 successes in 5 trials?

$P(X=3) = \binom{5}{3}(0.95)^3(0.05)^2$

2. Continuous PDF (Normal Distribution):

If a variable follows a normal distribution with a mean of 50 and a standard deviation of 5, the PDF is:

$f(x) = \frac{1}{\sqrt{2\pi} \cdot 5} e^{-\frac{(x-50)^2}{2 \cdot 5^2}}$