

The Fluxes and the Equations of Change

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§B.1 NEWTON'S LAW OF VISCOSITY

$$[\tau = -\mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^t) + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})\delta]$$

Cartesian coordinates (x, y, z):

$$\tau_{xx} = -\mu \left[2 \frac{\partial v_x}{\partial x} \right] + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \quad (\text{B.1-1})^a$$

$$\tau_{yy} = -\mu \left[2 \frac{\partial v_y}{\partial y} \right] + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \quad (\text{B.1-2})^a$$

$$\tau_{zz} = -\mu \left[2 \frac{\partial v_z}{\partial z} \right] + (\frac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v}) \quad (\text{B.1-3})^a$$

$$\tau_{xy} = \tau_{yx} = -\mu \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right] \quad (\text{B.1-4})$$

$$\tau_{yz} = \tau_{zy} = -\mu \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right] \quad (\text{B.1-5})$$

$$\tau_{zx} = \tau_{xz} = -\mu \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right] \quad (\text{B.1-6})$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \quad (\text{B.1-7})$$

^a When the fluid is assumed to have constant density, the term containing $(\nabla \cdot \mathbf{v})$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.

§B.1 NEWTON'S LAW OF VISCOSITY (continued)

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu \left[2 \frac{\partial v_r}{\partial r} \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-8})^a$$

$$\tau_{\theta\theta} = -\mu \left[2 \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-9})^a$$

$$\tau_{zz} = -\mu \left[2 \frac{\partial v_z}{\partial z} \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-10})^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (\text{B.1-11})$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu \left[\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right] \quad (\text{B.1-12})$$

$$\tau_{zr} = \tau_{rz} = -\mu \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right] \quad (\text{B.1-13})$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \quad (\text{B.1-14})$$

^a When the fluid is assumed to have constant density, the term containing $(\nabla \cdot \mathbf{v})$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.

Spherical coordinates (r, θ, φ):

$$\tau_{rr} = -\mu \left[2 \frac{\partial v_r}{\partial r} \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-15})^a$$

$$\tau_{\theta\theta} = -\mu \left[2 \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-16})^a$$

$$\tau_{\phi\phi} = -\mu \left[2 \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta \cot \theta}{r} \right) \right] + \left(\frac{2}{3}\mu - \kappa \right) (\nabla \cdot \mathbf{v}) \quad (\text{B.1-17})^a$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \quad (\text{B.1-18})$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu \left[\frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right] \quad (\text{B.1-19})$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu \left[\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \right] \quad (\text{B.1-20})$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \quad (\text{B.1-21})$$

^a When the fluid is assumed to have constant density, the term containing $(\nabla \cdot \mathbf{v})$ may be omitted. For monatomic gases at low density, the dilatational viscosity κ is zero.