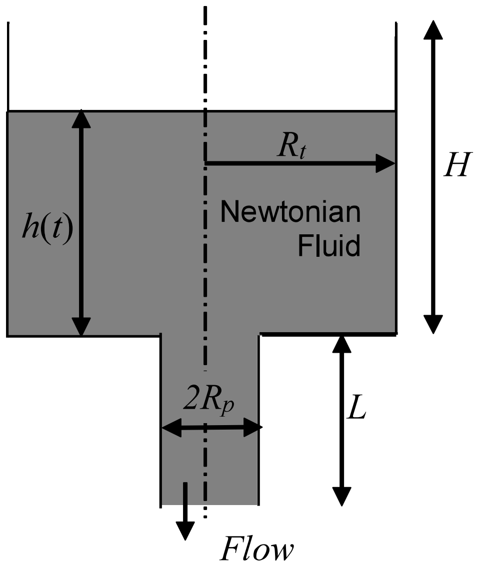
Transport Phenomena (CL 203) Problem Set 4 <u>Newtonian Flow</u> ChE, IITB *Sep* 2022

1) A cube of side L is immersed in a static fluid of density ρ with its top surface horizontal. The top surface of the cube is at a depth h from the air-fluid surface. (a) From Cauchy's 1st law, obtain the stress tensor in the fluid ($\underset{\approx}{\mathbf{T}}$) (b) Calculate the force acting on each surface of the cube. (c) Show that the net force exerted by the fluid on the cube equals the weight of the fluid displaced.

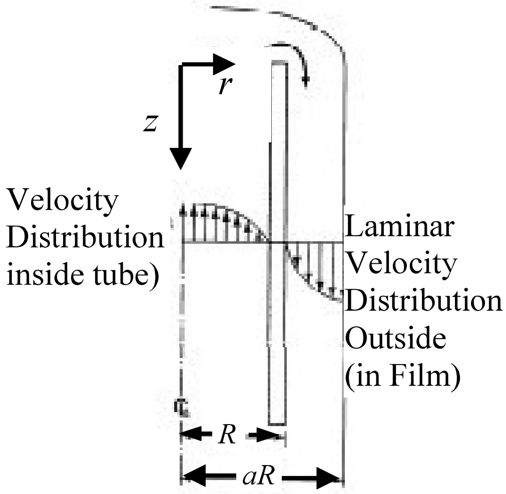
2) A cylindrical open tank of radius R_t and height H, and its drainpipe of length L (L~H) and radius R_p (<< R_t) <u>are completely filled</u> with a Newtonian heavy oil (properties: μ, ρ) (Drawing is not to scale). At time $t = 0$, the valve at the bottom of the drainpipe is opened to the atmosphere. **<u>Use</u>** the <u>*pseudo-steady-state*</u> approach; i.e., at any time t, when the fluid height in the tank is $h(t)$, carry out a <u>steady-state momentum balance</u>, for Newtonian flow in the pipe. **<u>Indicate</u>** the instantaneous pressure at the inlet of the pipe at time t. **<u>How</u>** does it affect the velocity profile in the pipe at time t? The outflow from the pipe causes a slow decrease in $h(t)$; i.e., in the <u>tank</u> the v_z is <u>slow and uniform</u> over r; **<u>draw</u>** the instantaneous control volume at time t. Thus, **<u>how long</u>** will it take to drain the tank? (integrate time-dependent volumetric flow rate corresponding to obtain total volume loss from the tank volume. Ignore the volume of the pipe). **<u>Given</u>**: in pipe *for the instant*, $\left(\dfrac{\partial v_z}{\partial t} = 0\right)$.

Also: $(\nabla \cdot \mathbf{v}) = \dfrac{1}{r}\dfrac{\partial}{\partial r}(r v_r) + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial \theta} + \dfrac{\partial v_z}{\partial z}$;

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$
$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$



3) **<u>Film Flow on the outside of a circular tube</u>** (see Fig.). In a gas absorption experiment a viscous fluid flows upward through a small circular tube and then downward on the outside.



a) Show that the velocity distribution (neglecting end effects) in the falling film is

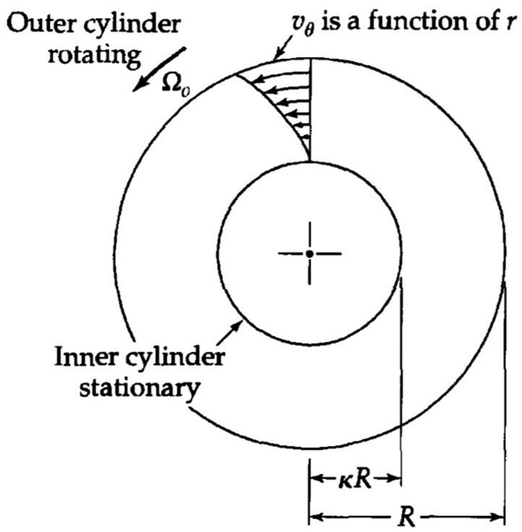
$$v_z = \frac{\rho g R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2 + 2a^2\ln\left(\frac{r}{R}\right)\right]$$

b) Derive an expression for the flow rate as a function of system parameters.

c) Obtain the mass rate of flow in the film.

d) Show that the result in (b) simplifies to flow down an inclined plate $\left(\rho^2 g W \delta^3 \cos\theta\right)/(3\mu)$ or vertical plate (θ=0), if the film thickness is very small.

4) **Couette Flow: Cylindrical vs Cartesian:**



Given tangential flow (Fig) of an incompressible Newtonian fluid between 2 cylinders, of radii κR (inner cylinder) and R. The outer cylinder rotates with an angular velocity, Ω_0. The velocity profile is illustrated.

a) **Now consider** a case **different from the figure**, where the **INNER** cylinder rotates with angular velocity, Ω_i. For this case, **first draw** a schematic of the velocity profile.

b) **Simplify** the governing component of the Navier Stokes' equation:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right)$$

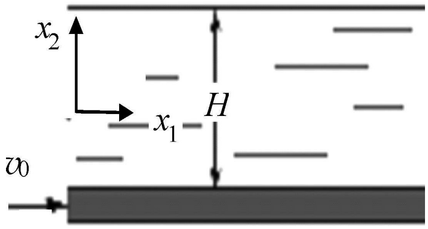
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

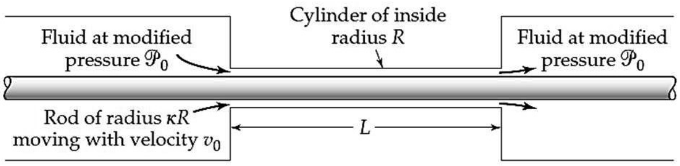
c) At steady state, prove

$$v_\theta \times \left(\frac{1}{\kappa} - \kappa\right) = \Omega_i \kappa R\left(\frac{R}{r} - \frac{r}{R}\right).$$

d) Now consider the case that the annular gap is very small; i.e., 1- κ << 1. This is similar to a fluid between flat plates, where the top plate is stationary, and bottom plate moves with velocity v_0. Find v_0 and H, from the figure and relate to the tangential flow parameters. Begin with the equation in part c



5) **Annular flow with inner cylinder moving axially (Application in Coating of Wires):**



A cylindrical rod of radius κR, moves at velocity V, co-axially through a pipe of radius R and length L. The flow is Newtonian, laminar, and the pipe is at uniform modified pressure $\mathcal{P}_0$.

a) Show that at steady state $\dfrac{v_z}{V} = \dfrac{\ln\left(r/R\right)}{\ln(k)}$

b) If $\varepsilon = 1-\kappa$ is small, what is the velocity profile? Use $r = R\times(1-y)$ Given: Taylor's expansion, $\ln(1-y) = -\sum\left(y^i/i\right)$

c) Determine the volumetric flow rate.

d) Determine the force required to pull the rod, in order to maintain the velocity.