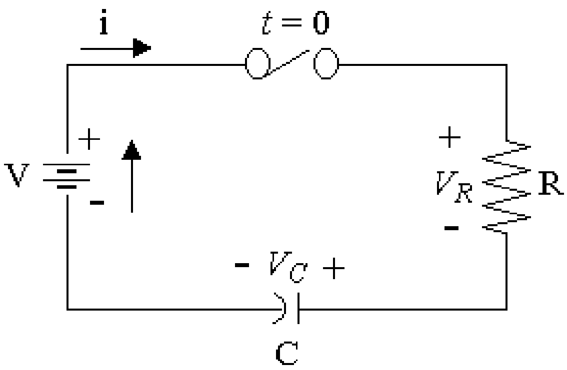
6. Application: Series RC Circuit



An RC series circuit

In this section we see how to solve the differential equation arising from a circuit consisting of a resistor and a capacitor. (See the related section Series RL Circuit in the previous section.)

In an RC circuit, the **capacitor** stores energy between a pair of plates. When voltage is applied to the capacitor, the charge builds up in the capacitor and the current drops off to zero.

Case 1: Constant Voltage

The voltage across the resistor and capacitor are as follows:

$$V_R = Ri$$

and

$$V_C = \frac{1}{C}\int i\ dt$$

Kirchhoff's voltage law says the total voltages must be zero. So applying this law to a series RC circuit results in the equation:

$$\boxed{Ri + \frac{1}{C}\int i\ dt = V}$$

One way to solve this equation is to turn it into a **differential equation**, by differentiating throughout with respect to t:

$$\boxed{R\frac{di}{dt} + \frac{i}{C} = 0}$$

Solving the equation gives us:

$$\boxed{i = \frac{V}{R}e^{-t/RC}}$$

Proof

Important note: We are assuming that the circuit has a **constant voltage** source, V. This equation does not apply if the voltage source is **variable**.

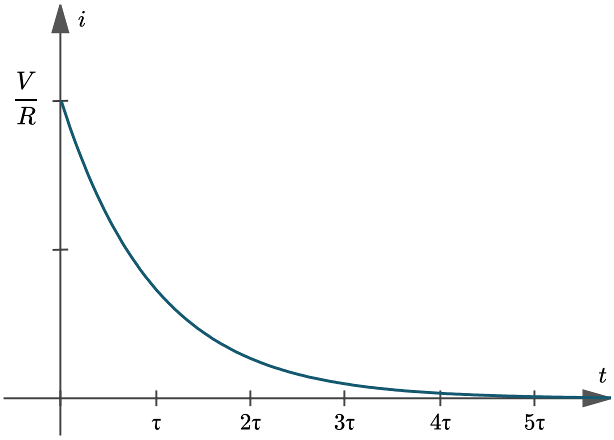
The **time constant** in the case of an RC circuit is:

$$\tau = RC$$

The function

$$i = \frac{V}{R} e^{-t/RC}$$

has an **exponential decay** shape as shown in the graph. The current stops flowing as the capacitor becomes fully charged



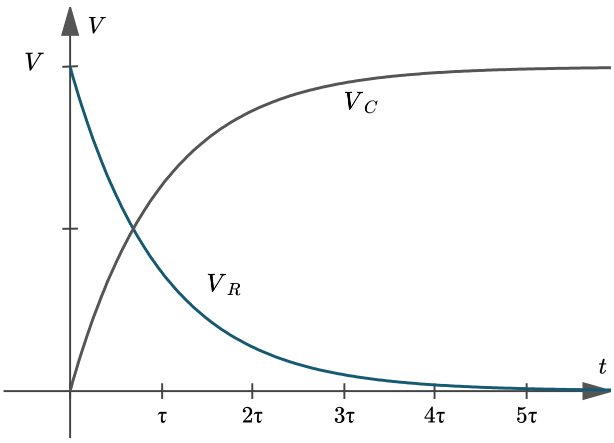
Graph of $i = \frac{V}{R} e^{-(t/RC)}$, an exponential decay curve.

Applying our expressions from above, we have the following expressions for the voltage across the resistor and the capacitor:

$$V_R = Ri = Ve^{-t/RC}$$

$$V_C = \frac{1}{C}\int i\ dt = V\left(1 - e^{-t/RC}\right)$$

While the voltage over the resistor drops, the voltage over the capacitor rises as it is charged:



Graphs of $V_R = Ve^{-t/RC}$ (in green) and $V_C = V\left(1 - e^{-t/RC}\right)$ (in gray).

Case 2: Variable Voltage and 2-mesh Circuits

We need to solve variable voltage cases in q, rather than in i, since we have an integral to deal with if we use i.

So we will make the substitutions:

$$i = \frac{dq}{dt}$$

and

$$q = \int i\ dt$$

and so the equation in i involving an integral:

$$Ri + \frac{1}{C}\int i\ dt = V$$

becomes the differential equation in ***q***:

$$R\frac{dq}{dt} + \frac{1}{C}q = V$$

Example 1

A series RC circuit with R = 5 W and C = 0.02 F is connected with a battery of E = 100 V. At t = 0, the voltage across the capacitor is zero.

(a) Obtain the subsequent voltage across the capacitor.

(b) As $t \to \infty$, find the charge in the capacitor.

Show answer

Example 2

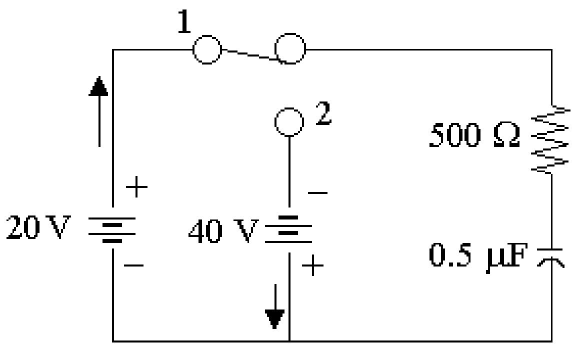
Find the charge and the current for $t > 0$ in a series RC circuit where $R = 10\ \text{W}$, $C = 4 \times 10^{-3}\ \text{F}$ and $E = 85\ \cos\ 150t\ \text{V}$.

Assume that when the switch is closed at $t = 0$, the charge on the capacitor is -$0.05\ \text{C}$.

Show answer

Example 3

In the RC circuit shown below, the switch is closed on position 1 at $t = 0$ and after $1\ \tau$ is moved to position 2. Find the complete current transient.



Show answer

Do not try this next one at home!

Here's a great Java-based RLC simulator (on an external site). He is actually making a coil gun. You can play with each of \
R, L and C and see the effects. Play and learn :-)

RLC Simulator

Sorry, it won't work on a mobile device.