APPENDIX

A. Proof of Theorem 1

Proof. Now we consider a $GBA(U_{B1})$ where poison values exist on both sides. Let V_{B1}^{L} denote the set of poison values on $[D_L, O]$, and V_{B1}^{R} denote those on $[O, D_R]$.

Without loss of generality, consider the case where the mean of the left side is larger. Let

$$C_0 = \Sigma(V_{B1}^{L} - O) + \Sigma(V_{B1}^{R} - O) < 0. \tag{11}$$

Let us choose the largest poison value $\mathbf{y_r}$ in V_{B1}^{R}, and an arbitrary subset of poison values $\mathcal{Y_L}$ from V_{B1}^{L}, such that the following two formulas are satisfied concurrently:

$$\Sigma(\mathcal{Y_L} - O) + \mathbf{y_r} - O \leq 0, \tag{12}$$

and for any $\mathbf{y_l} \in \mathcal{Y_L}$,

$$\Sigma(\mathcal{Y}'_{\mathcal{L}} - O) + \mathbf{y_r} - O > 0, \tag{13}$$

where $\mathcal{Y}'_{\mathcal{L}}$ is the set of elements in $\mathcal{Y_L}$ excluding $\mathbf{y_l}$.

This is always available, because

$$\Sigma(O - V_{B1}^{L}) > \Sigma(V_{B1}^{R} - O) \geq \mathbf{y_r} - O.$$

Add $\mathbf{y_l} - O$ on both sides of Equ. 13, we then have

$$\begin{aligned} \Sigma(\mathcal{Y}'_{\mathcal{L}} - O) + \mathbf{y_l} - O + \mathbf{y_r} - O \ &> \mathbf{y_l} - O \\ \Sigma(\mathcal{Y_L} - O) + \mathbf{y_r} - O &> \mathbf{y_l} - O \end{aligned} \tag{14}$$

Let

$$\mathbf{y}'_{\mathbf{l}} - O = \Sigma(\mathcal{Y_L} - O) + \mathbf{y_r} - O \tag{15}$$

According to Equ. 12 and Equ. 14, we have

$$\mathbf{y_l} - O < \mathbf{y}'_{\mathbf{l}} - O \leq 0$$

Apparently, $\mathbf{y}'_{\mathbf{l}} \in [D_L, O]$. Let us remove $\mathbf{y_r}$ from V_{B1}^{R}, $\mathcal{Y_L}$ from Y_L, and add $\mathbf{y}'_{\mathbf{l}}$ into V_{B1}^{L}. Such an operation eliminates a poison value on the right hand without changing C_0 according to Equ. 11 and Equ. 15. Hence, the generated distribution can be reduced to the initial one.

Repeating the same operation until V_{B1}^{R} is empty, we finally obtain a $V'_{B2} = V_{B1}^{L}$. This is achievable, as both V_{B1}^{L} and V_{B1}^{R} are finite, and $C_0 < 0$. We finally obtain a $BBA(U_{B2})$ reporting V'_{B2} where poison values are on the left hand. □

B. Proof of Theorem 2

Proof. The true mean O can be obtained by removing the effect of Y from V':

$$O = \frac{\sum\limits_{v'_i \in V'} v'_i - \sum\limits_{v'_j \in V'_B} v'_j}{1 - \gamma}. \tag{16}$$

Likewise, we can obtain O' from removing the effect of T:

$$O' = \frac{\sum\limits_{v'_i \in V'} v'_i - \sum\limits_{v'_j \in T} v'_j}{1 - \gamma_{sup}}. \tag{17}$$

Since the values in T are the largest γ_{sup} in Y, so we have

$$\sum_{v'_i \in T} v'_i \geq \sum_{v'_j \in V'_B} v'_j$$

Compare Equ. 16 and Equ. 17, we have $O' \leq O$. □

C. Proof of Theorem 5

Proof. Let $\mathcal{B} = \{B_{y_1}, ..., B_{y_t}\}$, and $\overline{\mathcal{B}} = \{B_{y_{t+1}}, ..., B_{y_{d'}}\}$. We start from the state where all buckets hold non-zero values, i.e., $B_{y_i} \neq 0, i \in \{1, ..., d'\}$, and reconstruct the frequency histogram for poison values in $[-C, C]$. The likelihood estimator in Equ. 1 becomes

$$\begin{aligned} l(F) &= \sum_{i=1}^{N} ln(\sum_{k=1}^{d} \hat{x_k} Pr[v'_i|v_i \in B_{x_k}] + \sum_{j=1}^{d'} \hat{y_j} Pr[v'_i|v_i \in B_{y_j}]) \\ &= \sum_{t=1}^{d'} c_t ln(\sum_{k=1}^{d} \hat{x_k} M_{b_t x_k} + \sum_{j=1}^{d'} \hat{y_j} M_{b_t y_j}). \end{aligned}$$

Note that $\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{d'} \hat{y_j} = 1$, we employ the Lagrangian multiplier method to derive the extremum. The Lagrangian function can be written as:

$$L(F) = l(F) + \lambda(\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{d'} \hat{y_j} - 1).$$

Let all first-order partial derivatives of L w.r.t. $\hat{x_k}$ and $\hat{y_j}$ equal zero

$$\begin{aligned} \frac{\partial L(F)}{\partial \hat{x_k}} &= \sum_{t=1}^{d'} c_t \frac{M_{b_t x_k}}{\sum_{k=1}^{d} \hat{x_k} M_{b_t x_k} + \hat{y_t}} + \lambda = 0, \ k \in \{1, ..., d\} \\ \frac{\partial L(F)}{\partial \hat{y_j}} &= \sum_{t=1}^{d'} c_t \frac{M_{b_t y_j}}{\sum_{k=1}^{d} \hat{x_k} M_{b_t x_k} + \hat{y_t}} + \lambda = 0, \ j \in \{1, ..., d'\}, \end{aligned}$$

we have:

$$\hat{x_k} = 0, \ k \in \{1, ..., d\}, \ \ \hat{y_j} = \frac{c_j}{N}, \ j \in \{1, ..., d'\}, \ \lambda = -N.$$

This result shows all collected values converge to poison values if no bucket is suppressed, and we can obtain:

$$(\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_i \neq 0, i \in \{1,...,d'\}} = \sum_{j=1}^{t} \frac{c_j}{N}.$$

When we suppress the bucket $B_{y_{d'}}$ (by setting $\hat{y_{d'}} = 0$) and carry out EMF, the collected values in $B'_{b_{d'}}$ can only converge to $B_{x_k} (k \in \{1, ..., d\})$, but not $B_{y_j} (j \in \{1, ..., d'\})$. Hence, every $\hat{x_k}$ will increase. Therefore, suppressing $B_{y_{d'}}$ leads to the increase of all $\hat{x_k}$, which in turn results in the decrease of all $\hat{y_j}$. However, since the decrease of $\hat{y_j} (j \in \{1, ..., t\})$ is a part of increment of $\hat{x}$, we can figure out $(\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_i \neq 0, i \in \{1,...,d'\}} \leq (\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_{d'} = 0}$. Suppress all buckets one by one in $\overline{\mathcal{B}}$ similarly, we have:

$$\begin{aligned} &(\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_i \neq 0, i \in \{1,...,d'\}} \leq (\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_{d'} = 0} \\ &\leq ... \leq (\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_{d'} = 0, ..., y_{t+1} = 0}. \end{aligned}$$

When the number of suppressed buckets in $\overline{\mathcal{B}}$ increases, the corresponding interference of $\overline{\mathcal{B}}$ decreases. Therefore, the collected values more accurately converge to the buckets that they should belong to, and thus achieve a better convergence result.

After suppressing all buckets in $\overline{\mathcal{B}}$, all collected values will convergence to normal values and poison values in $B_{y_j}(j \in 1,...,t)$ and we can infer that$(\sum_{k=1}^{d} \hat{x_k} + \sum_{j=1}^{t} \hat{y_j})\Big|_{y_{d'}=0,...,y_{t+1}=0} = 1$, which is the optimal case where none of the collected values will converge to buckets in $\overline{\mathcal{B}}$. $\square$