MTH 291 Quiz #2   (5,6)   Name: Shomiron Ghose

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Directions: Write your answers and show your calculations under each question.
Since you can insert material, you have as much space as you need.
Save your completed quiz on your computer and attach a copy to an e-mail message to me.
Be sure that you rename your quiz file: YourName291Q2.nb.
The .nb file extension is automatically inserted. Simply insert YourName at the beginning of the quiz file name.

1. (**1a**) Show that the differential equation is exact and then find the implicit solution of the diffferential equation.

$$(6x^2y - 4xy^2 + 5x^5)dx + (2x^3 - 4x^2y)dy = 0$$

So lets put this in the following form:

$$5x^5 + 6x^2y - 4xy^2 + (2x^3 - 4x^2y)\frac{dy}{dx} = 0$$

In the style of Paul ' s Notes, assume

$m = \psi_m = 5x^5 + 6x^2 y - 4xy^2$

$n = \psi_n = (2x^3 - 4x^2 y)$

assuming all the nice continuity stuff ~ and from what I can see,
assuming a nice space on which Clairaut' s and
Young' s theorems hold. As a result the second derivatives of ψ should be symmetric.

$m_y = 6x^2 - 8x = 6x^2 - 8x = n_x$

Since they are, thedifferentialequationisexact.Soby partial integration you get

2 x^3 y - 2 x^2 y^2 + (5 / 6) x^6 + g (y) and
2 x^3 y - 2 x^2 y^2 + f (x) by integrating m and n respectively
with the appropriate variable (for m, by x and for n, by y).
givingthat
ψ = 2 x^3 y - 2 x^2 y^2 + (5 / 6) x^6

(**1b**) Check your solution by differentiating it implicitly.

by differentiating I get $(6x^2)\, y - (4x)\, y^2 + 5x^5 + \frac{(2x^3 - 4x^2 y)\, dy}{dx} = 0$

2. **2a**) Classify the differential equation $\frac{dy}{dx} + y = x$ with respect to order and linearity.

The highest derivative is dy/dx = y' , therefore this is first order. Both the variables and their derivatives are multiplied by the constant 1 therefore the equation is linear! The equation is also non-homogeneous I believe.

2b) Solve the DE.

So (y - x) + (1) (dy/dx) = 0
M = (y - x)
N = (1)
My = 1
Nx = 0
~~ this is not an exact differential equation, however it can be solved the typical way using integrating factors.

p = 1 = the coefficient of y.
g = x
so multiplying both sides by an integrating factor μ which should satify the condition $\mu p = \mu'$, you get
$\mu\,(dy/dx) + \mu y = \mu x$. and since $\mu p = \mu'$
$\mu\,(dy/dx) + \mu' y = \mu x$
since $\mu * (dy/dx) + \mu' y = (\mu y)'$
$(\mu y)' = \mu x$
by then integrating by x in this case, as it is the dependent variable for μ.

$$\mu y + c = \int \mu x \, dx$$

$$y = \frac{\int \mu x \, dx - c}{\mu}$$

so to find μ :

$$\frac{\mu'}{\mu} = p$$

$(\mathrm{Ln}[\mu])' = p$
integrating then gives
$\mu = e^{\int p \, dx + k}$
$\mu = k e^{x}$

$$y = \frac{\int k e^{x} * x - c}{k e^{x}}$$

Integrate[k*Exp[x]*x, x]

$e^{x} k (-1 + x)$

So that gives $y = \left(k^{-1} e^{-x}\right) (e^{x} k (-1 + x) - c)$. constant ^2 is another constant. constant * constant = constant, so k can stay as k. L
ets say c is the superior one that stays.

$(-1 + x) - k e^{-x} c = (-1 + x) - c e^{-x} = -1 + x + c e^{-x}$.
it should be noted that quite a few of these steps could have been skipped since we can use the generalized $\mu = e^{\int p \, dx}$ relationship, though that would have been slightly more loosely with the coefficients.

3b) Use *Mathematica* to solve the differential equation.

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DSolve[y'[x] + y[x] == x, y[x], x]
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$\{\{y[x] \to -1 + x + e^{-x} C[1]\}\}$