NON-LINEAR CONTROL THEORY
LAB 2
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Truong Giang Vo
Subodh Mishra

1 Model 1 - Linear State Decoupling Control

1.1 System Definition

$$\begin{aligned} \ddot{x} &= -\sin(\theta)u_1 + \epsilon\cos(\theta)u_2 \\ \ddot{z} &= \cos(\theta)u_1 + \epsilon\sin(\theta)u_2 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \tag{1}$$

Where ϵ is consider very small, $\epsilon = 10^{-3}$. Output of the system:

$$\begin{aligned} &\begin{cases} y_1 &= x \\ y_2 &= z \end{cases} \\ \Rightarrow &\begin{cases} \ddot{y}_1 &= \ddot{x} = -\sin(\theta)u_1 + \epsilon\cos(\theta)u_2 & \text{Relative degree 2} \\ \ddot{y}_2 &= \ddot{z} = \cos(\theta)u_1 + \epsilon\sin(\theta)u_2 - 1 & \text{Relative degree 2} \end{cases} \end{aligned} \tag{2}$$

State vector of the system: $X = \begin{bmatrix} x & \dot{x} & z & \dot{z} & \theta & \dot{\theta} \end{bmatrix}^T$.
Since the dimension of state vector $\dim(X) = 6$ is greater than the total relative degrees (4), there is an internal dynamic in the system. The outputs must stabilize at 0, so an auxiliary control input w can be designed such that:

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} -k_{11}\dot{y}_1 - k_{12}y_1 \\ -k_{21}\dot{y}_2 - k_{22}y_2 \end{bmatrix} \tag{3}$$

Since this is a second order system, Eqn. (3) can be written as:

$$\begin{cases} \ddot{y}_1 + 2\zeta_1\omega_{n1}\dot{y}_1 + \omega_{n1}^2 y_1 &= 0 \\ \ddot{y}_2 + 2\zeta_2\omega_{n2}\dot{y}_2 + \omega_{n2}^2 y_2 &= 0 \end{cases}$$

In order to have no oscillations in the time response of y, the damping coefficient ζ must be greater or equal to 1. It is also proven that when $\zeta = 1$, the response time of the system is 1sec when $\zeta\omega_n = 5$. If one designs a controller with such response, consequently, the gains of the controller are:

$$\begin{aligned} k_{11} = k_{21} = 2\zeta\omega_n = 10 \\ k_{12} = k_{22} = \omega_n^2 = 25 \end{aligned}$$

From Eqn. (2) and Eqn. (3), we have:

$$\begin{aligned} w = \ddot{y} &= \begin{bmatrix} -\sin\theta & \epsilon\cos\theta \\ \cos\theta & \epsilon\sin\theta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ &= \beta u + \alpha \\ \Rightarrow \quad u &= \beta^{-1}(w - \alpha) \end{aligned} \tag{4}$$

1.2 Modeling

Figures 1.1 below describe the overview of the Simulink model of the system:

- The `Reference` block outputs $y_{1_{\text{ref}}} = \dot{y}_{1_{\text{ref}}} = y_{2_{\text{ref}}} = \dot{y}_{2_{\text{ref}}} = 0$
- The `Controller` block employs Eqn. (4) to find $u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$
- The `System` blocks uses Eqn. (1) to find $\ddot{y}_1$ and $\ddot{y}_2$. They are then integrated twice to find y_1 and y_2.
- The initial value of θ, x, z are set as $\theta_0 = 0$, $x_0 = 1$ and $z_0 = 2$, respectively.
- The simulation was carried out using Dormand-Prince (ode5) solver with step of 0.001s in 10s.

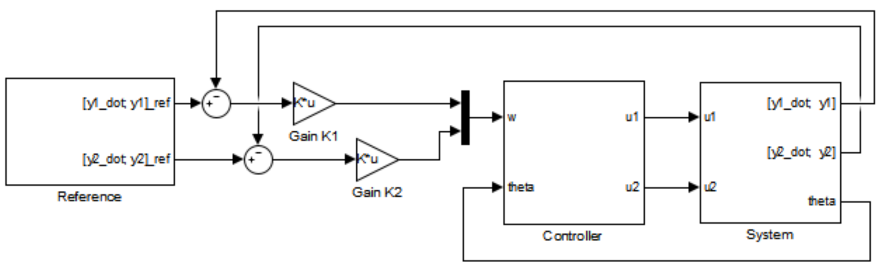

Figure 1.1: Global view of the Simulink model of the system

The simulation of the model produce the response shown in Figure 1.2 below:

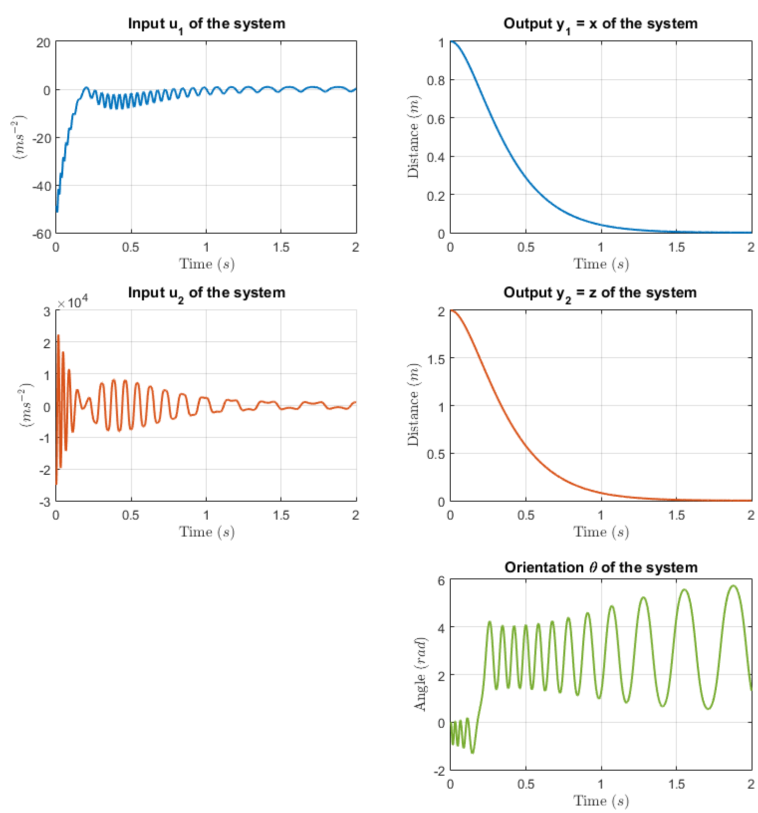

Figure 1.2: Input - Output Response and the orientation of the system

The outputs indeed converged to 0 with response time approximately at 1s, and settling time 1.5s. The internal dynamic θ exhibits oscillating response, which means it is not stable. The instability of θ can be proven. When the system has become stable, y_1 and y_2 converged to 0. Thus $\lim_{t\to\infty} w = 0$

$$\begin{aligned}
\Rightarrow \quad & u = -\beta^{-1}\alpha \\
\Rightarrow \quad & u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} -\sin\theta & \cos\theta \\ \frac{\cos\theta}{\epsilon} & \frac{\sin\theta}{\epsilon} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\
& = \begin{bmatrix} \cos\theta \\ \frac{\sin\theta}{\epsilon} \end{bmatrix}
\end{aligned}$$

From Eqn. (1) we have $\ddot{\theta} = u_2$, thus:

$$\ddot{\theta} - \frac{\sin(\theta)}{\epsilon} = 0$$

The equation above resemble that of an undamped pendulum. By plotting the phase portrait of the orientation, we can determine the stability of the internal dynamic:

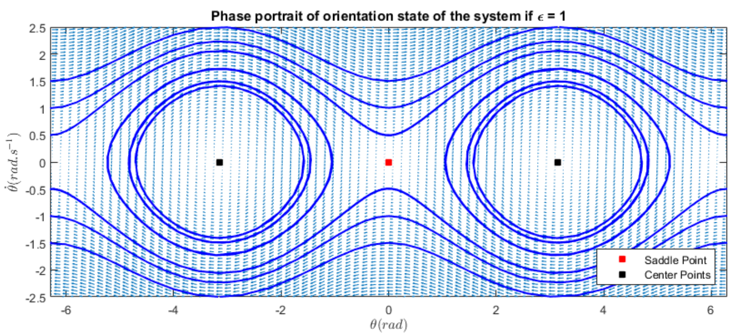

Figure 1.3: Phase portrait of the orientation ($\dot{\theta}$ vs θ) with $\epsilon = 1$

From Figure 1.3 above, we observe that:

- The initial condition at which $(\theta, \dot{\theta})$=(0,0) is a saddle point, which means it is unstable.
- The initial condition at which $(\theta, \dot{\theta})$=($\pm\pi$,0) are center points, which means it is stable but not asymptotically stable.
- For other initial conditions, the orientation is not stable, it will either rotate forever in one direction, or back and forth (bounded oscillation).

Hence it can be concluded that $\forall\ \theta(0),\ \dot{\theta}(0)$, the internal dynamic θ is unstable.

2 Model 2 - Dynamic State Decoupling Control

2.1 System Definition

Consider that $\epsilon = 0$ and there is uncertainty appears on θ, characterized by function $\delta(t)$. The equation of the system becomes:

$$\begin{aligned}
\ddot{x} &= -u_1 \sin\theta \\
\ddot{z} &= u_1 \cos\theta - 1 \\
\ddot{\theta} &= u_2 + \delta(t)
\end{aligned} \tag{5}$$

Assume $\delta(t) = 0$ for now and the output of the system are still $y_1 = x$ and $y_2 = z$, we have:

$$\begin{aligned} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} &= \begin{bmatrix} -u_1 \sin\theta \\ u_1 \cos\theta - 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -\sin\theta & 0 \\ \cos\theta & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ &= \alpha + \beta u \end{aligned}$$

In order to implement a dynamic state feedback controller, it is necessary to consider u_1 and $\dot{u}_1$ as new state variables. Hence, now $\dim(X) = 8$. New control input of the system is $\bar{u}_* = \begin{bmatrix} \ddot{u}_1 & u_2 \end{bmatrix}^T$. We have:

$$\begin{aligned} \begin{bmatrix} y_1^{(3)} \\ y_2^{(3)} \end{bmatrix} &= \begin{bmatrix} -\dot{u}_1 \sin\theta - u_1 \cos(\theta)\dot{\theta} \\ \dot{u}_1 \cos\theta - u_1 \sin(\theta)\dot{\theta} \end{bmatrix} \\ \begin{bmatrix} y_1^{(4)} \\ y_2^{(4)} \end{bmatrix} &= \begin{bmatrix} -\ddot{u}_1 \sin\theta - 2\dot{u}_1 \cos(\theta)\dot{\theta} + u_1 \sin(\theta)\dot{\theta}^2 - u_1 \cos(\theta)\ddot{\theta} \\ \ddot{u}_1 \cos\theta - 2\dot{u}_1 \sin(\theta)\dot{\theta} - u_1 \cos(\theta)\dot{\theta}^2 - u_1 \sin(\theta)\ddot{\theta} \end{bmatrix} \\ &= \begin{bmatrix} -2\dot{u}_1 \cos(\theta)\dot{\theta} + u_1 \sin(\theta)\dot{\theta}^2 \\ -2\dot{u}_1 \sin(\theta)\dot{\theta} - u_1 \cos(\theta)\dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} -\sin\theta & -u_1\cos\theta \\ \cos\theta & -u_1 \sin\theta \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ u_2 \end{bmatrix} \qquad \begin{matrix} \text{rel.deg. } 4 \\ \text{rel.deg. } 4 \end{matrix} \\ &= \alpha_* + \beta_* \bar{u}_* \end{aligned} \tag{6}$$

From Eqn. (6) above, we have the total relative degrees of the system is 8, which is equal to the dimension of the state vector, therefore, there is no internal dynamic within the system. State variable θ is now stable and observable. By introducing an auxiliary input w such that:

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -k_{11} y_1^{(3)} - k_{12}\ddot{y}_1 - k_{13}\dot{y}_1 - k_{14} y_1 \\ -k_{21} y_2^{(3)} - k_{22}\ddot{y}_2 - k_{23}\dot{y}_2 - k_{24} y_2 \end{bmatrix} \tag{7}$$

We have:

$$\begin{aligned} & w = \alpha_* + \beta_* \bar{u}_* \\ \Rightarrow \quad & \bar{u}_* = \beta_*^{-1}(w - \alpha_*) \end{aligned} \tag{8}$$

Eqn. (6) and (7) shows that when $u_1 = 0$, β_* is not invertible. Thus in order to avoid singularity, u_1 must not be 0. Also, since it is a fourth order system $y^{(4)} + k_1 y^{(3)} + k_2\ddot{y} + k_3\dot{y} + k_4 y = 0$, the characteristic equation is:

$$s^4 + k_1 s^3 + k_2 s^2 + k_3 s + k_4 = 0$$

In order for the system to be asymptotically stable, the roots of the characteristic equation must have negative real part. For simplicity, consider it has equal roots $s_{1,2,3,4} = -s_0$ ($s_0 > 0$). The equation can be rewritten as:

$$\begin{aligned} & (s + s_0)^4 = 0 \\ \Rightarrow \quad & s^4 + 4s_0 s^3 + 6s_0^2 s^2 + 4s_0^3 s + s_0^4 = 0 \\ \Leftrightarrow \quad & s^4 + k_1 s^3 + k_2 s^2 + k_3 s + k_4 = 0 \end{aligned}$$

Therefore, instead of tuning 8 individual gains, it is possible to tune the value of s_0 until a desirable response and performance is achieved.

When the system has become stable, let x_s, z_s and θ_s be the converged value of x, z and θ, respectively. From Eqn. (5) we have:

$$x_s = -u_{1s} \sin\theta_s = 0 \quad \Rightarrow \quad \sin\theta_s = 0 \ (u_1 \neq 0 \text{ to avoid singularity})$$

On the other hand: $z_s = u_{1s}\cos\theta_s - 1 = 0 \;\Rightarrow\; \cos\theta_s = 1/u_{1s}$

Hence it can be concluded that the stabilized value of u_1 and θ are:

$$(u_{1s} = 1,\ \theta_s = k2\pi) \quad \text{or} \quad (u_{1s} = -1,\ \theta_s = (2k+1)\pi)$$

2.2 Modeling

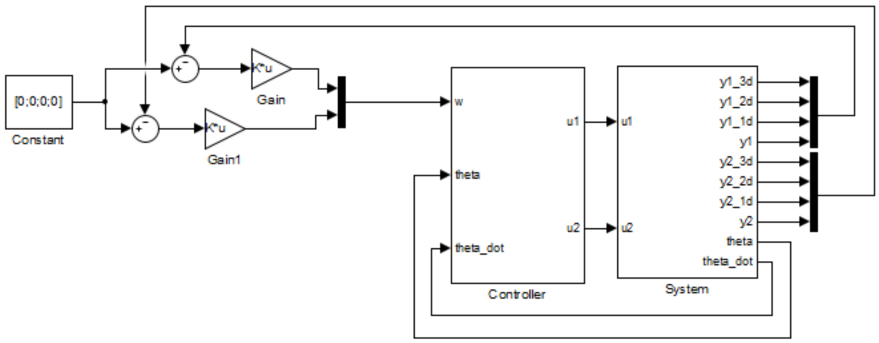

Figure 2.1: Global view of the Simulink model of the system, using dynamic state feedback

- `Controller` block uses Eqn. (8) to find $\bar{u}_*$ and then u from inputs w, θ and $\dot{\theta}$.
- `System` blocks uses Eqn. (5) to compute the outputs $y_1 = x$ and $y_2 = z$.
- `Gain` and `Gain1` blocks are row vectors contains the values of $\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \end{bmatrix}$ and $\begin{bmatrix} k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix}$
- The initial values are $x_0 = 1$, $\dot{x}_0 = 0$, $z_0 = 2$, $\dot{z}_0 = 0$, $\theta_0 = 0$ and $\dot{\theta}_0 = 0$.

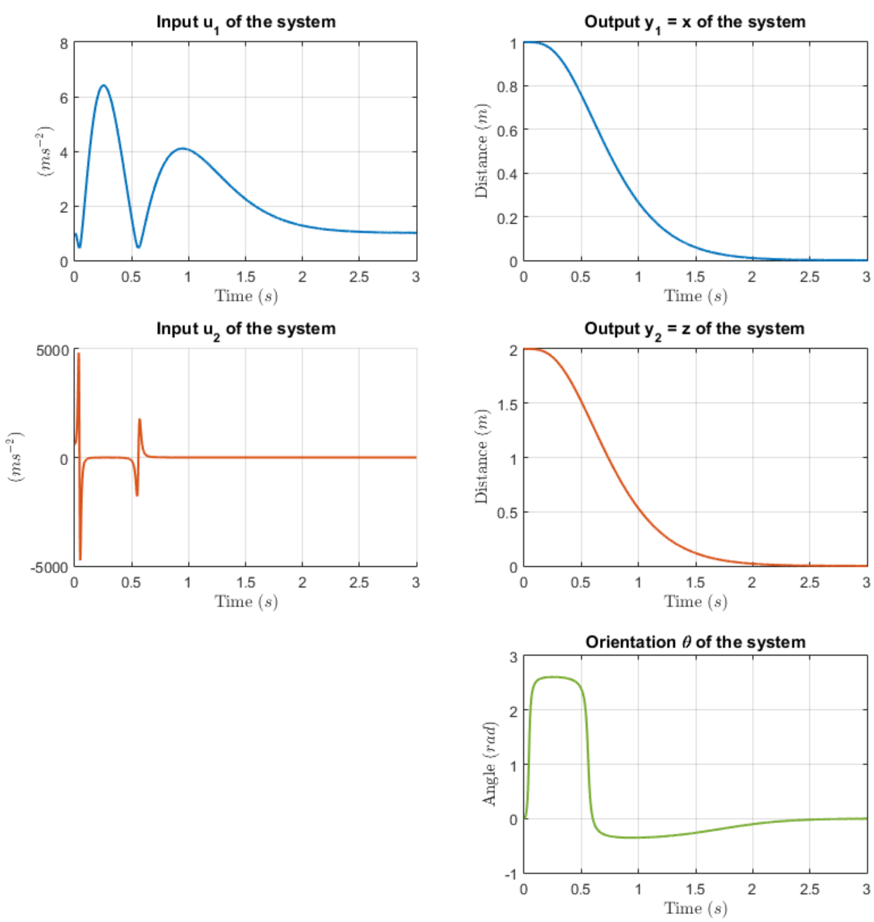

Figure 2.2: Response of the system when $\delta(t) = 0$

From Figure 2.2, it can be observed that by using dynamic state feedback control without any pertubation, the system is asymptotically stable. Variable x, z and θ stabilize around 0 while u_1 converges to $1(m.s^{-2})$. Control input u_2 also converges to 0 due to the fact that $\theta_s = 0$ and $\ddot{\theta} = u_2$.

In the cases where $\delta(t) \neq 0$, the pertubation affect negatively on the stability and performance of the system. When $\delta(t) = 200$, the system becomes unstable. In the case of $\delta(t) = 200\sin(t)$, the outputs are bounded but oscillating, which means the system is not asymptotically stable. Therefore, it can be concluded that dynamic state feedback control is very sensitive to pertubation and might not but suitable for pratical usage. In order to control the system with the presence of pertubation, it is necessary to use sliding mode control to achieve a desirable degree of robustness and better performance.

3 Model 3 - Sliding Mode Control

The first step is to choose a sliding variable which has a relative degree one with respect to the system inputs. As Eqn. (6) involves 4^{th} order derivative, the sliding variable should be:

$$\begin{aligned} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} &= \begin{bmatrix} x^{(3)} \\ z^{(3)} \end{bmatrix} + \lambda_1 \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} + \lambda_2 \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} + \lambda_3 \begin{bmatrix} x \\ z \end{bmatrix} \\ \Rightarrow \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} &= \begin{bmatrix} x^{(4)} \\ z^{(4)} \end{bmatrix} + \lambda_1 \begin{bmatrix} x^{(3)} \\ z^{(3)} \end{bmatrix} + \lambda_2 \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} + \lambda_3 \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} \end{aligned} \tag{9}$$

By using Eqn. (6), we have $\dot{s}$ when $\ddot{\theta} = u_2(\delta(t) = 0)$ is:

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} \sin(\theta)\dot{\theta}^2 u_1 - 2\cos(\theta)\dot{\theta}\dot{u}_1 \\ -\cos(\theta)\dot{\theta}^2 u_1 - 2\sin(\theta)\dot{\theta}\dot{u}_1 \end{bmatrix} + \lambda_1 \begin{bmatrix} x^{(3)} \\ z^{(3)} \end{bmatrix} + \lambda_2 \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} + \lambda_3 \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -\sin\theta & -u_1\cos\theta \\ \cos\theta & -u_1\sin\theta \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ u_2 \end{bmatrix}$$

Define α_{sm} and β_{sm} such that:

$$\begin{aligned} \alpha_{\text{sm}} &= \begin{bmatrix} \sin(\theta)\dot{\theta}^2 u_1 - 2\cos(\theta)\dot{\theta}\dot{u}_1 \\ -\cos(\theta)\dot{\theta}^2 u_1 - 2\sin(\theta)\dot{\theta}\dot{u}_1 \end{bmatrix} + \lambda_1 \begin{bmatrix} x^{(3)} \\ z^{(3)} \end{bmatrix} + \lambda_2 \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} + \lambda_3 \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} \\ \beta_{\text{sm}} &= \begin{bmatrix} -\sin\theta & -u_1\cos\theta \\ \cos\theta & -u_1\sin\theta \end{bmatrix} \end{aligned} \tag{10}$$

And taking into account the pertubation $\delta(t) \neq 0$ so that $\ddot{\theta} = u_2 + \delta(t)$, we have:

$$\dot{s} = \alpha_{\text{sm}} + \beta_{\text{sm}}\bar{u}_* + \Delta(t) \tag{11}$$

It can be noted that the sliding variable has a relative degree 1 with respect to the inputs $[\ddot{u}_1\ u_2]^T$.
The control objective is to ensure: $s = 0$
If one uses Lyapunov candidate function $V(s) = \frac{1}{2}s^2$, the attractivity condition is:

$$\dot{V} = s\dot{s} < 0$$

Imposing

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} -K_1\text{sign}(s_1) \\ -K_2\text{sign}(s_2) \end{bmatrix} + \Delta(t) \qquad K_1 > 0,\ K_2 > 0 \tag{12}$$

For sliding to occur along $s = 0$, the controller must not only linarize the system defined by Eqn. (11) but also nullify the effects of the disturbance $\Delta(t)$. In other words, K_1 and K_2 must be sufficiently larger than the maximum amplitude of the disturbance $|\Delta_M|$. This is considered a demerit to this approach because it

is necessary to have priori knowledge about the disturbance.
From Eqn. (14) and (12) we have:

$$\bar{u}_* = \beta_{\text{sm}}^{-1} \left(-\alpha_{sm} + \begin{bmatrix} -K_1 \text{sign}(s_1) \\ -K_2 \text{sign}(s_2) \end{bmatrix} \right) \qquad K_1, K_2 \gg |\Delta_M| > 0 \tag{13}$$

Also, when $s = 0$, Eqn. (9) become a third order differential equation which has the characteristic equation: $r^3 + \lambda_1 r^2 + \lambda_2 r + \lambda_3 = 0$. In order for the system to be asymptotically stable, the coefficients must be chosen such that the roots of characteristic equation have negative real part. An easy and practical method is to choose all the poles to be real, equal and negative $r_{1,2,4} = -r_0$ ($r_0 \in \mathbb{Z}^+$). By choosing $r_0 = 5$ we have:

$$\begin{aligned} (r + r_0)^3 &= r^3 + 3r_0 r^2 + 3r_0^2 r + r_0^3 \\ &= r^3 + \lambda_1 r^2 + \lambda_2 r + \lambda_3 \\ &= r^3 + 15r^2 + 75r + 125 \end{aligned}$$

3.1 Modeling

- `Controller` block takes feedback output from `System` block in order to calculate α_{sm}, β_{sm}^{-1} and finally the value of u. Similar to the dynamic state feedback control, the input $\ddot{u}_1$ will be integrated twice to get u_1, with $u_1(0) = 1$.
- `System` block does not change compared to the previous section. We use the initial values $x_0 = 1$, $z_0 = 2$ and $\theta_0 = 0$.

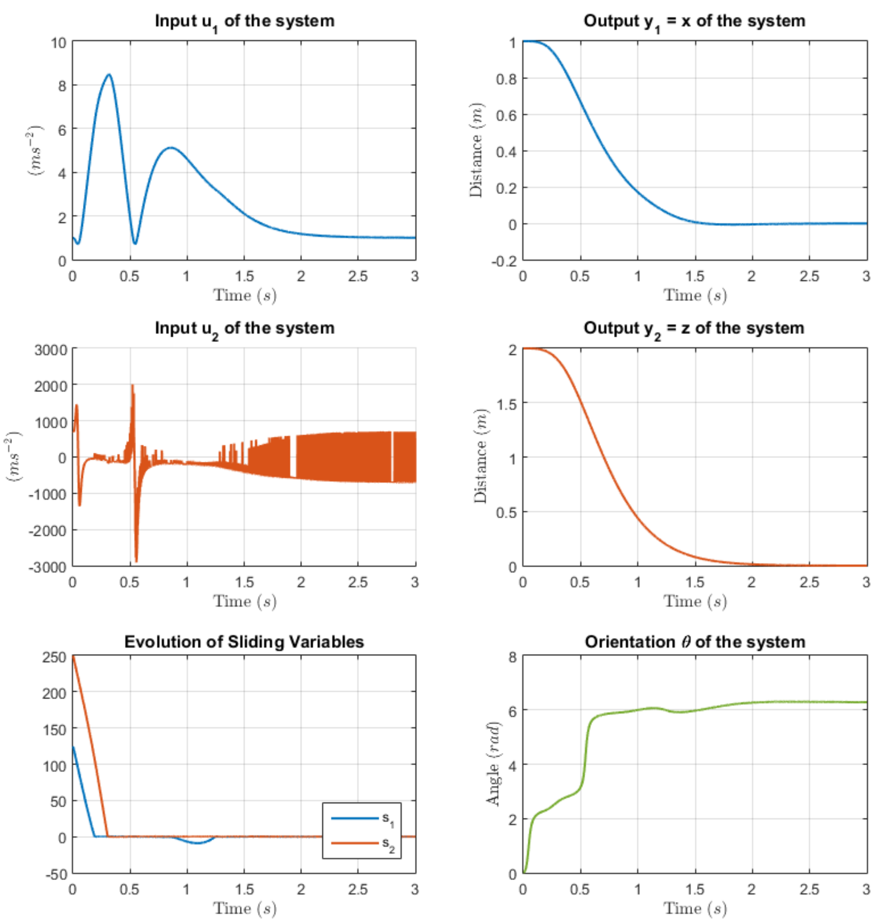

Figure 3.1: Response of the system by using Sliding Mode Control with $K_1 = 700$, $K_2 = 700$ and pertubation $\delta(t) = 200 \sin t$

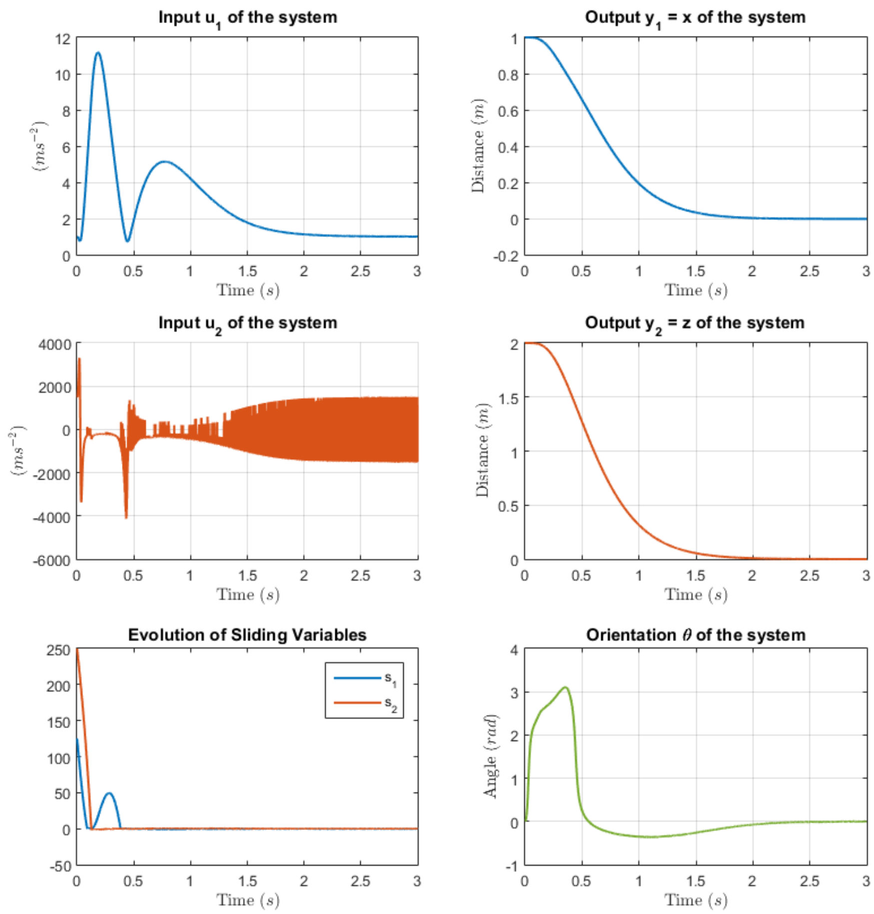

Figure 3.2: Response of the system by using Sliding Mode Control with $K_1 = 1500$, $K_2 = 1500$ and pertubation $\delta(t) = 200$

As shown in Figures 3.1 and 3.2 , by using sliding mode control, the outputs of the system stabilized around equlibrium point (0,0) after approximately 2.5s for both the sinusoid and the constant disturbance. The orientation θ of the system also converged to 0. Both sliding variables s_1 and s_2 surfaced after apporximately 0.45s. On the other hand, since the sign() functions were used, chattering effect with high magnitude appeared in control input u_2 of the system. It must also be noted that the gains K_1 and K_2 must be very large for the system to be stable and the value of these gains needs to be higher for a constant disturbance of the same amplitude.

4 Model 4 - Adaptive Sliding Mode Control

In this section, the gains will be adapted dynamically online w.r.t. the establishment (or not) of the sliding motion. The algorithm reads as:

$$\begin{aligned} w_1 &= -K_1(t)\text{sign}(s_1) \\ w_2 &= -K_2(t)\text{sign}(s_2) \end{aligned} \tag{14}$$

Where $K_i(t)$ $(i = 1, 2)$ are functions of time and:

$$\dot{K}_i = \begin{cases} \bar{K}_i|s_i|\text{sign}(|s_i| - \mu_i) & K_i > \eta_i \\ \eta_i & K_i \leq \eta_i \end{cases} \tag{15}$$

Where $K_i(0) > 0$, $\bar{K}_i > 0$, $\eta_i > 0$ and $\mu_i > 0$. μ_i are small compared to other variables.

4.1 Algorithm

Model 3 discussed in previous section uses constant gains with very high magnitudes. Furthermore, for such model to work, the K chosen must be sufficiently high to counter the effect of disturbances or perturbations. Such high gains may not be realizable in practice owing to actuator constraint and other non linearities in the system. Besides this, there is a major caveat to this approach, the amplitude of the perturbation must be known to decide the minimum value of the gain K.
The adaptive sliding mode control is an attempt to overcome these demerits.
According to Eqn. (15):

- When s_i is outside of the boundary ($|s_i| > \mu_i$), $\text{sign}(|s_i| - \mu_i)$ is positive, $\dot{K}_i > 0$ so K_i will increase and this shall force a very fast approach to the $s = 0$ surface.
- When s_i is within the boundary ($|s_i| \leq \mu_i$), $\dot{K}_i < 0$, hence K_i will decrease.
- But the gain cannot decrease indefinitely even if $|s_i| \leq \mu_i$, if the sliding variable goes below η, then $\dot{K}_i = \eta > 0$, so the gain increases again.

The basic idea of this approach is to gradually reduce the gain once the sliding surface is approached, but maintain a suffciently large value that is enough to counter the effects of disturbances and uncertainities. It also reduces the chattering effect on the system. The parameter η_i is introduced as a lower bound for K_i, which means even when K is declining, it would always remain positive, greater or equal to predefined η_i.
note: Unless otherwise specified, the simulations are done in `Ode5 Dormand-Prince` fixed step solver with a fixed step (Δt) of 0.001 s.

4.2 Tuning μ

The parameter μ needs to be tuned properly to ensure that the results are accurate, robust and stable.

- If the parameter μ is too small then the μ band will be very narrow , if, at the same time, the gain and the sampling period is relatively high, then the trajectory of s will be such that it will never stay within the μ band, consequently, $\dot{K}_i$ will always be positive, resulting in increase in K_i to infinitely large values. Such high gains will induce larger oscillations and result in divergence. In this case, the solution could be, reduction of the sampling time or the gain (by reducing $\bar{K}_i$ and η_i) or both. Another solution could be, increasing μ_i.
- If the parameter μ is too large, it implies that the μ band is too large and this means that the gain will start reducing even is the sliding variable is far away from the sliding surface. This is detrimental because the controller accuracy will not be as good as required.

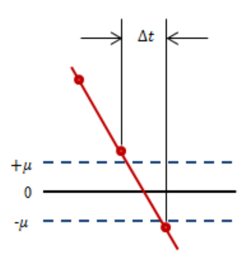

Figure 4.1: Illustration of the effect of high gains and sampling time with respect to low μ

a) Tuning for sinusoid disturbance: $\delta(t) = 200 \sin t$

The parameters values are: $\eta_i = 250$, $\bar{K}_i = 100$ and $K_i(0) = 100$

- **Tuning with $\mu_i = 0.3$:** The effect of very low μ can be seen in 4.2. The simulation is run for a longer duration to illustrate the increase in gains and the control inputs, though the x,y and θ are stabilized.

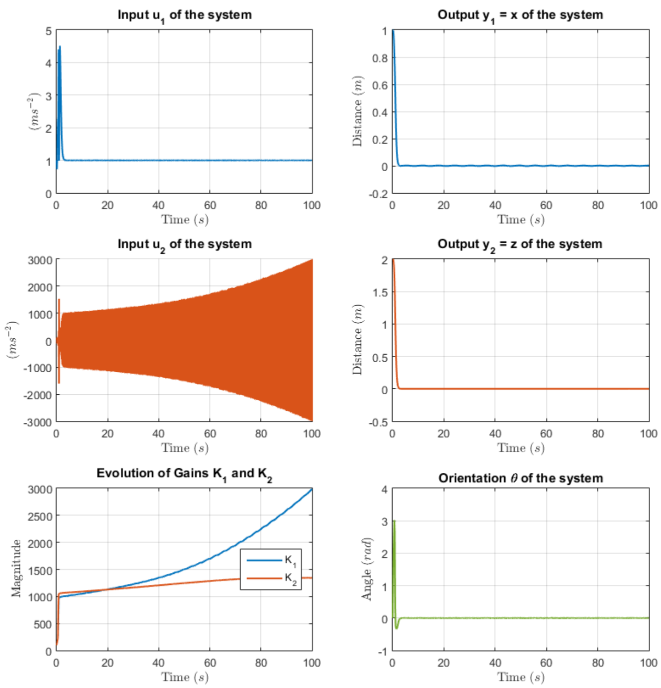

Figure 4.2: Response of the system with $\Delta t = 0.001s$, $\mu = 0.3$, $\eta = 250$, $\bar{K} = 100$, $K(0) = 100$ and pertubation $\delta(t) = 200\sin(t)$

- **Tuning with $\mu_i = 2$:** A higher μ, not only stabilizes x,y and θ, but also ensures that gains and the control inputs decrease over time or remain constant.

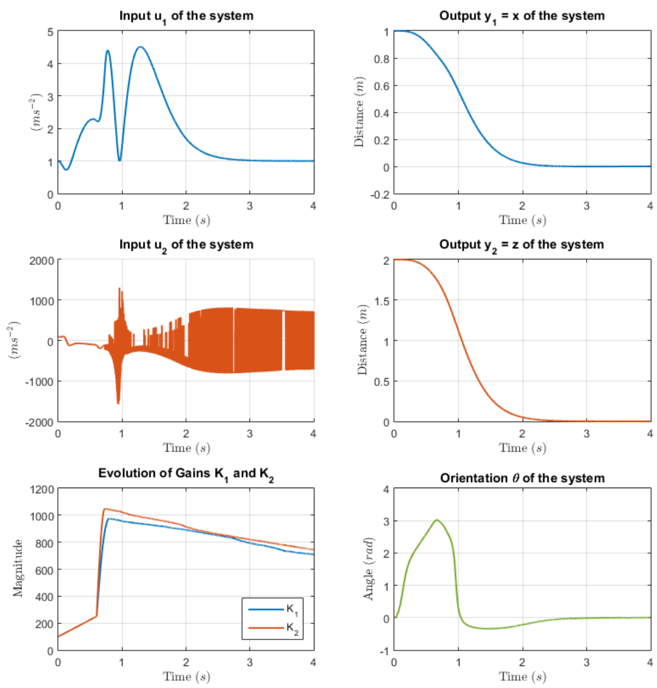

Figure 4.3: Response of the system with $\Delta t = 0.001s$, $\mu = 2$, $\eta = 250$, $\bar{K} = 100$, $K(0) = 100$ and pertubation $\delta(t) = 200\sin(t)$

- **Tuning with $\mu_i = 0.3$ and reduced gains and sampling time**: Figure 4.4 shows that a low μ is admissible for low values of gains and sampling time. This implies that the minimum value of μ which ensures system stability depends on the values of gains and sampling time. The simulation is run for a longer duration to illustrate the gradual decrease in system gains.

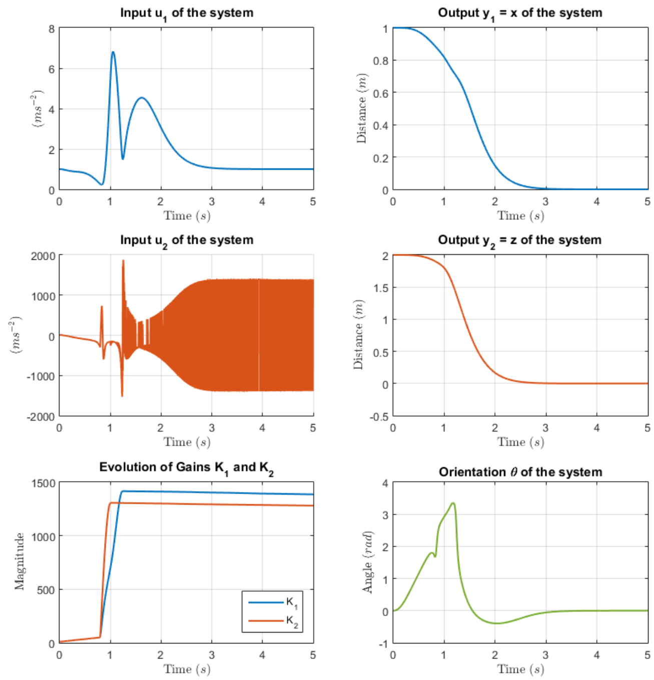

Figure 4.4: Response of the system with $\Delta t = 0.0001s$, $\mu = 0.3$, $\eta = 50$, $\bar{K} = 50$, $K(0) = 10$, and pertubation $\delta(t) = 200\sin t$

b) Tuning for constant disturbance: $\delta(t) = 200$

The parameters values are: $\eta_i = 250$, $\bar{K}_i = 100$ and $K_i(0) = 100$

- **Tuning with $\mu_i = 0.3$:** The effect of very low μ can be seen in 4.5. The simulation is run for a longer duration to illustrate the increase in gains and the control inputs, though the x,y and θ are stabilized.

- **Tuning with $\mu_i = 5$:** A higher μ, not only stabilizes x,y and θ, but also ensures that gains and the control inputs decrease over time or remain constant.

- **Tuning with $\mu_i = 0.3$ and reduced gains and sampling time**: Figure 4.7 shows that a low μ is admissible for low values of gains and sampling time. This implies that the minimum value of μ which ensures system stability depends on the values of gains and sampling time. The simulation is run for a longer duration to illustrate the gradual decrease in system gains.

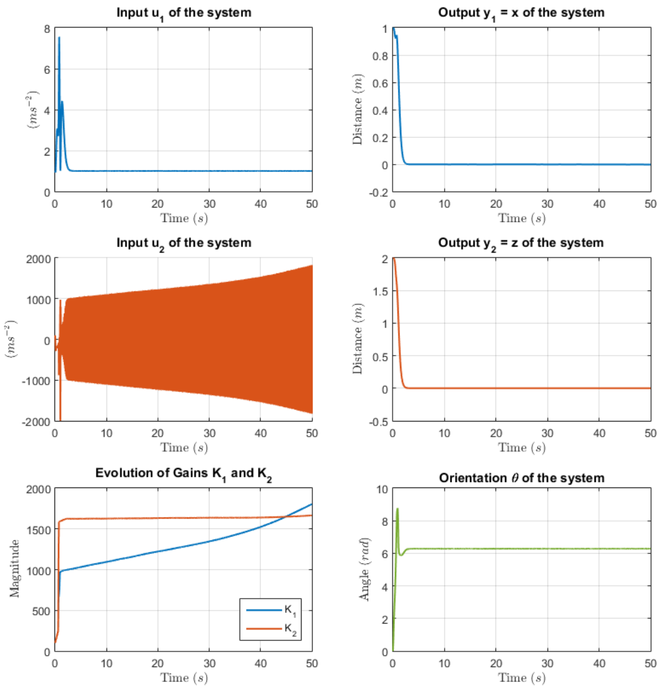

Figure 4.5: Response of the system with $\Delta t = 0.001s$, $\mu = 0.3$, $\eta = 250$, $\bar{K} = 100$, $K(0) = 100$ and pertubation $\delta(t) = 200$

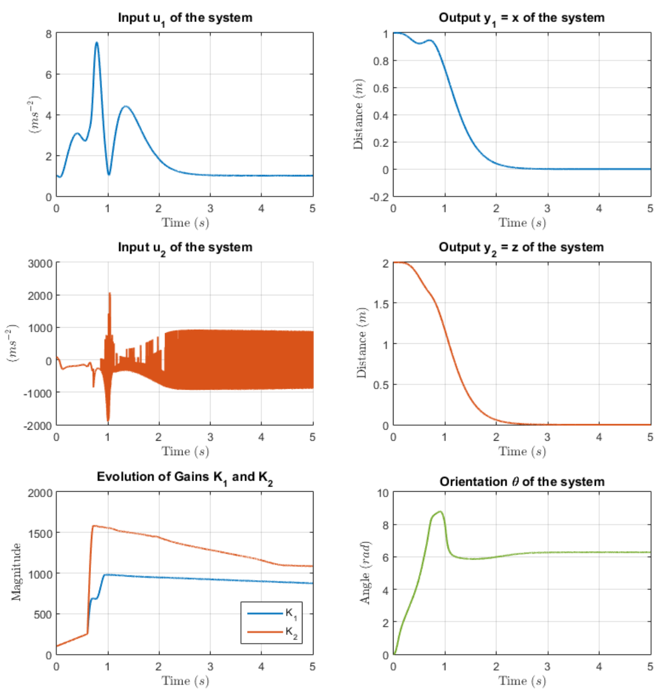

Figure 4.6: Response of the system with $\Delta t = 0.001s$, $\mu = 5$, $\eta = 250$, $\bar{K} = 100$, $K(0) = 100$ and pertubation $\delta(t) = 200$

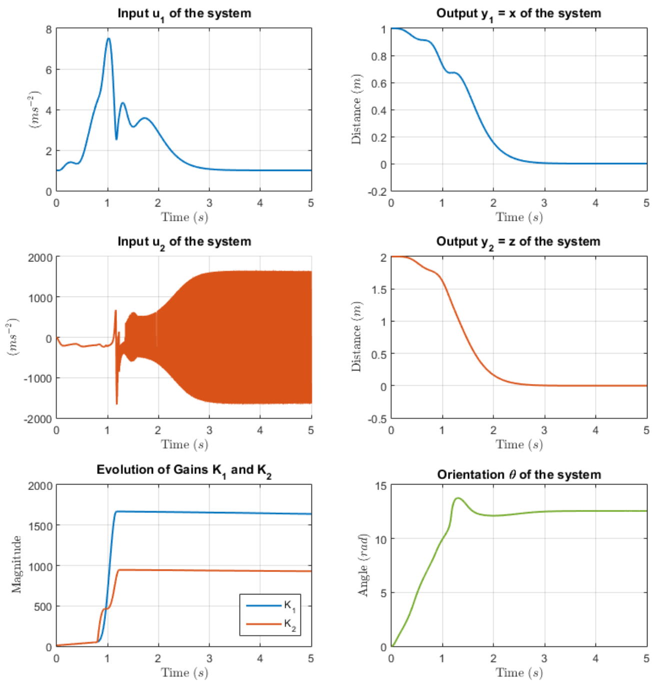

Figure 4.7: Response of the system with $\Delta t = 0.0001s$, $\mu = 0.3$, $\eta = 50$, $\bar{K} = 50$, $K(0) = 10$, and pertubation $\delta(t) = 200$

5 Conclusion

From four types of controller discussed above, it can be concluded that:

- **Static State Feedback Control** can only control the specified outputs, there is unstable internal dynamic in the system.
- **Dynamic state Feedback Control** makes sure there is no internal dynamic, all state variables are observable. However, this control law is sensitive to pertubation, which negatively affect the stability of the system.
- **Sliding Mode Control** provides a certain degree of robustness and insensitivity to pertubation. However, priori knowledge of the pertubation is required. Also, sliding mode control only works in ideal system where sliding variables s can reach 0. It is usually not the case in practice.
- **Adpative Sliding Mode Control** allows adapting the gains dynamically. Thus there is no need to know about the nature of pertubation before hand. It also guarantees that the controlling system will work even with discrete-time measurement.

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