# Master EMARO - ARIA

## NOLCO Exam

### Duration 1h30 - Open book

**2 pages**

**QUESTIONS (no computation is required)**

**1.** Explain the "idea" of the algorithm allowing to check the accessibility of a nonlinear system - in particular, detail what is searched at each step.
**2.** Explain what is the main feature of an observable system.
**3.** What is the interest of second order sliding mode with respect to a first order sliding mode ?

**EXERCICE.**

Consider the following nonlinear system ($[x_1\ x_2\ x_3]^T$ being the state vector and $u$ the control input)

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \cdot x_3 \\ \dot{x}_2 & = & x_3 + \cos(x_2) \cdot u + \delta \\ \dot{x}_3 & = & -x_1 \cdot x_3 \end{array} \quad (1)$$

with the output (for control) $y = x_1$. The control objective is to force $y$ to 0 in spite of the presence of the perturbation $\delta$. This perturbation is such that $|\delta| \leq \delta_M$.

<u>**One supposes firstly that $\delta = 0$.**</u>

**Structural analysis.**

1. Analyze the accessibility of the system.
2. Analyze the observability of the system if the variable $x_3$ is measured. Precise eventually the observability singularities.

**Control design.**

3. Evaluate the relative degree of the system (1) versus the output $y$.
4. Does it exist internal dynamics (not controlled) ? Justify your response (do not analyze its stability).
5. By a general point-of-view, what is the problem that could be induced by the internal dynamics ?
6. Calculate a control input $u$ which allows to linearize, by an input-output point-of-view, the system. Specify, if it is the case, control singularities.
7. Propose a controller such that the input-output behavior of the system is equivalent to a system with a damping coefficient $\zeta$ and a proper pulsation $\omega$. Write the controller as a function of $x$.

**One supposes now that $\delta \neq 0$.**

8. Design the sliding variable $\sigma$ allowing to control $x_1$. Write $\sigma$ as a function of $x$.
9. Suppose that the control input reads as $u = -K \cdot \text{sign}(\sigma)$. Give the condition ensuring the establishment of a sliding mode with respect to $\sigma$. Comment.
10. Propose now a control solution which allows to linearize the $\sigma$-dynamics when there is *NO* uncertainty.
11. Consider now the system with uncertainties. The previous controller is then completed by a sliding mode one in order to counteract the effects of uncertainties. Give the conditions on each controller gain ensuring the convergence of the sliding variables to 0.
12. Comment the values of the gains obtained in questions 9 and 11.
13. Which controller could be used in order to ensure the finite time convergence of $x_1$ to zero ? Justify the response.