ANCOS LAB
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1 Introduction

The report is divided into four parts. First one deals with an input-output linearization. Second one, deals with stabilization of all parts of the system. Third part, talks about sliding mode control. And finally the last part deals with adaptive sliding mode control proposed by (Plestan et al. 2010).

2 System definition

System definition is :

$$\begin{gathered}\ddot{x}=-\sin(\Theta)u_1+\epsilon\cos(\Theta)u_2\\ \ddot{z}=\cos(\Theta)u_1+\epsilon\cos(\Theta)u_2-1\\ \ddot{\Theta}=u_2\end{gathered} \tag{1}$$

Ne assume that the parameter is small, $\varepsilon=10^{-3}$

First, the output of the system are declared as :

$$\begin{gathered}y1=x\\ y2=z\end{gathered} \tag{2}$$

Then, the system is rewritten as:

$$\begin{gathered}x_1=x\\ x_2=\dot{x}\\ x_3=z\\ x_4=\dot{z}\\ x_5=\Theta\\ x_6=\dot{\Theta}\end{gathered} \tag{3}$$

Next, the system becomes:

$$\begin{aligned}&\dot{x}_1=x_2\\ &\dot{x}_2=-\sin(x_5)u_1+\epsilon\cos(x_5)u_2\\ &\dot{x}_3=x_4\\ &\dot{x}_4=\cos(x_5)u_1+\epsilon\sin(x_5)u_2-1\\ &\dot{x}_5=x_6\\ &\dot{x}_6=u_2\\ &y=(x_1 x_3)^T\end{aligned} \tag{4}$$

3 Open Loop simulation

The system is

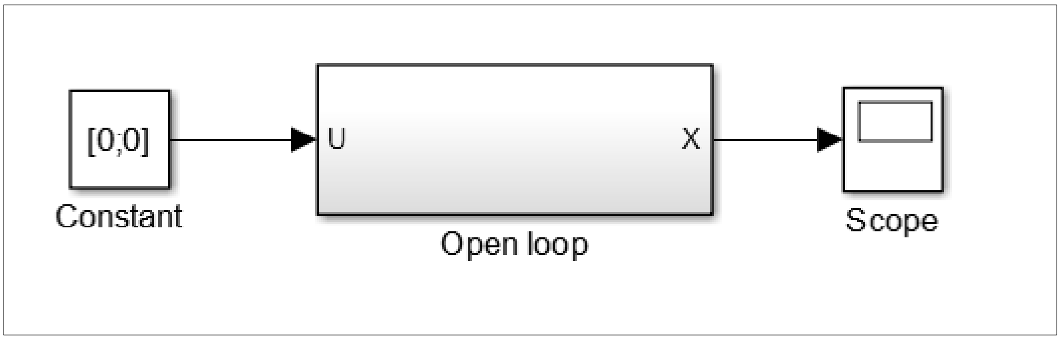


Illustration 1: Open Loop system with Simulink.

simulated with Simulink, our simulation has shown us that the system is instable.

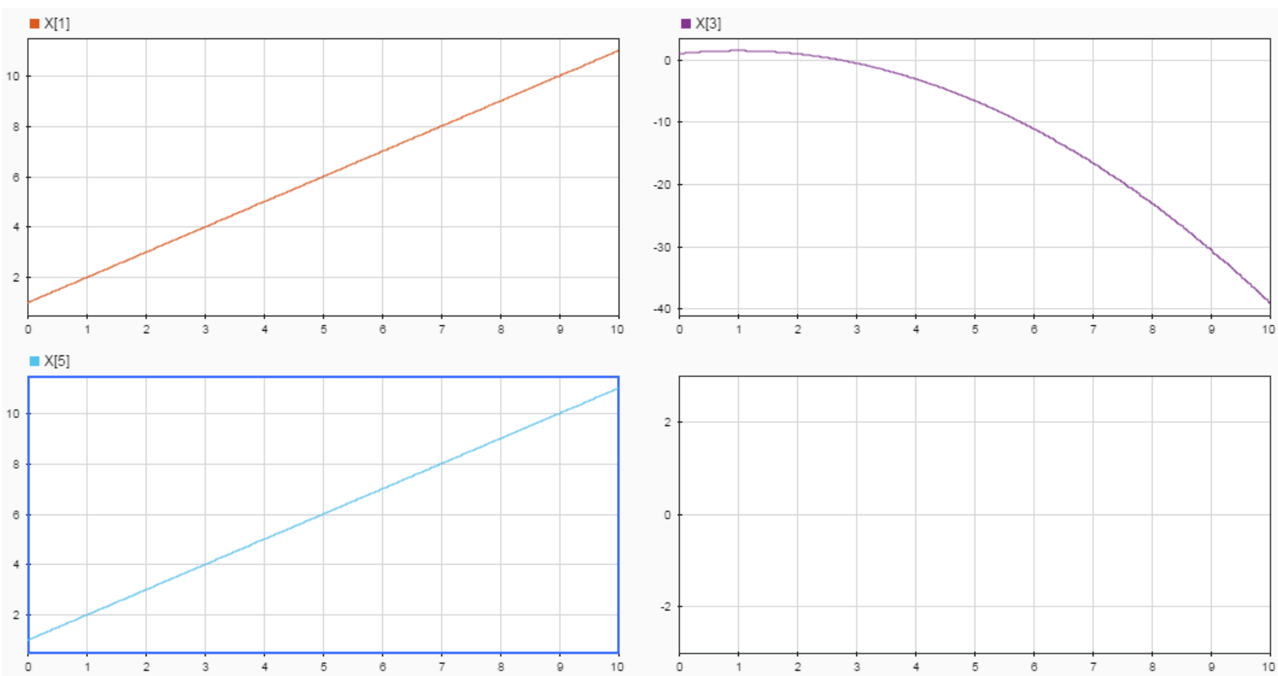


Illustration 2: System states.

We can see from Illustration 2 that the system is instable.

4 Part.1 : decoupling and linearization of the system

4.1.1 Decoupling and linearization

Decoupling and linearization equations are :

$$Y=\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\begin{pmatrix} x \\ z \end{pmatrix} \tag{5}$$

And

$$\dot{Y}=\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix}=\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} \tag{6}$$

Thus,

$$\ddot{Y}=\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix}=\begin{pmatrix} -\sin(\Theta)u_1+\epsilon\cos(\Theta)u_2 \\ \cos(\Theta)u_1+\epsilon\cos(\Theta)u_2-1 \end{pmatrix} \tag{7}$$

Thus,

$$\ddot{Y}=\begin{pmatrix} -\sin(\Theta) & \epsilon\cos(\Theta) \\ \cos(\Theta) & \epsilon\sin(\Theta) \end{pmatrix}u+\begin{pmatrix} 0 \\ -1 \end{pmatrix}=B_0u+\begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{8}$$

B0 is equal to :

$$B_0=\begin{pmatrix} -\sin(\Theta) & \epsilon\cos(\Theta) \\ \cos(\Theta) & \epsilon\sin(\Theta) \end{pmatrix} \tag{9}$$

This system can be decoupled because B0 is invertible. There is an internal dynamic because the relative degree[1] is lower than the degree of the system.

Decoupling and linearization can be done if we take:

$$u=\underbrace{B_0^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\alpha(x,z)}+\underbrace{B_0^{-1}}_{\beta(x,z)}\omega \tag{10}$$

Then, it can be written as:

1 The degree relative is equal to 2 for y1 and 2 for y2

$$\ddot{Y}=\omega \rightarrow \begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \tag{11}$$

Remark, we have two systems decoupled and linear.

4.1.2 Static state feedback

The first system can be written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_1 \end{pmatrix} \tag{12}$$

The static state feedback is :

$$\omega_1 = -k_1 x_1 - k_2 x_2 = -k_1 y_1 - k_2 y_2 \tag{13}$$

Pole placement design is:
We calculate the characteristic polynomial :

$$s^2 + k_2 s + k_1 = s^2 + 2\xi\omega_0 s + \omega_0^2 \tag{14}$$

It yields

$$\begin{aligned} k_1 &= 2\xi\omega_0 \\ k_2 &= \omega_0^2 \end{aligned} \tag{15}$$

Then

$$\xi = 1 \tag{16}$$

And

$$\omega_0 = 2 \tag{17}$$

We get :

$$k_1 = k_2 = 2 \tag{18}$$

4.1.3 Simulation

The system is coded in Simulink :

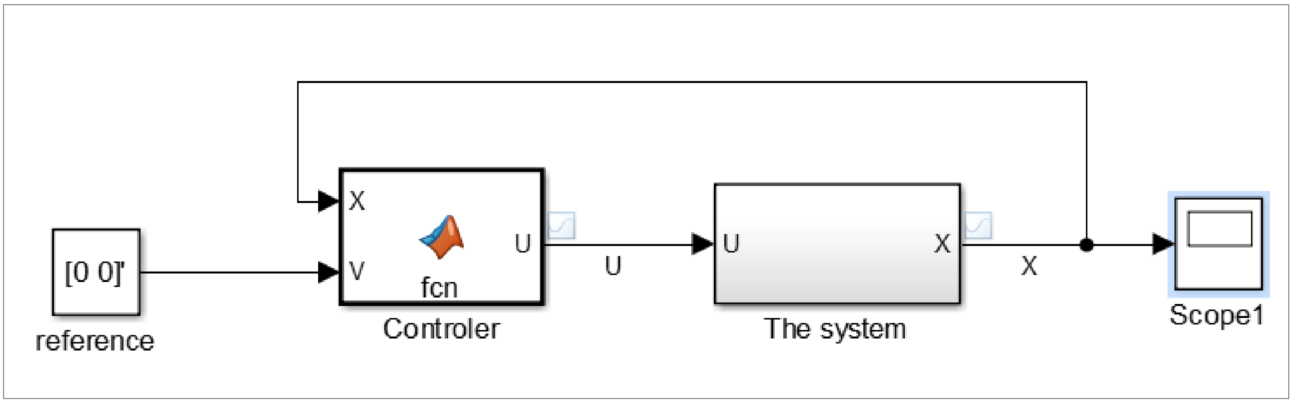


Illustration 3: Simulation of the closed loop system

Simulation results are :

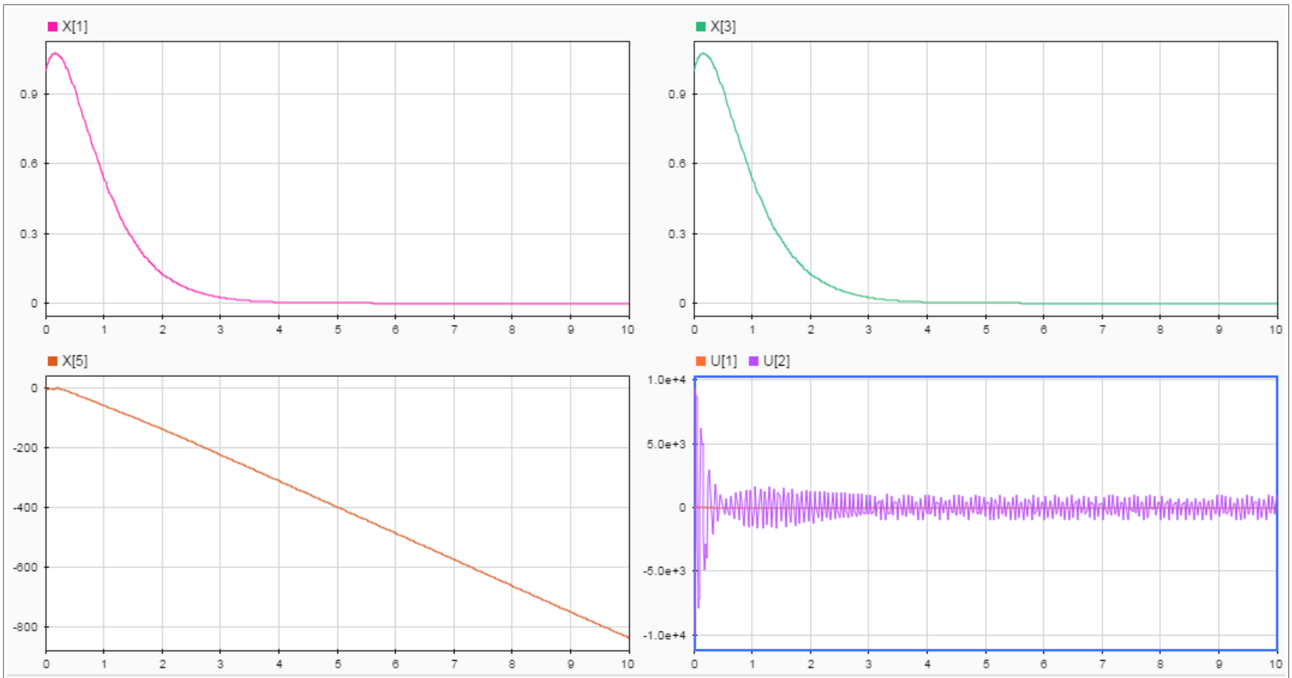


Illustration 4: States simulation

We note that the feedback control has ensured the stability of x1 and x3. However the internal dynamic x5 (Θ) is instable.

5 Part.2 : stabilization of all states x, z and Θ

In order to simulate the system, states variables of the system are :

$$X_a=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\\x_8\end{pmatrix}=\begin{pmatrix}x\\\dot{x}\\z\\\dot{z}\\\Theta\\\dot{\Theta}\\u_1\\\dot{u}_1\end{pmatrix} \tag{19}$$

And

$$\dot{X}_a=\begin{pmatrix}\dot{x}_1=x_2\\\dot{x}_2=-\sin(x_5)u_1\\\dot{x}_3=x_4\\\dot{x}_4=\cos(x_5)u_1\\\dot{x}_5=x_6\\\dot{x}_6=u_2\\\dot{x}_7=x_8\\\dot{x}_8=\ddot{u}_1\end{pmatrix} \tag{20}$$

After four derivation of the outputs, the system is equal to:

$$Y=\begin{pmatrix}y_1\\y_2\end{pmatrix} \tag{21}$$

$$\dot{Y}=\begin{pmatrix}\dot{y}_1\\\dot{y}_2\end{pmatrix} \tag{22}$$

$$\ddot{Y}=\begin{pmatrix}\ddot{y}_1\\\ddot{y}_2\end{pmatrix}=\begin{pmatrix}-x_7\sin(x_5)\\x_7\cos(x_5)-1\end{pmatrix} \tag{23}$$

$$\dddot{Y}=\begin{pmatrix}\dddot{y}_1\\\dddot{y}_2\end{pmatrix}=\begin{pmatrix}-x_8\sin(x_5)-x_7\cos(x_5)x_6\\x_8\cos(x_5)-x_7\sin(x_5)x_6\end{pmatrix} \tag{24}$$

$$\begin{pmatrix} y_1^{(4)} \\ y_2^{(4)} \end{pmatrix} = \alpha + \beta \tilde{u} \tag{25}$$

Such that:

$$\alpha = \begin{pmatrix} -2\dot{u}_1 \cos\theta\dot{\theta} + u_2 \sin(\theta)\dot{\theta}^2 \\ -2\dot{u}_1 \sin\theta\dot{\theta} - u_2 \cos(\theta)\dot{\theta}^2 \end{pmatrix} \tag{26}$$

And

$$\beta = \begin{pmatrix} -\sin(\theta) & -u_1 \cos(\theta) \\ \cos(\theta) & -u_1 \sin(\theta) \end{pmatrix} \tag{27}$$

Remark:

- the system can be decoupled as long as u1 is different from 0;
- the system has no internal dynamic because its relative degree = 8. Which is the same of the system ;

In order to have a decoupled system, the input is equal to:

$$\tilde{u} = \beta^{-1}\omega - \beta^{-1}\alpha \tag{28}$$

The system will be :

$$\begin{pmatrix} y_1^{(4)} \\ y_2^{(4)} \end{pmatrix} = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \tag{29}$$

Remark : we have two linear decoupled systems.

The state space representation of each sub-systems is:
We denote that:

$$y_1 = q_1 ; \dot{y}_1 = q_2 ; \ddot{y}_1 = q_3 ; \dddot{y}_1 = q_4 \tag{30}$$

The system can be written as :

$$\dot{Q}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} Q_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \omega_1 \tag{31}$$

With

$$Q_1=[q_1 \quad q_2 \quad q_3 \quad q_4]^T \tag{32}$$

We take $\omega_1 = - k * Q_1$ with

$$K=[k_1 \quad k_2 \quad k_3 \quad k_4]=acker(A, B, P) \tag{33}$$

We have found K equals to:

$$K=[10000 \quad 4000 \quad 600 \quad 40] \tag{34}$$

With poles equals to:

$$P=[-10 \quad -10 \quad -10 \quad -10] \tag{35}$$

We have chosen $\omega_2 = -k * Q_2$ such as :

$$Q_2=[z \quad \dot{z} \quad \ddot{z} \quad \dddot{z}]^T \tag{36}$$

Simulink blocs are written:

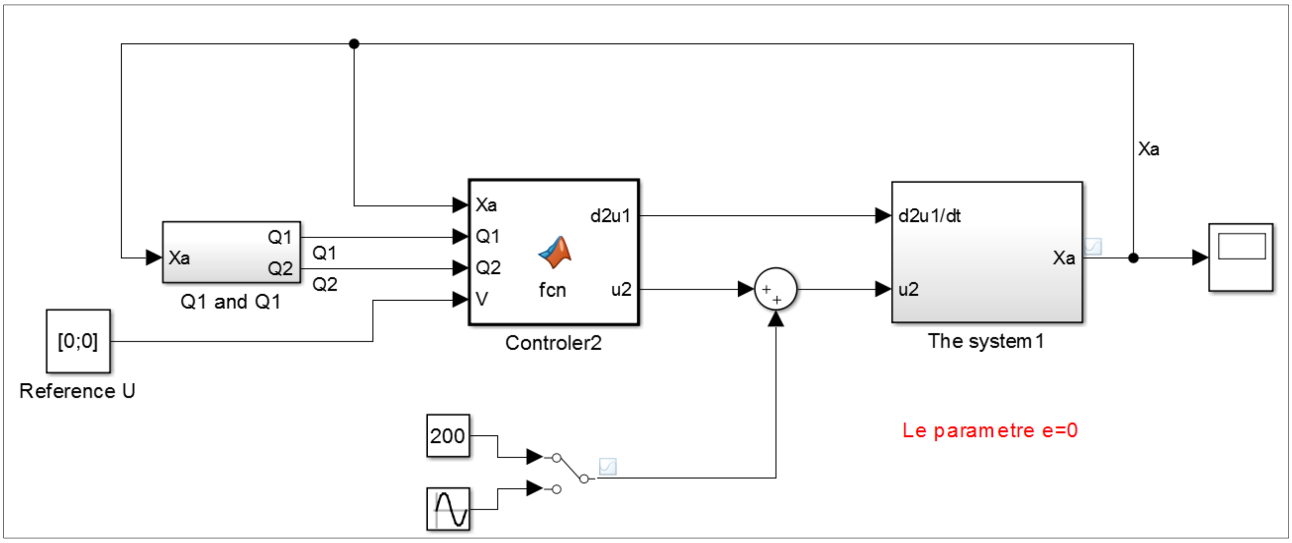


Illustration 5: Simulink blocs.

Simulation results are:

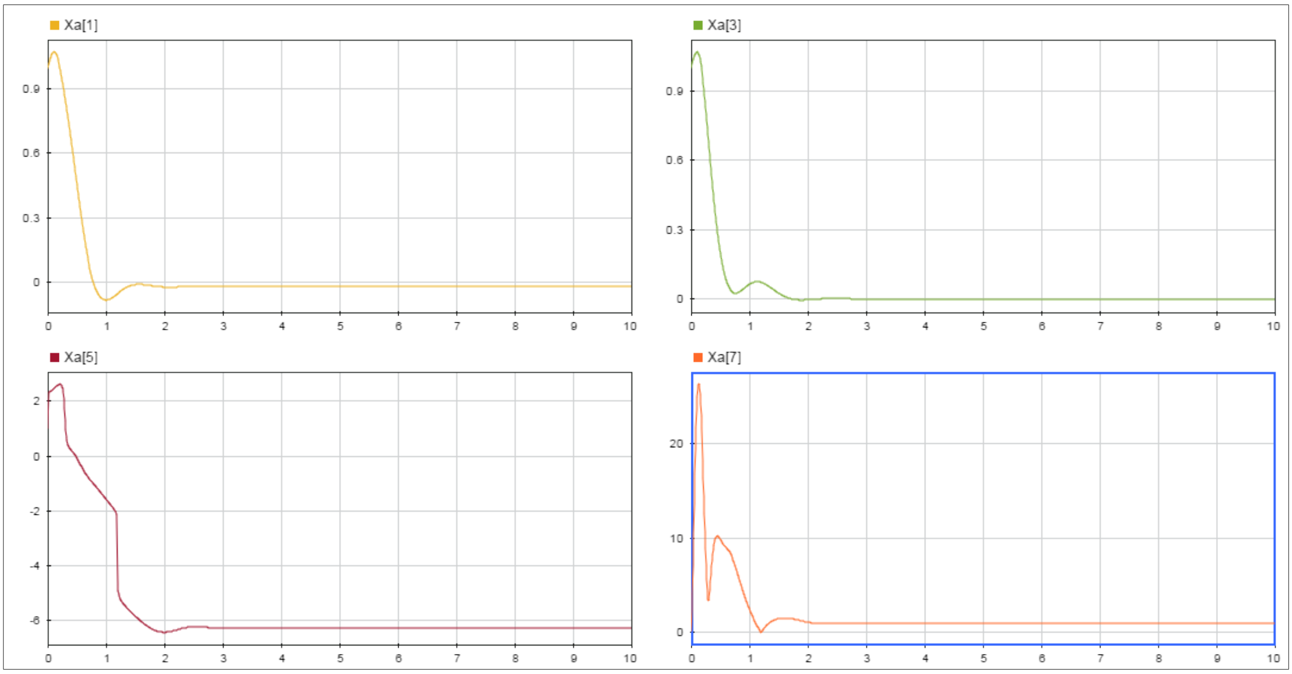


Illustration 6: State variables with a step (δ(t)= 200) as perturbation with

Xa[1] = x; Xa[3] = z; Xa[5] = θ; Xa[7] = u1

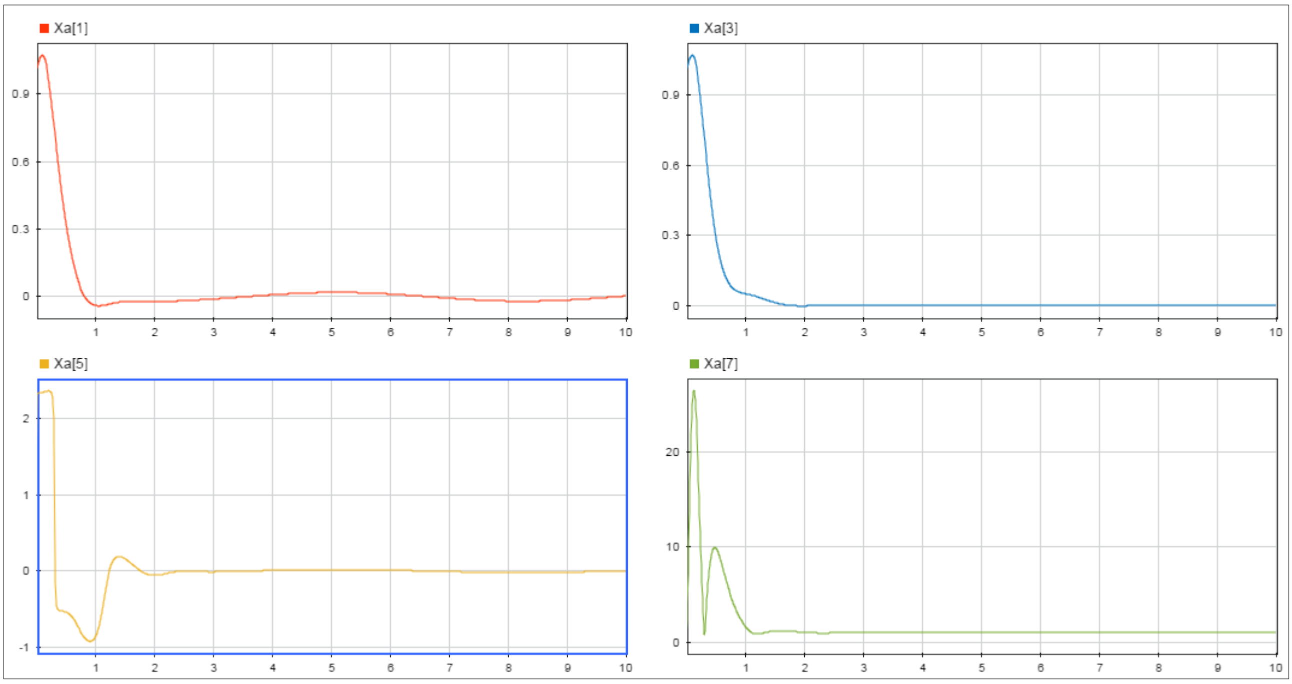


Illustration 7: State variables with sinusoidal (δ(t)= 200 sin(t)) as perturbation with

Xa[1] = x; Xa[3] = z; Xa[5] = θ; Xa[7] = u1

Remarks:

- The system is stable however we note that this is not tracking of the reference because of the perturbations. Which means the controller is not robust against the perturbations. However we can enhance the performance and robustness in taking smaller pols (goes to - ∞);

6 Part 3 : Sliding Mode control

6.1.1 Sliding mode surface

The decoupled system used is:

$$\begin{matrix} \dot{Q}_1 \\ \dot{Q}_2 \end{matrix} = \begin{matrix} AQ_1 + B\omega_1 \\ AQ_2 + B\omega_2 \end{matrix} \tag{37}$$

Such as:

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \alpha + \beta \begin{pmatrix} \ddot{u}_1 \\ u_2 \end{pmatrix} \tag{38}$$

And

$$Q_1 = \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{pmatrix} \tag{39}$$

$$\dot{Q}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} Q_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \omega_1 \tag{40}$$

The sliding variable is chosen as:

$$S_1 = \lambda Q_1 = \dddot{x} + \lambda_2 \ddot{x} + \lambda_1 \dot{x} + \lambda_0 x \tag{41}$$

Such that:

$$\lambda = [\lambda_0 \quad \lambda_1 \quad \lambda_2 \quad 1] \tag{42}$$

The condition to have a linear system is to get S1=0. Sliding Mode Variable derivative is calculated:

$$\dot{S}_1 = x^{(4)} + \lambda_2 \dddot{x} + \lambda_1 \ddot{x} + \lambda_0 \dot{x} \tag{43}$$

It is equal to:

$$\dot{S}_1 = \omega_1 + \bar{\lambda} Q_1 \tag{44}$$

With

$$\bar{\lambda} = [0 \quad \lambda_0 \quad \lambda_1 \quad \lambda_2] \tag{45}$$

We need $\dot{S}_1 = -K\, sign(S_1)$ in order to get $\dot{S}_1 \rightarrow 0$. Then ω_1 becomes:

$$\omega_1 = -K\, sign(S_1) - \bar{\lambda} Q_1 \tag{46}$$

When S=0, the characteristic polynomial is equal to:

$$S^3 + \lambda_2 S^2 + \lambda_1 S + \lambda_0 \tag{47}$$

$\bar{\lambda}$ are calculated with identification from (s-P)^3 = $s^3 - 3PS^2 + 3P^2 s - P^3$. then it is:

$$\lambda = [8 \quad 2 \quad 6 \quad 1] \tag{48}$$

With P equals to -2.

Remark:

- This Sliding Mode Surface does not ensure the tracking.

6.1.2 Sliding mode surface that ensure the tracking

In order to ensure the tracking we consider the error. Which is:

$$e = x - r_1 \tag{49}$$

Such that, r_1 is reference for x. The surface should be:

$$S_1 = \lambda Q_1 = \dddot{e} + \lambda_2 \ddot{e} + \lambda_1 \dot{e} + \lambda_0 e \tag{50}$$

Thus derivative surface is equal to:

$$\dot{S}_1 = e^{(4)} + \lambda_2 \dddot{e} + \lambda_1 \ddot{e} + \lambda_0 \dot{e} \tag{51}$$

And

$$\omega_1 = -K_1\, sign(S_1) + r_1^{(4)} - \bar{\lambda}(Q_1 - R_1) \tag{52}$$

$$\omega_2 = -K_2\, sign(S_2) + r_2^{(4)} - \bar{\lambda}(Q_2 - R_2) \tag{53}$$

With

$$R_1 = \begin{pmatrix} r_1 \\ \dot{r}_1 \\ \ddot{r}_1 \\ \dddot{r}_1 \end{pmatrix} \tag{54}$$

$$R_2 = \begin{pmatrix} r_2 \\ \dot{r}_2 \\ \ddot{r}_2 \\ \dddot{r}_2 \end{pmatrix} \tag{55}$$

r_1 and r_2 are the references for x and z.

Simulink is used to simulate the system:

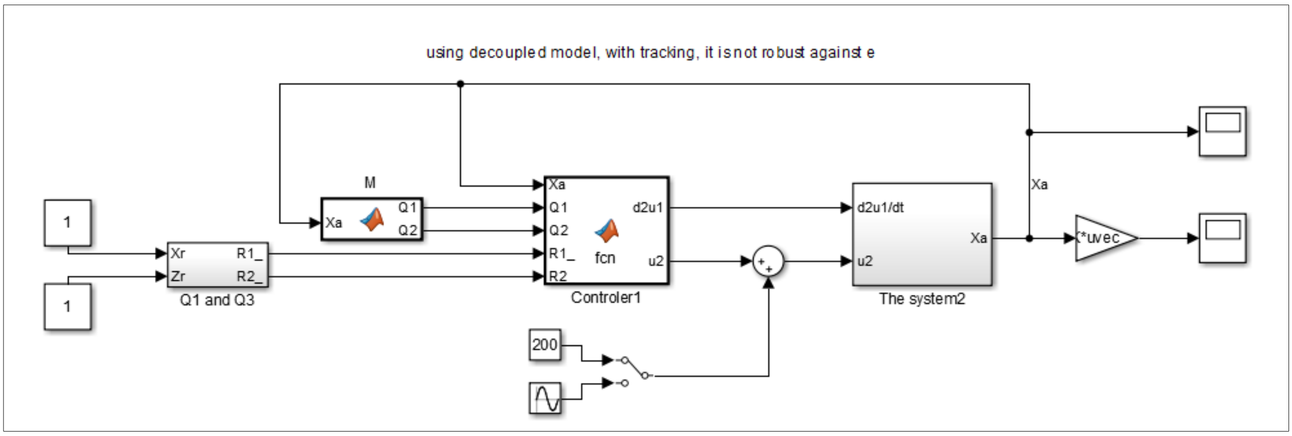


Illustration 8: Simulink blocs with Sliding Mode Control.

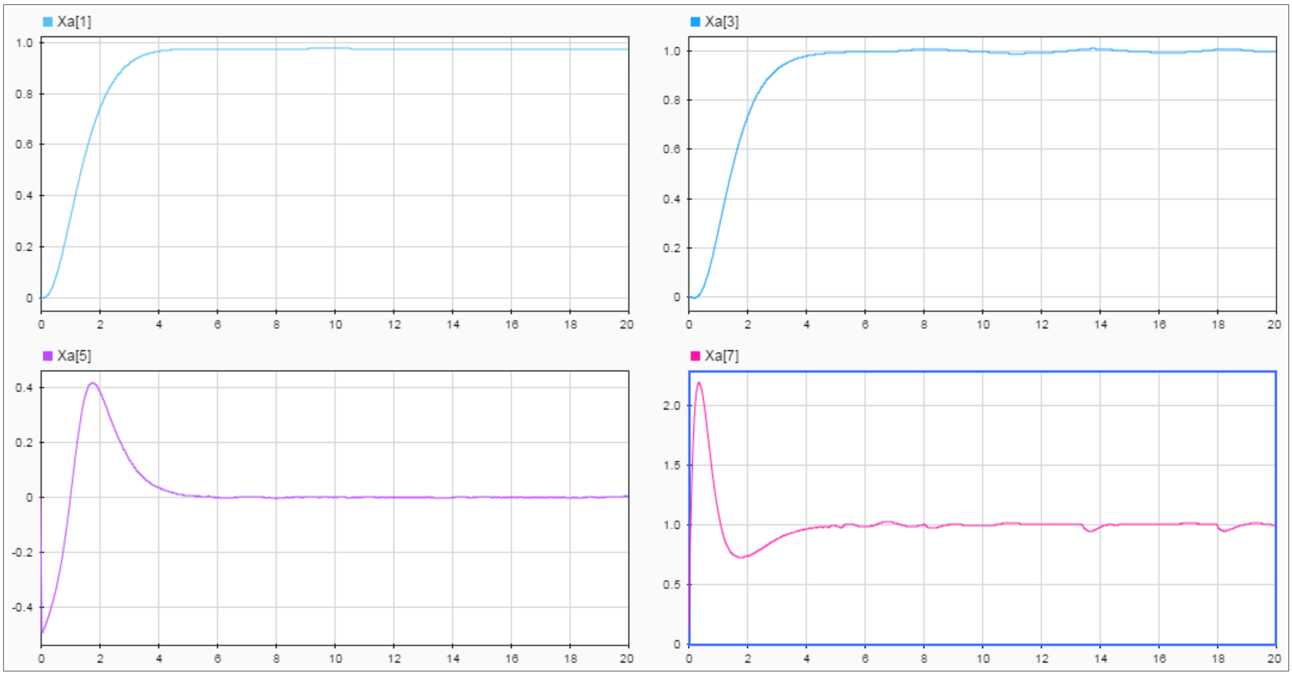


Illustration 9: State variable of the system.

Xa[1] = x; Xa[3] = z; Xa[5] = θ; Xa[7] = u1

The input u_2 is:

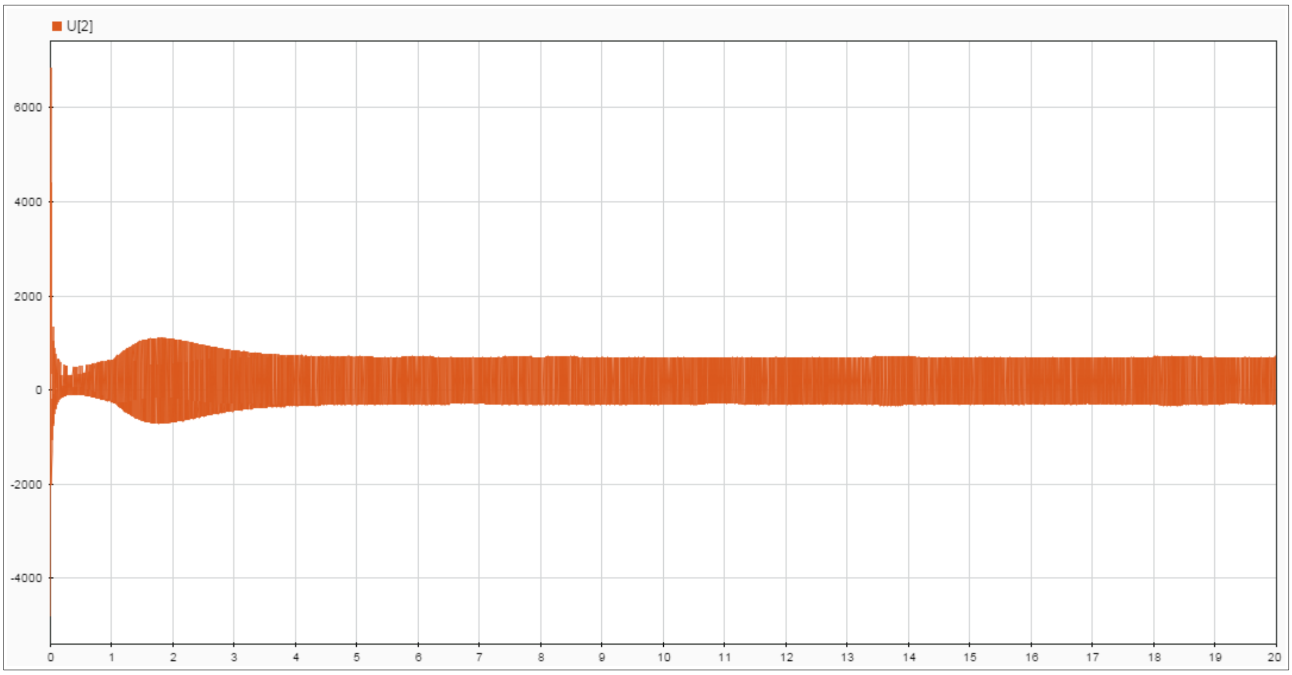


Illustration 10: Input u2.

Remark:

- The system is rock stable. We can see also that the system cannot track exactly the references ($r_1 = r_2 = 1$)when we have big perturbation (δ=200);
- We can see that the input u_2 presents the chattering effect;
- The problem of the chattering still remain even if there is no perturbation;

7 Part 3 : Adaptive Sliding Mode control with variable K

In this part, the variable K is defined as:

$$(\dot{K}_i)=\begin{Bmatrix} \bar{K}|S_i|sign(|S_i|-\mu_i) & if\ K_i>\eta_i \\ \eta_i & if\ K_i\leqslant\eta_i \end{Bmatrix} \tag{56}$$

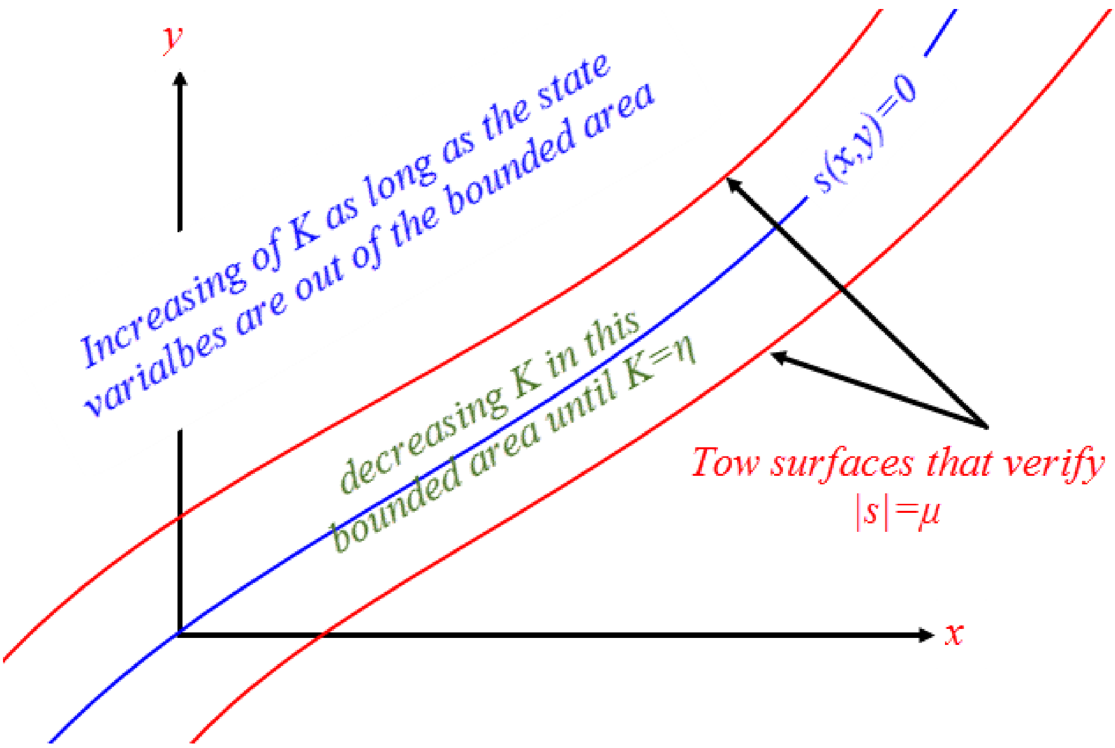
With i equal 1 and 2.

The role of the controller is to reduce the chattering. According to (Plestan et al. 2010) the controller has the purpose of:

- “Uncertainties/perturbation bounds exist but are not known.”
- “The gain-adaptation law does not overestimate uncertainties/perturbations magnitude and then, the obtained control magnitude is reasonable”.

The control algorithm work as:

- The first control algorithm can adjust the control gain. When the surface is not reach the gain K is high. When surface is reached the gain K is reduced. The chattering decreased when the sliding surface is reached;
- The second algorithm proposed a upper bound of the gain K in order to keep K positive;
- The purpose of μ_i is illustrated in the following figure:



Simulink is used to simulate the adaptive sliding mode :

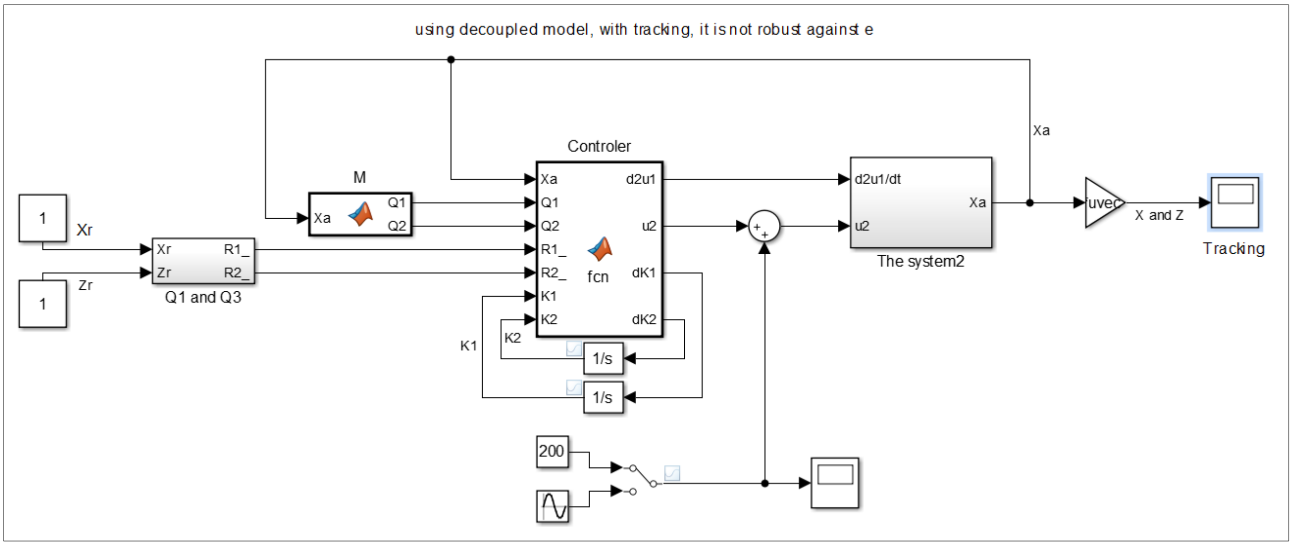


Illustration 11: Simulink blocs.

The simulation results are:

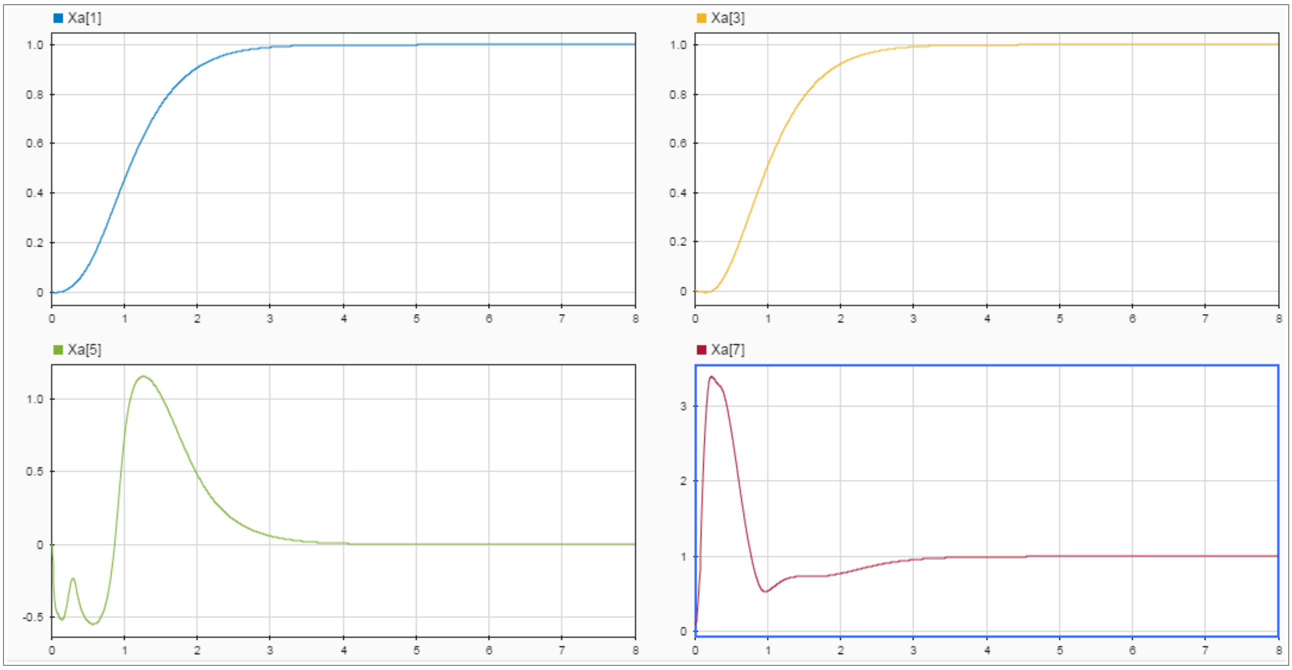


Illustration 12: State variable of the system.

Xa[1] = x; Xa[3] = z; Xa[5] = θ; Xa[7] = u1

The command u_2 is equal to :

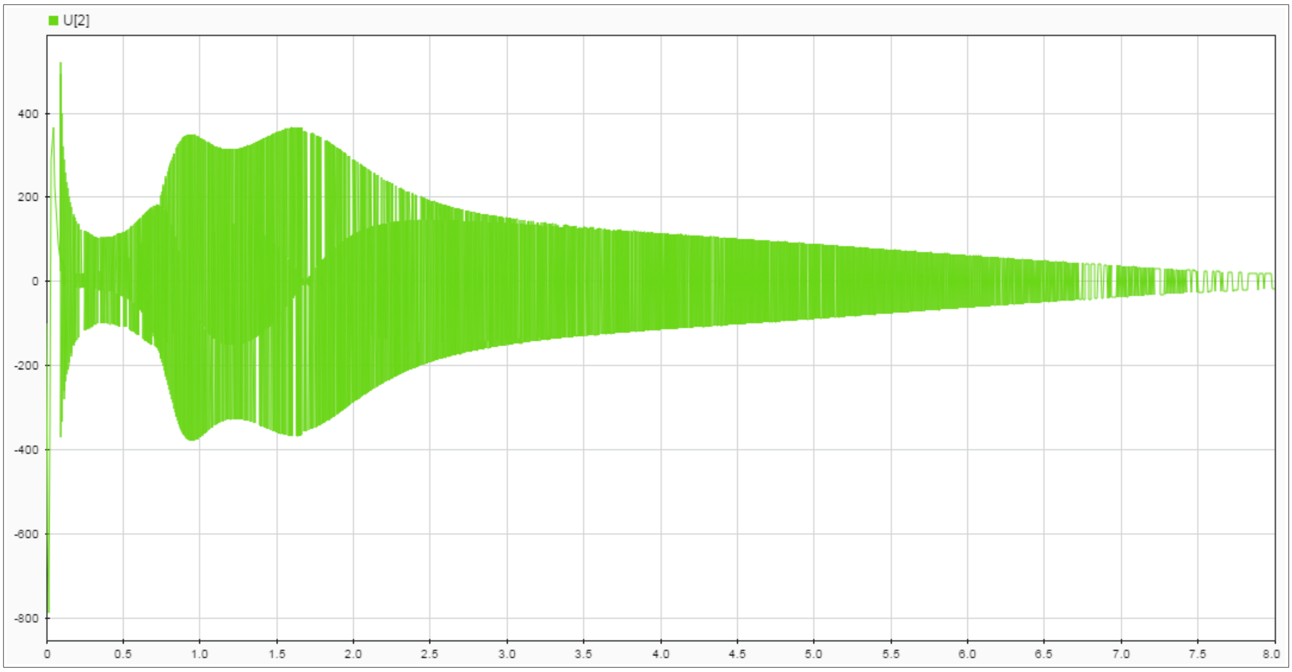


Illustration 13: Command u_2.

Remark:

- When S → 0 (S between the bounds) the chattering effect is reduced;
- The system is stable, and the robust against the perturbations and uncertainties ;
- The system outputs reach the inputs references with high certainty;

According to (Plestan et al. 2010), μ_i tunning need to be adequate because a wrong one could provide instability and control gain increasing to infinity. μ_i tunning effect are:

- If μ_i is too small, then K is increasing and S never stays lower than μ_i ;
- If μ_i is too big, S evolves around μ_i and the controller accuracy is not as good as possible.

The best value of μ_i is given by (Plestan et al. 2010) equals to:

$$\mu_i = 4K(t)T_e \tag{57}$$

With Te is the sample time.

8 Conclusion

This lab has given the opportunity to explore the knowledge seen in ANCOS lectures, sum up as:

- Input and output linearization. We have seen that a internal dynamic could be instable.

- The system has been stabilized with taking u1 and it derivation as augmented state variable;

- Sliding mode control has been simulates with Simulink. We have seen the robustness and chattering effect;
- Adaptive sliding mode control has been simulated and we have seen the reduction of chattering effect keeping the robustness and accuracy.

9 References

Plestan, Franck, Yuri Shtessel, Vincent Bregeault, and Alex Poznyak. 2010. “New Methodologies for Adaptive Sliding Mode Control.” *International Journal of Control* 83 (9): 1907–1919.