# Master EMARO - ASP

## NOLCO Exam

### Duration 1h00 - Open book

**EXERCICE 1.**

One considers the following nonlinear system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \cos(x_1)x_4 \\ \dot{x}_3 & = & -x_3 + x_1^2 \\ \dot{x}_4 & = & -x_3 + u \end{array} \tag{1}$$

with the output (for control) $y = x_1$. The control objective is to force $y$ to 0.

**1.1** What is the relative degree of the output $y$ ?

**1.2** Analyze the stability of the internal dynamics.

**1.3** Determine a state feedback control law such that the closed-loop system has a linear input-output representation (thanks to the input-output linearization approach):

- Show that the input-output representation can be reduce as a chain of integrators controlled by a “new” control input $v$.
- Propose a linear solution for $v$ allowing to stabilize this chain of integrators.
- If it is the case, give the singularities appearing in the control $u$.

**EXERCICE 2.**

Consider the following nonlinear system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & x_3 \\ \dot{x}_3 & = & \delta(t) + \cos(x_1)u \end{array} \tag{2}$$

with $[x_1\ x_2\ x_3]^T$ the state vector, and $u$ the control input. The control objective is to force the output $y = x_1$ tracking a reference $y_{ref}(t)$. The term $\delta(t)$ is a perturbation such that

$$|\delta(t)| \leq \delta_M.$$

One also supposes that $x_1$ is such that

$$|x_1| \leq \frac{\pi}{3}$$

**2.1** The objective consists in designing a first order sliding mode control law. Define the sliding variable with respect to the control objective ; justify the choice - in case of use of parameters, specify the way to choose these ones.

**2.2** Compute the “nominal” control law allowing to linearize (by an input-output point-of-view, the input being $u$ and the output being the sliding variable) the system when there is NO uncertainty or perturbation. Recall the key role of this pre-feedback.

**2.3** Give the condition on the discontinuous control gain, in order to ensure the convergence to the sliding surface and the establishment of a sliding motion in spite of perturbation.