C2Syst2E - Master ARIA

ANCOS Exam

Duration 1h30 - Open book

EXERCICE 1.

Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \\ \dot{x}_3 &= x_3 \cdot u \end{aligned} \tag{1}$$

with x_1, x_2, and x_3 the state variables and u the control input.

1. Consider the output $y = \ln(x_3) - x_2$. Compute the relative degree of the system.
2. Check the accessibility of the system. Conclusion, and comment with respect to the first question.

EXERCICE 2.

Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= \frac{m(x_2+1)^2}{1+m(x_2+1)^2} \cdot u_1 \end{aligned} \tag{2}$$

describing the dynamics of a one-legged hopping robot.

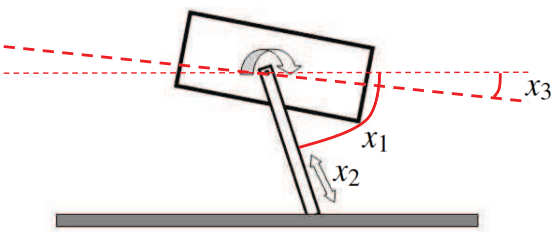

Figure 1: Scheme of a one-legged hopping robot.

x_1, x_2 and x_3 are the state variables, u_1 and u_2 the control inputs, and y_1, y_2 the both outputs defined as $y_1 = x_1$ and $y_2 = x_3$. m is a constant parameter. The control objective consists in forcing y_1 and y_2 to 0.

1. Prove that the system can not be decoupled by a static state feedback.

2. Suppose now that the control will be made through $[\dot{u}_1 \;\; u_2]^T$. Prove that the input-output representation of the system reads as

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \beta \cdot \begin{bmatrix} \dot{u}_1 \\ u_2 \end{bmatrix}$$

   What is the name of the matrix β ? Is there some singularity ?

3. Define a state feedback control law such that the closed-loop system has a linear input-output representation (thanks to the input-output linearization approach): show that the input-output representation can be reduced as a chain of integrators controlled by a "new" control input $v = [v_1 \; v_2]^T$. Then, propose a linear solution for v allowing to stabilize the system.

4. Prove that the system is observable if only x_1 and x_3 are measured. Precise the values of the observability indices and the singularities.

EXERCICE 3.

Consider the following nonlinear system describing the dynamics of a CO_2-laser

$$\begin{aligned} \dot{x}_1 &= k_0 x_2 - k_0 u + \delta \\ \dot{x}_2 &= -x_2 - 2k_0 x_2 e^{x_1} \end{aligned}$$

with x_1 a variable which is proportional to the logarithm of intensity, and x_2 lied to the population of the laser state. In the operating domain, one has $0 < x_1 < x_{1M}$ and $0 < x_2 < x_{2M}$. Furthermore, δ is an uncertain term such that $|\delta| < \delta_M$. k_0 is a constant, and u the control input.

The objective consists in designing a robust controller in order to force x_2 towards 0. Then, design a sliding mode controller thanks to the following steps

1. design the sliding variable from the control objective - if parameters are used, precise their signs and the way to tune them;

2. design the control composed by a continuous (equivalent) term and a discontinuous one;

3. propose a tuning for the gain of the discontinuous part; justify the choice.