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NOLCO Exam

Duration 1h30 - Open book

EXERCICE 1.

Consider the following nonlinear system ($[x_1\ x_2\ x_3]^T$ being the state vector and u the control input)

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \cdot \cos(x_1) + x_3^2 \\ \dot{x}_2 & = & x_1 \cdot u + \sin(x_3) \\ \dot{x}_3 & = & -x_1 \cdot x_3 \end{array} \tag{1}$$

with the output (for control) $y = x_1$. The control objective is to force y to 0. The single measurement is x_3.

Structural analysis.

1. Analyze the accessibility of the system.
2. Analyze the observability of the system. Precise, if necessary, the singularities.

Control design.

3. Evaluate the relative degree of the system (1) versus the output y.
4. Does it exist internal dynamics (not controlled) ? Justify your response. Do not analyze its stability.
5. Calculate a control input u which allows to linearize, by an input-output point-of-view, the system. Specify, if it is the case, control singularities.
6. Propose a controller such that the input-output behavior of the system is equivalent to a system with a damping coefficient ζ and a proper pulsation ω.

EXERCICE 2.

Consider the following uncertain system ($[x_1\ x_2]^T$ being the state vector and u being the control input)

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \gamma(t) + \beta(t) \cdot u \end{array} \tag{2}$$

Functions $\gamma(t)$ and $\beta(t)$ are unknown such that

$$|\gamma(t)| \leq \gamma_M \quad \text{and} \quad 0 < \beta_m \leq \beta(t) \leq \beta_M .$$

1. Recall the main advantages/drawbacks of the sliding mode control strategy.
2. Design the sliding variable σ allowing to force x_1 towards a reference strategy $x_1^r(t)$, thanks to a first order sliding mode controller, in spite of the uncertainties. Justify the choice.

3. Give the condition on K ensuring that the previous objective can be reached thanks to the control $u = -K\text{sign}(\sigma)$.

4. Suppose now that the control law strategy is based on *twisting* algorithm. In this case, how to define the sliding variable if the objective is to ensure that x_1 reaches $x_1^r(t)$ in a finite time ?

5. What would be the interest of the *super-twisting* algorithm ? In this case, is it possible to get a finite time convergence of x_1 towards $x_1^r(t)$? Justify.

Consider now the system ($[x_1\ x_2\ x_3]^T$ being the state vector and $[u_1\ u_2]^T$ being the control input vector)

$$\begin{aligned} \dot{x}_1 &= x_2 \cdot \cos(x_1) \\ \dot{x}_2 &= x_3^2 \cdot u_1 + u_2 \\ \dot{x}_3 &= x_1 \cdot u_2 - u_1 + x_3^2 \end{aligned} \tag{3}$$

The objectives are to force x_1 and x_3 to 0. Propose a controller allowing to decouple and to give, in closed-loop, at x_1 the behavior of a second order system with a damping coefficient ζ and a proper pulsation ω, and at x_3 the behavior of a first order system with a response time equal to t_r.
Is there an internal dynamics ? Justify.