Master EMARO - ARIA

NOLCO Exam

Duration 1h30 - Open book

QUESTIONS.

In some few words, **without any computation or any equation**, could you explain what physically characterizes the following concepts

- the relative degree ;
- an infinite relative degree ;
- the observability.

EXERCICE 1.

Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4^2 - \frac{k}{m x_1^2} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -2\frac{x_2 x_4}{x_1} + \frac{u}{m x_1} \end{aligned} \tag{1}$$

with x_1, x_2, x_3 and x_4 the state variables, u the control input and k, m constant parameters.

1. Analyze the accessibility of this system.
2. Analyze the observability of this system by supposing that only x_1 is measured.
3. Suppose now that all the state variables are available. Supposing that the objective is to force x_1 at a desired value $x_{1d} \neq 0$, evaluate the relative degree.
4. Does it exist internal dynamics ? Justify the response, but without trying to evaluate the zero dynamics. What could be the problem with such dynamics ?
5. Compute a control law u such that the closed-loop system has a linear input-output relation.
6. Design this control law in order to force x_1 to x_{1d}. Detail how to compute the gains of the controller, in order to reach this objective.

EXERCICE 2.

Consider the following nonlinear system

$$\begin{array}{rcl} \dot{x}_1 & = & k_0 x_2 - u \\ \dot{x}_2 & = & -x_2 - x_2 e^{x_1} \end{array} \tag{2}$$

with x_1 and x_2 the state variables, u the control input. k_0 is a parameter. The control objective consists in forcing x_2 to x_{2d}, by using a sliding mode controller.

1. Define the sliding variable σ in order to fulfill the control objective.

2. By supposing that all the state variables are measured and the parameter k_0 perfectly known, design a controller in order to get a linear behavior for the sliding variable dynamics (this part of the control u is called "nominal" part).

3. In order to force the sliding variable towards 0, propose to complete the previous linearizing control law by a sliding mode part (this part is called "discontinuous" part). Detail the choice of its gain, thanks to the sliding condition.

4. Suppose now that the parameter k_0 is uncertain and reads as $k_0 = k_{0N} + \delta k_0$. Propose a sliding mode controller with a similar structure as previously (a "nominal" part and a "discontinuous" one) in order to ensure the convergence of the sliding variable to x_{2d}, in spite of the uncertainties on k_0. Give the condition on the gain of the discontinuous part of the controller.