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Advanced Control Strategies of Nonlinear Systems
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Introduction

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All the slides are available on hippocampus.ec-nantes.fr/ANCOS

OUTLINE

- Mathematical preliminaries, state representation
- Controllability, accessability
- Input-output linearisation and static decoupling
- Inversion and dynamic decoupling
- Input-state linearization and flatness
- Observability and observers
- Robust control and stability
- Sliding mode control (first, second, adaptive)
- Backstepping approach

How are we going to proceed ?

- During the lectures, a lot of examples and exercices
- Practical works

Main topics of research

- Nonlinear control and observer design: sliding mode control (discontinuous)
- Time delay systems, multi-agent systems
- Fields of applications : electropneumatic systems, flying systems, renewable energy systems

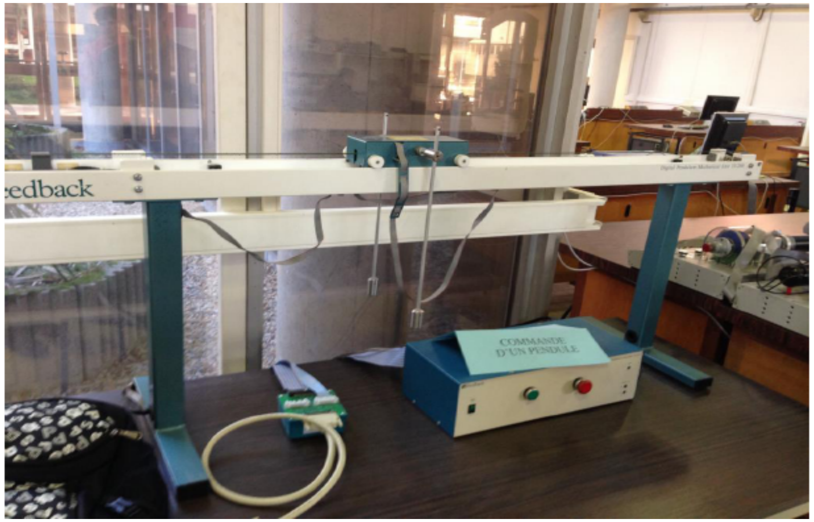
Some references.

- J.J. Slotine, W. Li, Applied Nonlinear Systems, Prentice-Hall, 1991.
- A. Isidori, Nonlinear Control Systems- 3rd edition, Springer, 1996.
- H. Khalil, Nonlinear systems – 3rd edition, Prentice Hall, 2002.
- V. Utkin, J. Gurdner, J. Shi, Sliding Mode Control in Electromechanical Systems, CRC, 2009.
- Y. Shtessel, C. Edwards, L. Fridman, A. Levant, Sliding mode control and observation, Springer, 2016
- A. Glumineau, Lecture slides, École Centrale de Nantes, 2016.

Linear / nonlinear systems

- Linear systems have been studied since a very long time
    - **An idea** : is it possible to « transform » nonlinear systems into linear systems in order to use « linear solutions » ?
        - « Approached » linearized system
        - Exact linearization (by state feedback or by state transformation)
    - **An example** : rigid arm …

Some examples **Inverted pendulum**



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{M + m - \dfrac{m^2 l^2}{J + ml^2}} \left[m\sin(x_3)(l{x_4}^2 - \frac{ml^2 g}{J + ml^2}\cos(x_3)) + u_1 - f_c x_2 \right]$$

$$\dot{x}_3 = x_4$$

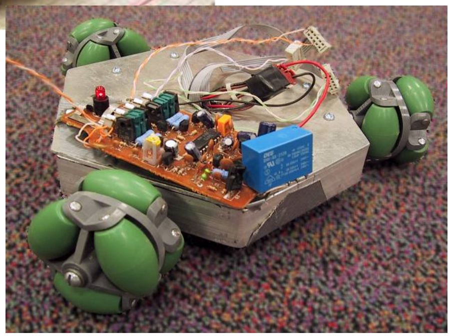
$$\dot{x}_4 = \frac{1}{\dfrac{J + ml^2}{ml} - al\cos^2(x_3)}$$

$$\left[-al\sin(x_3)\cos(x_3){x_4}^2 + g\sin(x_3) - \frac{a}{m}\cos(x_3)(u_1 - f_c x_2) \right]$$

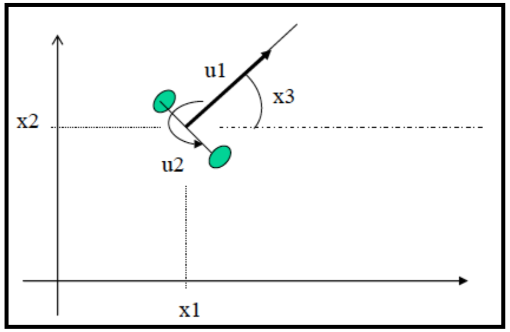
The inverted pendulum model reads as

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases}$$

Be careful ! Internal dynamics can be instable.



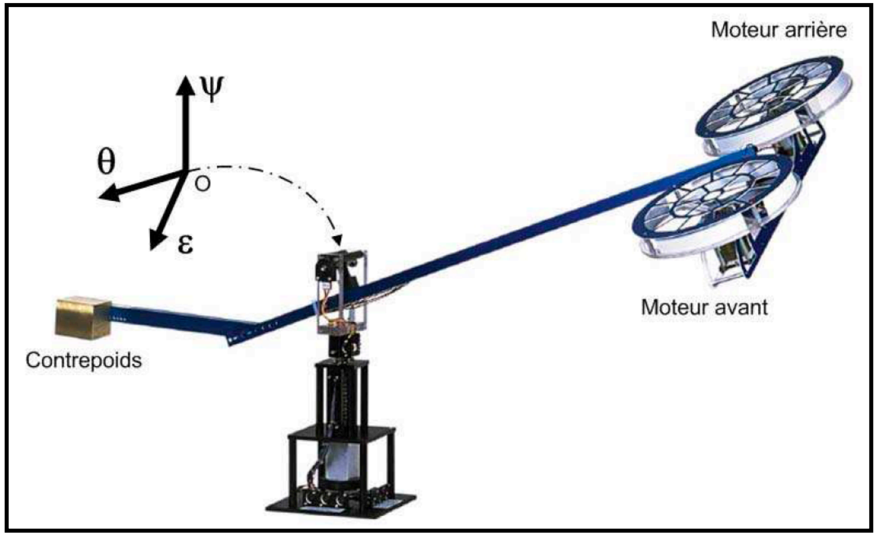
Mobile Robot

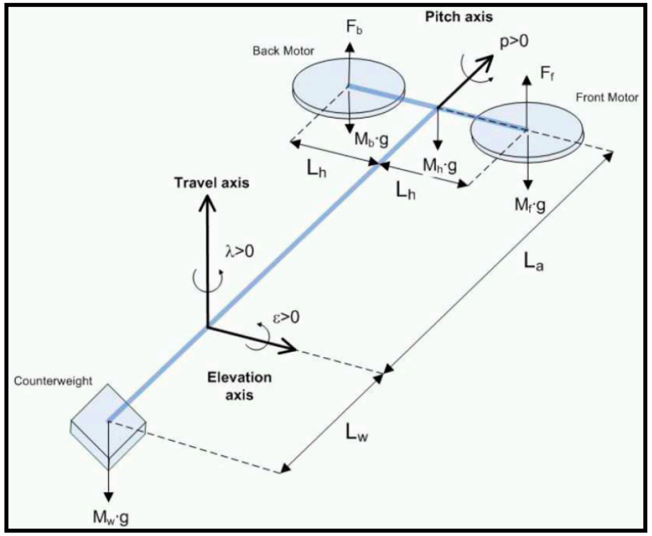


$$\dot{x}_1 = (\cos x_3)u_1$$
$$\dot{x}_2 = (\sin x_3)u_1$$
$$\dot{x}_3 = u_2$$

Non holonomous system : constraints - All the trajectories are not allowed!

3DOF Helicopter

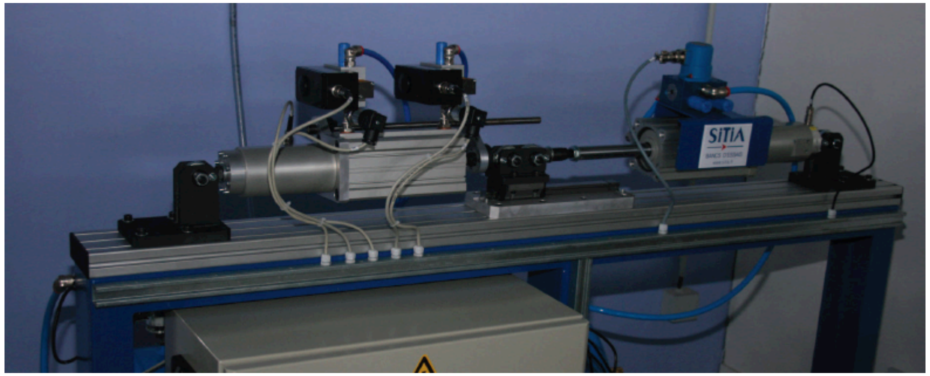


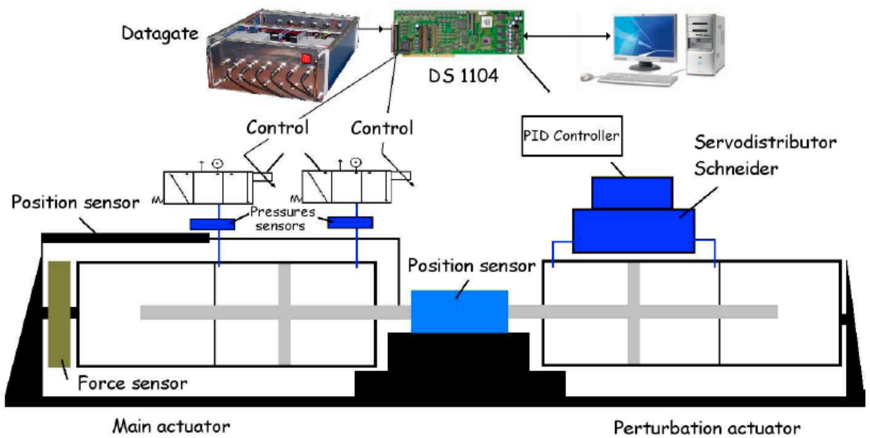


$$
\begin{aligned}
J_\varepsilon \ddot{\varepsilon} &= -M_h g cos(\varepsilon) L_a + M_w g cos(\varepsilon) L_w + K_f.(V_f + V_b).cos(\theta).L_a \\
J_\theta \ddot{\theta} &= K_f (V_f - V_b) L_h \\
J_\psi \ddot{\psi} &= K_f (V_f + V_b) sin(\theta) L_a
\end{aligned}
$$

Not controllable when $\theta = \pm \pi/2$!

Electropneumatic system





$$\frac{\mathrm{d}p_P}{\mathrm{d}t} = -k\frac{p_P}{V_P(y)}\frac{\mathrm{d}V_P(y)}{\mathrm{d}t} + \frac{krT}{V_P}q_m(u_P, p_P)$$

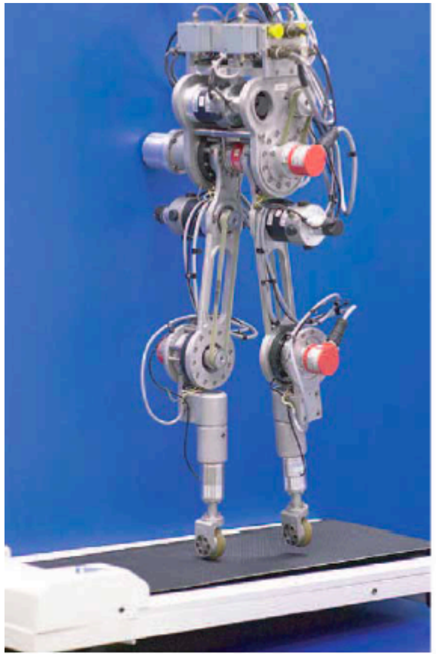
$$\frac{\mathrm{d}p_N}{\mathrm{d}t} = -k\frac{p_N}{V_N(y)}\frac{\mathrm{d}V_N(y)}{\mathrm{d}t} + \frac{krT}{V_N}q_m(u_N, p_N)$$

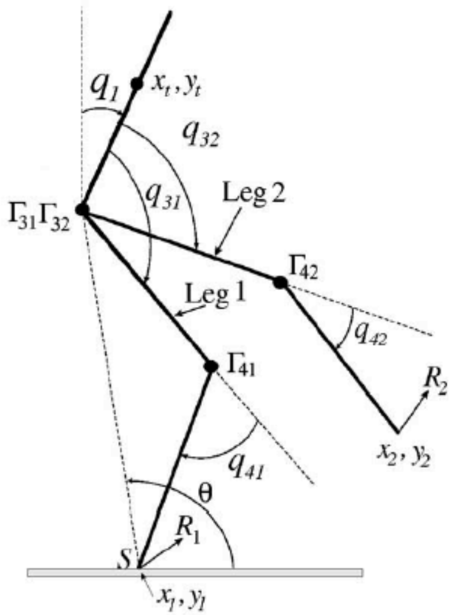
$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{M}\left[S(p_P - p_N) - F_f - b_v v - F\right]$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = v$$

- MIMO systems
- Estimation of state variables ? Observability.

Walking robot *RABBIT*





Swing phase

$$\boxed{\begin{aligned}\dot{x} &= \begin{bmatrix} \dot{q} \\ D^{-1}(-H - G + B\Gamma) \end{bmatrix} \\ &= f(x) + g(q_{\text{rel}}) \cdot \Gamma \end{aligned}}$$

$$q = \left[q_{\text{rel}}^T q_1\right]^T = [q_{31}\ q_{41}\ q_{32}\ q_{42}\ q_1]^T$$

$$q_{\text{rel}} := [q_{31}\ q_{32}\ q_{41}\ q_{42}]^T$$

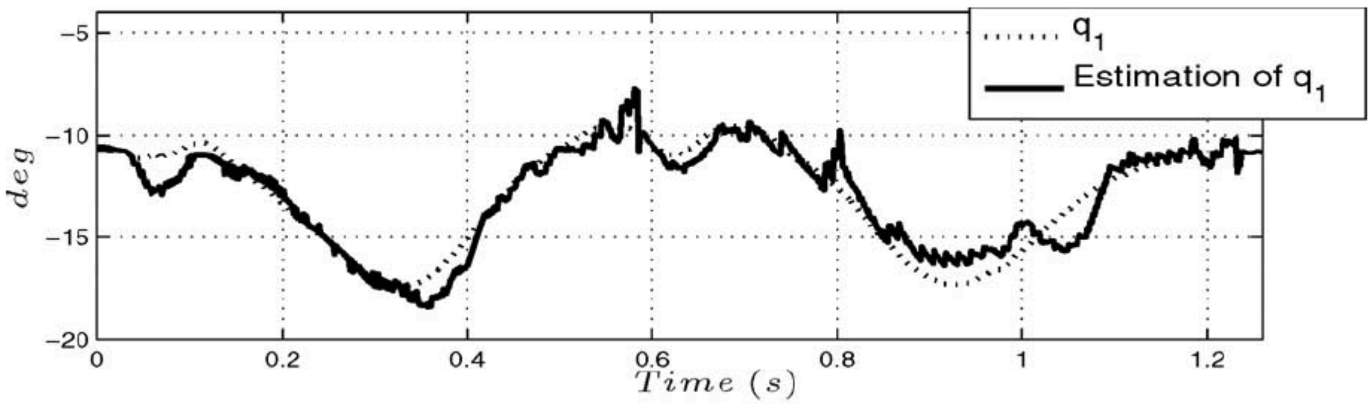
Impact phase

$$x^+ = \Delta(x^-)$$

➡ Nonlinear system with impulse effect

Practical problem : measurement of posture (torso angle)

- Finite of a step : necessity to estimate in a finite time
- Identification = hard task : necessity to use a robust observer



Difference between linear and nonlinear systems

- No frequency approach (transfer function)
- No transfer matrix

Nonlinear state space system

$$\dot{x} = f(x) + g(x)u \qquad x \in \mathbb{R}^n \quad u \in \mathbb{R}^m$$
$$y = h(x) \qquad y \in \mathbb{R}^p$$

What are the questions that we have to answer ?

- Controllability ?
- Input-output linearization ?
- Decoupling ?
- Observability ? Which kind of observers ?
- Robust control ? sliding mode ? Backstepping ?