Assignment 4

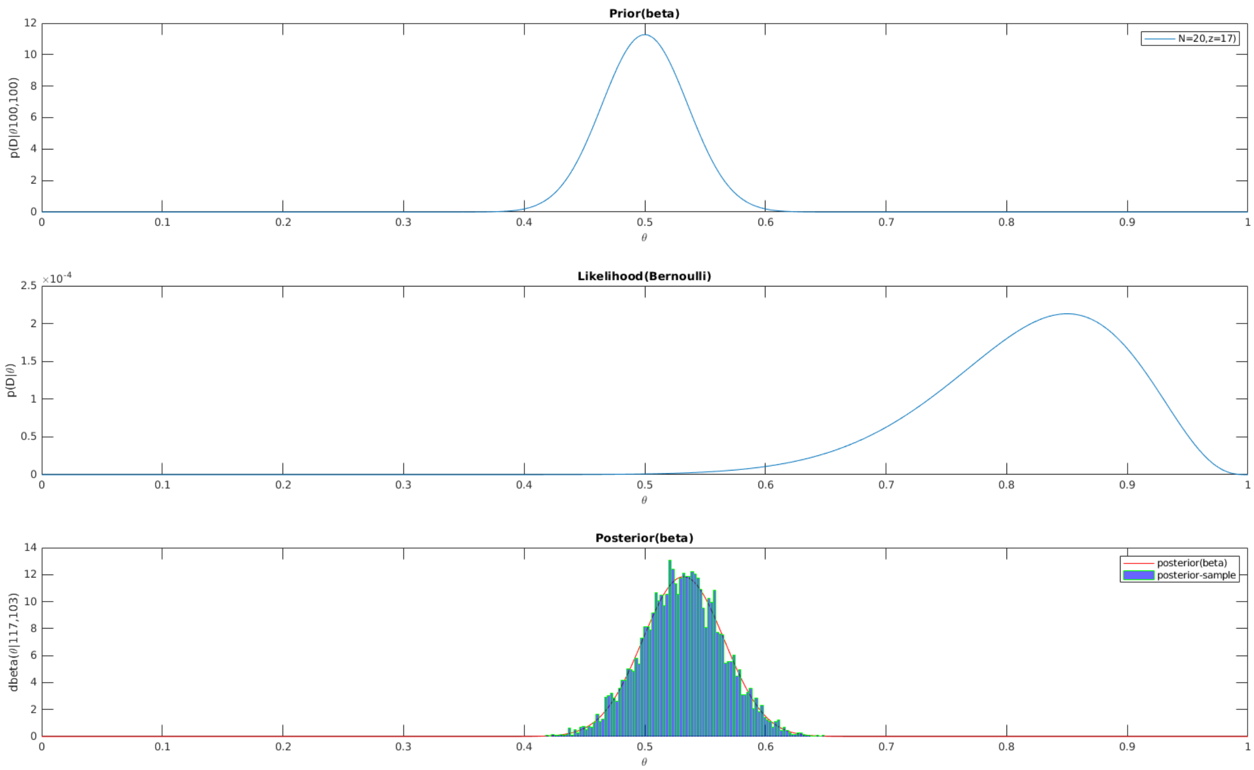
TaskA:

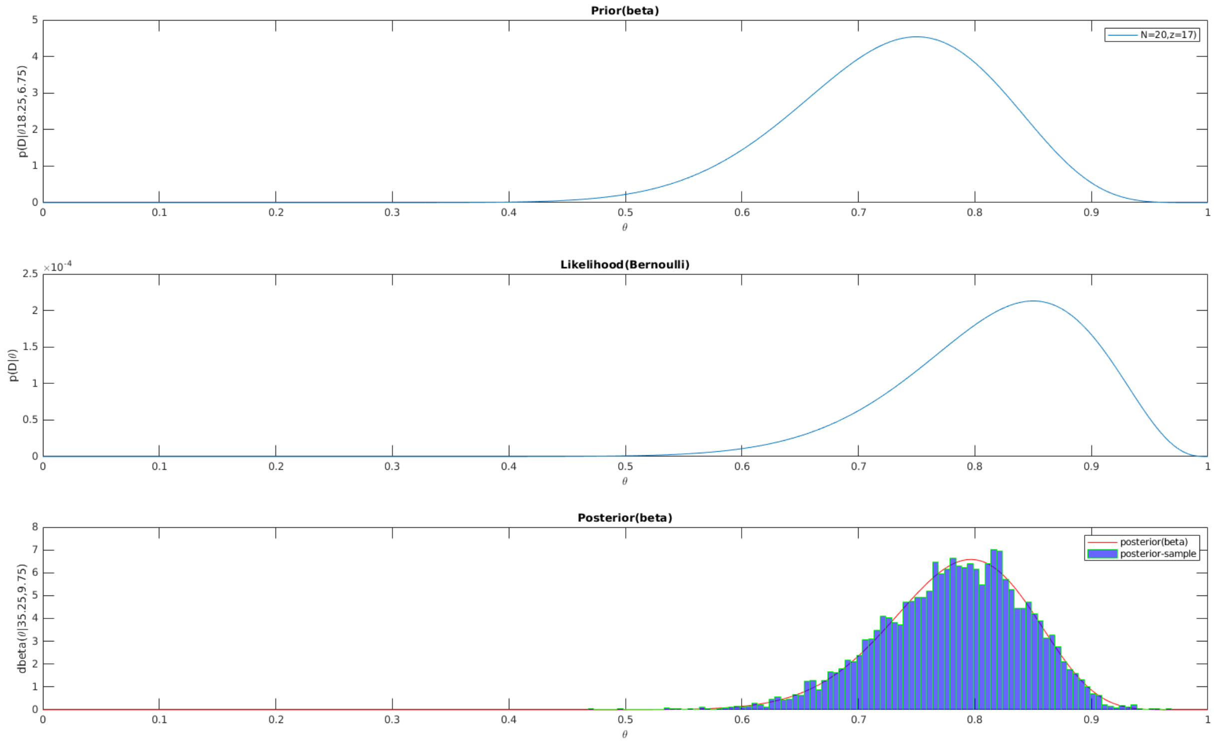
1. In this task you will recreate Figure 6.4, just as you did in Assignment 3. This can be done in STAN using the build in MCMC procedure (read Ch 7), or using MATLAB, python and julia and e.g. slice or Metropolis sampling as MCMC procedure.
   Additional information:
   To implement slice sampling by yourself, follow this link
   https://projecteuclid.org/download/pdf_1/euclid.aos/1056562461
   It rather easy to implement your self and the article is easy to follow. A description is also available in wikipedia but I think Neal's article and his pseudo code is easier to follow. Slice sampling mimics Gibbs sampling (Ch 7) and if the parameters are very correlated, you can get bad mixing, more on how to fix this later in the course.

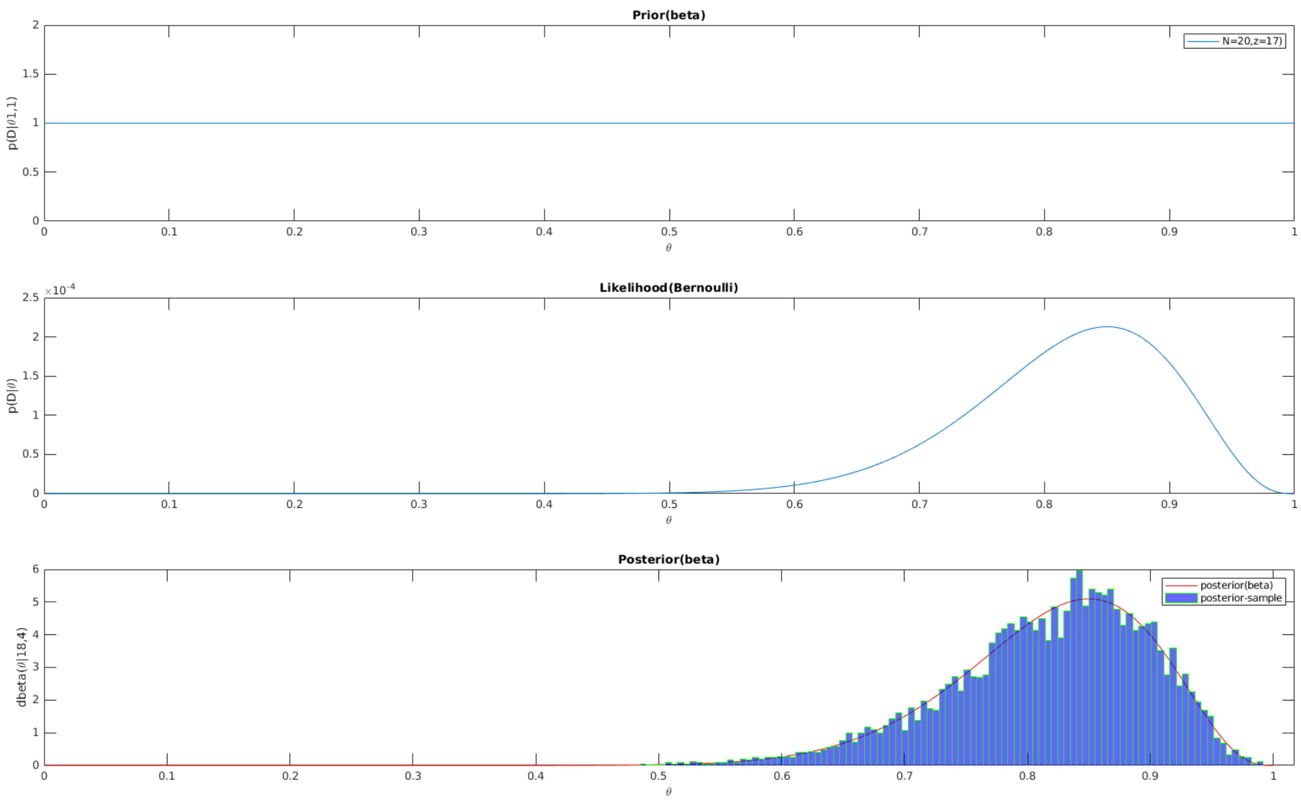
   - Python users: Slice sampling is available from the python script I delivered to you in the previous emails.
   - MATLAB users: Slice sampling is available as a toolbox or make your own implementation. You can also peek at my julia implementation below.
   - Julia users: My julia implementation of slice sampling and the following script defining a Bernoulli posterior and sampling from it.

   To get things running quickly for you, I provide this python example in both STAN and using my slice sampling. It does MCMC sampling from the posterior, for the example with bernoulli coin flips.

   - You will need put both the python example script and my slice sampling python script in the same folder to have it running.
   - For the STAN alternative in my example you do not need anything other than STAN.


Prior(beta)
p(D|θ100,100)
N=20,z=17)
θ
Likelihood(Bernoulli)
×10^-4
p(D|θ)
θ
Posterior(beta)
dbeta(θ|117,103)
posterior(beta)
posterior-sample
θ


Prior(beta)
p(D|θ18.25,6.75)
N=20,z=17)
θ
Likelihood(Bernoulli)
×10^-4
p(D|θ)
θ
Posterior(beta)
dbeta(θ|35.25,9.75)
posterior(beta)
posterior-sample
θ



Prior(beta)
N=20,z=17)
p(D|θ1,1)
θ
Likelihood(Bernoulli)
×10^-4
p(D|θ)
θ
Posterior(beta)
posterior(beta)
posterior-sample
dbeta(θ|18,4)
θ

TaskB:

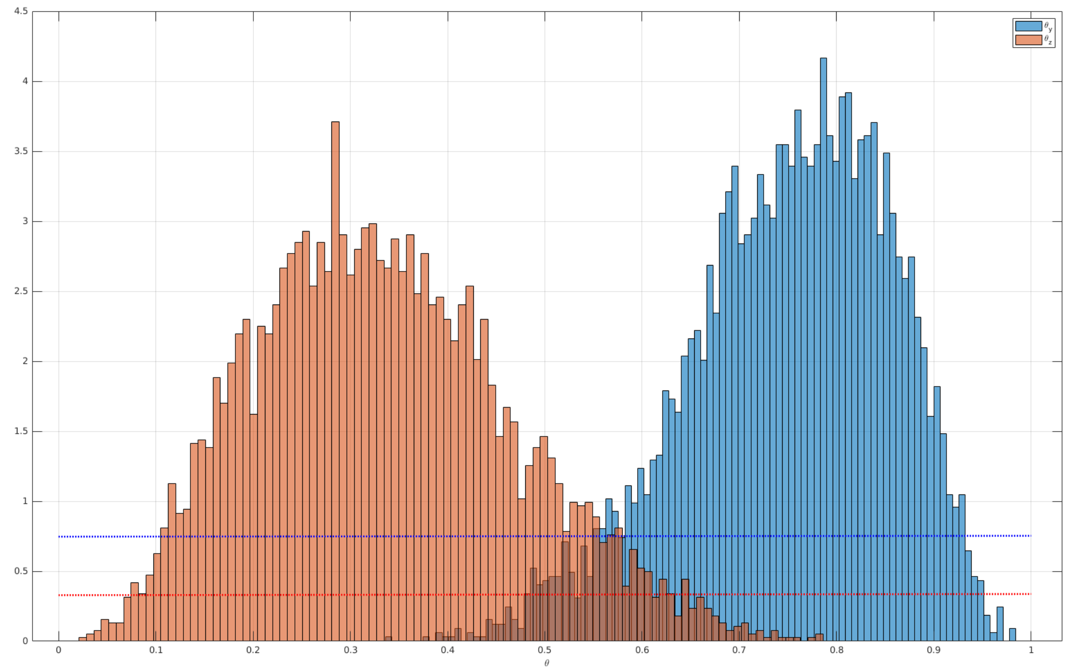
Once you get things running with the example code and you manage to recreate Figure 6.4, answer the following questions:

1. Given the following measurements $y = [1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 1]$:
    1. What is the expected probability of getting a head?
       Give a 95% credible interval of this probability (use either equal-tailed or HDI or both).
    2. What is the probability that $\theta > 0.5$?

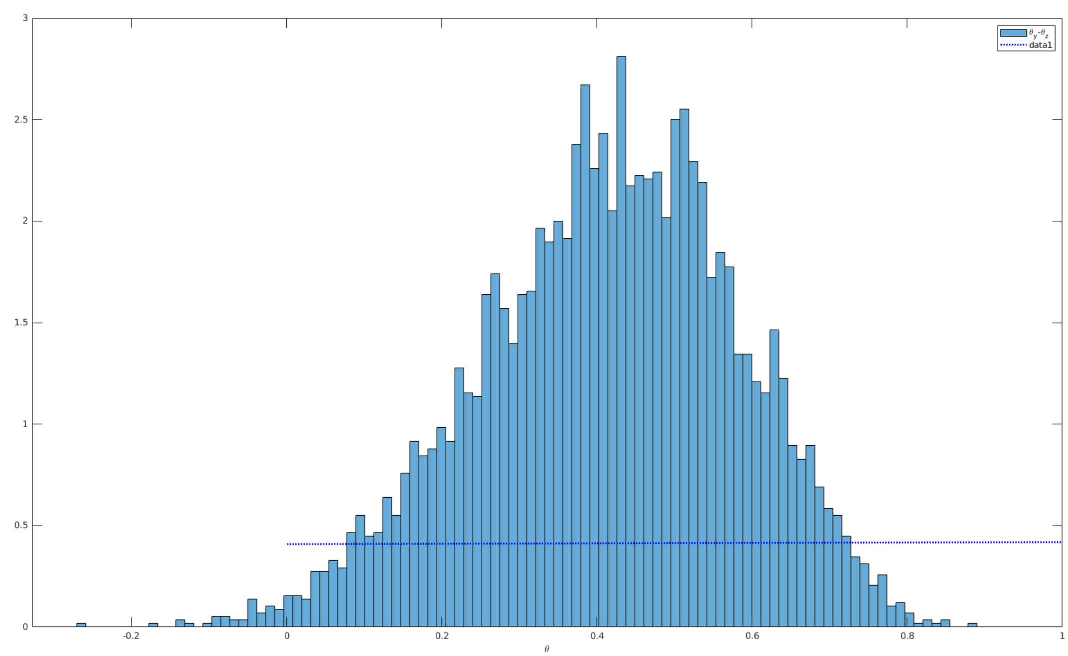
Ans: p_y: 0.9782, p_z: 0.1214

3. Given an additional set of measurements $z = [1, 0, 0, 0, 0, 0, 0, 1, 1, 0]$, are y and z measurements from the same coin? You may answer this in different ways:
    1. Answer this by calculating the probability that $\theta_y > \theta_z$ given the measurements y, z.
    2. Answer this by creating a new variable $d\theta = \theta_y - \theta_z$ and calculating a 95% credible interval.

Plot a histogram representing $p(d\theta|y, z)$. Is it beta distributed? Motivate your answer.



θ_y
θ_z
θ



θ_y-θ_z
data1
θ