$$F(x) = \begin{cases} 0, & -\pi \le x < 0 \\ x, & 0 \le x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(x)\, dx$$

$$a_0 = \frac{1}{2\pi} \left(\int_{-\pi}^{0} 0\, dx + \int_{0}^{\pi} x\, dx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos(nx)\, dx$$

$$a_n = \frac{1}{\pi} \int_{0}^{\pi} x \cos(nx)\, dx$$

Integración por partes

$$\int x \cos(nx)\, dx = \frac{x\, sen(nx)}{n} + \frac{\cos(nx)}{n^2} \Big|_0^{x}$$

$$= \frac{\pi\, sen(n\pi)}{n} + \frac{\cos(n\pi)}{n^2} - \frac{0}{n^2} = \frac{(-1)^n}{n^2} - \frac{1}{n^2}$$

$$a_n = \frac{2((-1)^n - 1)}{\pi n^2}$$

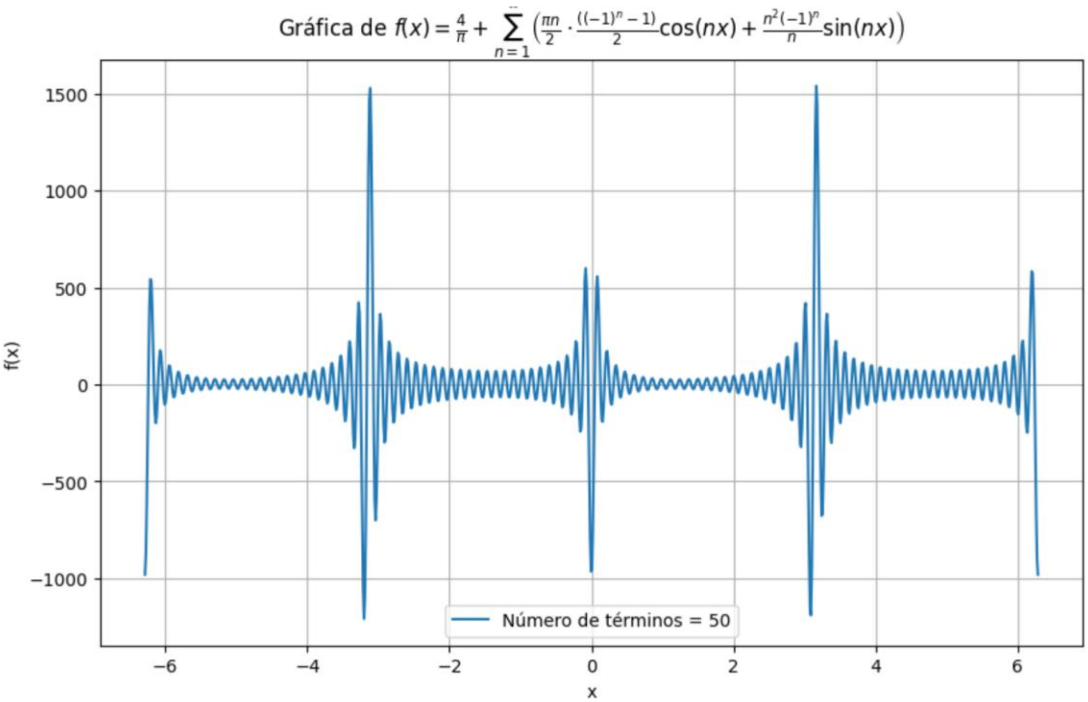
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\, sen(nx)\, dx$$

$$b_n = \frac{1}{\pi} \int_{0}^{\pi} x\, sen(nx)\, dx$$

$$\int x\, sen(nx)\, dx = \frac{-x\cos(nx)}{n} + \frac{sen(nx)}{n^2} \Big|_0^{\pi} =$$

$$= \frac{-\pi\cos(n\pi)}{n} + 0 = \frac{-\pi(-1)^n}{n} = \quad b_n = \frac{2(-1)^n}{n}$$

$$\boxed{F(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \left(\frac{2((-1)^n - 1)}{\pi n^2} \cos(nx) + \frac{2(-1)^n}{n} sen(nx) \right)}$$


Gráfica de $f(x) = \frac{4}{\pi} + \sum_{n=1}^{\infty} \left(\frac{\pi n}{2} \cdot \frac{((-1)^n - 1)}{2} \cos(nx) + \frac{n^2(-1)^n}{n} \sin(nx) \right)$
f(x)
1500
1000
500
0
−500
−1000
−6
−4
−2
0
2
4
6
x
Número de términos = 50

$$f(x)\begin{cases} x & -\frac{\pi}{2} \le x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x < \frac{3\pi}{2} \end{cases}$$

a_0

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\pi - x)\,dx\right)$$

$$a_0 = \frac{1}{2\pi}\left(\frac{x^2}{2}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \pi x\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} - \frac{x^2}{2}\Big|_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}\right)$$

$$a_0 = \frac{1}{2\pi}\left(\frac{\pi^2}{4} + \frac{\pi^2}{4}\right) = \frac{\pi}{4}$$

a_n

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$a_n = \frac{1}{\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos(nx)\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\pi - x\cos(nx)\,dx\right)$$

Integración por partes

$$\int x\cos(nx)\,dx = \frac{x\,\mathrm{sen}(nx)}{n} + \frac{\cos(nx)}{n^2}$$

$$\int (\pi - x)\cos(nx)\,dx = \pi\int \cos(nx)\,dx - \int x\cos(nx)\,dx$$

$$a_n = 0$$

b_n

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\mathrm{sen}(nx)\,dx$$

$$b_n = \frac{1}{\pi}\left(\int x\sin(nx)\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\pi - x)\,\mathrm{sen}(nx)\,dx\right)$$

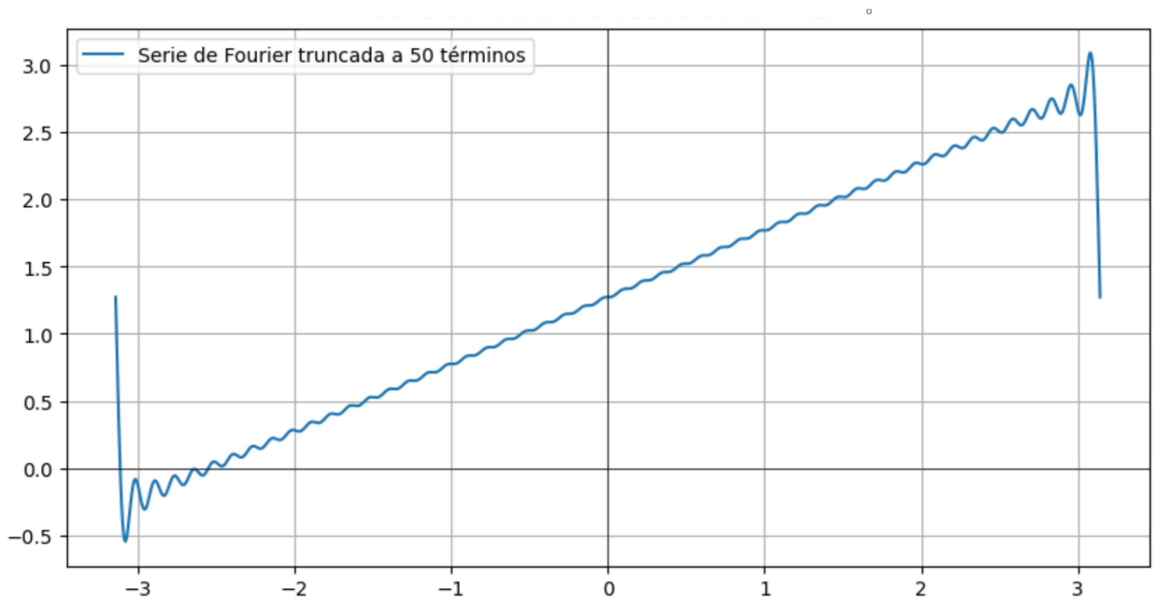
Integración x partes

$$\int x\,\mathrm{sen}(nx)\,dx = -\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}$$

$$\int (\pi - x)\sin(nx)\,dx = \pi\int \sin(nx)\,dx \int x\sin(nx)\,dx$$

$$b_n = \frac{2(-1)^{n+1}}{n}$$

$$F(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin(nx)$$



$$F(x) = \begin{cases} 1 & 0 \le x \le 2 \\ -1 & 2 \le x \le 4 \end{cases}$$

Parte a)

a_0

$$a_0 = \frac{1}{4} \int_0^4 F(x)\, dx$$

$$a_0 = \frac{1}{4} \left(\int_0^2 1\, dx + \int_2^4 -1\, dx \right)$$

$$a_0 = \frac{1}{4} (2 - 2) = 0$$

a_n

$$a_n = \frac{1}{2} \int_0^4 F(x) \cos \left(\frac{n \pi x}{2} \right) dx$$

$$a_n = \frac{1}{2} \left(\int_0^2 \cos \left(\frac{n \pi x}{2} \right) dx + \int_2^4 - \cos \left(\frac{n \pi x}{2} \right) dx \right)$$

$$a_n = \frac{1}{\pi n} \left[\operatorname{sen} \left(\frac{n \pi x}{2} \right) \Big|_0^2 - \operatorname{sen} \left(\frac{n \pi x}{2} \right) \Big|_2^4 \right]$$

$$a_n = \frac{2}{\pi n} \left[\operatorname{sen}(n\pi) - \operatorname{sen}(0) \right] = 0$$

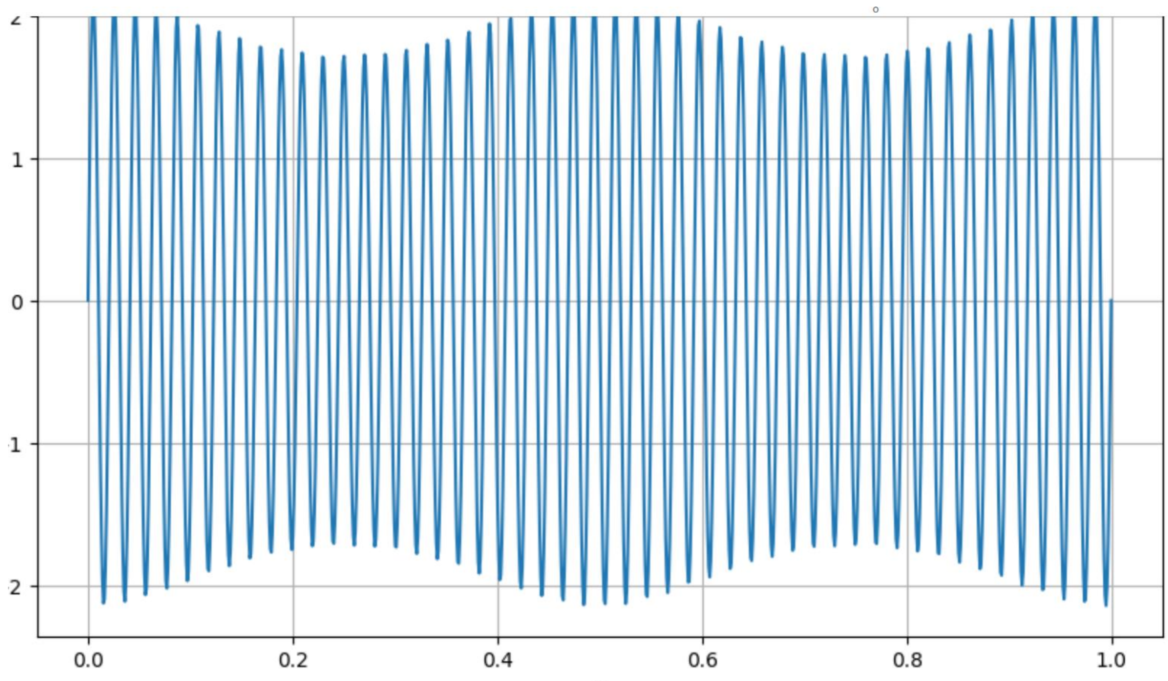
b_n

$$b_n = \frac{1}{2} \int_0^4 F(x) \operatorname{sen} \left(\frac{n \pi x}{2} \right) dx$$

$$b_n = \frac{1}{2} \left(\int_0^2 \operatorname{sen} \left(\frac{n \pi x}{2} \right) dx + \int_2^4 - \operatorname{sen} \left(\frac{n \pi x}{2} \right) dx \right)$$

$$b_n = \frac{-2}{\pi n} \left[1 - (-1)^n \right]$$

$$F(x) = \sum_{n=1}^{\infty} \frac{-2}{\pi n} \left(1 - (-1)^2 \right) \operatorname{sen} \left(\frac{n \pi x}{2} \right)$$


2
1
0
-1
-2
0.0
0.2
0.4
0.6
0.8
1.0

Parte b)

$$f(x) = \begin{cases} -x, & -2 \leq x < 0 \\ x & 0 \leq x < 2 \end{cases}$$

a_0

$$\frac{1}{4}\int_{-2}^{2} f(x)\,dx =$$

$$= \frac{1}{4}\left(\int_{2}^{0} -x\,dx + \int_{0}^{2} x\,dx\right) =$$

$$= \frac{1}{4}\left(\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{2}\right) = 0$$

a_n

$$a_n = \frac{1}{2}\int_{2}^{2} f(x)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$a_n = \frac{1}{2}\left(\int_{-2}^{0} -x\cos\left(\frac{n\pi x}{2}\right)dx + \int_{0}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx\right)$$

Integración por partes

$a_n = 0$

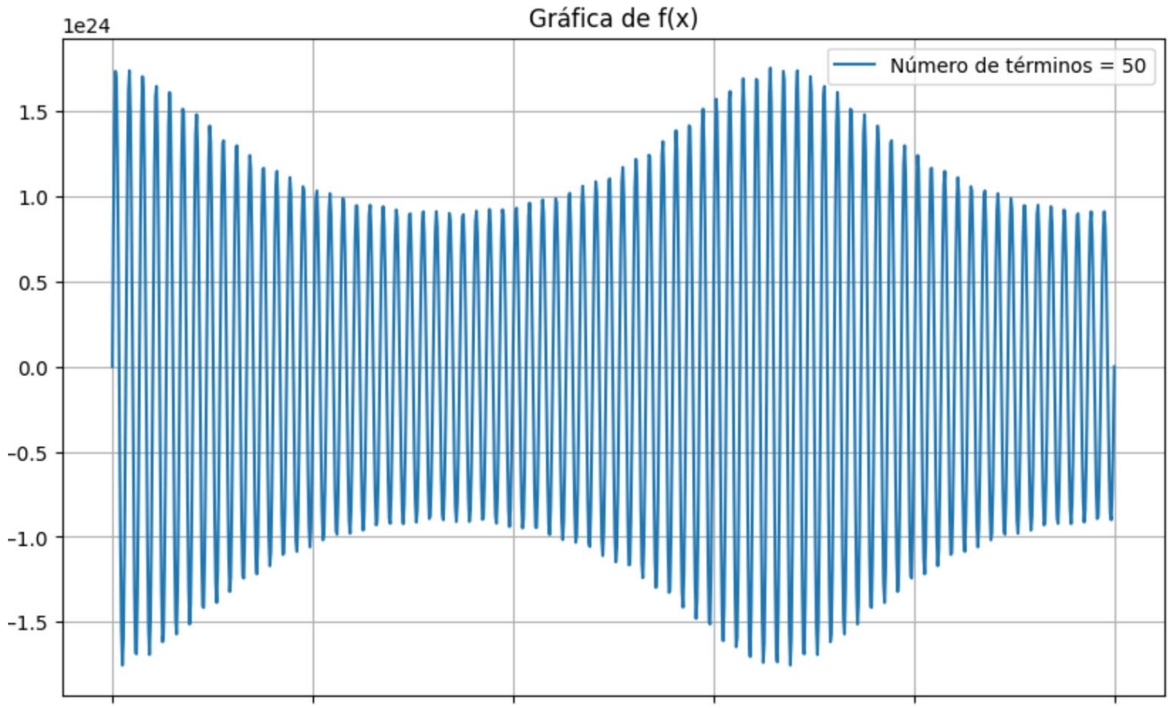
b_n

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\,\text{sen}\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \frac{1}{2}\left(\int_{-2}^{0} -x\,\text{sen}\left(\frac{n\pi x}{2}\right)dx + \int_{0}^{2} x\,\text{sen}\left(\frac{n\pi x}{2}\right)\right)dx$$

$$b_n = \frac{-2}{\pi n}\left(1-(-1)^n\right)$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{\pi n}\left(1-(-1)^n\right)\text{sen}\left(\frac{n\pi x}{2}\right)$$


Gráfica de f(x)
1e24
Número de términos = 50
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5

Parte C

$F(x) = x$ para $0 \leq x \leq 6$

a_0

$$a_0 = \frac{1}{6}\int_0^6 x \, dx =$$

$$= \frac{1}{6} \cdot \frac{x^2}{2}\Big|_0^6 = \frac{1}{6} \cdot 18 = 3$$

$$a_n = \frac{1}{3}\int_0^6 x \cos\left(\frac{n\pi x}{3}\right) dx$$

Integratió por partes

$$a_n = 0$$

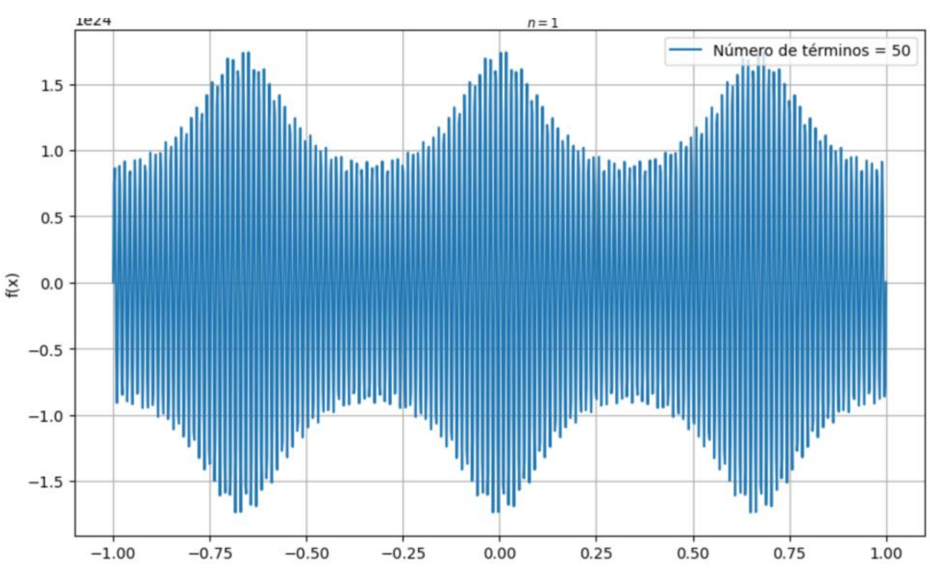
b_n

$$b_n = \frac{1}{3}\int_0^6 x \,\text{sen}\left(\frac{n\pi x}{3}\right) dx$$

Integración por partes

$$\boxed{b_n = \frac{6}{\pi n}}$$

$$F(x) = 3 + \sum_{n=1}^{\infty} \frac{6}{\pi n} \,\text{sen}\left(\frac{n\pi x}{3}\right)$$



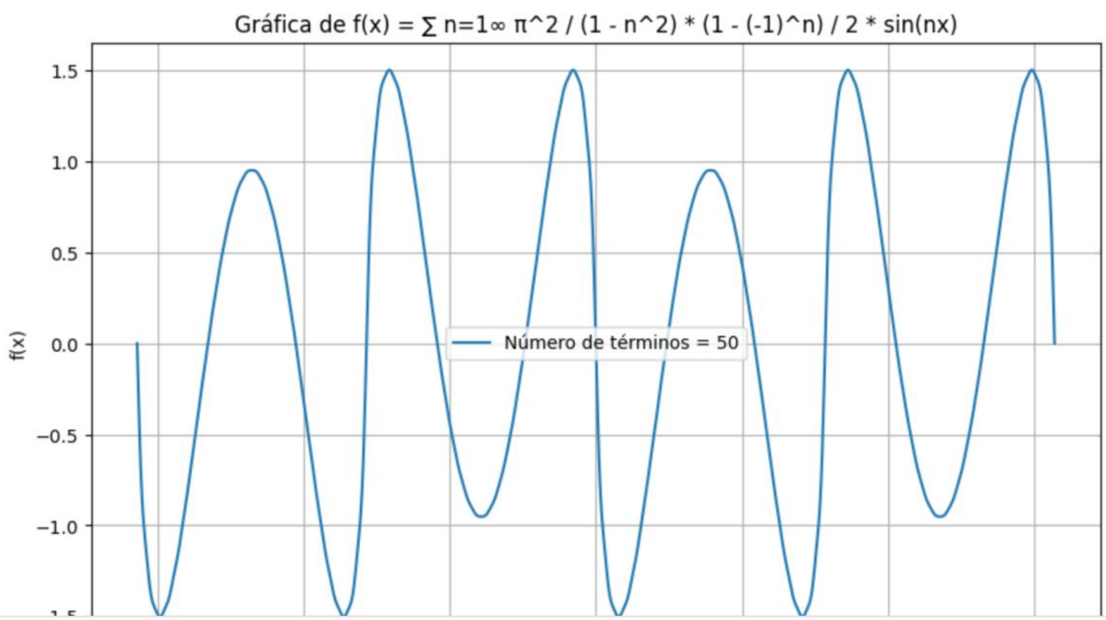
Desarrolla $f(x) = \cos(x)$ en una serie de Fourier de senos en $0 < x < \pi$

$$f(x) = \sum_{n=1}^{\infty} b_n \operatorname{sen}(nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} \cos(x) \operatorname{sen}(nx)\, dx$$

$$b_n = \frac{2}{\pi} \cdot \frac{1-(-1)^n}{1-n^2}$$

$$f(x) \sum_{n=1}^{\infty} \frac{2}{\pi} \cdot \frac{1-(-1)^n}{1-n^2} \operatorname{sen}(nx)$$



Desarrolla $f(x) = x$ en una serie de Fourier de medio rango en $0 < x < \pi$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$b_n = \frac{2}{n}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n} \sin(nx)$$