Multi-Robot Path Planning with Due Times

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Outline

Introduction: Multi-Robot Path Planning

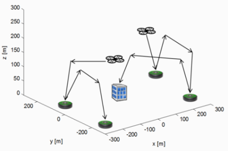
- multi-robot path planning (MRPP)[1] or multi-agent path finding (MAPF)[2] is an enabling research field for practical deployment of multi-robot systems

(a)

(b)

(c)

(d)



(e)

[1] Lynne E. Parker. "Multiple Mobile Robot Teams, Path Planning and Motion Coordination in". In: *Encyclopedia of Complexity and Systems Science* (2009), pp. 5783–5800. DOI: 10.1007/978-0-387-30440-3_344.

[2] Roni Stern. "Multi-Agent Path Finding – An Overview". In: *Artificial Intelligence*. Springer International Publishing, 2019, pp. 96–115. DOI: 10.1007/978-3-030-33274-7_6.

Introduction: Solutions

- MRPP or MAPF can be solved by various methodologies:
  - reduction-based (SAT, ILP, AST...)
  - search-based, optimal (CBS, ICBS, M*...)
  - search-based, bounded suboptimal (ECBS, CBS-HWY...)
  - search-based, unbounded suboptimal (HCA*,WHCA*,co-WHCA*...)
  - rule-based (Push&Swap, Push&Rotate, traffic rules...)
  - decoupled (prioritized planning, path coordination...)
  - sensor-based or reactive (ORCA...)
  - learning-based (PRIMAL,PRIMAL2,MAGAT...)
  - ...

Motivation

- in practice, multi-robot systems have to accomplish tasks with due times can be anticipated
- multi-robot path planning with time and distance-based objectives:[3]
  - makespan
  - total travel times
  - maximum distance
  - total distances
- multi-agent path finding with deadline-based objectives:[4]
  - the number of successful agents
- none of existing problem formulations could reflect soft temporal constraints of tasks[5]

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[3] Jingjin Yu and Steven M. LaValle. "Optimal Multirobot Path Planning on Graphs: Complete Algorithms and Effective Heuristics". In: *IEEE Transactions on Robotics* 32.5 (2016), pp. 1163–1177. ISSN: 15523098. DOI: 10.1109/TRO.2016.2593448. arXiv: 1507.03290.

[4] Hang Ma et al. "Multi-agent path finding with deadlines". In: *IJCAI International Joint Conference on Artificial Intelligence* 2018-July (2018), pp. 417–423. ISSN: 10450823.

[5] Ernesto Nunes et al. "A taxonomy for task allocation problems with temporal and ordering constraints". In: *Robotics and Autonomous Systems* 90 (2017), pp. 55–70. ISSN: 09218890. DOI: 10.1016/j.robot.2016.10.008.

Problem Formulation: MRPP-DT

- inputs:
  - underlying graph $G = (V, E)$
  - robot set R composed of n robots
  - start vertices V_s
  - goal vertices V_g
  - due times D
- outputs:
  - n feasible collision-free paths $P = \{p_1, \cdots, p_n\}$
- three due time-related objectives:
  - minimization of maximum lateness L_{max}
  - minimization of total tardiness $\sum_{i=1}^{n} H_i$
  - minimization of total unit penalties $\sum_{i=1}^{n} U_i$

Problem: Multi-Robot Path Planning with Due Times

Given (G, R, V_s, V_g, D) as is described previously, find an optimal solution $P = \{p_1, \cdots, p_n\}$ composed of n feasible collision-free paths for all robots with objective 1, 2, or 3.

Examples and Problem Analysis

Examples

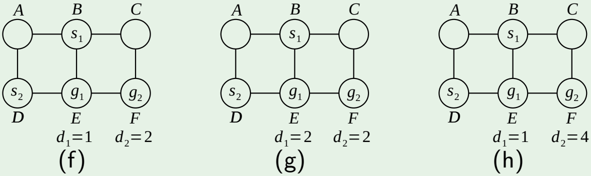


Figure: Three examples with the same start and goal vertices, but different due times. The underlying graph has 6 vertices.

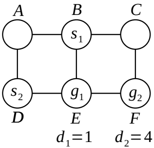
Intractrability

The MRPP-DT problem is NP-hard.

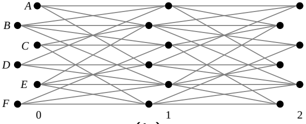
Parato Structure

Any two of seven objectives cannot be optimized simultaneously.

Reduction-based Algorithms



(a)



(b)

$$\min : l_{max}$$

$$\min : nT - \sum_{i=1}^{n} d_i - \sum_{i=1}^{n}\sum_{t=1}^{T} z_i^t y_i^t$$

$$\min : n - \sum_{i=1}^{n} \sum_{e_j \in \delta^-(v_t)} x_{i,j}, v_t \in \mathcal{S}^*$$

(c)

Completeness Theorem

MinMaxLat, MinTotalTard, MinTotalUP are all complete.

Instance Design

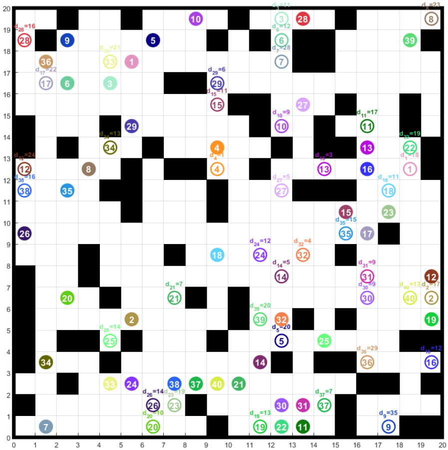


Figure: An MRPP-DT instance with 40 robots on a $20 \times 20 \times 20\%$ grid map.

Results of Objective 1

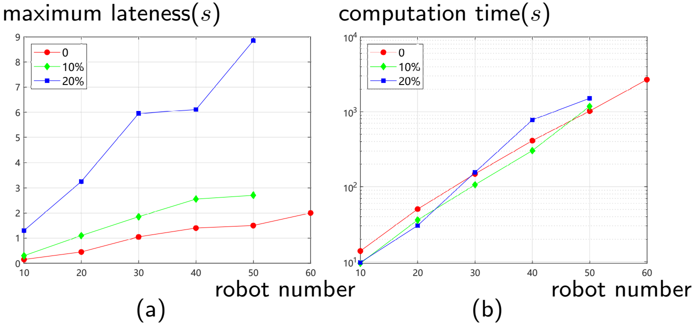


Figure: Performance of MinMaxLat over MRPP-DT instances.

Results of Objective 2

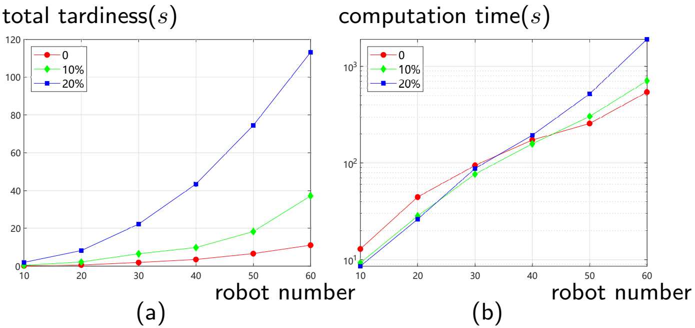


Figure: Performance of MinTotalTard over MRPP-DT instances.

Results of Objective 3

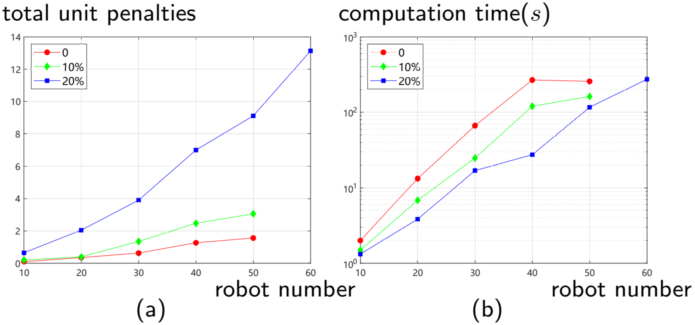


Figure: Performance of MinTotalUP over MRPP-DT instances.

Conclusions and Future Work

- conclusions:
  - MRPP-DT is formulated to address task awareness in path coordination level
  - complete reduction-based algorithms are developed
- future work:
  - develop fast algorithms
  - apply related ideas in practical applications, i.e., logistics
  - combined task and path planning

Key References

[1] Lynne E. Parker. "Multiple Mobile Robot Teams, Path Planning and Motion Coordination in". In: *Encyclopedia of Complexity and Systems Science* (2009), pp. 5783–5800. DOI: 10.1007/978-0-387-30440-3_344.

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