Exercise 9:Hypothesis Tests: Single Population

1. Write the null and alternative hypotheses for each of the following examples. Determine if each is a case of a two-tailed, a left-tailed, or a right-tailed test.
   a. To test if the mean amount of time spent per week watching sports on television by all adult men is different from 9.5 hours.
   b. To test if the mean amount of money spent by all customers at a supermarket is less than RM 105
   c. To test whether the mean starting salary of college graduates is higher than RM 39,000 per year.
   d. To test if the mean waiting time at the drive-through window at a fast-food restaurant during rush hour differs from 10 minutes.
   e. To test if the mean hours spent per week on house chores by all housewives is less than 30.

2. Find the *p*-value for each of the following hypothesis tests.
   a. $H_0 : \mu = 23, H_1 : \mu \neq 23, n = 50, \bar{x} = 21.25, \sigma = 5$
   b. $H_0 : \mu = 15, H_1 : \mu < 15, n = 80, \bar{x} = 13.25, \sigma = 5.5$
   c. $H_0 : \mu = 38, H_1 : \mu > 38, n = 35, \bar{x} = 40.25, \sigma = 7.2$

3. Consider $H_0 : \mu = 29$ versus $H_1 : \mu \neq 29$. A random sample of 25 observations taken from this population produced a sample mean of 25.3. The population is normally distributed with $\sigma = 8$.
   a. Calculate *p*-value.
   b. Considering the *p*-value of part a, would you reject the null hypothesis if the test were made at the significance level of .05?
   c. Considering the *p*-value of part a, would you reject the null hypothesis if the test were made at the significance level of .01?

4. For each of the following examples of tests of hypotheses about μ, show the rejection and non-rejection regions on the sampling distribution of the sample mean assuming that it is normal.
   a. A two-tailed test with $\alpha = .05$ and $n = 40$
   b. A left-tailed test with $\alpha = .01$ and $n = 20$
   c. A right-tailed test with $\alpha = .02$ and $n = 55$

5. Consider the null hypothesis $H_0 : \mu = 50$. Suppose a random sample of 24 observations is taken from a normally distributed population with $\sigma = 7$. Using $\alpha = .05$, show the rejection and nonrejection regions on the sampling distribution curve of the sample mean and find the critical value(s) of z when the alternative hypothesis is
   a. $H_1 : \mu < 50$
   b. $H_1 : \mu \neq 50$
   c. $H_1 : \mu > 50$

6. A consumer advocacy group suspects that a local supermarket's 10-ounce packages of cheddar cheese actually weigh less than 10 ounces. The group took a random sample of 20 such packages and found that the mean weight for the sample was 9.955 ounces. The population follows a normal distribution with the population standard deviation of .15 ounces. Find the p-value for the test of hypothesis with the alternative hypothesis that the mean weight of all such packages is less than 10 ounces. Will you reject the null hypothesis at α =.01?

7. Test the hypothesis of part a using the critical-value approach and α =.01. A study claims that all adults spend an average of 14 hours or less on chores during a weekend. A researcher wanted to check if this claim is true. A random sample of 200 adults taken by this researcher showed that these adults spend an average of 14.65 hours on chores during a weekend. The population standard deviation is known to be 3.0 hours.
   a. Find the p-value for the hypothesis test with the alternative hypothesis that all adults spend more than 14 hours on chores during a weekend. Will you reject the null hypothesis at α =.01?
   b. Test the hypothesis of part a using the critical-value approach and α =.02.

8. For each of the following examples of tests of hypotheses about μ, show the rejection and nonrejection regions on the ***t*** distribution curve.
   a. A two-tailed test with $\alpha = .02$ and n = 20
   b. A left-tailed test with $\alpha = .01$ and n = 16
   c. A right-tailed test with $\alpha = .05$ and n = 18

9. A random sample of 25 observations taken from a population that is normally distributed produced a sample mean of 58.5 and a standard deviation of 7.5. Find the ranges for the p-value and the critical and observed values of t for each of the following test of hypothesis using $\alpha = .01$.
   a. $H_0 : \mu = 55$ versus $H_1 : \mu > 55$
   b. $H_0 : \mu = 55$ versus $H_1 : \mu \neq 55$

10. The president of a university claims that the mean time spent parting by all students at this university is not more than 7 hours per week. A random sample of 40 students taken from this university showed that they spent an average of 9.50 hours partying the previous week with a standard deviation of 2.3 hours. Test at the 2.5% significance level whether the president's claim is true. Explain your conclusion in words.

11. A soft-drink manufacturer claims that its 12-ounce cans do not contain, on average, more than 30 calories. A random sample of 14 cans of this soft drink, which were checked for calories, contained a mean of 32 calories with a standard deviation of 3 calories. Does the sample information support the alternative hypothesis that the manufacturer's claim is false? Use a significance level of 5%. Find the range for the p-value for this test. What will your conclusion be using this p-value and $\alpha = .05$

Null hypothesis: A claim about a population parameter that is assumed to be true until proven otherwise.

Alternative hypothesis: A claim about a population parameter that will be true of the null hypothesis is false.

$\boldsymbol{\alpha}$: The significance level of a test of hypothesis that denotes the probability of rejecting a null hypothesis when it actually is true. (The probability of committing a Type I error).

$\boldsymbol{\beta}$**:** The probability of not rejecting a null hypothesis when it actually is false. (The probability of committing Type II error).

One-tailed test: A test in which there is only one rejection region, either in the left tail or in the right tail of the distribution curve.

Two-tailed test: A test in which there are two rejection regions, one in each tail of the distribution curve.

Left-tailed test: A test in which the rejection region lies in the left tail of the distribution curve.

Right-tailed test: A test in which the rejection region lies in the right tail of the distribution curve.

Significance level: The value of α that gives the probability of committing a Type I error.

Critical value or critical point (z_{cri} or t_{cri}): One or two values that divide the whole region under the sampling distribution of a sample statistic into rejection or non-rejection regions.

Observed value of z or t (z_{test} or t_{test}): The value of z or t calculated for a sample statistics such as the sample mean.

***p*-value**: The smallest significance level at which a null hypothesis can be rejected.