

GLOBAL STAR FORMATION REVISITED

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ABSTRACT

A general treatment of disk star formation is developed from a dissipative multiphase model, with the dominant dissipation due to cloud collisions. The Schmidt–Kennicutt (SK) law emerges naturally for star-forming disks and starbursts. We predict that there should be an inverse correlation between Tully–Fisher law and SK law residuals. The model is extended to include a multiphase treatment of supernova feedback that leads to a turbulent pressure-regulated generalization of the star formation law and is applicable to gas-rich starbursts. Enhanced pressure, as expected in merger-induced star formation, enhances star formation efficiency. An upper limit is derived for the disk star formation rate in starbursts that depends on the ratio of global ISM to cloud pressures. We extend these considerations to the case where the interstellar gas pressure in the inner galaxy is dominated by outflows from a central active galactic nucleus (AGN). During massive spheroid formation, AGN-driven winds trigger star formation, resulting in enhanced supernova feedback and outflows. The outflows are comparable to the AGN-boosted star formation rate and saturate in the super-Eddington limit. Downsizing of both SMBH and spheroids is a consequence of AGN-driven positive feedback. Bondi accretion feeds the central black hole with a specific accretion rate that is proportional to the black hole mass. AGN-enhanced star formation is mediated by turbulent pressure and relates spheroid star formation rate to black hole accretion rate. The relation between black hole mass and spheroid velocity dispersion has a coefficient (Salpeter time to gas consumption time ratio) that provides an arrow of time. Highly efficient, AGN-boosted star formation can occur at high redshift.

Key words: galaxies: ISM – galaxies: jets – galaxies: starburst – ISM: clouds – quasars: general – stars: formation

1. INTRODUCTION

Supernova feedback is considered to be a crucial element for negative feedback in star formation in disk galaxies. The star formation history in massive spheroids requires, according to the prevalent view, negative feedback from active galactic nucleus (AGN). Whether this is sufficient to explain the observed downsizing is far from clear. Here, we reassess the Schmidt–Kennicutt (SK) star formation law and develop a simple multiphase model in terms of the porosity formalism applied to disk galaxies (Silk 2001). We extend the model to incorporate AGN-triggered star formation and provide an application to spheroid formation and ultraluminous starbursts.

A cloud collision model of the SK law has been previously presented by Tan (1999), who uses galactic shear to compute the cloud collision rate. One advantage of this approach is that it provides a natural explanation for the low star formation rates observed in the outer parts of disk galaxies and complements an alternative explanation which appeals to UV background radiation-controlled H_2 suppression in the dust-deprived outer disk (Schaye 2004). We provide a simplified reformulation below, that we will apply in the context of a multiphase medium to incorporate star formation and supernova feedback (Section 2). In Section 3, we explore regulation of star formation by turbulent pressure and set an upper limit on the disk surface brightness due to star formation. Section 4 builds on the AGN feedback model (Silk 2005) and applies AGN triggering to star formation in protospheroids. Scaling laws are derived for the black hole growth rate and the star formation rate. Downsizing of both supermassive black holes and stellar mass is found to be a natural consequence of Bondi accretion-fed black holes and AGN-induced star formation.

2. DISK STAR FORMATION RATE: CLOUD COLLISION MODEL

Consider cloud collisions in the disk as a trigger of star formation. Cloud formation and collisions are driven by the non-axisymmetric gravitational instability of a cold self-gravitating gas-rich disk. Let a typical cloud have pressure p_{cl} and surface density Σ_{cl} . We expect star-forming clouds to be marginally self-gravitating and also to be confined by ambient gas pressure. Clouds form this way, and may be maintained if the cloud covering factor is of order unity, this condition guaranteeing that collisions occur on a local dynamical timescale. If the clouds are strongly bound, it is difficult to avoid a short lifetime, collapse, and star formation. Our description is a statistical one where we are assuming a steady-state ensemble of clouds although the clouds are being formed and reformed all the time in competition with cloud destruction and dispersal processes such as star formation and collisions. For typical parameters in the Galaxy, the disk crossing time normal to the disk and the cloud lifetimes are similar both of the order of 10 Myr, although we assume in general a statistically steady-state cloud population in this analysis.

The following relations then apply:

$$p_g = \rho_g \sigma_g^2 = \pi G \Sigma_g \Sigma_{tot}, \quad (1)$$

assuming equal scale-heights for clouds and stellar mass. If the clouds are self-gravitating, then $p_{cl}^{sg} = \pi \chi G \Sigma_{cl}^2$, where $\chi \sim 10$ is an estimate of the pressure enhancement due to self-gravity of interstellar clouds. We redefine $p_{cl}^{sg} = \chi p_{cl}$, and can now write $\Sigma_{cl} = (p_{cl}/p_g)^{1/2} (\Sigma_{tot}/\Sigma_g)^{1/2}$. The covering factor S_{cl} of clouds in the disk is directly inferred to be $S_{cl} = (\Sigma_g/\Sigma_{cl}) f_{cl}$, where f_{cl} is the gas fraction in clouds. We

rewrite this as $S_{\text{cl}} = f_{\text{cl}}(p_g/p_{\text{cl}})^{1/2}(\Sigma_g/\Sigma_{\text{tot}})^{1/2}$. Here, Σ_g is the total (cloud plus diffuse) gas surface mass density. The cloud collision timescale is $t_{\text{coll}} = (\Sigma_{\text{cl}}H)/(\Sigma_g f_{\text{cl}}\sigma_g)$, where the scale height $H^{-1} = (\pi G \Sigma_{\text{tot}})/\sigma_g^2$, and σ_g is the cloud velocity dispersion. The collision time can also be expressed as $t_{\text{coll}} = S_{\text{cl}}^{-1}t_{\text{cross}}$ with $t_{\text{cross}} = H/\sigma_g$, which becomes $t_{\text{coll}} = f_{\text{cl}}^{-1}(p_{\text{cl}}/p_g)^{1/2}(\Sigma_{\text{tot}}/\Sigma_g)^{1/2}(H/\sigma_g)$. More generally, inclusion of more realistic three-dimensional cloud kinematics (compare Tasker & Tan 2008) yields correction factors of order unity.

We now assume the disk star formation rate is self-regulated by supernova feedback which drives the cloud velocity dispersion. While this assumption has a long history (for example, see Firmani & Tutukov 1992), it remains controversial. Numerical simulations certainly demonstrate that supernovae provide negative feedback to star-forming clouds by driving turbulence (Joung & Mac Low 2006; Tasker & Bryan 2006; Koyama & Ostriker 2009a, 2009b; Kim & Ostriker 2007; Joung et al. 2008). Turbulent pressure plays an important role in regulating star formation, via controlling the porosity of supernova remnant-driven bubbles (Silk 2001) as well as the molecular hydrogen fraction (Blitz & Rosolowsky 2006). At the same time, global shear also plays a role in controlling cloud peculiar velocities, especially for massive clouds (Gammie et al. 1991). Since global gravitational instabilities ultimately drive cloud formation, and hence control star formation, the common origin of competitive turbulence drivers means that effects of shear and supernovae in self-regulating cloud turbulence are not easily separated in two-dimensional models (Shetty & Ostriker 2008). However, fully three-dimensional high-resolution models of self-consistent star-forming disks embedded within dark halos demonstrate that nonaxisymmetric gravitational instabilities dominate the observed turbulence of $\sim 10 \text{ km s}^{-1}$ at low star formation rates, but that supernova feedback will be important via the intermediary of the hot gaseous phase at a star formation rate in excess of $10^{-3} M_{\odot} \text{ kpc}^{-2} \text{ yr}^{-1}$ (Agertz et al. 2009; Tamburro et al. 2009).

Let m_{SN} be the mass in stars formed in order to result in a Type II supernova. This is just a function of the adopted initial mass function (IMF). Momentum balance gives

$$\dot{\Sigma}_*(E_{\text{SN}}/(m_{\text{SN}}v_c)) = f_c \Sigma_g \sigma_g / t_{\text{coll}}. \quad (2)$$

Here, f_c is the cloud volume filling factor, which can be expressed in terms of porosity Q as $f_c = e^{-Q}$. Also, E_{SN} is the kinetic energy of a SNe II and v_c is the velocity at the onset of strong cooling of the SNe II remnant. Canonical numbers used throughout are $m_{\text{SN}} = 150 M_{\odot}$ (for a Chabrier IMF) and $v_c = 400 \text{ km s}^{-1}$.

We can rewrite the star formation rate per unit volume as

$$\dot{\rho}_* = \epsilon_{\text{SN}} f_c f_{\text{cl}} \sqrt{G \rho_g \rho_g} \quad (3)$$

with $\epsilon_{\text{SN}} = (m_{\text{SN}}v_c\sigma_g)E_{\text{SN}}^{-1}(p_g/p_{\text{cl}})^{1/2}$. This formulation is commonly used as a star formation rate prescription in semianalytical modeling of galaxy formation. It may be more relevant to rewrite this formulation for disks:

$$\begin{aligned} \dot{\Sigma}_* &= f_c f_{\text{cl}} G (\pi \Sigma_{\text{tot}})^{1/2} \left(\frac{m_{\text{SN}}v_c}{E_{\text{SN}}} \right) [p_g/p_{\text{cl}}]^{1/2} \Sigma_g^{3/2} \\ &= \epsilon_{\text{SN}} f_c f_{\text{cl}} \sqrt{f_g(R/H)^{1/2} \Sigma_{\text{gas}}}, \end{aligned} \quad (4)$$

where $\Sigma_{\text{tot}} = \Sigma_g + \Sigma_*$. Here, the disk gas fraction is $f_g \sim 0.1$, and we use the disk scale-height-to-radius relation $H/R = (\sigma_g/v_r)^2$

for a disk rotating at v_r with $\Omega^2 = G \Sigma_{\text{tot}}/R$. Remarkably, although the preceding formula ignores the multiphase nature of the interstellar medium and the possibility of gas outflows (see below), one nevertheless manages to fit the SK relation.

We write the observed SK law as $\dot{\Sigma}_* = C_{\text{SK}} \Sigma_g^{3/2}$, and obtain the SK law coefficient

$$C_{\text{SK}} = \pi^{1/2} G f_c f_{\text{cl}} (m_{\text{SN}}v_c/E_{\text{SN}}) [p_g/p_{\text{cl}}]^{1/2} \Sigma_{\text{tot}}^{1/2}. \quad (5)$$

Inserting typical parameter values, we find that

$$\dot{\Sigma}_* \approx 0.02 \left(\frac{\epsilon_{\text{SN}}}{0.02} \right) \left(\frac{f_c}{0.3} \right) f_{\text{cl}} f_g \left(\frac{0.1R}{H} \right)^{1/2} \Sigma_g \Omega. \quad (6)$$

This demonstrates that we get the correct normalization at, say, 3 kpc, the scale length of the molecular gas in the Milky Way, where the scale-height is around 100 pc, the gas fraction is around 0.2, and the molecular gas covering fraction around 30%. The observed star formation efficiency in inner spiral disks is found to be fairly robust and for H_2 alone amounts to $5.25 \pm 2.510^{-10} \text{ yr}^{-1}$ (Leroy et al. 2008).

This compares well with the Kennicutt law, both locally and at $z \sim 2$ in shape ($\dot{\Sigma}_* \propto \Sigma_{\text{gas}}^{3/2}$) and in normalization (for $\epsilon_{\text{SN}} \approx 0.02$; Bouché et al. 2007). For the luminous starbursts at $z \sim 2$, the turbulence is enhanced ($\sigma_g \sim 40 \text{ km s}^{-1}$), but the scale height is thickened. One reason is that $\epsilon_{\text{SN}} \propto \sigma_g$ and $(R/H)^{1/2} \propto 1/\sigma_g$ for disks with varying amounts of turbulence, as might be induced by minor mergers. If the covering factor increases, it is not obvious if the star formation rate in a cloud collision model would increase. To lowest order, these effects all cancel at fixed Σ_{tot} , and we can hence understand how starbursts remain on the local SK law. Supernova feedback effectively keeps star formation inefficient.

Of course, we need to better understand how starbursts satisfy the same scaling law as quiescent disks. One hint is that while the gas velocity dispersion may vary in starbursts depending on the merging history, Σ_{tot} satisfies Freeman's law and is approximately constant for star-forming disk galaxies. The observed dispersion in the SK law may arise from the dispersion in total surface density and molecular as well as total gas fraction f_g .

2.1. Tully–Fisher Relation

The Tully–Fisher (TF) relation is also controlled by disk surface density. We use the empirical I -band TF relation: $L_* = C_{\text{TF}} v_r^\alpha$, with $\alpha \approx 4$ in the K band (Masters et al. 2008) and v_r the maximum rotation velocity, and where the virial theorem requires that $C_{\text{TF}} = (3/4\pi)G^{-2}\Sigma_{\text{tot}}^{-1}(L_*/M_{\text{tot}})$. We find using Equation (5) that

$$C_{\text{SK}} = \frac{3}{4} C_{\text{TF}}^{-1/2} f_c f_{\text{cl}} \left(\frac{m_{\text{SN}}v_c}{E_{\text{SN}}} \right) [p_g/p_{\text{cl}}]^{1/2} \left(\frac{L_*}{M_{\text{tot}}} \right)^{1/2}. \quad (7)$$

We infer that the SK law residuals should anticorrelate with the TF law residuals. The TF normalization is correct, by assumption: what is new is the predicted inverse correlation between SK and TF law residuals.

2.2. Gas-Dominated Disks

The global star formation law can be applied to regions that are gas-dominated. When gas dominates the self-gravity, the cloud collision model suggests that $\dot{\Sigma}_* \propto \Sigma_g^{3/2} \Sigma_{\text{tot}}^{1/2} \propto \Sigma_{\text{gas}}^2$, and the KS law steepens. There are indications of such a steepening in several environments.

1. The extended H I spiral structure in NGC 6946 (Boomsma 2007) shows that global gravitational instability is not a sufficient condition for forming stars. In the case of M83, the H I disk extends to more than twice the optical scale. Deep UV imaging reveals very low level star formation in the outer H I disk, well below the SK threshold. The cloud collision model provides a possible explanation of these phenomena, although the observed radial dependence of star formation rate is too steep to be explained by the simplest models (Leroy et al. 2008).
2. Individual young star complexes in M51 fall on the SK law, although with increased dispersion (Kennicutt et al. 2007) and a slightly steeper slope.
3. Damped Lyman alpha systems at $z \sim 2$ (Wolfe & Chen 2006) underproduce stars by up to a factor of 10 in star formation rate as predicted from the SK law.
4. Steepening also occurs in the inner regions of disks at extreme star formation rates. This is found in intensely star-forming galaxies at high redshift (Gao et al. 2007). Krumholz & Thompson (2007) account for the linear relation found for the local HCN data in terms of the critical density for excitation of the HCN transition in dense gas, that effectively samples only the densest molecular clouds and thereby bypasses the sensitivity to dynamical timescale.

Steepening in a cloud collision model is not a unique explanation for any of these phenomena. For example, the outer parts of disks are more thermally stable (Schaye 2004), and the star formation rate in DLAs could be suppressed because of the low H₂ content due to a combination of a low dust content plus a high radiation field.

3. PRESSURE-REGULATED STAR FORMATION AND STARBURSTS

Turbulent pressure-regulated star formation is especially likely to be important in starbursts. In disks, atomic cooling provides an effective thermostat for the turbulent velocity dispersion. Feedback operates via the hot phase venting into the halo. Gas may cool and fall back into the disk, as in the galactic fountain model, or escape in a wind, as happens for dwarf starburst galaxies. The simple porosity description of supernova feedback in a multiphase ISM (Silk 2001) provides an expression for the star formation rate in which porosity-driven turbulence is the controlling factor: $\dot{\rho}_* = Q m_{\text{SN}} (4\pi/3 R_a^3 t_a)^{-1}$, where the shell reaches a final size R_a before break-up, determined by the ambient pressure at expansion time t_a . The shell evolution is generally described by (Cioffi et al. 1988)

$$t = t_0 E_{51}^{3/14} n_g^{-4/7} (v_c/v)^{10/7} \quad (8)$$

and

$$R = R_0 E_{51}^{2/7} n_g^{-3/7} (t/t_0)^{3/10}, \quad (9)$$

where $v_0 = 413 \text{ km s}^{-1}$, $R_0 = 14 \text{ pc}$, $t_0 = 1.3 \times 10^4 \text{ yr}$. Here, cooling becomes significant at shell velocity $v_c = 413 E_{51}^{1/8} n_g^{1/4} \lambda^{3/8} \text{ km s}^{-1}$ where the cooling timescale within a SN-driven shell moving at velocity v_c is $t_c = v_c/\lambda\rho$,

$$\lambda^{-1} = 3m_p^{3/2} k^{1/2} T^{1/2} / \Lambda_{\text{eff}} \quad (10)$$

and $\Lambda_{\text{eff}}(T)$ is the effective cooling rate ($\propto t^{-1/2}$ over the relevant temperature range $10^4 < T < 10^6 \text{ K}$ associated with cooling shock velocities $< 100 \text{ km s}^{-1}$).

The SNR expansion is limited by the ambient turbulent pressure to be $\rho_g \sigma_g^2$, and we identify v_a (shell velocity at time t_a) with σ_g . We obtain

$$\dot{\rho}_* = Q \sqrt{G \rho_g} \rho_g (\sigma_g / \sigma_{\text{fid}})^{19/7}, \quad (11)$$

where

$$\sigma_{\text{fid}} = (c_0 G^{1/2} m_p^{3/2} v_0^{19/7} E_{51}^{62/49} m_{\text{SN}}^{-1})^{7/19} n_g^{-1/14} \quad (12)$$

$$\approx 20 n_g^{-1/14} m_{\text{SN},100}^{-0.37} E_{51}^{0.47} \text{ km s}^{-1} \quad (13)$$

and $c_0 = \frac{4\pi}{3} R_0^3 t_0$.

The dependence of star formation rate on turbulent velocity is reminiscent of Barnes's model for star formation in the Mice galaxies, an ongoing merger (Barnes 2004). A turbulence prescription is required to reproduce the observed spatially extended stellar distribution, which is inconsistent with the simple density-dependent SK law.

We apply the cloud collision model of Section 2 to compute Q . The star formation rates derived via porosity and via cloud collisions can be set equal. Comparison with the star formation rate derived from cloud collisions yields

$$Q = \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7} f_c f_{\text{cl}} \frac{m_{\text{SN}} v_c \sigma_{\text{fid}}}{E_{\text{SN}} \pi^{3/2}} \left(\frac{p_g}{p_{\text{cl}}} \right)^{1/2}, \quad (14)$$

then using $f_c = e^{-Q}$, we find

$$Q e^Q = f_{\text{cl}} \epsilon_{\text{SN},\text{fid}} \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7}. \quad (15)$$

This is an explicit expression for the porosity as a function of the turbulent velocity. The star formation efficiency is evaluated here at the fiducial velocity dispersion so that

$$\epsilon_{\text{SN},\text{fid}} = (m_{\text{SN}} v_c \sigma_{\text{fid}}) E_{\text{SN}}^{-1} (p_g / p_{\text{cl}})^{1/2}. \quad (16)$$

There are two regimes: $Q \ll 1$ and $Q \gg 1$, where approximate solutions can be found and the real solution joins them smoothly. For canonical values, with $p_{\text{cl}} \sim p_g$ and $\sigma_g \sim \sigma_{\text{fid}}$ the right-hand side of the above equation for Q is small (due to the 1% efficiency of star formation calculated previously) and is of the order of $\epsilon_{\text{SN},\text{fid}} \sim 10^{-2}$ and thus

$$Q \sim f_{\text{cl}} \epsilon_{\text{SN},\text{fid}} \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7}. \quad (17)$$

For completeness we give here the case where $Q \gg 1$, namely,

$$Q \sim \ln \left[f_{\text{cl}} \epsilon_{\text{SN},\text{fid}} \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7} \right]. \quad (18)$$

An approximate formula encompassing both regimes $Q \gg 1$ and $Q \ll 1$ is

$$Q \sim \ln \left[1 + f_{\text{cl}} \epsilon_{\text{SN},\text{fid}} \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7} \right]. \quad (19)$$

Generally, we find that the star formation efficiency is

$$\epsilon_{\text{SN}} = Q^{-7/12} e^{-7Q/12} (\epsilon_{\text{SN},\text{fid}})^{19/12} f_{\text{cl}}^{7/12}. \quad (20)$$

A desirable feature is that the star formation rate vanishes at very large Q , and becomes larger for small Q . There is no minimum in the star formation efficiency as a function of Q but the above features suggest that Q asymptotically becomes constant, of order unity, and self-regulation occurs since as Q exceeds unity it depends only logarithmically on the velocity dispersion. One can better understand why starbursts lie on the same SK law if we assume that local physics specifies the gas fraction converted into stars, in effect ϵ_{SN} , as in the model of Krumholz et al. (2006). This is plausible for individual molecular cloud complexes. A constant star formation fraction (equivalently, efficiency) is also expected globally in quiescent disks. Since v_{turb} self-regulates at $\sigma_g \sim 10 \text{ km s}^{-1}$, then if porosity also self-regulates, the efficiency or fraction of gas converted into stars per dynamical time is constant and small. Then, the higher turbulence in a starburst means that the porosity is low. In turn, low porosity guarantees inefficient feedback and runaway star formation.

In merger-driven starbursts, the porosity is small since $\epsilon_{\text{SN}} \approx \sigma_g$, whence $Q \propto \sigma_g^{-12/7}$. Small Q suggests a nuclear starburst, whereas large Q regulates global feedback and the star formation rate in a disk. This is complicated by the dependence of ϵ_{SN} on v_{turb} which is compensated by the increase in scale-height with enhanced turbulence.

In a quiescent star-forming disk galaxy, we might expect the porosity to self-regulate and be of order unity. This is the case, for example, for the Milky Way, where the supernova feedback regulates the global star formation rate. Even in this case, the star formation is not monotonically decreasing with time, as the SK law would suggest. Numerical simulations (Slyz et al. 2005) suggest that mini-starbursts occur stochastically, on a scale of the order of 1 kpc, with the mean global value decreasing as the gas supply is reduced. Observational evidence for a nonmonotonic star formation history in the solar neighborhood comes from surveys of chromospheric age indicators (Rocha-Pinto et al. 2000). Evidence for a series of starbursts is found in the inner disk star-forming regions of spiral galaxies (Allard et al. 2006).

Another aspect is the extreme pressure induced by gas dissipation. This must play a role in regulating star formation. An explicit case for pressure regulation is made by Blitz & Rosolowsky (2006) who propose a modified Kennicutt law: $\dot{\Sigma}_* \propto \Sigma_g p_g^{0.9}$. Our SN-regulated law using Equation (3) is

$$\dot{\Sigma}_* = G^{1/2} (m_{\text{SN}} v_c / E_{\text{SN}}) [p_g / p_{\text{cl}}]^{1/2} p_g^{1/2} \Sigma_g, \quad (21)$$

becoming

$$\dot{\Sigma}_* = \pi^{-1/2} m_{\text{SN}} v_c (E_{\text{SN}} \Sigma_{\text{cl}})^{-1} p_g \Sigma_g. \quad (22)$$

This is close to a pressure-regulated star formation law, if all star-forming clouds have a threshold column density.

3.1. Outflows

If the porosity is high, as may happen transiently in starbursts, we assume that disk outflows occur. These may be winds from dwarf galaxies or fountains in the case of more massive disks. Numerical simulations of star formation in the multiphase interstellar medium of a disk galaxy are able to model the disk outflows and global star formation history (Tasker & Bryan 2006). It is useful however to provide an analytic formulation. Let f_L be the hot gas loading factor. It is measured to be around 10 for the outflow from NGC 1569 (Martin 2005).

Now the outflow from the disk is

$$\dot{M}_{\text{out}} = (1 - e^{-Q}) f_L \dot{M}_* \approx Q f_L \dot{M}_*. \quad (23)$$

This tells us that $\dot{M}_{\text{out}} / \dot{M}_* = f_L Q \approx \sigma_g^{-1.7}$. Low porosity suppresses outflows so that outflows are suppressed in massive potential wells.

We can also express the outflow rate as

$$\dot{M}_{\text{out}} \approx Q^2 f_L (\sigma_g / \sigma_{\text{fid}})^{2.7} f_g^{1/2} M_g / t_d. \quad (24)$$

This shows that outflows could indeed occur from massive potential wells if porosity can somehow be maintained. We argue below that AGN-triggered star formation fulfills this role.

Outflows are important for dwarf galaxies, but are seen to be quenched in deep potential wells as well as at extreme porosity. If σ_g is high, the porosity is low, feedback is suppressed and supernova-driven winds are quenched. Even if σ_g is low, outflows may be suppressed if the gas is dense, leading to low porosity. Dwarf galaxy outflows play an important role in IGM enrichment at high redshift. The mass ejected during dwarf formation is comparable to the mass retained in stars formed. Nearby starbursts display this trend, suggesting that the effect may be generic to powerful starbursts in dwarf galaxies.

3.2. Upper Limit on the Disk Star Formation Rate

Meurer et al. (1997) and Hathi et al. (2008) report a bolometric upper limit on the disk surface brightness in starbursts of $2 \times 10^{11} L_{\odot} \text{ kpc}^{-2}$ over $0.1 \lesssim R_e \lesssim 10 \text{ kpc}$. We interpret this upper limit on disk luminosity for our model galaxy in terms of radiation pressure and mechanical pressure from limiting the gas surface density (compare Thompson et al. 2005).

To avoid lift-off via application of the Eddington condition requires:

1. for radiation pressure:

$$\Sigma_L < (\pi/2) G c \Sigma_{\text{tot}} \Sigma_g \quad (25)$$

or $\dot{\Sigma}_* < c \pi G f_l^{-1} \Sigma_{\text{tot}} \Sigma_g / 2$, where $f_l = \epsilon_l c^2 \approx 10^{-3} - 10^{-4} c^2$ for massive stars. Let us evaluate this expression after using the earlier expression for $\dot{\Sigma}_*$ to eliminate Σ_g . We find using Equation (25) that star formation is reduced at low gas surface density and lift-off can be avoided if

$$\left[\left(\frac{\epsilon_l m_{\text{SN}} c^2}{E_{\text{SN}}} \right) \left(\frac{v_c}{c} \right) \right]^2 \left[\frac{p_g}{p_{\text{cl}}} \right] \leq \Sigma_{\text{tot}} / \Sigma_g. \quad (26)$$

2. For mechanical energy input: we have instead no lift-off provided that

$$\left[\left(\frac{\epsilon_m m_{\text{SN}} c^2}{E_{\text{SN}}} \right) \left(\frac{v_c}{v_w} \right) \right]^2 \left[\frac{p_g}{p_{\text{cl}}} \right] \leq \frac{\Sigma_{\text{tot}}}{\Sigma_g}. \quad (27)$$

Here, the symbol ϵ_m denotes the efficiency of conversion of rest mass energy to mechanical energy in massive stars. For the lift-off limits to be reached the left-hand side of the above two equations must be greater than unity since $\Sigma_g \leq \Sigma_{\text{tot}}$. For the radiation case, unity is not normally reached on the left-hand side unless some of the parameters such as ϵ_l are significantly different from their normal values. This is what led Thompson et al. (2005) to argue for an AGN luminosity driving this Eddington limit for disks. For mechanical winds, the left side is boosted over the equivalent radiation driving term by a factor of $(c/v_w)^2$ making it reasonable that such mechanically driven

feedback occurs. For completeness and because of our poor knowledge of star formation at high redshift we will keep both the radiation and mechanical estimates in our calculations. This upper limit on Σ_g corresponds to an upper limit on global star formation of $45 M_\odot \text{ kpc}^{-2} \text{ yr}^{-1}$. By writing the disk surface brightness as $\Sigma_L = \epsilon_l c^2 \dot{\Sigma}_*$, we find using Equation (26) that the disk surface brightness satisfies

$$\Sigma_L < \left(\frac{\pi}{2}\right)^2 G c \Sigma_{\text{tot}}^2 \left[\frac{E_{\text{SN}}}{m_{\text{SN}} c^2 \epsilon_l} \left(\frac{c}{v_c}\right) \right]^2 \left(\frac{p_{\text{cl}}}{p_g}\right). \quad (28)$$

For mechanical input we have a similar equation using Equation (27)

$$\Sigma_L < \left(\frac{\pi}{2}\right)^2 G c \Sigma_{\text{tot}}^2 \left[\frac{E_{\text{SN}}}{m_{\text{SN}} c^2 \epsilon_m} \left(\frac{v_w}{v_c}\right) \right]^2 \left(\frac{p_{\text{cl}}}{p_g}\right). \quad (29)$$

We see that the absolute maximum global star formation-driven surface brightness, $\Sigma_{L,\text{max}}$, is

$$\Sigma_{L,\text{max}} = (\pi/2) G c \Sigma_{\text{tot}}^2. \quad (30)$$

This expression can be derived directly from the previous equation by setting $\Sigma_g = \Sigma_{\text{tot}}$. Note that in this case all the photon energy is used up to support the disk and the photon bubbles can occur with the signature of light and dark patches on the disk. This suggests that saturation of the star formation rate occurs at high surface density. Inserting typical numbers, $\Sigma_L < 10^{11} L_\odot \text{ kpc}^{-2} E_{51}^2 v_{c,400}^{-2} \epsilon_{n,-3}^{-2} \Sigma_{\text{tot},1000}^2$. Note that Komugi et al. (2005) report an offset roughly equivalent to an effective steepening of the SK law for ultra-luminous starbursts, as do Gao et al. (2007). We now find that for an extreme starburst at the Eddington luminosity with Thomson scattering, σ_T , as the dominant opacity the relation between surface brightness and mass surface density is

$$\Sigma_L = 4\pi G c \Sigma_{\text{tot}} \left(\frac{m_p}{\sigma_T}\right), \quad (31)$$

where m_p is the proton mass. From Equations (28) and (29) we deduce that the total surface density, which is essentially all gas in initial situations, satisfies

$$\begin{aligned} \Sigma_{\text{tot}} > \Sigma_* &\equiv \left(\frac{16}{\pi}\right) \left[\frac{m_{\text{SN}} \epsilon_l c^2}{E_{\text{SN}}} \left(\frac{v_c}{c}\right) \right]^2 \left(\frac{p_g}{p_{\text{cl}}}\right) \left(\frac{m_p}{\sigma_T}\right) \\ &\sim 10^3 M_\odot \text{ pc}^{-2}, \end{aligned} \quad (32)$$

and similarly for mechanical input

$$\Sigma_{\text{tot}} > \Sigma_* \equiv \left(\frac{16}{\pi}\right) \left[\frac{m_{\text{SN}} \epsilon_m c^2}{E_{\text{SN}}} \left(\frac{v_w}{v_c}\right) \right]^2 \left(\frac{p_g}{p_{\text{cl}}}\right) \left(\frac{m_p}{\sigma_T}\right). \quad (33)$$

A direct and simple way to derive these limits is to take Equations (31) and (26) and find that

$$\Sigma_{\text{tot}} \geq 8 \left(\frac{m_p}{\sigma_T}\right) \sim 4 \times 10^5 M_\odot \text{ pc}^{-2} \quad (34)$$

which is a derivation of Fish's Law. The densest galactic molecular clouds, traced by H_2O masers, have a similar surface density (Plume et al. 1997). By $z \sim 2$, there is

a substantial population of galaxies with star formation rates of $500\text{--}1000 M_\odot \text{ yr}^{-1}$, ultraluminous infrared galaxies (ULIRGs) and submillimeter galaxies (SMGs). A recent study of SMGs at 0.5 arcsec resolution (Tacconi et al. 2008) finds that they are gas-rich (molecular gas fraction $\sim 30\%$) with the gas in compact disks at a density of $\sim 10^4 M_\odot \text{ pc}^{-2}$, and are undergoing major mergers.

4. APPLICATION TO ACTIVE GALACTIC NUCLEI

AGN outflows overpressure the interstellar medium. They can deplete the gaseous environment by driving a wind. AGN outflows are the principal element in semianalytical modeling of massive ellipticals that helps quench recent star formation. The energetics are as follows: the specific energy per baryon from supernovae is $E_{\text{SN}}/m_{\text{SN}} \approx 10^{-5} c^2 \text{ erg gm}^{-1}$, whereas AGN outflows provide $\sim 10^{-4} c^2 \text{ erg gm}^{-1}$ per unit spheroid mass, for an assumed efficiency of 0.1 and a supermassive black hole (SMBH)-to-spheroid mass ratio of 10^{-3} . We argue below that AGN outflows have global impact by driving overpressurized cocoons into the inhomogeneous ISM.

Useful formulae are as follows.

1. The Eddington luminosity

$$L_{\text{Edd}} = 4\pi c G M_{\text{BH}} m_p / \sigma_T, \quad (35)$$

2. the Salpeter timescale

$$t_S = \eta c^2 M_{\text{BH}} / L_{\text{Edd}} = \eta \sigma_T c (4\pi G m_p)^{-1}, \quad (36)$$

and

3. the self-regulating feedback mass

$$M_{\text{SR}} = f_g \frac{\sigma_T}{m_p} \frac{\sigma^4}{\pi G^2}. \quad (37)$$

Blow-out by radiation pressure occurs (assuming a homogeneous interstellar medium) if $L = L_{\text{Edd}}$ at $M_{\text{BH}} = M_{\text{SR}}$ (Silk & Rees 1998; Fabian 1999). This will lead to a wind, deplete the baryon reservoir, and quench black hole growth by gas accretion. With $f_g \sim 0.1$, as expected initially in the protogalactic core, this simple relation fits the mean of the observed relation over 3 orders of magnitude in black hole mass. Numerical simulations generally confirm these simplistic estimates. We further define a critical AGN luminosity L_{SR} for star formation-boosted AGN outflow by the Eddington luminosity associated with the critical black hole mass M_{SR} that corresponds to the balance between Eddington luminosity and protospheroid self-gravity,

$$L_{\text{SR}} = L_{\text{Edd}} M_{\text{SR}} / M_{\text{BH}} = 4\sigma^4 c / (G f_g). \quad (38)$$

4.1. AGN Triggering of Star Formation

Negative feedback helps account for the black hole mass- σ correlation (Di Matteo et al. 2008) and for the luminosity function of massive galaxies (Bower et al. 2006; Croton et al. 2006). More physics must be added however to account for downsizing and efficient star formation in massive galaxies. The key may be AGN outflows that can trigger star formation by compressing dense clouds. These would precede the outflow phase which in this case is due to the combined effect of AGN outflow and triggered SNe. A prior phase of positive feedback is a possible new ingredient in feedback modeling and is motivated by evidence (admittedly sparse but compelling: see, e.g., Feain et al. 2007) for AGN triggering of star formation.

The following model is necessarily schematic pending fully three-dimensional simulations of jet propagation into a clumpy protogalactic interstellar medium. We speculate that the triggering works as follows. Jet propagation into a clumpy medium develops into an expanding, overpressurized cocoon (Saxton et al. 2005; Antonuccio-Delogu & Silk 2008). This adds a potentially large multiplier to the efficacy of the BH-driven outflow for the following reason. The jet-driven plasma-filled radio lobe drives a cocoon that expands into the hot virialized component of the protogalaxy at a speed v_{co} that is much larger than the velocity field associated with the gravitational potential well. Protogalactic clouds that are above or near the Jeans mass will be induced to collapse. The cloud overpressuring and resulting triggered star formation occurs at a rate much larger, by 1–2 orders of magnitude, than is associated with normal gravitationally driven fueling, as would be appropriate to a star-forming disk galaxy. We show below that the star formation rate enhancement amount to a factor $\sim v_{\text{co}}/\sigma$. Hence central massive black holes that have not yet grown by accretion to the limiting mass controlled by radiation outflow should still have a considerable impact on the evolution of the protogalaxy core. Incidentally, this early phase of black hole feedback makes observational confirmation difficult, as discussed below.

Numerical studies of radiative shock-induced cloud collapse reveal the complex interplay of hot and cold gas (Mellema et al. 2002; Fragile et al. 2004). Here, we focus on the implications for star formation via analytic considerations. For a simple ansatz, suppose that the triggered star formation occurs over the cocoon propagation time, t_{co} that is much less than the dynamical time, t_d , $t_{\text{co}} \ll t_d$. We show below that jet triggering of the cocoon expansion gives a star formation rate enhancement factor $t_d/t_{\text{co}} \sim v_{\text{co}}/\sigma$.

Although we shall generally assume v_{co} is a parameter of the wind, consider the case in which the injected momentum flux is L/c for a wind that is mechanically driven but is originated via radiation pressure. We expect the mechanical jet luminosity to be a fraction $v_w/c \sim 0.1$ of the luminosity. More precisely, the ratio $(L_{\text{mech}}/v_w)/(L/c)$ is equal to the optical depth τ , due to a combination of Thomson scattering, line opacity and/or dust. To estimate τ , an effective Rosseland mean opacity can be defined from these opacity contributions. This is valid if the jet is radiation pressure-driven. Now

$$v_{\text{co}} = \sigma \tau^{1/2} f_g^{-1/2} \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/2}, \quad (39)$$

where we have set $\rho_g = \sigma^2/2\pi Gr^2$. The previously derived star formation rate can be generalized for round systems with velocity dispersion σ to

$$\dot{M}_* = (\epsilon_{\text{SN}}/\sigma) f_c f_{\text{cl}} M_g (G p_g)^{1/2}. \quad (40)$$

Thus, if the gas pressure is replaced by the AGN-driven pressure, we can see how a central AGN can boost the star formation rate by writing the pressure as

$$p_{\text{AGN}} = L_{\text{AGN}} 4\pi v_{\text{co}} r^{2-1}. \quad (41)$$

The star formation rate is now given generally using Equations (40) and (41) by

$$\dot{M}_*^{\text{AGN}} \approx \left[f_c f_{\text{cl}} \epsilon_{\text{SN}} \left[\frac{M_g}{t_d} \right] \right] f_g^{-1/2} \left(\frac{c}{V_{\text{AGN}}} \right)^{1/2} \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/2}. \quad (42)$$

Here, we have left V_{AGN} as an arbitrary variable to be applied to the appropriate situation such as jet, wind, outflow, etc. In the radiation-driven case discussed above $V_{\text{AGN}} = v_{\text{co}}$, the cocoon velocity, and using Equations (39) and (42) we find

$$\dot{M}_*^{\text{AGN}} \approx \left[f_c f_{\text{cl}} \epsilon_{\text{SN}} \left[\frac{M_g}{t_d} \right] \right] f_g^{1/4} \tau^{-1/4} \left(\frac{c}{\sigma} \right)^{1/4} \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/4}. \quad (43)$$

The AGN-driven enhancement factor is $(p_{\text{AGN}}/p_g)^{1/2} \approx (v_{\text{co}}/\sigma) \tau^{1/2}$. Note that $\epsilon_{\text{SN}} \propto \sigma$, so that the star formation efficiency coefficient (fraction of stars formed per dynamical time) is boosted considerably for spheroids relative to disks. The AGN luminosity explicitly drives star formation. It is the AGN-triggered star formation multiplier rather than the AGN itself that drives the feedback. The boost effect is generally important in the innermost spheroid, and globally important for super-Eddington AGN luminosities.

Note that we must be careful that the AGN pressure does not blow away the ISM completely. For a SNe-regulated ISM, the pressure relates to Q and includes details of the SNe bubble evolution. But when the AGN jet provides the pressure it will overpressure the clouds and turn them into stars with some efficiency. We need to relate the star formation from the triggered clouds to the AGN accretion rate and this would then affect the jet luminosity and pressure. Essentially, we raise the external pressure (here using the AGN) on the star-forming clouds. This allows us to regulate Q for the SN bubbles. If the porosity is maintained to be constant by the triggering of massive star formation, we have achieved the desired self-regulation between AGN activity and star formation. If Q is given by other means (and is of order unity), then star formation-regulated feedback follows naturally. To show this point explicitly in our formulation of AGN-driven star formation we return to the pressure-regulated model of the structure of the ISM, and instead of calculating the final size of SNeII-driven remnant bubbles by setting the remnant's velocity, v_a equal to σ_g we set the external pressure $\rho_g \sigma_g^2$ equal to the mechanical pressure driven by the AGN, $L/4\pi r^2 v_{\text{co}}$. Assuming, as above, that $\rho = \sigma_g^2/Gr^2$ with an associated pressure $p_g = \sigma_g^4/Gr^2$ and replacing this with the AGN pressure, p_{AGN} given by $p_{\text{AGN}} = L_{\text{AGN}}/4\pi V_{\text{AGN}} r^2$ we find that the required transformation to replace gas pressure with AGN pressure is

$$\left(\frac{\sigma_g}{\sigma_{\text{fid}}} \right) = \left(\frac{c}{V_{\text{AGN}}} \right)^{1/4} \left(\frac{L_{\text{AGN}}}{L_{\text{SR, fid}}} \right)^{1/4}. \quad (44)$$

It then follows directly that

$$Q e^Q = f_{\text{cl}} \epsilon_{\text{SN, fid}} \left(\frac{V_{\text{AGN}}}{c} \right)^{1/4} \left(\frac{L_{\text{SR, fid}}}{L_{\text{AGN}}} \right)^{3/7}, \quad (45)$$

and for $Q \ll 1$ we find

$$Q \sim f_{\text{cl}} \epsilon_{\text{SN, fid}} \left(\frac{V_{\text{AGN}}}{c} \right)^{1/4} \left(\frac{L_{\text{SR, fid}}}{L_{\text{AGN}}} \right)^{3/7}. \quad (46)$$

For $Q \gg 1$

$$Q \sim \ln \left[f_{\text{cl}} \epsilon_{\text{SN, fid}} \left(\frac{V_{\text{AGN}}}{c} \right)^{1/4} \left(\frac{L_{\text{SR, fid}}}{L_{\text{AGN}}} \right)^{3/7} \right], \quad (47)$$

with a good fit to the range $Q \ll 1$ up to $Q \gg 1$ given by

$$Q \sim \ln \left[1 + f_{\text{cl}} \epsilon_{\text{SN, fid}} \left(\frac{V_{\text{AGN}}}{c} \right)^{1/4} \left(\frac{L_{\text{SR, fid}}}{L_{\text{AGN}}} \right)^{3/7} \right]. \quad (48)$$

4.2. Cocoon Overpressure and the Bonnor–Ebert Condition

Following the simple cocoon model of Begelman & Cioffi (1989), we examine the effect of the power flow in the canonical two bidirectional jets diverted into a small nuclear cocoon and then via the cocoon pressure acting back on the central nucleus where gravitational instability and enhanced collapse and accretion may occur if the Bonnor–Ebert (BE) critical pressure is reached. For an isothermal gas distribution with velocity dispersion σ as used throughout this paper, a jet opening angle Θ_J , an approximately ellipsoidal cocoon with axes a and b with $a > b$ for simplicity powered by two thin jets with luminosity, L_J , we find that the cocoon pressure is given by

$$P_{\text{co}} = \left(\frac{L_J \sigma^2 v_J \Theta_J}{G} \right)^{1/2} \left(\frac{a}{b} \right)^2 \quad (49)$$

and the ratio of the cocoon pressure to the gas pressure in the central region (putting $a \sim b$) for simplicity is

$$\frac{P_{\text{CO}}}{P_g} = \left(\frac{L_J}{L_{\text{SR}}} \right)^{1/2} \left(\frac{v_J c}{\sigma^2} \right)^{1/2} \Theta_J^{1/2}. \quad (50)$$

Now for the central isothermal core of gas pressured by the cocoon pressure, the ratio of the BE mass to the core mass is given by

$$\frac{M_{\text{BE}}}{M} = 1.18 \left(\frac{L_J}{L_{\text{SR}}} \right)^{-1/4} \left(\frac{v_J c}{\sigma^2} \right)^{-1/4} \Theta_J^{-1/4}. \quad (51)$$

Thus, if $v_J \sim 0.1c$, the core can be over-pressured by the cocoon if $\Theta_J > 10(\sigma/c) \sim 10^{-2}$ when $L_J \sim L_{\text{SR}}$. This may be another way to look at the feedback process involving the growth of black holes along the $M_{\text{BH}}-\sigma$ line since at the Eddington luminosity $L_J \sim L_{\text{SR}}$ implies $M \sim M_{\text{SR}}$.

4.3. AGN Winds

The global mass loss for AGN-driven wind is given by

$$\dot{M}_{\text{out}}^{\text{gal}} = \frac{L_{\text{AGN}}}{V_{\text{AGN}}^2} \quad (52)$$

and for the radiation-driven AGN-generated wind case

$$\dot{M}_{\text{out}}^{\text{gal}} = \frac{L_{\text{AGN}}}{c v_{\text{co}}} \approx \left(\frac{\sigma_g^3}{G} \right) \tau^{-1/2} \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/2}. \quad (53)$$

The outflow rate is proportional to the spheroid velocity dispersion and to the square root of the AGN luminosity. The scaling of the outflow rate with regard to AGN luminosity implies that outflows saturate. It may be compared with observations of broadened features that demonstrate the presence of massive winds in ultraluminous star-forming infrared and radio galaxies.

Making use of the AGN-enhanced star formation rate, we can express the outflow as a ratio using Equations (42) and (53) generally as

$$\frac{\dot{M}_{\text{out}}^{\text{gal}}}{\dot{M}_{*}^{\text{AGN}}} = \left(\frac{f_g^{1/2}}{f_c f_{\text{cl}} \epsilon_{\text{SN}} (c/\sigma)} \right) \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/2} \left(\frac{c}{V_{\text{AGN}}} \right)^{3/2}. \quad (54)$$

It follows that the outflow rate from AGN is always of the order of the AGN-boosted star formation rate for the protospheroid. We may also compare the star formation-boosted global outflow

rate with the AGN outflow. The AGN mass outflow rate is $\eta c \dot{M}_{\text{acc}}/(v_{\text{out}})$, with $v_{\text{AGN}} \sim 0.1c$. Hence it is of the order of the SMBH accretion rate. As expected, the mass flux associated with the galaxy outflow dominates that from the AGN. The mass flux ratio is

$$\frac{\dot{M}_{\text{out}}^{\text{gal}}}{\dot{M}_{\text{acc}}} = \eta f_w^{-1} \left(\frac{c}{\sigma} \right) \left(\frac{L_{\text{SR}}}{L_{\text{AGN}}} \right)^{1/2}. \quad (55)$$

Hence $\dot{M}_{\text{out}}^{\text{gal}}/\dot{M}_{\text{acc}} \sim 100$ for AGN at the Eddington luminosity and $\kappa \sim 10^3 \sigma_T/m_p$. In order to allow for dust, if a factor τ^{-1} is incorporated into the definition of L_{SR} , this ratio is seen to be inversely proportional to the square root of the adopted (dust) opacity. The momentum flux ratio is

$$\frac{\dot{M}_{\text{out}}^{\text{gal}} \sigma}{\dot{M}_{\text{acc}} v_{\text{AGN}}} = \eta f_w^{-1} \frac{c}{v_{\text{AGN}}} \left(\frac{L_{\text{SR}}}{L_{\text{AGN}}} \right)^{1/2}. \quad (56)$$

We see that the momentum ejected from the AGN dominates over that in the global outflow by a factor of a few for AGN near the Eddington luminosity $L_{\text{AGN}} \sim L_{\text{SR}}$.

4.4. Downsizing

The piston model enables downsizing of AGN and spheroids by coupling their growth. For some fiducial AGN energy conversion efficiency η (~ 0.1), we note that $L^{\text{AGN}} = \eta c^2 \dot{M}_{\text{acc}}$ is a measure of the BH accretion rate. Since L^{AGN} controls the star formation rate and is itself controlled by the black hole accretion rate, we infer that black hole growth and star formation triggering downsize together, provided Q is approximately constant due to AGN triggering of SN. The AGN driving of star formation overcomes the pressure suppression of porosity in the absence of the AGN. A large porosity also results in a wind. The required turbulent velocity field controls the accretion rate and might be specified by other physics, such as a merger, or even be due to the AGN itself. Let us try to make these assertions more quantitative.

The AGN is the ultimate driver of the porosity. We need to connect AGN-induced star formation and outflows to the black hole growth rate via the AGN luminosity. The global outflow rate is

$$\dot{M}_{\text{out}}^{\text{gal}} = Q f_L \epsilon_{\text{SN}} M_g / t_d. \quad (57)$$

By momentum conservation, this must equal the global AGN-boosted outflow rate $L^{\text{AGN}}/(c v_c)$.

4.4.1. Downsizing for Porosity-Regulated Star Formation

Incorporating the effects of porosity-driven star formation means that the outflows must satisfy

$$\dot{M}_{\text{out}}^{\text{gal}} = Q^2 f_L (\sigma_g / \sigma_{\text{fid}})^{2.7} f_g^{-1/2} \sigma_g^3 / G. \quad (58)$$

The AGN luminosity is controlled by the accretion rate onto the central black hole, \dot{M}_{acc} . Our next step is to evaluate the black hole growth rate, \dot{M}_{acc} . This is the key to explaining downsizing.

To reproduce the downsizing phenomenon, observed for AGN (Hasinger et al. 2005) and their massive host galaxies (Kriek et al. 2007) to occur almost coevally, we need to understand why massive SMBH and spheroids form before their less massive counterparts. The required scaling for L^{AGN} or \dot{M}_{acc} is reminiscent of the scaling found for protostellar jets. The magnetically regulated disk phenomenon plausibly obeys a universal scaling law that could equally apply to jets and

outflows from disks around SMBH. The protostellar scaling is (Mohanty et al. 2005) $\dot{M}_{\text{acc}} \propto M^2$. Allen et al. (2006) find that for the black holes that power the AGNs in massive ellipticals, the Bondi accretion rate is approximately proportional to the jet power. The connection with outflows and jets that are magnetically guided by the wound-up field in the accretion disk proposed by Banerjee & Pudritz (2006) is a generic scaling in their study of protostellar jets, $\dot{M}_{\text{wind}}^{\text{AGN}} = f_w \dot{M}_{\text{acc}}$, with $f_w \sim 0.1$, for outflows associated with central objects that range from brown dwarfs to supermassive black holes.

The Bondi accretion formula therefore regulates SMBH growth and, implicitly, outflow. It yields $\dot{M}_{\text{BH}} = \pi G^2 (p_g/\sigma^5) M_{\text{BH}}^2$. A simple interpretation of this scaling is that for Bondi accretion,

$$\dot{M}_{\text{out}}/f_w = \dot{M}_{\text{acc}} = 4\pi(GM/\sigma^2)^2 \rho v \propto (\rho/\sigma^3) M^2, \quad (59)$$

in combination with adiabatic compression, so that $\rho \propto \sigma^3$. For the AGN case, we write $\dot{M}_{\text{acc}} = \alpha \dot{M}_{\text{BH}}^2$ with $\alpha \propto G^2 \rho/\sigma^3$. Rewriting this we see that

$$\frac{dM_{\text{BH}}}{dM_*} = Q f_L f_w^{-1} \quad (60)$$

Therefore if Q reaches a self-regulating constant and f_L and f_w are also constant then the black hole and galaxy growth move together on a fixed trajectory in the Magorrian plane as discussed below. *It is this fixed trajectory that forces downsizing.*

4.4.2. Downsizing for SN Energy Injection

Substituting further the relevant quantities for the case of SN energy input we find for the case without AGN feedback

$$\frac{d \ln M_{\text{BH}}}{d \ln M_*} = f_g^{1/2} \left(\frac{\bar{t}_S}{t_d} \right) = (G f_g \rho \bar{t}_S^2), \quad (61)$$

where

$$\bar{t}_S = \left(\frac{t_S \sigma}{\eta_{\text{SNC}} f_c f_{\text{cl}}} \right) = \beta t_S. \quad (62)$$

Thus the critical parameter determining the logarithmic slope in the Magorrian plane is \bar{t}_S . There is a critical density $\rho_{\text{crit}} = (G f_g \bar{t}_S^2)^{-1}$ above which black hole growth dominates and below which star formation dominates. This can be rewritten in terms of a critical velocity dispersion if one takes ρ_{crit} to be the density at the edge of the Bondi accretion sphere (the sphere of influence of the black hole, R_{BH}), namely, $\rho \sim (M/r^3) \sim (M_{\text{BH}}/r_{\text{BH}}^3) \sim (G^{-3} \sigma^6 M_{\text{BH}}^{-2})$ giving then an equivalent σ_{crit}

$$\sigma_{\text{crit}} = \left(\frac{M_{\text{SR}}}{M_{\text{BH}}} \right) f_g^{1/2} f_c^{-1} f_{\text{cl}}^{-1} \left(\frac{p_{\text{cl}}}{p_g} \right)^{1/2} \left(\frac{E_{\text{SN}}}{m_{\text{SN}} v_c} \right), \quad (63)$$

which is a satisfying combination of black hole galaxy and ISM properties. Continuing to the case with AGN feedback, we find

$$\frac{d \ln M_{\text{BH}}}{d \ln M_*} = f_g^{1/2} \left(\frac{\bar{t}_S}{t_d} \right) \left(\frac{v_{\text{AGN}}}{c} \right)^{1/2} \left(\frac{L_{\text{SR}}}{L_{\text{AGN}}} \right)^{1/2} \quad (64)$$

and in the radiation-driven case

$$\frac{d \ln M_{\text{BH}}}{d \ln M_*} = f_g^{1/4} \left(\frac{\bar{t}_S}{t_d} \right) \left(\frac{\sigma}{c} \right)^{1/2} \tau^{1/4} \left(\frac{L_{\text{SR}}}{L_{\text{AGN}}} \right)^{1/4}. \quad (65)$$

Corresponding expressions for the critical density and velocity dispersion in the AGN case can be readily obtained. At higher redshifts, galaxy systems can be denser, although much of the physics of the nuclear regions depends on local physics, and thus the dominant black hole growth phase may be more easily entered at higher redshift. The Bondi accretion formula can be rewritten as

$$\left(\frac{t_{\text{BH}}}{t_S} \right) = f_g^{-1} \left(\frac{t_d}{t_S} \right)^2 \left(\frac{\eta c}{\sigma} \right) \left(\frac{M_{\text{SR}}}{M_{\text{BH}}} \right). \quad (66)$$

Using the equation balancing inflow and outflow and the equation for the logarithmic slope in the Magorrian plane, we find

$$\left(\frac{M_*}{M_{\text{BH}}} \right) \left(\frac{Q f_L}{\beta f_w} \right) = \left(\frac{t_S}{t_d} \right) \left(\frac{v_{\text{AGN}}}{c} \right)^{1/2} \left(\frac{L_{\text{SR}}}{L_{\text{AGN}}} \right)^{1/2}. \quad (67)$$

Eliminating (t_S/t_d) we find

$$\left(\frac{Q f_L}{\beta f_w} \right)^2 \left(\frac{t_{\text{BH}}}{t_S} \right) = \left(\frac{\eta c}{\sigma} \right) \left(\frac{v_{\text{AGN}}}{c} \right) \left(\frac{L_{\text{EDD}}}{L_{\text{AGN}}} \right) \left(\frac{M_{\text{SR}}}{M_*} \right)^2 \quad (68)$$

and in this case we have used the momentum balance equation with momentum injection from SN balancing dissipation. For the radiation-driven case we find

$$\left(\frac{Q f_L}{\beta f_w} \right)^2 \left(\frac{t_{\text{BH}}}{t_S} \right) = \tau^{1/2} \left(\frac{L_{\text{EDD}}^2}{L_{\text{AGN}} L_{\text{SR}}} \right)^{1/2} \left(\frac{M_{\text{SR}}}{M_*} \right)^2. \quad (69)$$

In both the above cases there is no evidence for downsizing even with constant Q . However, if there is another way to deduce Q for these turbulent AGN-driven multiphase media and the momentum injection for the medium is taken up by the AGN then, as we shall show in the following, downsizing can occur naturally as a consequence of the turbulent ISM properties.

4.5. Constraints on Evolutionary Tracks

There is an obvious but powerful constraint on the behavior of the evolutionary tracks in the observed mass $\ln[M_{\text{BH}}] - \ln[M]$ plane. We first emphasize that this concept of tracks is implicit in our model and that there is a flow of points in the mass plane with evolutionary arrows all pointing in the direction of black hole growth. The slope $p = d \ln[M_{\text{BH}}]/d \ln[M]$ of the track cannot be negative since: (1) black holes can only grow in mass; and (2) galaxies only grow in mass (ignoring their small fractional mass loss). On dwarf galaxy scales the fractional mass loss can be easily incorporated but even there it is much less than of order unity. Therefore, for example, tracks cannot loop back from above the mean line with any slope less than zero after overshooting the mean line on a trajectory originating from below. *Therefore, any nonpathological track will spend most of its life on a track with a slope close to the mean.* Thus, we can assume p is approximately constant; observationally p is of order unity. We now use this slope as a parameter in our timescale and evolutionary equations and find

$$\left(\frac{p}{\beta} \right)^2 \left(\frac{t_{\text{BH}}}{t_S} \right) = \left(\frac{\eta c}{\sigma} \right) \left(\frac{v_{\text{AGN}}}{c} \right) \left(\frac{L_{\text{EDD}}}{L_{\text{AGN}}} \right) \left(\frac{M_{\text{SR}}}{M_{\text{BH}}} \right)^2. \quad (70)$$

For the radiation-driven case we find

$$\left(\frac{p}{\beta} \right)^2 \left(\frac{t_{\text{BH}}}{t_S} \right) = \tau^{1/2} \left(\frac{L_{\text{EDD}}^2}{L_{\text{AGN}} L_{\text{SR}}} \right)^{1/2} \left(\frac{M_{\text{SR}}}{M_{\text{BH}}} \right)^2. \quad (71)$$

4.6. Why the M^2 Dependence of Accretion?

We see that the parameter controlling accretion α , which is proportional to the phase space density, and also measures the specific entropy s of the initial gas distribution, is specified by the physics of accretion. Specifically, $\alpha \propto s^{-3/2}$, where $s = kTn^{-2/3}$. In terms of a polytropic equation of state, α is constant for $\gamma = 5/3$.

4.6.1. Hot Phase Entropy

In any dissipative multiphase medium, the entropy cannot be strictly constant. However, to demonstrate that entropy is indeed slowly varying in the hot phase of the system, consider thermal balance between gravitational accretion heating, which is also proportional to the resulting power in the AGN outflow and associated heating, and atomic cooling. One obtains $c_1 v^5 / G = \rho^2 r^3 \lambda(v)$, where $\lambda(v)$ represents the cooling rate per atom per unit density. In the range of interest where hot gas dominates the gas pressure on galaxy and cluster scales, $\lambda(v)$ is weakly varying, e.g., $\lambda(v) \propto v$ for thermal bremsstrahlung cooling at $T \gtrsim 10^7$ K and is approximately constant over 10^6 – 10^7 K. In fact, to avoid fragmentation, a necessary condition for effective central black hole growth (Lodato & Natarajan 2006), one needs to be at $T \gtrsim 10^6$ K and have $\gamma > 4/3$. Above 10^7 K, appropriate to massive galaxies and clusters, $s \propto T^{1/3}$ is found to be slowly varying and this helps account for the central entropy “floor” in clusters. This results in the maximum accretion rate being in the core. Black hole growth by Bondi accretion indeed requires constant specific entropy flow, which explains why $\dot{M}_{\text{acc}} \propto \dot{M}_{\text{BH}}$. These arguments should apply on massive galaxy scales where the gas pressure is controlled by the ISM hot gas phase at the outer boundary of the flow.

4.6.2. Cold Phase Entropy

Similar arguments apply to a cold phase in a multiphase medium. Here, the cold clouds themselves, envisaged as bound self-gravitating entities that move on ballistic orbits, act like massive particles whose dynamics can be described by Bondi accretion. For supersonic turbulence there are three points to note.

1. The system is highly dissipative, and so momentum conservation not energy conservation is the rule.
2. The density as described in the probability density function (PDF) is essentially dimensionless and only measured in units of the square of the Mach number (\mathcal{M})².
3. Continuous energy and momentum input comes for the central source so that the PDF structure of the medium remains statistically robust. This means there will be the same number of clouds with the same mass function even though there is continuous creation and destruction. If a cloud is dissipated then another cloud will be created to take its place in the ensemble.

Finally, the M^2 dependence discussed above occurs over a very wide range of sources and environments both relativistic and nonrelativistic and with power sources ranging from protostars to microquasars to quasars. A clear invariant seems to be supersonic turbulence generated by a central source. For the cold supersonic phase $\rho \sim M^2$ and $\sigma \sim \mathcal{M}$ and thus we expect the phase space density to be $\rho/\sigma^3 \sim \mathcal{M}^{-1}$. Note that over a wide range of the systems discussed from protostar to quasars the value of $\mathcal{M} \sim 3$ – 10 is appropriate. In addition, the cool gas over a wide range of conditions is at similar temperatures. The

cool gas is carried in packets (in clouds) whose number density and mass function are properties of the supersonic turbulence and the Mach number. Thus, over this wide dynamic range the variation in the parameter α may not be large.

4.6.3. Multiple Phases

For cloud populations with different properties their effective temperature is associated with the velocity dispersion of the cloud ensemble, as opposed to the gas kinetic temperature. In this case, $s = \sum M_i \sigma_i^2 n_i^{-2/3}$, where we sum over hot and cold cloud components, and the Bondi accretion rate for a two-phase medium is now $\dot{M}_{\text{BH}} = \pi G^2 \sum (\rho_i / \sigma_i^3) \dot{M}_{\text{BH}}^2$.

4.7. From Bondi Accretion to Star Formation

We now develop the interplay between the Bondi accretion rate parameter α and the porosity-driven star formation rate. We show that constant Q implies constant α , and vice versa. This is the key to understanding coupled downsizing for spheroids and supermassive black holes. Define the black hole growth time by $t_{\text{BH}} = \dot{M}_{\text{BH}} / \dot{M}_{\text{acc}} = 1/(\alpha \dot{M}_{\text{BH}})$.

4.7.1. Significant Downsizing for Porosity-Regulated Star Formation

Using the above Equations (60) and (58) which relate \dot{M}_{out} and \dot{M}_{acc} , we find that

$$\left(\frac{Q^2 f_L}{f_w} \right)^2 \left(\frac{\eta c}{\sigma_{\text{fid}}} \right) \left(\frac{t_{\text{BH}}}{t_s} \right) = \left(\frac{M_{\text{BH}}}{M_{\text{SR}}} \right)^2 \left(\frac{M_{\text{BH}}}{M_g} \right) \left(\frac{\sigma_{\text{fid}}}{\sigma} \right)^{31/7} \quad (72)$$

which becomes

$$\begin{aligned} & \left(\frac{Q^2 f_L}{f_w} \right)^2 \left(\frac{\eta c}{\sigma_{\text{fid}}} \right) \left(\frac{t_{\text{BH}}}{t_s} \right) \\ &= \left(\frac{M_{\text{BH}}}{M_{\text{SR}}} \right)^2 \left(\frac{M_{\text{BH}}}{M_g} \right) \left(\frac{L_{\text{SR}, \text{fid}}}{L_{\text{SR}}} \right)^{31/28}. \end{aligned} \quad (73)$$

Alternatively using the p -parameterization discussed in Section 4.5 we find

$$(Qp)^2 \left(\frac{\eta c}{\sigma_{\text{fid}}} \right) \left(\frac{t_{\text{BH}}}{t_s} \right) = \left(\frac{M_g}{M_{\text{SR}}} \right) \left(\frac{M_{\text{BH}}}{M_{\text{SR}}} \right) \left(\frac{\sigma_{\text{fid}}}{\sigma} \right)^{31/7}. \quad (74)$$

Constant porosity therefore guarantees downsizing, since

$$\left(\frac{t_{\text{BH}}}{t_s} \right) \propto L_{\text{SR}}^{-31/28} \propto M_{\text{BH}}^{-31/28} \approx M_{\text{BH}}^{-1}. \quad (75)$$

For Eddington-limited accretion, $\dot{M}_{\text{BH}} = \dot{M}_{\text{SR}}$. The ratio of black hole growth time to Salpeter time decreases with increasing black hole mass at constant Q . Note also that

$$1/\alpha = t_{\text{BH}} \dot{M}_{\text{BH}} \propto Q^{-4} M_{\text{BH}}^2 M_{\text{SR}}^{-59/28} \propto M_{\text{BH}}^{-3/28}.$$

Hence constant porosity also favors Bondi accretion since $\alpha \approx$ constant. Since $\sigma \sim 10\sigma_{\text{fid}}$ for a massive spheroid, we also infer that for Eddington-limited accretion, the porosity must be of the order of 10% if the black hole growth time is of the order of the Salpeter time. Indeed, we can equate these timescales and infer that porosity depends weakly on black hole mass, $Q \propto M_{\text{BH}}^{-1/3}$. In the absence of AGN feedback, the derived star formation law yields a ratio of star-formation timescale to dynamical time that is proportional to σ_g and hence approximately constant, independently of galactic mass. This suggests that in starbursts when AGNs play no role, there should be no downsizing, as

would be expected if internal processes such as those associated with formation of massive star clusters were to dominate.

Also, the Eddington ratio can be written as

$$f_{\text{Edd}} \equiv \frac{L_{\text{AGN}}}{L_{\text{Edd}}} = \eta c \left(\frac{\sigma_T \alpha}{m_p G} \right) M_{\text{BH}}.$$

Hence constant α is consistent with the observed trend measured in the Eddington ratio (Alonso-Herrero et al. 2008). The Eddington ratio is found to be lower for AGN than for quasi-stellar objects (QSOs) due to a combination of reduced host galaxy (and SMBH) mass as well as AGN feeding.

4.8. Why Does Porosity Self-Regulate?

First, we give a qualitative argument for self-regulation. If the porosity Q is low, the jet is blocked, and the turbulent pressure is enhanced. Weak shocks propagate ahead of the cocoon and squeeze self-gravitating clouds over the BE stability limit. Star formation is triggered and the resulting supernovae drive up the pressure. Blow-out most likely occurs of the residual gas. This in turn increases $Q \sim 1$. However, dense clouds can now fall in unimpeded by intervening dense cold gas. The infall reduces Q , drives accretion and resurrects a strong jet.

Now, it is reasonable to assume that the cold phase is defined by a minimum density n_{cr} set by dissipative cooling and molecular formation, and that is not strongly dependent on metallicity and ionization fraction either in the molecular (Norman & Spaans 1997) or atomic (Wolfire et al. 2003) phases. The shape of the density PDF is well represented by a lognormal distribution (Wada & Norman 2007) with two parameters, dispersion of the distribution and its amplitude. When calculating the filling factor of either hot or cold gas, one integrates the lognormal PDF below or above n_{cr} . At fixed n_{cr} , the filling factor of either hot or cold gas depends only on one parameter, the dispersion. In general, for the isothermal case considered here, the dispersion is a linear function of $\ln \mathcal{M}$, for high Mach numbers (Krumholz & McKee 2005). In the adiabatic case the lognormal PDF dispersion is independent of the Mach number. The lognormal form is retained for supersonic turbulence both with and without star formation (Wada & Norman 2007). Even with significant feedback and increased Mach number, Q only varies logarithmically with Mach number, and therefore any change of filling factor in systems with developed supersonic turbulence is relatively slow as the turbulence is increased. In summary, the volume filling fractions of cold gas e^{-Q} and hot phase $1 - e^{-Q}$ depend only logarithmically on the Mach number. Hence the porosity is plausibly constant. Quantifying this argument we define the hot-phase filling factor in a turbulent supersonic medium to be the volume of the turbulent medium with a lower density than the mean by a factor $v_h < 1$. Using the usual lognormal PDF

$$f(n) = \frac{1}{\sqrt{2\pi}\sigma_t n} \exp \left[-\frac{(\ln n)^2}{2\sigma_t^2} \right], \quad (76)$$

where for supersonic turbulence we use

$$\sigma_t^2 = \ln[1 + \lambda_t \mathcal{M}^2] \quad (77)$$

with \mathcal{M} being the Mach number, and for the parameter λ_t we use $\lambda_t = 3/4$ (Krumholz & McKee 2005) for numerical estimates. We define

$$v_h = \frac{n_h}{n_0 [1 + \lambda_t \mathcal{M}^2]}, \quad (78)$$

where n_0 is a reference density and we find an expression for f_h

$$f_h = \text{erfc} \left[\frac{\ln \left[\frac{1}{v_h} \right]}{\sqrt{2}\sigma_t} \right], \quad (79)$$

where often the first term of the asymptotic expansion for $\text{erfc}[x] = \exp[-x^2]/x\sqrt{\pi} + \dots$ is a useful guide. For the Mach number, $\mathcal{M} = 3-10$ and for the under density parameter of the hot phase relative to the mean we use $v_h = 0.1$ and find the hot phase filling factor is $\sim 10\%-20\%$. As the Mach number increases, f_h increases in turn as the width of the PDF increases. Since $f_h = 1 - \exp -Q$ we find that

$$Q = \ln[1 - f_h] \quad (80)$$

which for $Q \ll 1$ becomes

$$Q = \ln[1 + f_h] \quad (81)$$

giving

$$Q \sim \ln \left[1 + \text{erfc} \left[\ln \left[\frac{1}{v_h} \right] \right] \right]. \quad (82)$$

This shows how Q increases as the Mach number increases. Proceeding further, we can generalize our previous expression for Q , including both the competing effects of the star formation and supersonic turbulence, to

$$Q \sim \ln \left[1 + \text{erfc} \left[\ln \left[\frac{1}{v_h} \right] \right] \right] + f_{\text{cl} \in \text{SN, fid}} \left(\frac{\sigma_{\text{fid}}}{\sigma_g} \right)^{12/7}. \quad (83)$$

Feedback from AGN is now readily incorporated by substituting $\mathcal{M} = (\sigma/\sigma_{\text{fid}})^2$ and then making the substitute, as before, for the multiphase medium under the action of the mechanical and radiative pressures originating in the central source giving

$$\mathcal{M}_{\text{AGN}}^2 = \left(\frac{c}{V_{\text{AGN}}} \right)^{1/2} \left(\frac{L_{\text{AGN}}}{L_{\text{SR, fid}}} \right)^{1/2} \quad (84)$$

and thus using Equations (77) and (84) we obtain

$$\sigma_{t, \text{AGN}}^2 = \ln \left[1 + \lambda_t \left(\frac{c}{V_{\text{AGN}}} \right)^{1/2} \left(\frac{L_{\text{AGN}}}{L_{\text{SR, fid}}} \right)^{1/2} \right]. \quad (85)$$

In the AGN case, it is now clearly quantified how the AGN pressure is reducing f_h by confining the hot SNR bubbles but on the other hand the AGN increases the Mach number of the turbulence and therefore broadens the PDF distribution, thus increasing f_h .

4.9. Star Formation Rate

Global gas consumption is dominated by star formation, and locally by SMBH growth. The two are connected via the piston model and suggest a possible self-regulation loop for both spheroid and SMBH growth. We now show that the time sequence underlying the Magorrian relation can be interpreted in terms of the ratio of spheroid to SMBH growth rates.

The star formation (or gas consumption) rate is from Equation (42)

$$\frac{1}{t_*} = \frac{\epsilon_{\text{SN}}}{t_d} \left(\frac{L_{\text{AGN}} \tau}{L_{\text{SR}}} \right)^{1/2} \quad (86)$$

$$= \bar{\epsilon}_{\text{SN}} G \Sigma_{\text{tot}} \left(\frac{L_{\text{AGN}}}{L_{\text{SR}}} \right)^{1/2} \quad (87)$$

$$\propto P_{\text{AGN}}^{1/2}, \quad (88)$$

where $\bar{\epsilon}_{\text{SN}} = \epsilon_{\text{SN}}/\sigma$ is a constant depending only on supernova properties and the IMF. This controls spheroid growth and demonstrates spheroid downsizing via the AGN-enhanced pressure and associated star formation rate.

Writing the AGN luminosity as $L_{\text{AGN}} = \eta \dot{M}_{\text{BH}} c^2$, the Eddington luminosity can be expressed in terms of the Salpeter time as $\eta M_{\text{BH}} c^2 / t_S$. We now rewrite the star formation rate expression and obtain

$$\frac{M_{\text{BH}}}{\sigma^4} = \left(\frac{t_S}{t_*} \right)^2 \left(\frac{m_p}{\bar{\epsilon}_{\text{SN}} \sigma_T \eta c \Sigma_{\text{tot}}} \right)^2. \quad (89)$$

The predicted normalization of the Magorrian relation agrees with the local value and slope for canonical parameter values ($\Sigma_{\text{tot}} \sim \Sigma_*$, $\eta \sim 0.001$, $\epsilon_{\text{SN}} \sim 0.1$). Of course, there is considerable uncertainty due to possible variations in the IMF, supernova energy, and star formation timescale.

In fact, the relevant SMBH measure in distant objects is L_{AGN} rather than L_{Edd} . Let us make use of L_{SR} as a fiducial luminosity, in effect a proxy for σ^4 . We now rewrite the preceding expressions to obtain

$$L_{\text{AGN}}/L_{\text{SR}} = \epsilon_{\text{SN}}^{-2} (f_g t_d / t_*)^2 \sim (t_d / t_*)^2. \quad (90)$$

We also have for the AGN-boosted star formation rate

$$\dot{M}_*^{\text{AGN}} \approx \epsilon_{\text{SN}} M_g \Omega (L_{\text{AGN}}/L_{\text{SR}})^{1/2} \sim M_g \Omega. \quad (91)$$

This is of course the optimal rate. We also see that $\dot{M}_*^{\text{AGN}} \propto \sigma^4$. This is not inconsistent with the observed dependence of outflow velocity on star formation rate (Martin 2005).

We may consider the case of a recently detected kiloparsec scale starburst at $z = 6.24$, hosted by a luminous quasar which has spatially resolved [C II] emission as well as a large reservoir of CO-detected molecular gas (Walter et al. 2009). Other similar high z objects, detected in CO, are believed to be super-Magorrian (Maiolino et al. 2007). This quasar host galaxy also has a star formation rate of $\sim 1000 M_\odot \text{ year}^{-1} \text{ kpc}^{-2}$, an order of magnitude larger than is typical of starbursts without luminous AGN. For comparison, Arp 220, a low-redshift starburst hosting an AGN of luminosity comparable to its starburst, has a similar surface brightness in star formation but only over a 100 pc scale. It is tempting to infer, admittedly with only two well mapped examples, that we may be viewing AGN boosting of star formation, with the phenomenon being greatly magnified at high redshift for the most massive objects.

We infer that if the SMBH mass is super-Magorrian, then $t_* < t_S$ and $t_* < t_d$, and star formation is very efficient. This seems to be the case at high redshift. The preponderance of data indeed suggests that the local relation becomes super-Magorrian prior to $z \sim 2$ (McLure et al. 2006; Woo et al. 2008) and indeed persists to $z \gtrsim 5$ (Maiolino et al. 2007). Coevolution of AGN accretion and the comoving star formation rate densities occurs to $z \sim 2$, but the accretion rate falls off relatively toward higher redshift (Silverman et al. 2008).

Comparison of the cosmic star formation history and AGN accretion rates in comoving number density as a function of luminosity suggests that the peak in massive black hole growth rate occurs several Gyr prior to the star formation peak and that downsizing at $z < 1$ is due to diminishing accretion rates (Babić et al. 2007). Submillimeter galaxies (SMGs) are an exception. SMGs at $z \sim 2$ contain SMBH that are under-massive relative to the Magorrian relation (Alexander et al. 2008). This is suggestive of triggered star formation, which reduces t_* , and may be appropriate in major mergers that generate dense central gas environments where porosity feedback is suppressed. Note also that the peak in the major merger rate also precedes the peak in cosmic star formation rate (Ryan et al. 2008), and is approximately consistent with the peak in comoving AGN accretion rate density.

5. SUMMARY AND CONCLUSIONS

In summary, a general and robust treatment of disk star formation is developed from a cloud collision model. The SK law emerges naturally for star-forming disks. We predict that there is an inverse relation between TF law and SK law residuals. A multiphase treatment of supernova feedback leads to a turbulent pressure-regulated generalization of the star formation law that is applicable to gas-rich starbursts. Negative feedback from star formation occurs in disks under turbulent pressure regulation. In combination with a cloud collision model, the SK law can be understood in diverse environments, spanning quiescent disks and starbursts. Enhanced pressure, as expected in merger-induced star formation, enhances star formation efficiency. An upper limit is derived for the disk star formation rate in starbursts that depends only on the IMF and on the ratio of global to cloud pressures. For clouds in approximate pressure with interstellar medium and a local IMF, we infer a limiting gas surface density of $\sim 1000 M_\odot \text{ pc}^{-2}$.

We extend these considerations to the case where the interstellar gas pressure in the inner galaxy is dominated by outflows from a central AGN. The star formation rate is pressure-driven and depends on the excess pressure applied by the AGN outflows. During massive spheroid formation, AGN-driven winds trigger star formation, resulting in enhanced supernova feedback and outflows. Downsizing is predicted to be a consequence of AGN-driven positive feedback. Our most important results refer to downsizing, for which we provide a new interpretation in terms of Bondi accretion feeding of the central black hole.

The specific accretion rate is proportional to the black hole mass. We found that Bondi accretion results in $M_{\text{BH}} \propto \sigma^{59/7} Q^4 (t_{\text{BH}}/t_S)$. This means that if porosity self-regulates to be constant, black hole growth proceeds rapidly until it saturates at the Magorrian relation $M_{\text{BH}} \propto \sigma^4$ due to blow-out. Black hole downsizing occurs if α is approximately constant. We clarify this as follows.

There are three specific rates that define our model. The Salpeter rate t_S^{-1} is constant, the black hole growth rate is $1/t_{\text{BH}} = (1/t_S)(L_{\text{AGN}}/L_{\text{Edd}})$, and the star formation rate is $1/t_* = \bar{\epsilon}_{\text{SN}} (G L_{\text{AGN}} \tau)^{1/2} (c \sigma^4 f_g)^{-1/2}$. Hence

$$\frac{t_{\text{BH}}}{t_*} = \frac{\epsilon_{\text{SN}} t_S}{f_g t_d} L_{\text{Edd}} \left(\frac{\tau}{L_{\text{AGN}} L_{\text{SR}}} \right)^{1/2} \sim \frac{t_S}{t_d} \tau^{1/2} \quad (92)$$

and we infer that

$$L_{\text{AGN}}/L_{\text{SR}} \sim (t_d/t_*)^2 \sim (t_S/t_{\text{BH}})^2. \quad (93)$$

This shows that the black hole growth rate and star formation rate are coupled. At given σ , there is a critical AGN luminosity, above which AGN-triggered star formation rates dominates over the black hole growth rate. This critical luminosity is found to be

$$\frac{L_{\text{AGN}}^{\text{cr}}}{L_{\text{Edd}}} = \eta \tau \frac{\epsilon_{\text{SN}}^2}{f_g} \frac{t_S}{t_d} \frac{M_{\text{BH}}}{M} \frac{c}{\sigma} \left(\frac{t_{\text{BH}}}{t_*} \right)^2 \sim \left(\frac{t_{\text{BH}}}{t_*} \right)^2. \quad (94)$$

At super-Eddington luminosities, AGN-triggered star formation dominates.

The model contains two characteristic luminosities which are functions of σ . The Eddington luminosity, if combined with the quenching assumption, scales as σ^4 . The AGN-triggered star formation luminosity is $\epsilon_i c^2 \bar{\epsilon}_{\text{SN}} \sigma^2 (L_{\text{AGN}} \tau)^{1/2} (G c f_g)^{-1/2}$. This is proportional to $\alpha^{1/2} M_{\text{BH}} \sigma^2$. Adopting $M_{\text{BH}} \propto \sigma^4$ and $\alpha \propto \sigma^{4/3}$ if Q is constant, we find that $L_{\text{AGN}}^{\text{cr}} \propto \sigma^{20/3}$. Hence we again infer a critical Eddington luminosity above which triggered star formation dominates the luminosity of the system. This guarantees efficient star formation for luminous AGN. Moreover, if M_{BH}/σ^4 increases with increasing redshift, as inferred from the downsizing of the black hole growth rate, spheroid star formation downsizes both in mass and in efficiency.

The ratio of AGN to star formation luminosity is

$$\frac{L_{\text{AGN}}}{L_{\text{AGN}}^{\text{cr}}} = \frac{\eta \dot{M}_{\text{BH}}}{\epsilon_n \dot{M}_{\text{AGN}}} \propto \frac{dM_{\text{BH}}}{dM_*}. \quad (95)$$

This yields an arrow of time in the form of tracks in the Magorrian diagram. If for each data point in the Magorrian plane, defined by black hole and bulge mass, one can separate star formation and AGN luminosity, then the ratio gives a vector and hence an arrow of time. This is of course the instantaneous trajectory of points in the Magorrian plane as viewed at any given epoch. However, statistically the vectors should provide the flow of galaxy bulges toward the Magorrian relation. It would be interesting to construct Magorrian flow diagrams binned over several redshift ranges. This would provide insight into the cosmological evolution of the flow of points in the Magorrian plane.

AGN-enhanced star formation is mediated by turbulent pressure and relates spheroid star formation rate to black hole accretion rate. As the AGN pressure is increased, via jet/cocoon pressure-driven interactions with the ISM, the induced star formation rate is correspondingly boosted. Downsizing for spheroid formation is a consequence of massive black hole downsizing. The observed relation between black hole mass and spheroid velocity dispersion is obtained, with a coefficient (the ratio of Salpeter time to gas consumption time) that provides an arrow of time. Highly efficient, AGN-boosted star formation is favored at high redshift. Outflows are of the order of the AGN-boosted star formation rate and saturate in the super-Eddington limit.

We end with some relevant questions for observers that are pertinent to our model. Is AGN activity correlated with the star formation rate? Was spheroid formation and black hole growth coeval and symbiotic? Which came first, if either? Is the reach of the AGN too localized to globally affect star formation? Is or was feedback significant in radio-quiet AGN? Are the youngest radio sources, notably the GPC sources, templates for the earliest stages of AGN feedback, and if so, is there associated triggering of star formation? If star formation is triggered by radio cocoons, where is the evidence for a post-starburst stellar population in old radio lobes? If the efficient

mode of star formation is due to coherent cocoon triggering as argued above, what is the evidence for spatially and temporally coordinated episodes of star formation in well studied examples such as the Antennae? Has the trigger of positive feedback in ultraluminous starbursts disappeared, due to a short duty cycle, or is it well and truly buried in embedded AGN nuclei? Are the associated outflow rates from AGN/starbursts of any significance for balancing the baryonic budget of the universe, and if so, where does the enriched debris end up? If the gas remains in the halos of massive galaxies, as essentially all current semianalytic simulations predict, why has not it been seen? And for the modelers (who are effectively observers of the computer), how can AGN feedback simulations possibly play any predictive role if one has to begin with massive seed black holes of dubious heritage and uncertain fate? Perhaps our analytic discussion will motivate more realistic recipes and subgrid physics prescriptions for future generations of feedback simulations.

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APPENDIX

LIST OF SYMBOLS

α	Bondi accretion rate parameter: phase space density
β	factor for black hole growth timescale
C_{SK}	Schmidt–Kennicutt law coefficient
C_{TF}	Tully–Fisher law coefficient
E_{SN}	kinetic energy of a SNeII
ϵ_{SN}	star formation efficiency factor
$\epsilon_{\text{SN, fid}}$	star formation efficiency at fiducial velocity
ϵ_l	energy release per unit rest mass
ϵ_m	mechanical energy release per unit rest mass
ϵ_n	nuclear burning efficiency per unit rest mass
$\bar{\epsilon}_{\text{SN}}$	modified star formation factor
Σ_g	gas mass surface density
σ_g	gas velocity dispersion
Σ_{cl}	cloud mass surface density
f_{cl}	gas fraction in clouds by mass
f_c	cloud volume filling factor ($= e^{-Q}$)
f_L	hot gas loading factor
f_l	luminosity per unit mass of massive star formation
f_w	fraction accreting material that flows out in wind
H	disk scale height of gas
L_*	I -band luminosity
Λ_{eff}	effective cooling function for SN bubbles
λ	cooling rate per atom per unit density
L_{SR}	self-regulating critical luminosity

L_{EDD} Eddington luminosity
 $L_{\text{SR, fid}}$ self-regulating critical luminosity at fiducial velocity dispersion
 L_{mech} mechanical luminosity
 L_J jet luminosity
 λ_t numerical constant
 m_{SN} mass formed in stars per SNeII
 \dot{M}_{out} mass outflow rate
 M_{SR} self-regulating critical mass
 M_{BH} black hole mass
 M_{BE} Bonnor–Ebert mass
 $\dot{M}_{\text{out}}^{\text{gal}}$ galaxy mass outflow rate
 \dot{M}_*^{AGN} AGN-enhanced star formation rate
 \mathcal{M} turbulent Mach number
 v_h hot phase underdensity parameter
 Ω angular rotation of the disk
 p logarithmic slope in the black hole mass versus galaxy plane
 p_g gas pressure
 p_{cl} pressure in a cloud
 P_{co} cocoon pressure
 Q porosity of the interstellar medium
 R radius of disk
 R_a maximum SN bubble radius
 R_0, t_0, v_0, c_0 defined constants for SN bubbles
 ρ_g gas mass density
 S_{cl} covering factor of clouds
 Σ_* mass surface density in stars
 Σ_{tot} total mass surface density of the disk
 σ_{fid} fiducial velocity dispersion for SN driven ISM
 σ velocity dispersion of system
 s specific entropy
 σ_t PDF dispersion
 t_* gas consumption time
 \bar{t}_S modified Salpeter time
 t_{coll} cloud collision timescale
 t_a time for SN bubble to reach maximum radius
 t_c cooling timescale of SN bubble
 t_d dynamical timescale
 t_{co} cocoon propagation timescale
 τ Rosseland mean optical depth
 t_{BH} black hole growth time
 t_S Salpeter timescale
 Θ_J jet opening angle
 v_c supernova velocity at strong cooling
 v_r rotation velocity of the disk
 v_{co} cocoon velocity
 v_w wind velocity
 V_{AGN} velocity of AGN outflow
 v_J jet velocity

χ interstellar cloud pressure enhancement due to self-gravity

REFERENCES

- Agertz, O., Lake, G., Teyssier, R., Moore, B., Mayer, L., & Romeo, A. B. 2009, *MNRAS*, **392**, 294
 Alexander, D. M., et al. 2008, *AJ*, **135**, 1968
 Allard, E. L., Knapen, J. H., Peletier, R. F., & Sarzi, M. 2006, *MNRAS*, **371**, 1087
 Allen, S. W., Dunn, R. J. H., Fabian, A. C., Taylor, G. B., & Reynolds, C. S. 2006, *MNRAS*, **372**, 21
 Alonso-Herrero, A., Pérez-González, P. G., Rieke, G. H., Alexander, D. M., Rigby, J. R., Papovich, C., Donley, J. L., & Rigopoulou, D. 2008, *ApJ*, **677**, 127
 Antonuccio-Delogu, V., & Silk, J. 2008, *MNRAS*, **389**, 1750
 Babić, A., Miller, L., Jarvis, M. J., Turner, T. J., Alexander, D. M., & Croom, S. M. 2007, *A&A*, **474**, 755
 Banerjee, R., & Pudritz, R. E. 2006, *ApJ*, **641**, 949
 Barnes, J. E. 2004, *MNRAS*, **350**, 798
 Begelman, M. C., & Cioffi, D. F. 1989, *ApJ*, **345**, L21
 Blitz, L., & Rosolowsky, E. 2006, *ApJ*, **650**, 933
 Boomsma, R. 2007, PhD thesis, Univ. Groningen
 Bouché, N., et al. 2007, *ApJ*, **671**, 303
 Bower, R. G., Benson, A. J., Malbon, R., Helly, J. C., Frenk, C. S., Baugh, C. M., Cole, S., & Lacey, C. G. 2006, *MNRAS*, **370**, 645
 Cioffi, D. F., McKee, C. F., & Bertschinger, E. 1988, *ApJ*, **334**, 252
 Croton, D. J., et al. 2006, *MNRAS*, **365**, 11
 Di Matteo, T., Colberg, J., Springel, V., Hernquist, L., & Sijacki, D. 2008, *ApJ*, **676**, 33
 Fabian, A. 1999, *MNRAS*, **308**, 39
 Feain, I. J., Papadopoulos, P. P., Ekers, R. D., & Middelberg, E. 2007, *ApJ*, **662**, 872
 Firmani, C., & Tutukov, A. 1992, *A&A*, **264**, 37
 Fragile, P. C., Murray, S. D., & Lin, D. N. C. 2004, *ApJ*, **617**, 1077
 Gammie, C. F., Ostriker, J. P., & Jog, C. J. 1991, *ApJ*, **378**, 565
 Gao, Y., Carilli, C. L., Solomon, P. M., & Vanden Bout, P. A. 2007, *ApJ*, **660**, L93
 Hasinger, G., Miyaji, T., & Schmidt, M. 2005, *A&A*, **441**, 417
 Hathi, N. P., Malhotra, S., & Rhoads, J. E. 2008, *ApJ*, **673**, 686
 Joung, M. K. R., & Mac Low, M.-M. 2006, *ApJ*, **653**, 1266
 Joung, M. R., Mac Low, M.-M., & Bryan, G. L. 2008, arXiv:0811.3747
 Kennicutt, R. C., Jr., et al. 2007, *ApJ*, **671**, 333
 Kim, W.-T., & Ostriker, E. C. 2007, *ApJ*, **660**, 1232
 Komugi, S., Sofue, Y., Nakanishi, H., Onodera, S., & Egusa, F. 2005, *PASJ*, **57**, 733
 Koyama, H., & Ostriker, E. C. 2009a, *ApJ*, **693**, 1316
 Koyama, H., & Ostriker, E. C. 2009b, *ApJ*, **693**, 1346
 Kriek, M., et al. 2007, *ApJ*, **669**, 776
 Krumholz, M. R., Matzner, C. D., & McKee, C. F. 2006, *ApJ*, **653**, 361
 Krumholz, M. R., & McKee, C. F. 2005, *ApJ*, **630**, 250
 Krumholz, M. R., & Thompson, T. A. 2007, *ApJ*, **669**, 289
 Leroy, A. K., Walter, F., Brinks, E., Bigiel, F., de Blok, W. J. G., Madore, B., & Thornley, M. D. 2008, *AJ*, **136**, 2782
 Lodato, G., & Natarajan, P. 2006, *MNRAS*, **371**, 1813
 Maiolino, R., et al. 2007, *A&A*, **472**, L33
 Martin, C. L. 2005, *ApJ*, **621**, 227
 Masters, K. L., Springob, C. M., & Huchra, J. P. 2008, *AJ*, **135**, 1738
 McLure, R. J., Jarvis, M. J., Targett, T. A., Dunlop, J. S., & Best, P. N. 2006, *MNRAS*, **368**, 1395
 Mellema, G., Kurk, J. D., & Rottgering, H. J. A. 2002, *A&A*, **395**, L13
 Meurer, G. R., Heckman, T. M., Lehnert, M. D., Leitherer, C., & Lowenthal, J. 1997, *AJ*, **114**, 54
 Mohanty, S., Jayawardhana, R., & Basri, G. 2005, *ApJ*, **626**, 498
 Norman, C. A., & Spaans, M. 1997, *ApJ*, **480**, 145
 Plume, R., Jaffe, D. T., Evans, N. J., II, Martin-Pintado, J., & Gomez-Gonzalez, J. 1997, *ApJ*, **476**, 730
 Rocha-Pinto, H. J., Scalco, J., Maciel, W. J., & Flynn, C. 2000, *A&A*, **358**, 869
 Ryan, R. E., Jr., Cohen, S. H., Windhorst, R. A., & Silk, J. 2008, *ApJ*, **678**, 751
 Saxton, C. J., Bicknell, G. V., Sutherland, R. S., & Midgley, S. 2005, *MNRAS*, **359**, 781
 Schaye, J. 2004, *ApJ*, **609**, 667

- Shetty, R., & Ostriker, E. C. 2008, [ApJ](#), **684**, 978
- Silk, J. 2001, [MNRAS](#), **324**, 313
- Silk, J. 2005, [MNRAS](#), **364**, 1337
- Silk, J., & Rees, M. J. 1998, [A&A](#), **331**, L1
- Silverman, J. D., et al. 2008, [ApJ](#), **679**, 118
- Slyz, A. D., Devriendt, J. E. G., Bryan, G., & Silk, J. 2005, [MNRAS](#), **356**, 737
- Tacconi, L. J., et al. 2008, [ApJ](#), **680**, 246
- Tamburro, D., Rix, H. W., Leroy, A. K., Mac Low, M. M., Walter, F., Kennicutt, R. C., Brinks, E., & de Blok, W. J. G. 2009, [arXiv:0903.0183](#)
- Tan, J. 2000, [ApJ](#), **536**, 173
- Tasker, E. J., & Bryan, G. L. 2006, [ApJ](#), **641**, 878
- Tasker, E. J., & Tan, J. C. 2008, [arXiv:0811.0207](#)
- Thompson, T. A., Quataert, E., & Murray, N. 2005, [ApJ](#), **630**, 167
- Wada, K., & Norman, C. A. 2007, [ApJ](#), **660**, 276
- Walter, F., Riechers, D., Cox, P., Neri, R., Carilli, C., Bertoldi, F., Weiss, A., & Maiolino, R. 2009, [Nature](#), **457**, 699
- Wolfe, A. M., & Chen, H.-W. 2006, [ApJ](#), **652**, 981
- Wolfire, M. G., McKee, C. F., Hollenbach, D., & Tielens, A. G. G. M. 2003, [ApJ](#), **587**, 278
- Woo, J.-H., Treu, T., Malkan, M. A., & Blandford, R. D. 2008, [ApJ](#), **681**, 925