# Homework 5

10/8/2021

## Question 1

```
n1 <- 10000
u1 <- runif(n1)
gx1<- exp((-u1^3)/3)/(1+u1^2)
theta.hat1 <- mean(gx1)
```

-Ans. $\hat{\theta} = 0.7399254$ vs. Actual Integral $= 0.73845....$

## Question 2 (Ex. 6.1)

```
n2 <- 10000
u2 <- runif(n2, 0, pi/3)
gx2<- sin(u2)
theta.hat2 <- (pi/3)*mean(gx2)
```

-Ans. $\hat{\theta} = 0.5020301$ vs. Actual Integral $= 0.5$.

## Question 3 (Ex. 6.4)

```
#Beta(3,3)
betasim <- function (n, alpha, beta, quantile){
  u <- runif(n, 0, quantile)
  gx <- (factorial(alpha+beta-1)/(factorial(alpha-1)*factorial(beta-1)))*(u^(alpha-1))*((1-u)^(beta-1))
  return (mean(gx)*quantile)
}
```

```
n3 <- 10000
alpha <- 3
beta <- 3

theta.hat3 <- numeric(9)
for (i in 1:9){
  theta.hat3[i] <- betasim(n3, alpha, beta, i/10)
}
```

```
p <- seq(.1, .9, .1)
cdf3 <- pbeta(p, alpha, beta)
round(rbind(p, cdf3, theta.hat3),3)
```

```
##             [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9]
## p          0.100 0.200 0.300 0.400 0.500 0.600 0.700 0.800 0.900
## cdf3       0.009 0.058 0.163 0.317 0.500 0.683 0.837 0.942 0.991
## theta.hat3 0.009 0.058 0.163 0.317 0.495 0.690 0.835 0.944 0.991
```

## Question 4

```
n4 <- 10000
u4 <- runif(n4, 0, 0.8)
gx4<- exp((-u4^2)/2)/(sqrt(2*pi))
theta.hat4 <- 0.8*mean(gx4)

r4 <- rnorm(n4)
gr4<- exp((-r4^2)/2)/(sqrt(2*pi))
theta.star4 <- mean(gr4)
```

-Ans.

| $\hat{\theta}$ | $\theta*$ | Actual Integral |
|---|---|---|
| 0.2880984 | 0.2815817 | 0.288145 |

## Question 5

```
#Gamma(3,2)
gammasim <- function (n, r, lambda, quantile){
  u <- runif(n, 0, quantile)
  gx <- ((lambda^r)/factorial(r-1))*(u^(r-1))*(exp(-lambda*u))
  return (mean(gx)*quantile)
}
```

```
n5 <- 10000
r <- 3
lambda <- 2

theta.hat5 <- numeric(9)
for (i in 1:10){
  theta.hat5[i] <- gammasim(n5, r, lambda, (i*2)/10)
}

p5 <- seq(.2, 2, .2)
cdf5 <- pgamma(p5, r, lambda)
round(rbind(p5, cdf5, theta.hat5),3)
```

```
##              [,1]  [,2]  [,3]  [,4]  [,5] [,6]  [,7] [,8]  [,9] [,10]
## p5          0.200 0.400 0.600 0.800 1.000 1.20 1.400 1.60 1.800 2.000
## cdf5        0.008 0.047 0.121 0.217 0.323 0.43 0.531 0.62 0.697 0.762
## theta.hat5 0.008 0.048 0.121 0.218 0.321 0.43 0.535 0.62 0.697 0.761
```