

## End Semester Exam

SUB: TOC

SUB CODE: CB2204

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Date: 25 May 2021

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(a) TRUE

A language is finite if it has a finite number of strings in it

$$\therefore |L| < \infty$$

We know that

① regular languages are context-free

② finite language is always regular

hence, finite language is context free.

(b) TRUE

Let  $G$  be a context free language such that

$$G = (V, \Sigma, R, S)$$

Let  $T$  be a parse tree generated by grammar

$G$ ,

when height of parse tree is 1 then

max yield = longest yield of  $T = \max(\text{length}(\alpha))$   
 $A \rightarrow \alpha \in R$

let it be 'P'

The maximum yield for next height is  $p^2$ ,

using mathematical induction, ~~the~~  
the max yield is  $p^h$  where  $h$  is the  
height of the tree

Hence there exists an upper bound.

(A) (C) FALSE

Pumping theorem for the class of regular language can be used to prove, that a language is not regular. Regular language is a subset of CFL. So this language may or <sup>may not</sup> be CFL. Hence we can't use pumping theorem for the class of regular language to prove that a language is CFL.

~~7(a)~~  
 ~~$\{w_1, w_2 : w_i \in \{a, b\}^*\}$~~

(1) (d)

FALSE

we know that regular languages are also context free and the intersection of class of regular lang. and CFLs are also context free.  $\Rightarrow$   
Hence there might be some combinations of 2 CFLs whose intersection is also context free



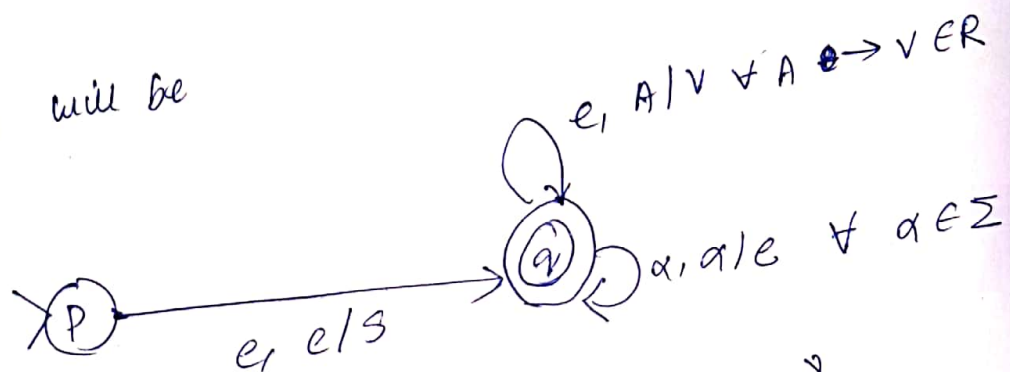
(2)  
(a)  $\{w_1, w_2 : w_1 \in \{a, b\}^*, w_2 \in \{c, d\}^* \text{ and } |w_1| = 2|w_2|$   
 that is length of  $w_1$  is twice that of  $w_2$

Rules are :-

$R = \{$   
 $S \rightarrow aaSc$   
 $S \rightarrow bbSc$   
 $S \rightarrow abSc$   
 $S \rightarrow baSc$   
 $S \rightarrow aaSd$   
 $S \rightarrow bbSd$   
 $S \rightarrow abSd$   
 $S \rightarrow baSd$   
 $S \rightarrow e \}$

$\Sigma = \{a, b, c, d\}$

PDA will be



the PDA  $M = \{K, \Sigma, \Gamma, \Delta, \delta, F\}$

$K = \{P, q\}$

$\Sigma = \{a, b, c, d\}$

$\Gamma = \{a, b, S, c, d\}$

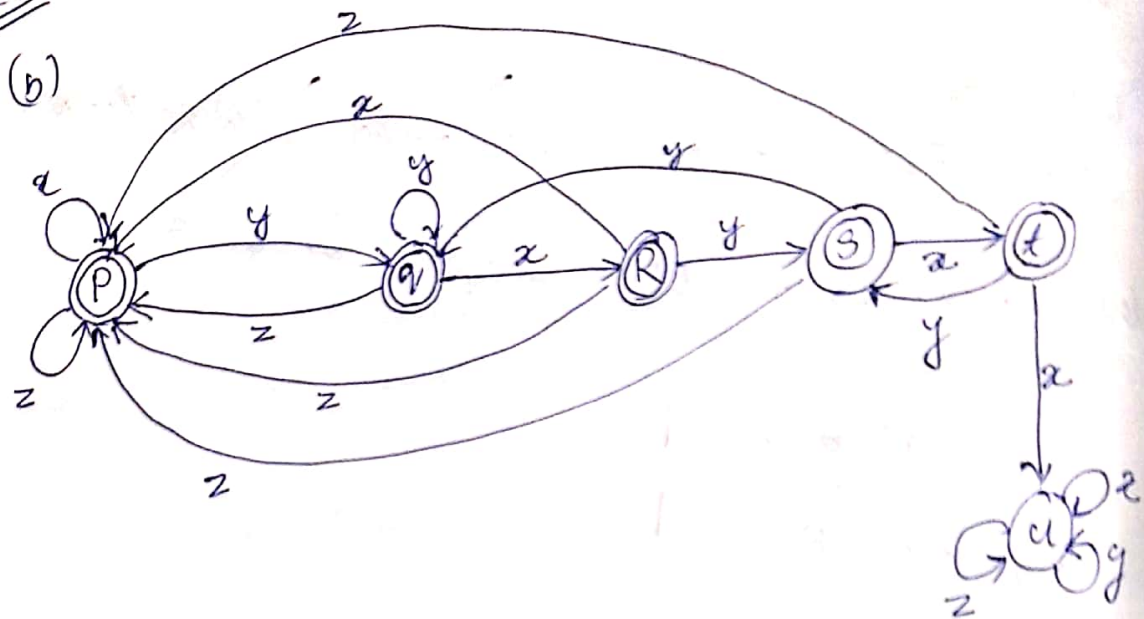
$\Delta = P$

$F = \{q\}$

$$\Delta = \{ (p, e, e, q, s), (q, e, s, q, aa, sc) \\ (q, e, s, q, bb, sc), (q, e, s, q, ab, sc) \\ (q, e, s, q, ba, sc), (q, e, s, q, aa, sd) \\ (q, e, s, q, bb, sd), (q, e, s, q, ab, sd) \\ (q, e, s, q, ba, sd), (q, a, a, q, e) \\ (q, b, b, q, e), (q, c, c, q, e) \\ (q, d, d, q, e), (q, e, s, q, e) \}$$

(2)

(b)



$$M = \{ K = \{ P, Q, R, S, T, U \}, Z = \{ x, y, z \} \}$$

$$\begin{aligned} \delta = & (P, y, Q), (P, x, P), (P, z, P) \\ & (Q, x, R), (Q, y, Q), (Q, z, P) \\ & (R, x, P), (R, y, S), (R, z, P) \\ & (S, x, T), (S, y, Q), (S, z, P) \\ & (T, x, U), (T, z, P), (T, y, S) \\ & (U, x, U), (U, y, U), (U, z, U) \end{aligned}$$

$$\$ = P$$

$$F = \{ P, Q, R, S, T \}$$

}



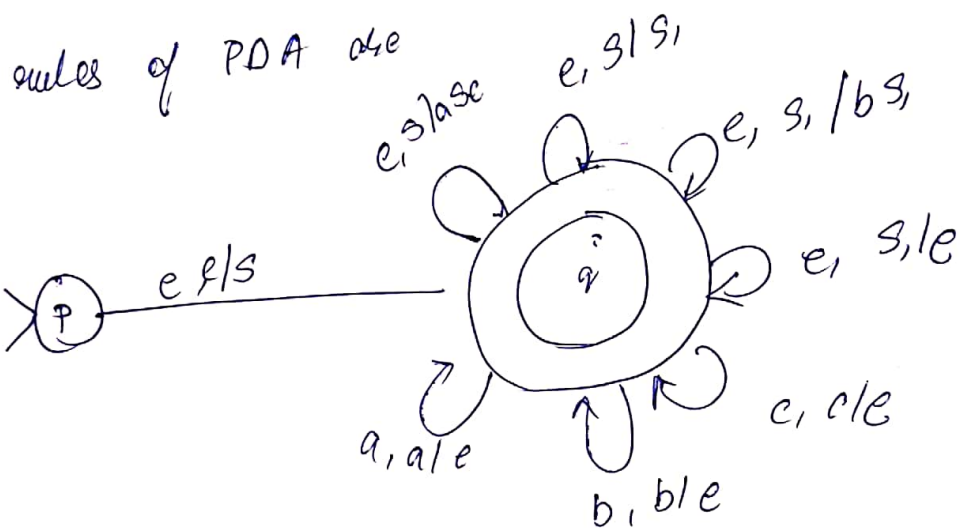
$$2(c) = \{a^i b^j c^i : i, j \geq 0\} \text{ [PDA]}$$

$$L = \{w \in \{a, b, c\}^* \mid a^i b^j c^i, i, j \geq 0\}$$

rules of CFG be

$$R = \left\{ \begin{array}{l} S \rightarrow aSc \\ S \rightarrow S_1 \\ S_2 \rightarrow bS_1 \\ S_2 \rightarrow e \end{array} \right\}$$

$$\Sigma = \{a, b, c\}$$



$$\text{the PDA} = (K, \Sigma, \Gamma, \Delta, q, F)$$

$$K = \{P, q\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, b, c, S_1, S_2\}$$

$$\Delta = \left\{ \begin{array}{l} (P, e, e, q, S_1), (q, e, S_1, q, aSc) \\ (q, e, S_1, q, S_2), (q, e, S_2, q, bS_1) \\ (q, e, S_1, q, e), (q, a, a, q, e) \\ (q, b, b, q, e), (q, c, c, q, e) \end{array} \right\}$$

$$q = P$$

$$F = \{q\}$$

(3)

(a) Given  $\{a^i b^j c^j d^i : i, j \geq 0\}$  [CFG]

the context free grammar  
 $G = (V, \Sigma, R, S)$  is constructed as:

$$R = \{ \begin{array}{l} S \rightarrow a S d \\ S \rightarrow S S_1 \\ S_1 \rightarrow b S_1 c \\ S_1 \rightarrow \epsilon \end{array} \}$$

$$V = \{ S, S_1, a, b, c, d \}$$

$$\Sigma = \{ a, b, c, d \}$$

$$S = S$$

This generates the language given above

(3) (b)  $\{a^i b^j c^i d^j : i, j \geq 0\}$  [06]  
 the rules for the unrestricted grammar are defined below.

~~$S \rightarrow S_1 S_2$~~   
 ~~$S_1 \rightarrow b S_2 d$~~   
 ~~$S_1 \rightarrow [$~~

~~$S \rightarrow S_1 S_2$~~   
 ~~$S_2 \rightarrow$~~

$R = \{$   
 $S \rightarrow S_1 S_2$   
 $S_1 \rightarrow a S_1 c$   
 $S_2 \rightarrow B S_2 d$   
 $S_1 \rightarrow [$   
 $S_2 \rightarrow ]$   
 $CB \rightarrow BC$   
 $c] \rightarrow ]c$   
 $[B \rightarrow b[$   
 $[ \rightarrow e$   
 $\}$

Let the unrestricted grammar be  $G = (V, \Sigma, S, R)$

$V = \{a, b, c, d, S, S_1, S_2, B, C, [, ]\}$

$\Sigma = \{a, b, c, d\}$

$S = S$

$R$  as written above.