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SUBJECT : THEORY OF COMPUTATION

HOME TASK 3

1) $L_1 = \{ ww \mid w \text{ in } \{0,1\}^* \}$

Let L_1 be regular.

$\therefore$ L_1 satisfies pumping property

Let k be the pumping constant

$S = 0^k 1 0^k 1$, $S \in L$

Since $s \in L$ and $|s| = 2k+2 \geq k$,

$S = uvw$ where $|uv| \leq k$,

$|v| \geq 1$

and $uv^iw \in L$

$\forall i \geq 1$

since $|uv| \leq k$, Let $v = 0^m$,

where $m \geq 1$

$\therefore$ $|uv^iw| = |uvwv^{i-1}| = 2k+2+(i-1)m$

$uv^iw = 0^{im} 0^{k-m} 1 0^k 1$

$= 0^{k+(i-1)m} 1 0^k 1$

for $i \geq 2$, $uv^iw \notin L_1$

$\therefore$ Pumping property not satisfied

$\therefore$ L_1 is not regular.

b) $L_2 = \{0^i 1^j \mid i > j\}$

Let L_2 be regular.

If L_2 is regular $\overline{L_2}$ is also regular (Closure Property)

$\overline{L_2} = \{0^i 1^j \mid i \leq j\}$

$\overline{L_2} = L_3(0^x 1^x) \cup L_4(0^x 1^y \mid x < y)$

We know $0^x 1^x$ is irregular.

$\therefore$ $\overline{L_2}$ is also irregular

$\because$ union of irregular with any language gives irregular language.

~~$\therefore \overline{L_2}$~~

$\therefore$ $\overline{L_2}$ is irregular, L_2 is also irregular ~~as be~~ as regular languages are closed under the complement operation.

$\therefore$ L_2 is irregular

c) $L_3 = \{ w \text{ in } \{0,1\}^* \mid w = w^R \}$

Let L_3 be regular.

$\therefore$ L_3 satisfies pumping property

Let k be pumping constant.

$S = 0^k 1 0^k$, $|S| = 2k+1 \geq k$

$S = uvw$, u = empty

$v = 0^m$, $m \geq 1$

$w = 0^{k-m} 1 0^k$

$uv^i w = 0^{im} 0^{k-m} 1 0^k$

$= 0^{k+(i-1)m} 1 0^k$

$\forall i \geq 2$, $uv^iw \notin L_3$.

$\therefore$ Pumping not satisfied.

$\therefore$ By contrapositive statement, L_3 is not regular

d) $L_4 = \{ (10)^p 1^q \mid p, q \in N, p \geq q \}$

$\overline{L_4} = \{ (10)^p 1^q \mid p, q \in N, p < q \}$

Let $\overline{L_4}$ be regular with pumping constant k.

$S = (10)^{k-1} 1^k$ $|S| = 3k-2 \geq k$.

$S = uvw$, u = empty

$v = 10$ [to mantain form of string when pumped]

$w = (10)^{k-2} 1^k$

$uv^iw = (10)^i (10)^{k-2} 1^k = (10)^{k+i-2} 1^k$

$uv^i w = (10)^{k+i-2} 1^k$

$\forall\, i \geq 3, \quad s \notin \overline{L_4}$

$\therefore \overline{L_4}$ is not regular as pumping property is not satisfied.

$\because \overline{L_4}$ is not regular, L_4 is aslo not regular

[Closure Property]