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SUBJECT : THEORY OF COMPUTATION

HOME TASK 1

Let a DFA, $M = (Q, \Sigma, \delta, q_0, F)$

$L(M)$: Language of machine M. (set of all strings accepted by the DFA)

$\therefore L(M) = \{ w \in \Sigma^* : M \text{ accepts } w \}$.

Let Σ be an alphabet and $A \subseteq \Sigma^*$ be a language over Σ.

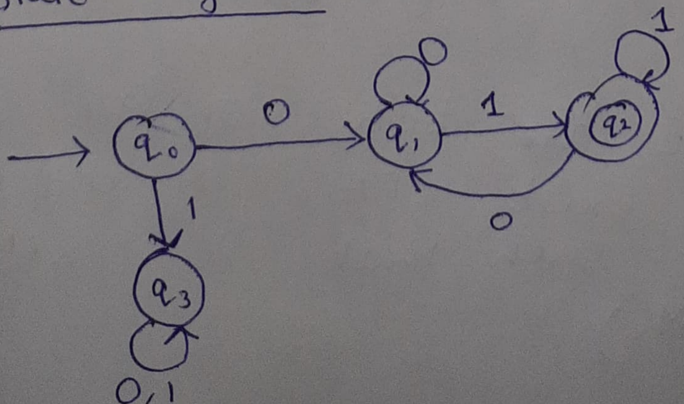
The language A is regular, if there exists a DFA such that $L(M) = A$

1a) $L_1 = \{ w \in \{0,1\}^* \mid w \text{ starts with 0 and ends with 1} \}$

If L_1 is a regular language then there exists a DFA M_1 such that $L(M_1) = L_1$, $\Sigma = \{0,1\}$

To design state digram of the DFA M_1, we define 4 states $\Rightarrow q_0, q_1, q_2, q_3$ where q_0 is the initial state.

$\therefore$ State Diagram :



q_2 = Final State

q_3 = Dead State

Since language L_1 has a corresponding DFA M_1 — such that $L(M_1) = L_1$, L_1 is a regular language

b) $L_2 = \{ w \in \{0,1\}^* \mid w \text{ contains } 1010 \text{ as substring} \}$

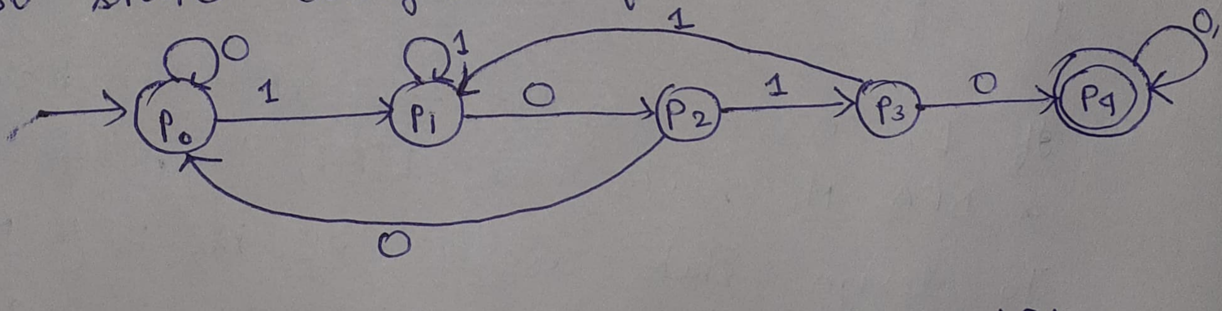
$\Sigma = \{0, 1\}$.

If L_2 is a language then there exists a DFA M_2, such that $L(M_2) = L_2$.

Let us consider 5 states of DFA as follows,

$P_0 \rightarrow$ seen nothing
$P_1 \rightarrow$ seen "1"
$P_2 \rightarrow$ seen "10"
$P_3 \rightarrow$ seen "101"
$P_4 \rightarrow$ seen "1010"

So state diagram of DFA M_2 $\Rightarrow$



Since L_2 has a corresponding DFA M_2 such that $L(M_2) = L_2$ — L_2 is a regular language.

i) L_1 and L_2 are both regular languages as there exists DFA's M_1 and M_2 respectively such that $L_1 = L(M_1)$ and $L_2 = L(M_2)$

ii) ~~To check~~ We have seen L_1 & L_2 are regular. We know regular languages are closed under union operation. $\therefore$ Since L_1 & L_2 are regular languages, $L_1 \cup L_2$ is also regular.

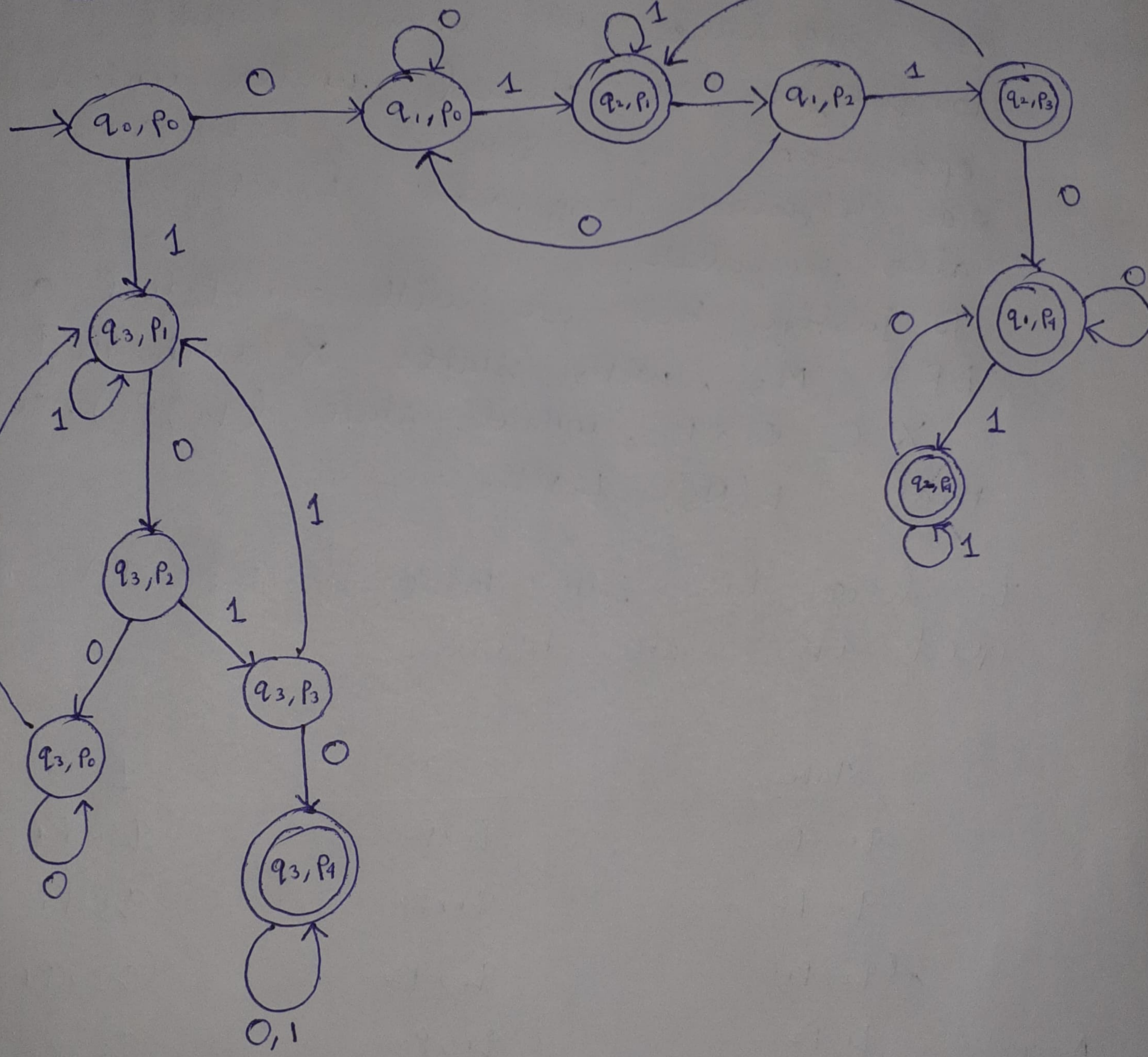
To justify this we will construct a DFA M_3, with states X such that $X \subseteq Q \times P$, initial state (q_0, p_0), such that $L(M_3) = L_1 \cup L_2$

To design the state table of the DFA we need its state table

Accepted by DFA

State	0	1
q_0, p_0	q_1, p_0	q_3, p_1
q_1, p_0	q_1, p_0	q_2, p_1
(q_2, p_1)	q_1, p_2	q_2, p_1
q_1, p_2	q_1, p_0	q_2, p_3
(q_2, p_3)	q_1, p_4	q_2, p_1
(q_1, p_4)	q_1, p_4	q_2, p_4
(q_2, p_4)	q_1, p_4	q_2, p_4
q_3, p_1	q_3, p_2	q_3, p_1
q_3, p_2	q_3, p_0	q_3, p_3
q_3, p_0	q_3, p_0	q_3, p_1
q_3, p_3	q_3, p_4	q_3, p_1
(q_3, p_4)	q_3, p_4	q_3, p_4

State Diagram:



∴ $L_1 \cup L_2$ is also a regular language.