## APPENDIX: LOCAL CONVERGENCE PROOF OF THE PSO METHOD

The update function of the moving velocity of the ellipse parameters can be expressed as:

$$\boldsymbol{v}_k^{t+1} = \omega \boldsymbol{v}_k^t + c_1 r_1 \left(\boldsymbol{x}_{k,pb} - \boldsymbol{x}_k^t\right) + c_2 r_2 \left(\boldsymbol{x}_{\mathrm{gb}}^i - \boldsymbol{x}_k^t\right) \tag{1}$$

where, $\boldsymbol{x}_k^t = \left\{E_1^t, \cdots, E_s^t; R_{\mathrm{o}}^t\right\}$ represents the ellipse parameters at time *t*, $\boldsymbol{x}_{k,pb} = \left\{E_1^k, \cdots, E_s^k; R_{\mathrm{o}}^k\right\}$ represents the optimized ellipse parameters of the *k*th particle, $\boldsymbol{x}_{\mathrm{gb}}^i = \left\{E_1^*, \cdots, E_s^*; R_{\mathrm{o}}^i\right\}$ represents the optimized ellipse parameters among all particles for the *i*th circular target, $\boldsymbol{v}_k^t$ represents the moving velocity of the ellipse parameters of the *k*th particle at time *t*, $\omega$ represents the inertia weight of the moving velocity of the ellipse parameters, $c_1$ and $c_2$ represent cognitive coefficient of $\boldsymbol{x}_{k,pb}$ and social coefficient of $\boldsymbol{x}_{\mathrm{gb}}^i$, respectively, and $r_1, r_2 = \mathrm{rand}(\ )$.

The update function of the optimized ellipse parameters can be expressed as:

$$\boldsymbol{x}_k^{t+1} = \boldsymbol{x}_k^t + \boldsymbol{v}_k^{t+1} \tag{2}$$

Combined (1) with (2), the following formulation can be introduced by:

$$\begin{aligned} \boldsymbol{x}_k^{t+1} &= \left[1 + \omega - \left(c_1 r_1 + c_2 r_2\right)\right] \boldsymbol{x}_k^t - \omega \boldsymbol{x}_k^{t-1} \\ &\quad + c_1 r_1 \boldsymbol{x}_{k,pb} + c_2 r_2 \boldsymbol{x}_{\mathrm{gb}}^i \end{aligned} \tag{3}$$

The expectation of (3) can be further simplified as:

$$\begin{aligned} E\left(\boldsymbol{x}_k^{t+1}\right) &= \left(1 + \omega - \frac{c_1 + c_2}{2}\right) E\left(\boldsymbol{x}_k^t\right) \\ &\quad - \omega E\left(\boldsymbol{x}_k^{t-1}\right) + \frac{c_1 \boldsymbol{x}_{k,pb} + c_2 \boldsymbol{x}_{\mathrm{gb}}^i}{2} \end{aligned} \tag{4}$$

The characteristic equation of (4) can be expressed as:

$$\lambda^2 - \left(1 + \omega - \frac{c_1 + c_2}{2}\right)\lambda + \omega = 0 \tag{5}$$

Assume that two solutions of (5) are $\lambda_1$ and $\lambda_2$. If $\left|\lambda_1\right|$ and $\left|\lambda_2\right|$ are both less than 1, then $E\left(\boldsymbol{x}_k^t\right)$ is convergent. And it is proved that the necessary and sufficient condition is $0 \le \omega < 1$ and $0 < c_1 + c_2 < 4(1+\omega)$, while $E\left(\boldsymbol{x}_k^t\right)$ is convergent to $\dfrac{c_1 \boldsymbol{x}_{k,pb} + c_2 \boldsymbol{x}_{\mathrm{gb}}^i}{c_1 + c_2}$.

The variance of (3) can be expressed as:

$$\begin{aligned} & D\left(\boldsymbol{x}_k^{t+2}\right) \\ &= \left(\psi^2 + R^t - \omega\right) D\left(\boldsymbol{x}_k^{t+1}\right) - \omega\left(\psi^2 + R^t - \omega\right) D\left(\boldsymbol{x}_k^t\right) \\ &\quad + R\left[\left(E\left(\boldsymbol{x}_k^{t+1}\right) - \mu\right)^2 + \omega\left(E\left(\boldsymbol{x}_k^t\right) - \mu\right)^2\right] \\ &\quad - 2T\left[E\left(\boldsymbol{x}_k^{t+1}\right) - \mu + \omega\left(E\left(\boldsymbol{x}_k^t\right) - \mu\right)\right] \\ &\quad + \omega^3 D\left(\boldsymbol{x}_k^{t-1}\right) + Q(1+\omega) \end{aligned} \tag{6}$$

where, $\upsilon = \dfrac{c_1 + c_2}{2}$, $\mu = \dfrac{c_1 \boldsymbol{x}_{k,pb} + c_2 \boldsymbol{x}_{\mathrm{gb}}^i}{c_1 + c_2}$, $\psi = 1 + \omega - \upsilon$, $R^t = c_1 r_1 + c_2 r_2 - \upsilon$, and $Q^t = \dfrac{c_1 c_2 \left(r_1 - r_2\right)\left(\boldsymbol{x}_{\mathrm{gb}}^i - \boldsymbol{x}_{k,pb}\right)}{c_1 + c_2}$.

The characteristic equation of (6) can be expressed as:

$$\lambda^3 - \left(\psi^2 + R^t - \omega\right)\lambda^2 + \omega\left(\psi^2 - R^t - \omega\right)\lambda - \omega^3 = 0 \tag{7}$$

Similarly, it is proved that the convergent condition is $0 \le \omega < 1$, $c_1 + c_2 > 0$, and $f(1) > 0$, while $D\left(\boldsymbol{x}_k^t\right)$ is convergent to $\dfrac{1}{6f(1)}\left(\dfrac{c_1 c_2}{c_1 + c_2}\right)^2 \left(\boldsymbol{x}_{\mathrm{gb}}^i - \boldsymbol{x}_{k,pb}\right)^2 (1+\omega)$, among them, $f(1) = -\left(c_1 + c_2\right)\omega^2 + c_1 + c_2 - \dfrac{1}{3}c_1^2 - \dfrac{1}{3}c_2^2 + \left(\dfrac{1}{6}c_1^2 + \dfrac{1}{6}c_2^2 + \dfrac{1}{2}c_1 c_2\right)\omega - \dfrac{1}{2}c_1 c_2$.

All in all, if and only if $0 \le \omega < 1$, $c_1 + c_2 > 0$, and $0 < f(1) < \dfrac{c_2^2(1+\omega)}{6}$, the PSO method is convergent to $\boldsymbol{x}_{\mathrm{gb}}^i$ with probability 1.