HW - 3

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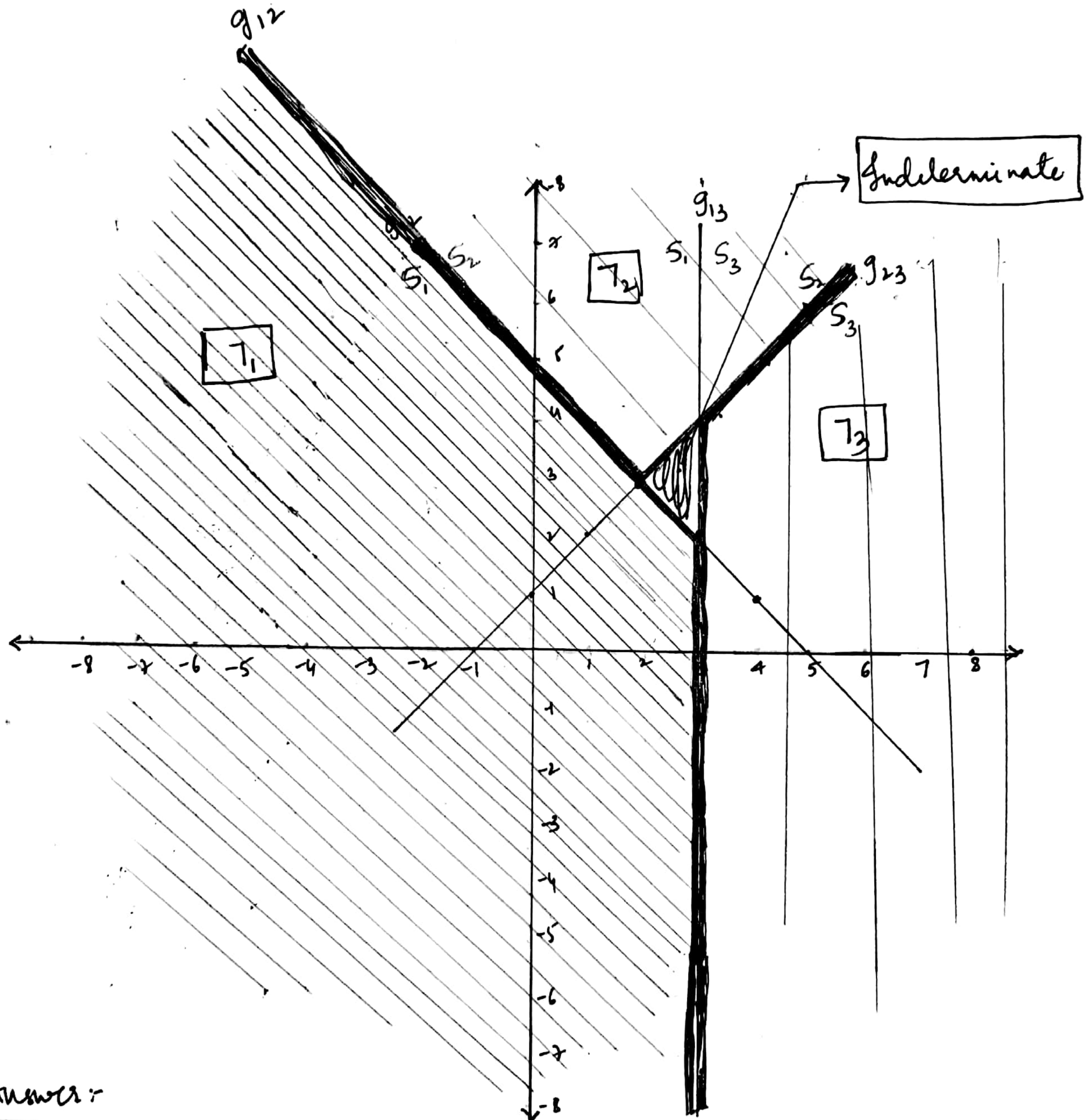
Problem 1:



Answer:-

$x = (4,1) \rightarrow$ Indeterminate because it lies on boundary of Γ_1 and Γ_3)

$x = (1,5) \rightarrow \Gamma_2 \rightarrow$ Class S_2

$x = (0,0) \rightarrow \Gamma_1 \rightarrow$ class S_1

Yes, there is an indeterminate region.
Point (2.5, 3.5) lies in that region.

Problem 3:-

a) For minimum - distance to square means,

$= \|x - \mu_i\|^2$

$= (x-\mu_i)^T (x-\mu_i)$

$= x^T x - 2x^T \mu_i + \mu_i^T \mu_i$

$-2x^T \mu_i + \mu_i^T \mu_i$ is minimized

by maximizing $x^T \mu_i - \frac{1}{2}\mu_i^T \mu_i$

when class = 2, c = 2,

$g_1(x) = x^T \mu_1 - \frac{1}{2}\mu_1^T \mu_1$

$g_2(x) = x^T \mu_2 - \frac{1}{2}\mu_2^T \mu_2$

$g(x) = g_1(x) - g_2(x)$

$= (x^T \mu_1 - \frac{1}{2}\mu_1^T \mu_1) - (x^T \mu_2 - \frac{1}{2}\mu_2^T \mu_2)$

Yes, it is linear and expression is

$$\boxed{g(x) = x^T(\mu_1 - \mu_2) - \frac{1}{2}(\mu_1^T \mu_1 - \mu_2^T \mu_2)}$$

b) w.k.t $\mu_1 = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$= x^T\left(\begin{bmatrix} 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) - \frac{1}{2}\left([0\ -2]\begin{bmatrix} 0 \\ -2 \end{bmatrix} - [0\ 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$= x^T\left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} - \frac{1}{2}(4-1)\right]$

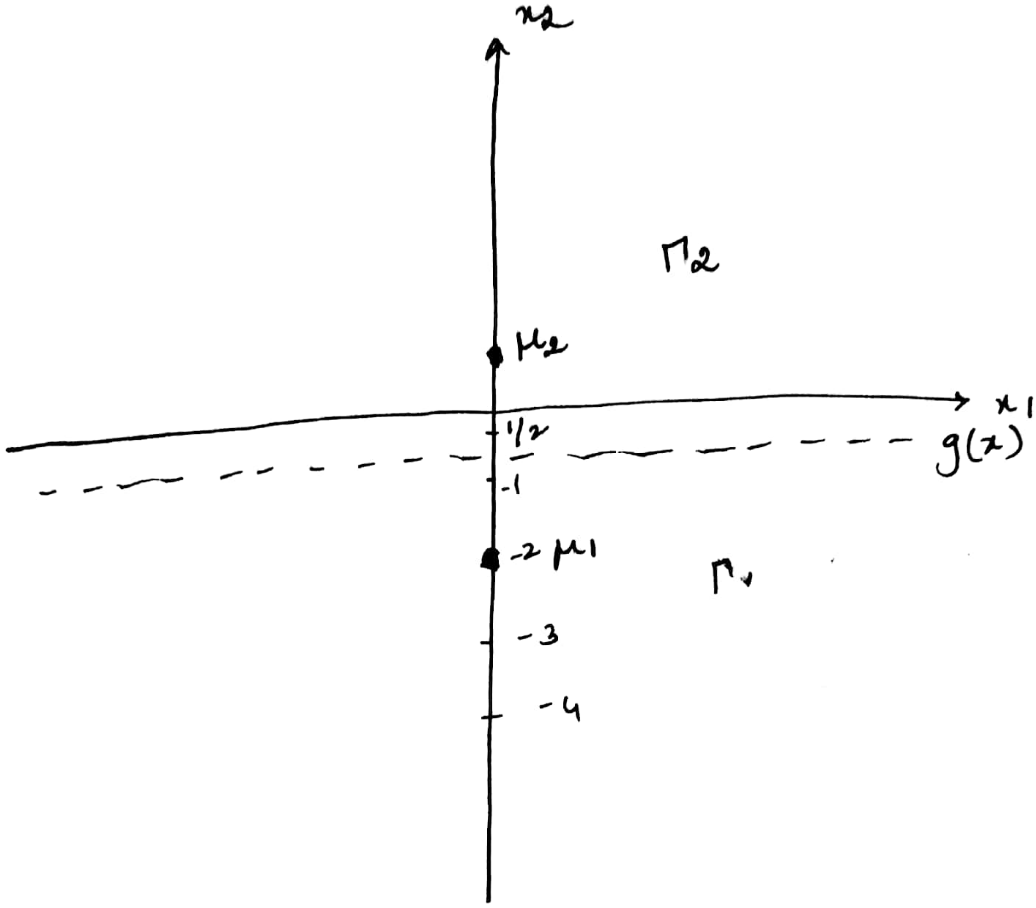
$= x^T\left(\begin{bmatrix} 0 \\ -3 \end{bmatrix} - \frac{1}{2}(3)\right]$

$= x^T\begin{pmatrix} 0 \\ -3 \end{pmatrix} - \frac{3}{2}$

$= (x_1\ x_2)\begin{pmatrix} 0 \\ -3 \end{pmatrix} - \frac{3}{2}$

$$\boxed{g(x) = -3x_2 - \frac{3}{2}}$$

Scanned by CamScanner



c) From (a),

$$g_1(x) = x^T \mu_1 - \frac{1}{2}\mu_1^T \mu_1$$

$$g_2(x) = x^T \mu_2 - \frac{1}{2}\mu_2^T \mu_2$$

$$g_3(x) = x^T \mu_3 - \frac{1}{2}\mu_3^T \mu_3$$

<u>$g_1(x)$</u>

$$g_1(x) = x^T \mu_1 - \frac{1}{2}\mu_1^T \mu_1$$

$$(x_1 \ \ x_2)\begin{pmatrix}0\\-2\end{pmatrix} - \frac{1}{2}(0 \ \ -2)\begin{pmatrix}0\\-2\end{pmatrix}$$

$$\boxed{g_1(x) = -2x_2 - 2}$$

<u>$g_2(x)$</u>

$$g_2(x) = x^T \mu_2 - \frac{1}{2}\mu_2^T \mu_2$$

$$= (x_1 \ \ x_2)\begin{pmatrix}0\\1\end{pmatrix} - \frac{1}{2}(0 \ \ 1)\begin{pmatrix}0\\1\end{pmatrix}$$

$$\boxed{g_2(x) = x_2 - \frac{1}{2}}$$

$$g_3(x) = (x_1 \;\; x_2)\begin{pmatrix}2\\0\end{pmatrix} - \frac{1}{2}(2 \;\; 0)\begin{pmatrix}2\\0\end{pmatrix}$$

$$= 2x_1 - \frac{1}{2}(4)$$

$$\boxed{g_3(x) = 2x_1 - 2}$$

h_{12}

$$g_1(x) = g_2(x)$$

$$2x_2 - 2 = x_2 - \frac{1}{2}$$

$$-3x_2 = \frac{-1}{2} + 2$$

$$-3x_2 = \frac{3}{2}$$

$$\boxed{x_2 = -1/2}$$

h_{13}

$$g_1(x) = g_3(x)$$

$$-2x_2 - 2 = 2x_1 - 2$$

$$-2x_2 = 2x_1$$

$$\boxed{x_2 = -x_1}$$

$(0,0)$, $(1,-1)$ $(-1,1)$
$(1,-2)$ $(-2,2)$

h_{23}

$$g_2(x) = g_3(x)$$

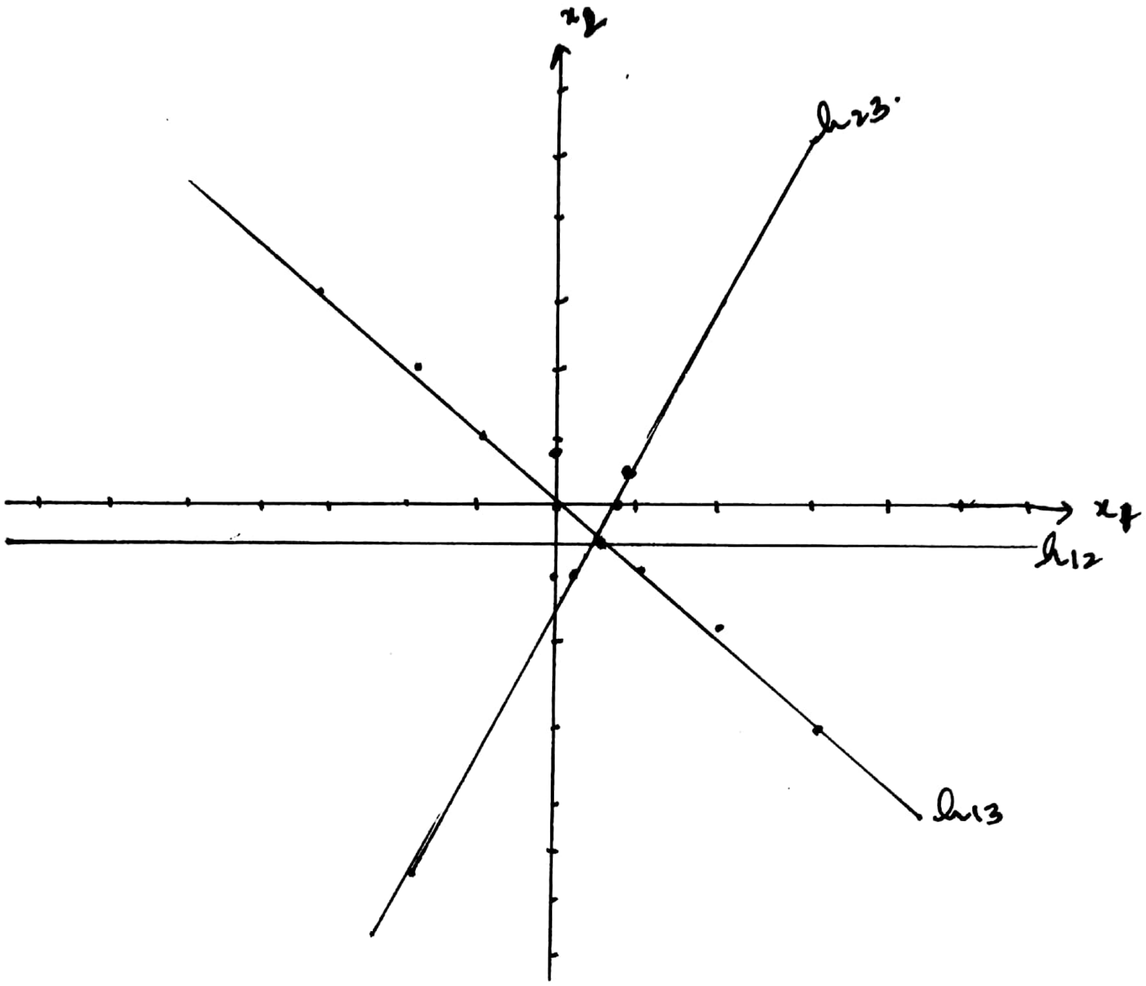
$$2x_1 - x_2 = -\frac{1}{2} + 2$$

$$2x_1 - x_2 = \frac{3}{2}$$

$$2x_1 = \frac{3}{2} - x_2$$

$$\boxed{x_2 = \frac{3}{2} - 2x_1}$$

$\left(\frac{3}{4}, 0\right)$, $\left(1, \frac{1}{2}\right)$ $\left(\frac{1}{4}, -1\right)$


x_2
h_{23}
x_1
h_{12}
h_{13}