PHIL 261 Final Essay

The question that drives this paper is 'does fictionalism offer an adequate philosophy of mathematics and can it adequately account for mathematical constraint and mathematical discovery'? Mathematical fictionalism developed in response to mathematical platonism[3]. Specifically, mathematical fictionalism developed as a means of providing an account of mathematical objectivity and mathematical truth without the additional complication of requiring a corresponding ontology[2]. Through questioning the motivation that drives the philosophy of mathematics itself and enlightening examples, Stephen Yablo's hermeneutic fictionalism provides greater ways to think about the purpose of mathematics (applicability over purity) and the practice of mathematics (best understood as metaphors while denouncing the existence of abstract objects). In this essay, I support Yablo's hermeneutic fictionalism perspective by defending his view from the critique of the three disanalogies between the fictional world and the mathematical world. Specifically, the disanalogies of mathematical constraint, mathematical discovery, and the quality of truth. I further show the strength of the hermeneutic fictionalism view by preserving it against a critique on the differences between converting language metaphors and mathematical metaphors into more literal forms. Finally, I alter the definition of 'abstract entities' through a novel variation of the epistemological argument against platonism. In doing so, I greatly reduce the distinction between platonism and fictionalism, and ultimately conclude that fictionalism proves equally adequate as platonism as a philosophy of mathematics.

I begin by providing a brief overview of mathematical fictionalism. Mathematical fictionalism maintains that mathematics is best understood as metaphor while also acknowledging and accepting the literal value of a metaphor's content as per, "numbers thus come to play a double role, functioning both as representational aids and things-represented"[1].

This perspective is clearly illustrated through the example of 'infinitesimal fluid element' which enables equations of fluid dynamics in partial differential form to be derived, irrespective of the fact that it is commonly known that fluids are discrete and not continuous[2]. And thus, mathematical fictionalism shows how mathematical metaphors can be used to approximate the real world as well as provide a representational tool for better understanding the real world.

While acknowledging the infinitely generative nature of our capacity for creating fictional worlds and mathematical worlds, it is important to begin by addressing how each is equally bound by constraint. Specifically, this paper will consider constraint as the degree to which a fictional world or mathematical world is both *consistent* and *non contradictory*. Firstly, I uphold that the fictional world and the mathematical world are accountable to holding the same degree of consistency, as both are ultimately "constrained by coherence with the story so far"[2]. Fictional worlds must be consistent as illustrated by fan faction. The extensions of fictional worlds (eg. Harry Potter, Sherlock Holmes) cannot alter the core tenets of the existing story[2]. For example, Harry Potter must retain his core elements. He cannot be arbitrarily converted into Kung Fu Panda, at least not in a legitimate work of fan fiction. Similarly, consistency in mathematical worlds must be maintained, such as adhering to prime number theory or algebraic topology[2]. Secondly, I argue that both fictional worlds and mathematical worlds are non contradictory. This line of reasoning challenges the view that fictional worlds can support contradictions without reducing the merits of the body of fictional work. This is notably seen in movie bloopers (eg. Dorothy's hair length changes in the Wizard of Oz). These errors occur due to the psychological phenomenon of inattentional blindness. However, I argue that these instances represent *mistakes in the practice of fiction* or *mistakes in the practice of mathematics* as opposed to fundamental system level contradictions. For example, mathematical errors in the practice of mathematics occur at all levels of study, from learning elementary multiplication tables to advanced calculus, without representing fundamental system level contradictions. Ultimately, both fictional worlds and mathematical worlds represent a form of "tracking reality"[1], and thus, they must maintain the principles of consistency and non contradiction in that process.

The disanalogy between fictional worlds and mathematical worlds as it pertains to discovery lies on the following frail premise. This premise argues that there is a bidirectional relationship between the real world and the mathematical world. Specifically, the mathematical world (eg. π is transcendental) is capable of influencing the real world (eg. you cannot square the circle)[2]. In addition, the real world (eg. Appel & Haken's program) is capable of influencing the mathematical world (eg. four colours suffice to colour a planar map)[2]. However, this line of reasoning holds that while the real world is capable of influencing the fictional world, the fictional world is incapable of influencing the real world, thus presenting a disanalogy. I challenge this claim on the grounds that numerous inventions first occurred in the fictional world or the imaginations of their inventors prior to the realization of those ideas in the real world. For example, Leonardo Da Vinci imagined a helicopter significantly prior to the development of helicopters. Further, this example supports the perspective that "make-believe games can make it easier to reason about such facts, to systematize them, to visualize them, to spot connections with other facts, and to evaluate potential lines of research"[1]. Thus, beyond disproving this disanalogy, these analogies strengthen the similarities between fictional worlds and mathematical worlds. Additionally, they show how both are representational tools that can be used for tracking the real world and influencing the real world by facilitating greater discovery.

Thirdly, it is important to address the disanalogy that alleges that there are varying qualities of truth found in the fictional world compared to the real world and the mathematical world. I will raise an objection to this disanalogy by putting forth the controversial idea that the mathematical world is as prone to being 'truth gappy' as the fictional world while the real world remains the only domain that can be considered to be 'truth dense'. Firstly, the disanalogy between the fictional world and real world is developed through an example of a carriage in the real world in contrast to a carriage in the fictional world of Proust's *In Search of Lost Time*[2]. It is argued that in the real world we can follow attributes or features of the carriage to their original source (eg. manufacturer) upon which we proceed to discover new knowledge about the carriage. This shows how the real world is truth dense, it remains available to answer numerous lines and directions of our curiosity. However, the same cannot be said for the fictional carriage in the work *In Search of Lost Time.* Proust has described certain features of the carriage as he deems them relevant to his story, beyond which we cannot discern further information about the carriage. Thus, this shows that fiction has a finite and limited amount of information that can be obtained about it's fictional universe. However, I argue that in the mathematical world, the amount of knowledge that can be known about a particular mathematical system can also be considered finite or limited. Firstly, this is supported by the fact that we have no universal theory of mathematics to date. Secondly, this occurs because according to the fictionalism viewpoint, mathematical worlds and fictional worlds, are fundamentally constrained metaphors for the real world. This constraint eliminates their capacity to achieve a truth density that rivals the real world. Ultimately, it remains to be seen how a representation of the real world such as fiction or a mathematical theory can possibly match the depth of truth available in the real world itself.

Mathematical fictionalism maintains that mathematics is best understood as metaphor, as it seeks to adeptly and accurately capture underlying phenomenon in the real world. However, one objection to the view of mathematics as metaphor argues that there exists the following discrepancy between the simple metaphors of language and mathematics as metaphor. In simple metaphors of language, if a person does not understand the metaphor, then it can be explained in more literal terms. For example, suppose that someone does not understand the metaphor, the classroom was a zoo. This can be explained in the more literal terms that perhaps the children were very rambunctious and the classroom was messy and disorganized. However, suppose a person does not understand the mathematical metaphor that π is transcendental. One line of reasoning argues that this metaphor cannot be explained in more literal terms but rather it can only be explained further by showing more mathematics[2]. I contest this line of reasoning by arguing that mathematical metaphors can exist at different layers of abstraction. Thus, if a person is shown more mathematics at a lower level of abstraction to further explain the metaphor that π is transcendental, this should be considered as providing a more literal interpretation. I argue that this is akin to the more literal explanations provided in the case of simple metaphors. I support the view that mathematics is rightly understood as metaphors, as some metaphors can be "better or worse in relation to how well they approximate their subject-matter or the light they shed on the portion of the world they're targeting"[2], and the same can be said for mathematical theories. There are simply some mathematical theories that are more robust and enduring than others. Further, metaphors are a strong analogy for mathematics as humans understand the world through storytelling which can be considered a form of metaphor , which can thus be said to encompass the field of mathematics.

Having debunked the disanalogies between fictional worlds and mathematical worlds, I argue that the difficulty in supporting platonism over fictionalism or fictionalism over platonism arises not because of their differences, but because of their similarities, and thus, there may be no true method of firmly distinguishing platonism from fictionalism. Platonism and fictionalism are aligned on semantic grounds while differing on the ontological thesis of the existence of abstract entities. I proceed to show how their differences on the existence of abstract entities can be resolved. I do this by altering the definition of abstract entities based on a novel line of reasoning of the epistemological argument against platonism, upon which platonism can be refined to be viewed as a form of fictionalism. While the epistemological argument objects to the nonspatial and nontemporal attributes of abstract entities to ultimately reject their existence, I accept all the attributes of abstract objects with the exception of the nonmental by arguing that we would not be capable of comprehending them if they were nonmental[3]. Because abstract entities are mental, nonspatial, nontemporal and noncausal, they can be considered fictional objects or metaphors for underlying phenomenon. For example, reading a book entails imagining characters in the mind. Thus, the characters are mental, nonspatial, nontemporal and noncausal . By successfully challenging the definition of abstract entities, and thus, altering the definition of abstract entities, it has been shown that there is significant difficulty in differentiating platonism and fictionalism. This furthers the authenticity of fictionalism and merits it's view greater consideration as a philosophy of mathematics.

In conclusion, this paper began by outlining Yablo's view of hermeneutic fictionalism and how it enables us to gain a greater understanding of the world. This fictionalism viewpoint was further supported by challenging the three disanalogies, namely, mathematical constraint, mathematical discovery, and quality of truth. In the refutation of these disanalogies, this paper showed greater similarity and stronger analogy between fictional worlds and mathematical worlds, and how they are successful tools for approximating the real world. Additionally, mathematical fictionalism was successfully defended from an attack that sought to show the differences between how simple metaphors and mathematical metaphors can be decomposed into their literal constituents. Further, this paper has shown that while platonists believe in abstract entities, abstract entities are in themselves fictional. Given the semantic similarities and alignment on the issue of abstract entities, newly refined as fictional entities or mathematical metaphors of platonism and fictionalism, this paper proposes a line of future investigation for examining the merits of platonism over fictionalism or vice versa. Namely, perhaps the distinction and merits between platonism and fictionalism rests upon their differing fundamental motivations (eg. purity vs applicability), and the utility and implications of these differing motivations to the philosophy of mathematics, and mathematics at large.

References

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