Game Architecture

Linear Algebra

Here’s looking at Euclid

Today’s Agenda

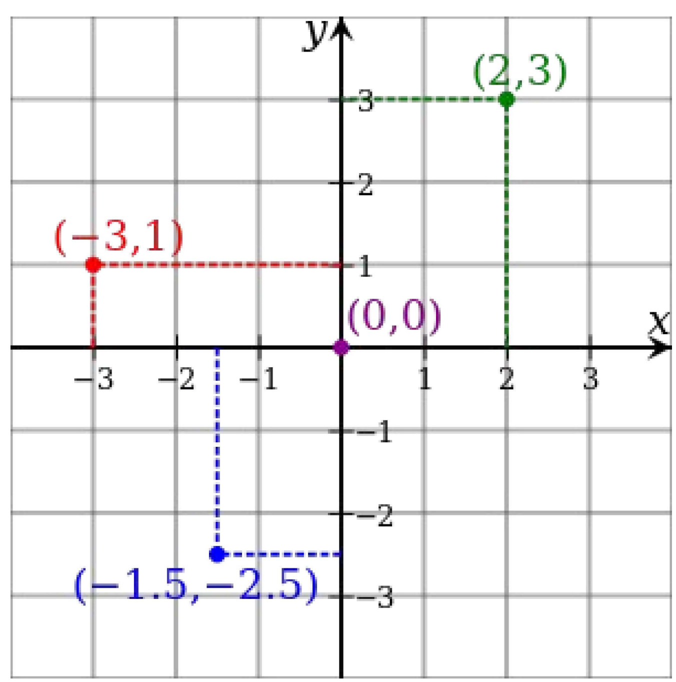
- Points and spaces
- Coordinate Systems
- Vector and vector operations
- Planes

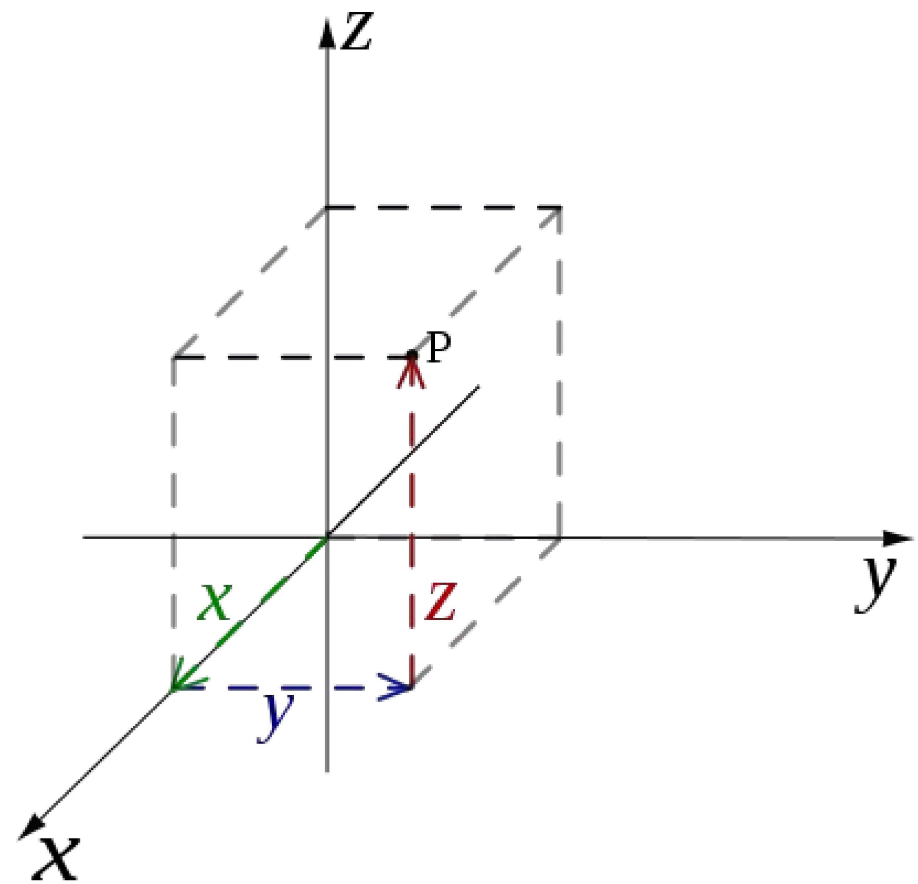
Points

- An element of some set called a space.
- Captures the idea of a unique location in that space.
- Represented by tuple of n terms where n is the dimension of the space.

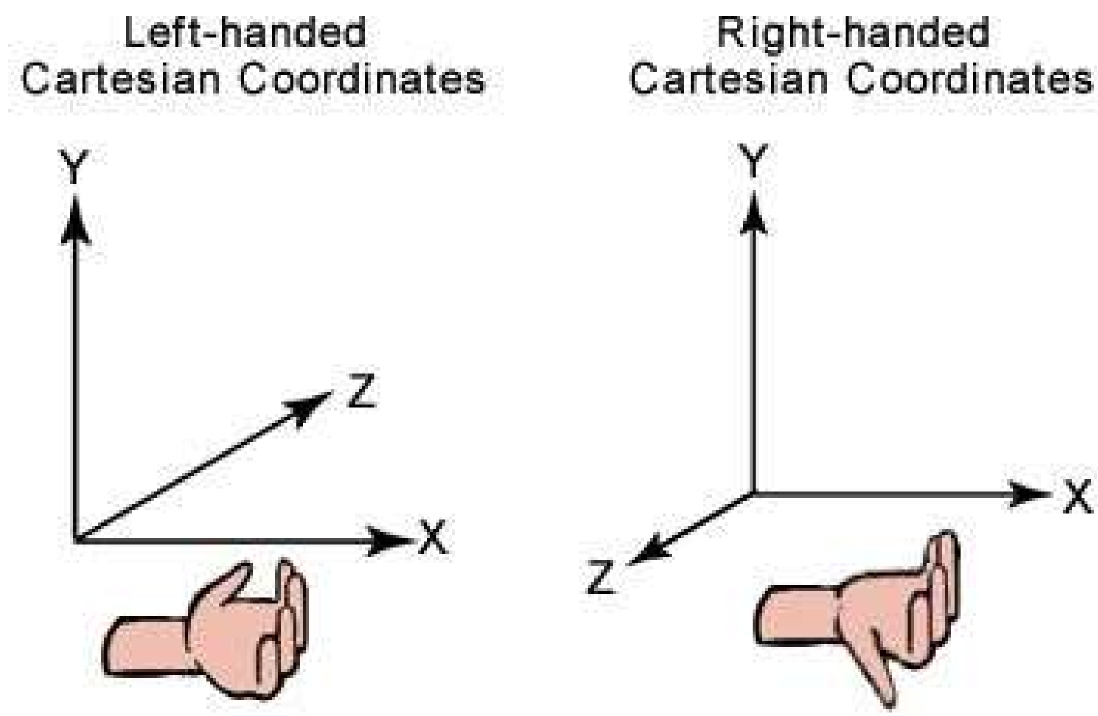
$$P = (P_x, P_y, P_z)$$

Cartesian Coordinate System

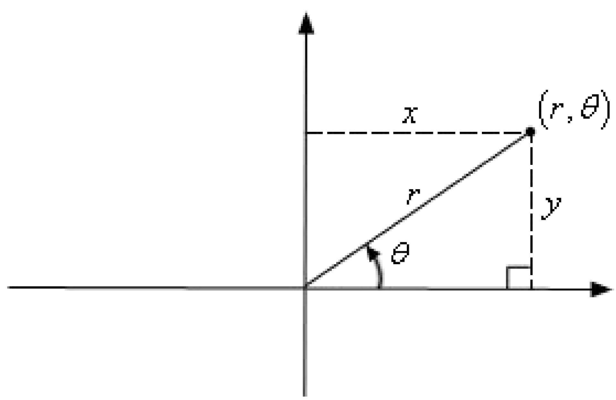


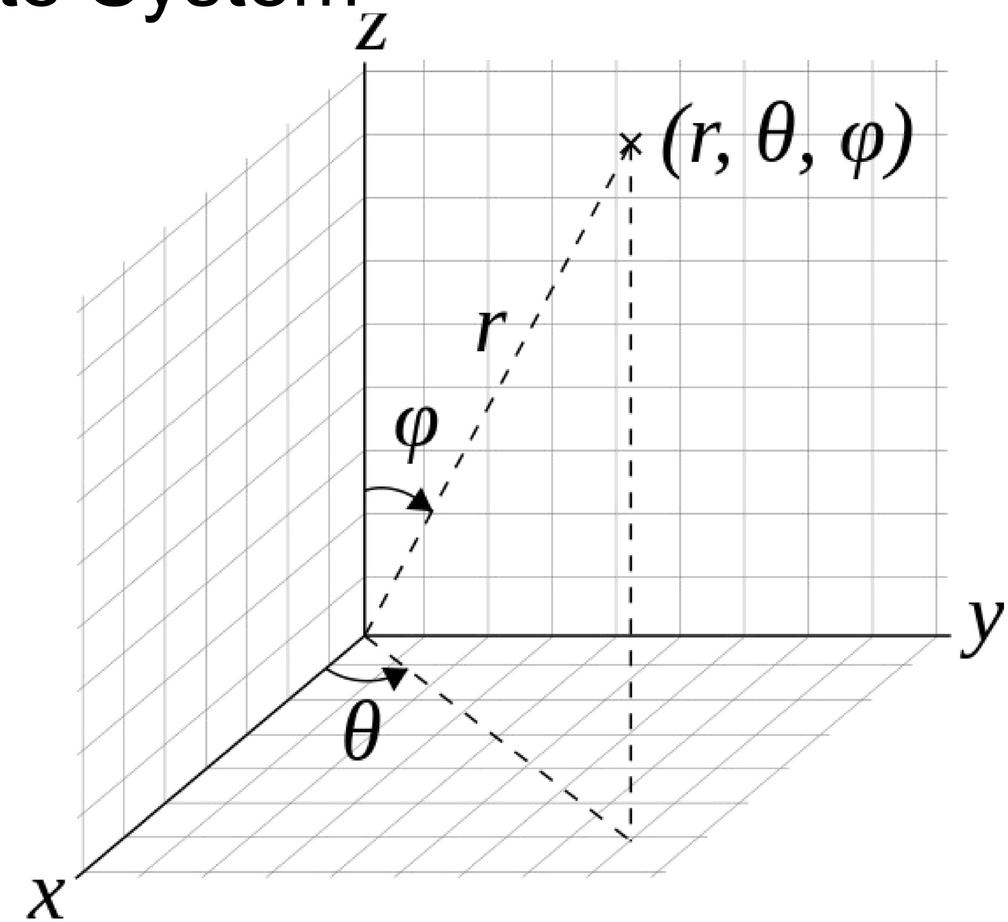


Handedness


Left-handed
Cartesian Coordinates
Y
Z
X
Right-handed
Cartesian Coordinates
Y
X
Z

Polar (Spherical) Coordinate System






I don't think this is what the professor meant by "polar coordinates"
igloos make a lot more sense in spherical coordinates, though.
2006
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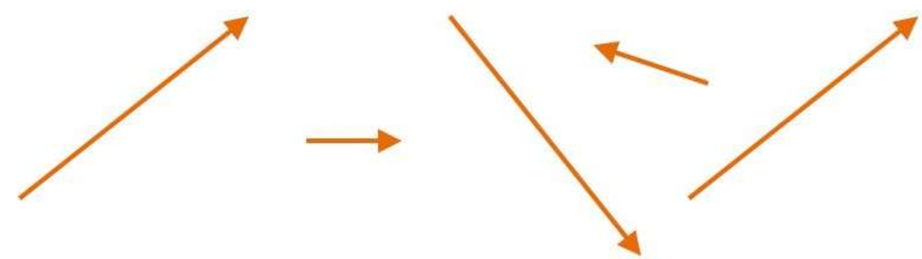
Vectors

- A geometric primitive with:
  - Direction
  - Magnitude (Length)
- Represented by tuple of n terms where n is the dimension of the space.

$$\mathbf{v} = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z)$$

Examples:

- Velocity
- Acceleration
- Force



Linear Operations

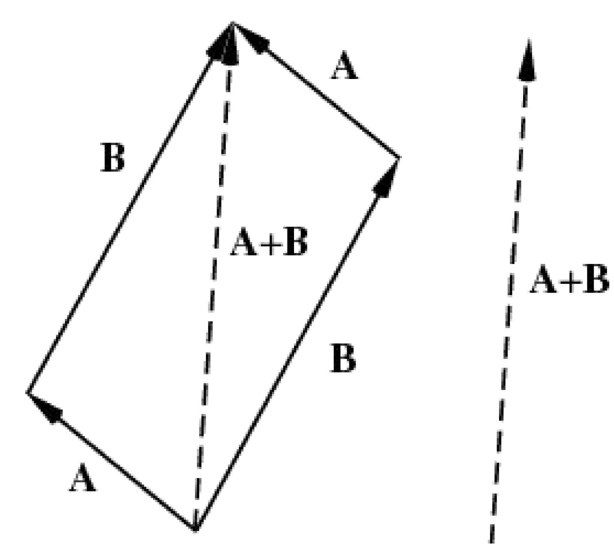
- Vector Addition
- Scalar Multiplication

Linear Operations

- **Vector Addition**

  $\mathbf{a} + \mathbf{b} = (\mathbf{a}_x + \mathbf{b}_x, \mathbf{a}_y + \mathbf{b}_y)$

- Scalar Multiplication

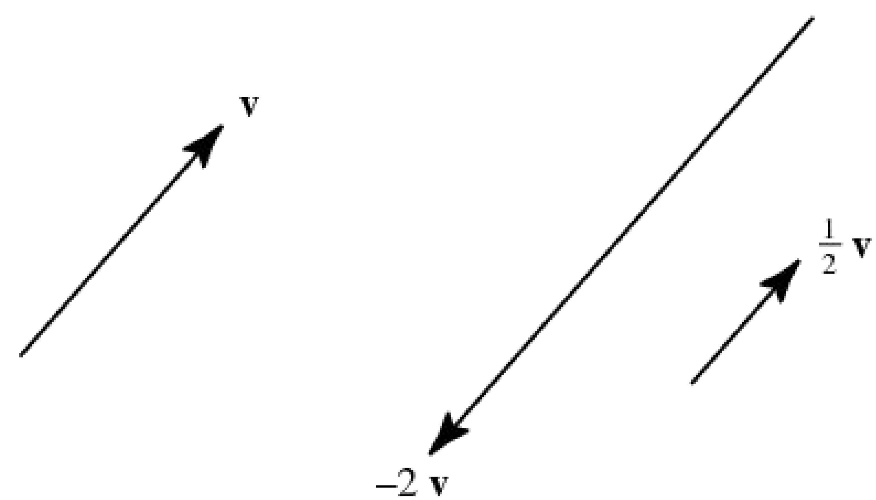


Linear Operations

- Vector Addition
- **Scalar Multiplication**

$$\mathbf{v}\alpha = (\mathbf{v}_x\alpha, \mathbf{v}_y\alpha, \mathbf{v}_z\alpha)$$

Length changes but direction does not (can go negative, but still same direction).



Vector Space

- A vector space is a set of vectors that is *closed* under linear operations (vector addition and scalar multiplication).
- A set is *closed* under an operation if the result of that operation on any members of the set is also a member of the set.

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Euclidean space ($\mathbb{R}^n$) is a vector space.

- Add two vectors together in $\mathbb{R}^n$ and get a third vector also in $\mathbb{R}^n$.
- Multiply a vector in $\mathbb{R}^n$ by a scalar in get another vector in $\mathbb{R}^n$.

Vector Basis

- A set of vectors form a *basis* if every vector in the vector space are a linear combination of the vectors in that set.

Standard Basis

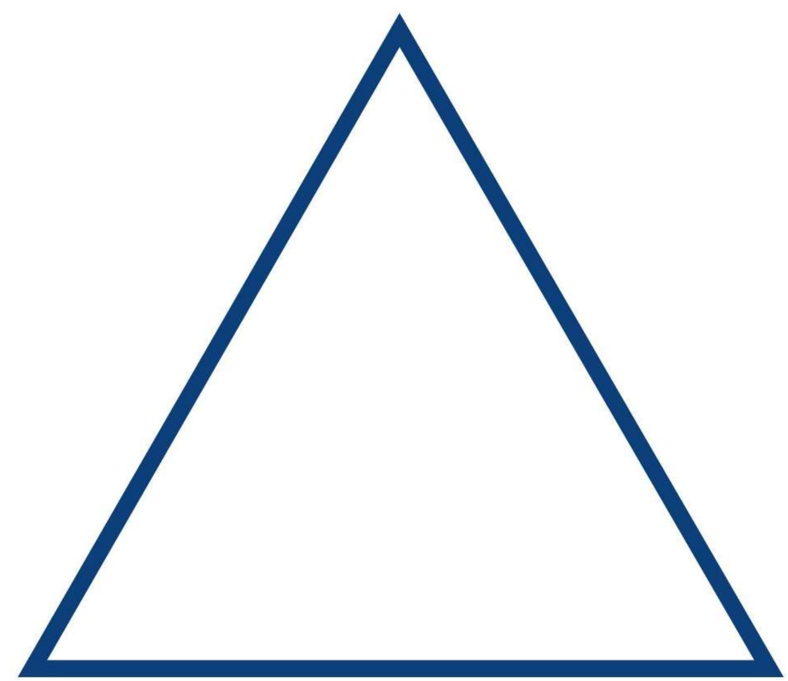
$\boldsymbol{i} = (1, 0, 0)$

$\boldsymbol{j} = (0, 1, 0)$

$\boldsymbol{k} = (0, 0, 1)$

Trigonometry

Branch of mathematics that studies relationships involving lengths and angles of triangles.

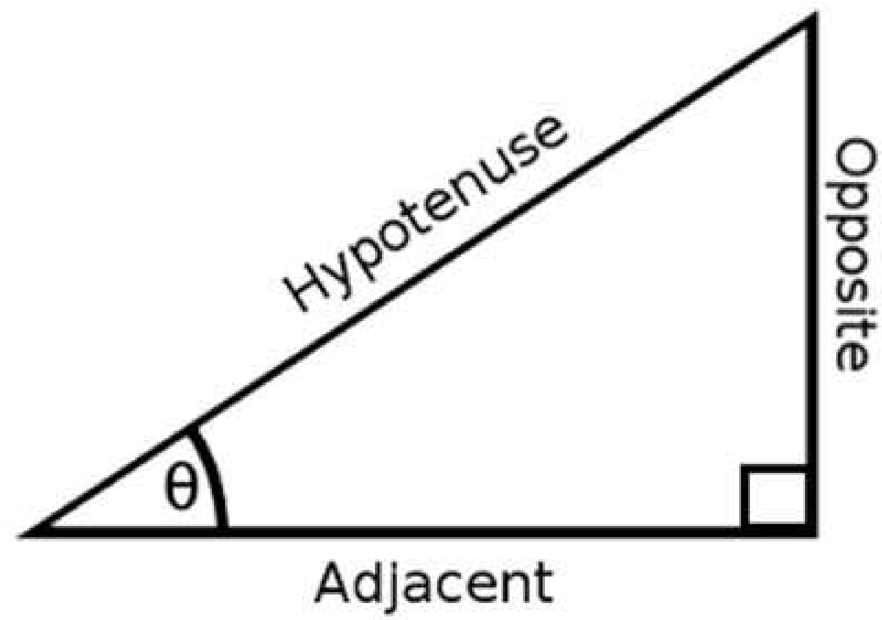


Trigonometry

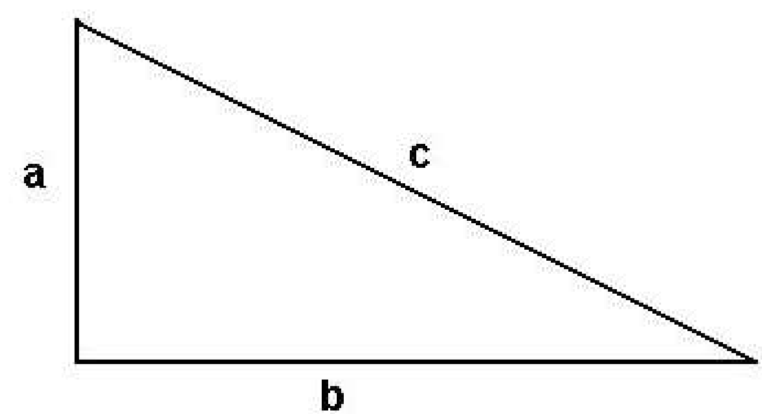
$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Opposite}{Adjacent}$$

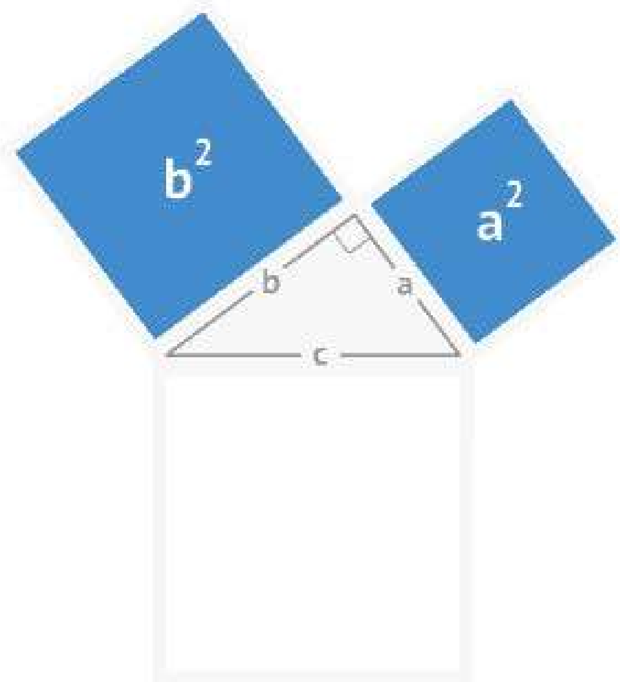


Pythagoras Theorem



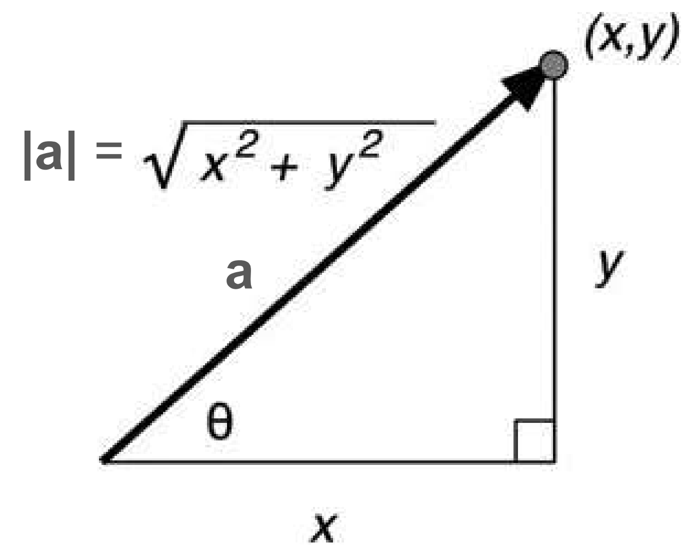
$$a^2 + b^2 = c^2$$

Pythagoras Theorem



Vector Length (Magnitude)

$$|\mathbf{a}| = \sqrt{\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}$$



Vector Normalization

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

The school of Pythagoras

The school of Pythagoras

- Odd numbers good. Even numbers evil.

Obviously.

The school of Pythagoras

- Odd numbers good. Even numbers evil.
- Beans home to souls of dead.

Do not eat the beans.

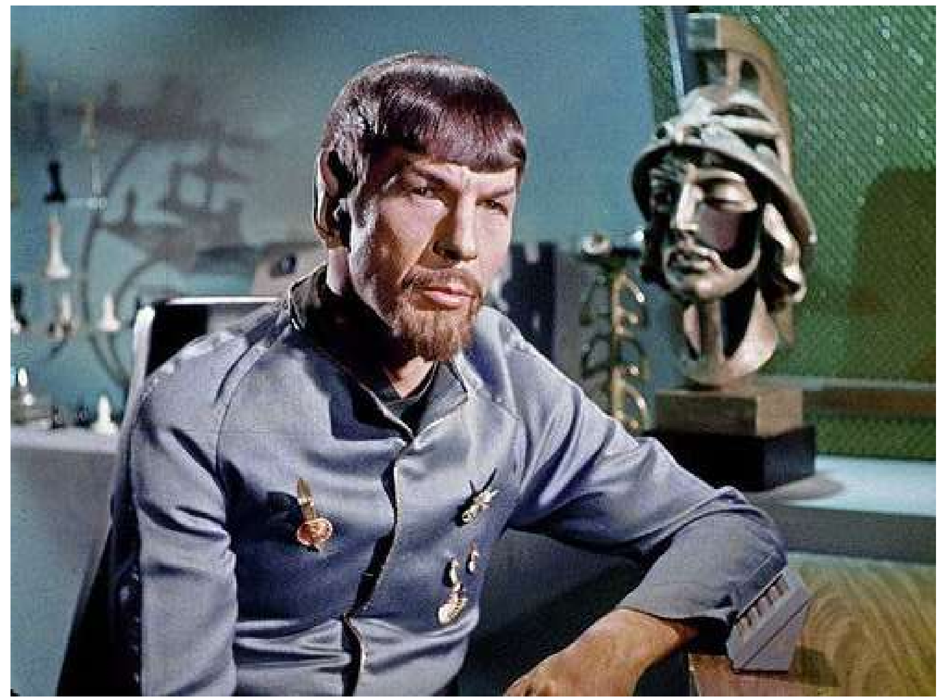
The school of Pythagoras

- Odd numbers good. Even numbers evil.
- Beans home to souls of dead.
- Wear pants.

This was a weird idea for the time.

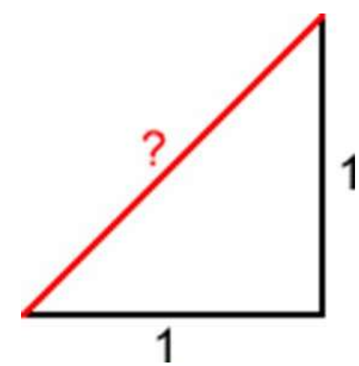
The school of Pythagoras

- Odd numbers good. Even numbers evil.
- Beans home to souls of dead.
- Wear pants.
- Do not look in mirrors.



The school of Pythagoras

- Odd numbers good. Even numbers evil.
- Beans home to souls of dead.
- Wear pants.
- Do not look in mirrors.
- All numbers expressed as ratio of whole numbers.



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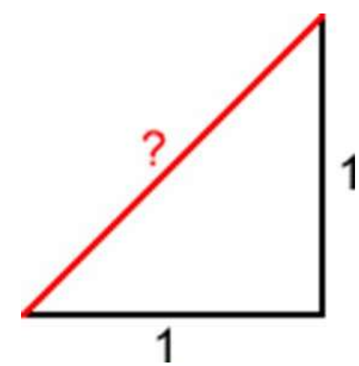

1. $1^2 + 1^2 = c^2$

The school of Pythagoras

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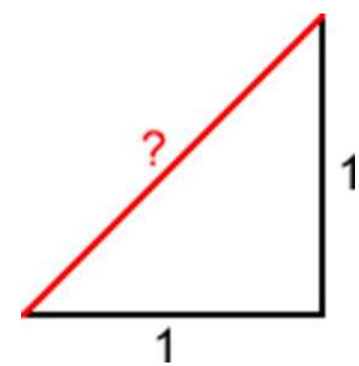
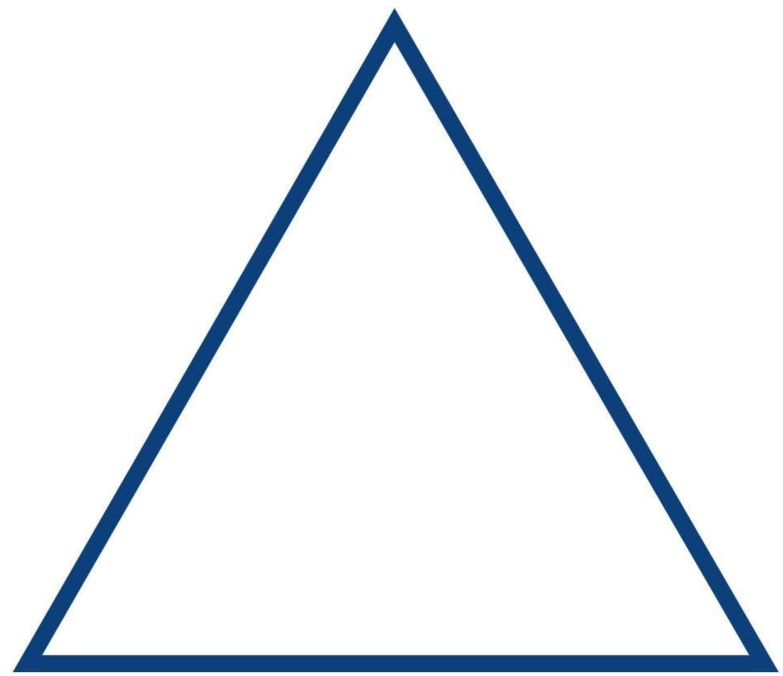


1. $1^2 + 1^2 = c^2$
2. $c^2 = 2$
3. $c = \sqrt{2}$
4. $a / b = \sqrt{2}$

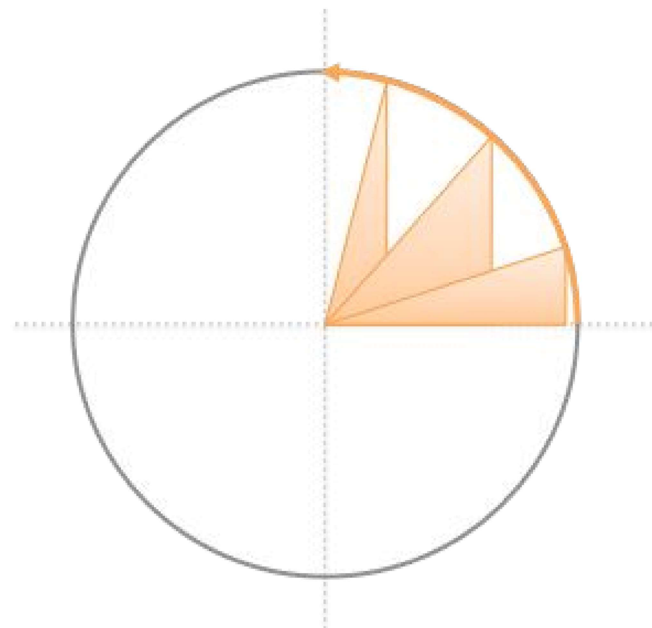
Trigonometry

Branch of mathematics that studies relationships involving lengths and angles of triangles.

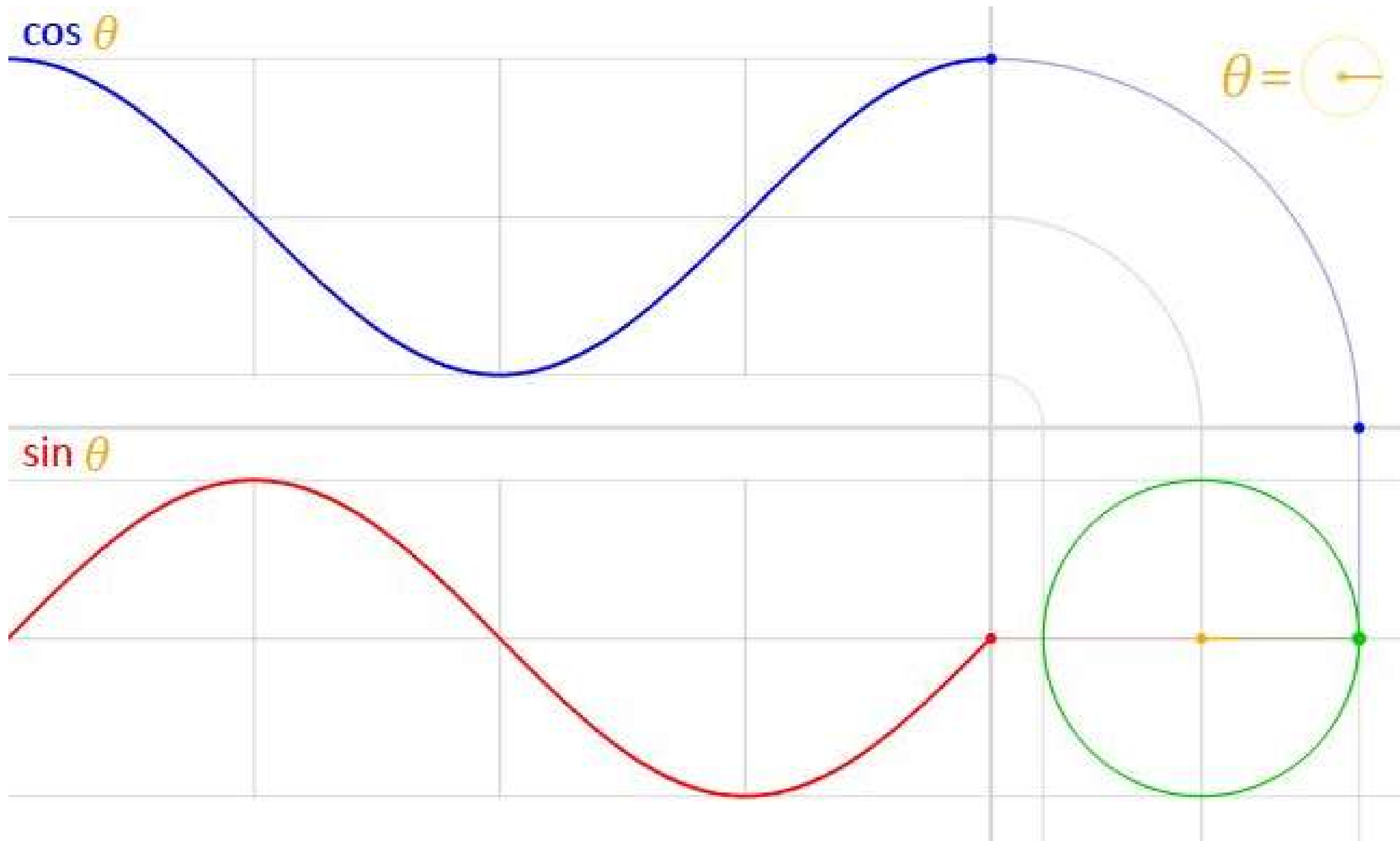


Trigonometry

Branch of mathematics that studies circles.



Geometry

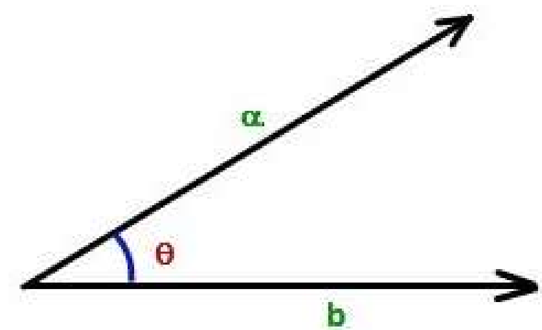


Dot Product

$$\mathbf{a}\cdot\mathbf{b} = \mathbf{a}_x\mathbf{b}_x + \mathbf{a}_y\mathbf{b}_y + \mathbf{a}_z\mathbf{b}_z$$

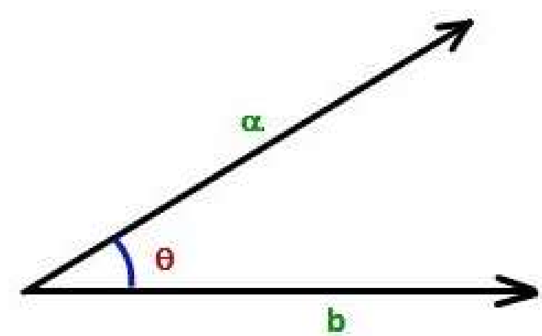
Dot Product

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$



Fun with Dot Product

- Angle between two vectors
- Vector length (squared)
- Vector projection and rejection
- Planes



Fun with Dot Product

- **Angle between two vectors**
- Vector length (squared)
- Vector projection and rejection
- Planes

Original definition:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Rearranging terms:

$$\theta = cos^{-}1 \frac{a \cdot b}{|a||b|}$$

Fun with Dot Product

- **Angle between two vectors**
- Vector length (squared)
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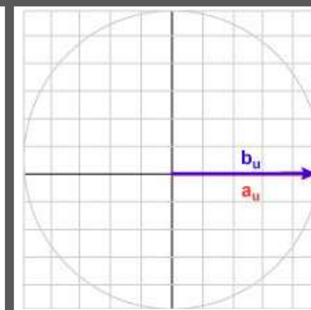
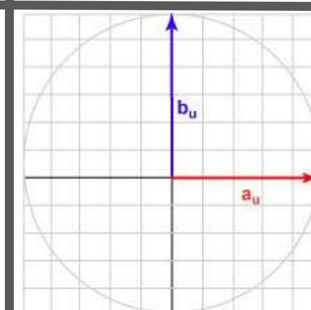
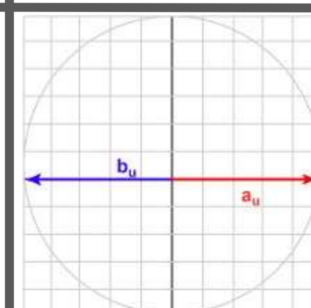
- Planes

θ	a·b	
0	1.0	
90	0.0	
180	-1.0	

Fun with Dot Product

- Angle between two vectors
- **Vector length (squared)**
- Vector projection
- Planes

Original definition:

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Substituting **b** for **a**:

$$\mathbf{a}\cdot\mathbf{a} = |\mathbf{a}||\mathbf{a}|\cos\theta$$

So θ is 0 and $\cos(0) = 1$:

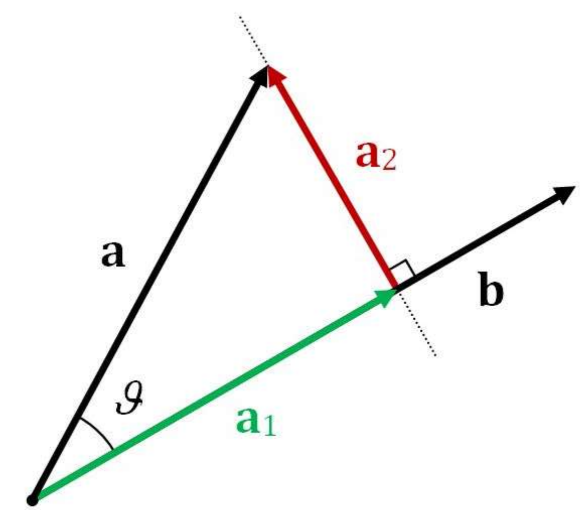
$$\mathbf{a}\cdot\mathbf{a} = |\mathbf{a}|^2$$

Fun with Dot Product

- Angle between two vectors
- Vector length (squared)
- **Vector projection**
- Planes

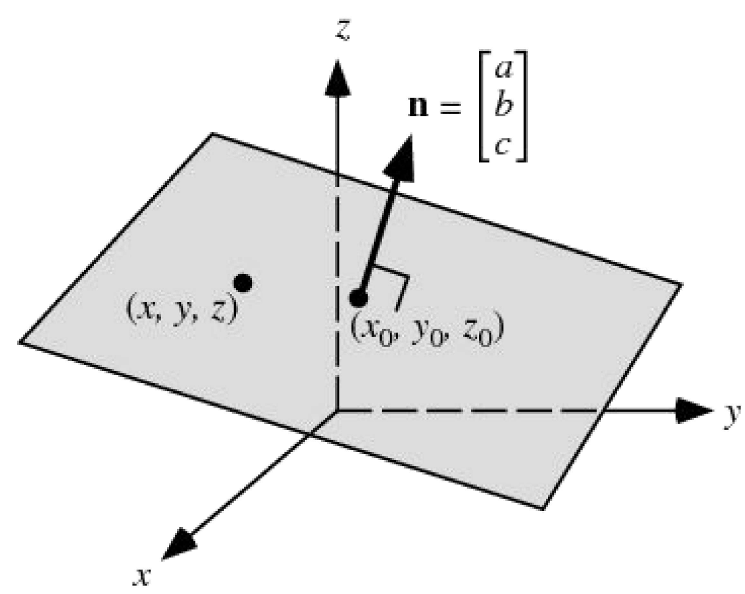
The projection of **a** onto **b**:

$$proj_b a = (a \cdot \hat{b})\hat{b}$$



Fun with Dot Product

- Angle between two vectors
- Vector length (squared)
- Vector projection
- **Planes**



$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{x_0}) = 0$$

or

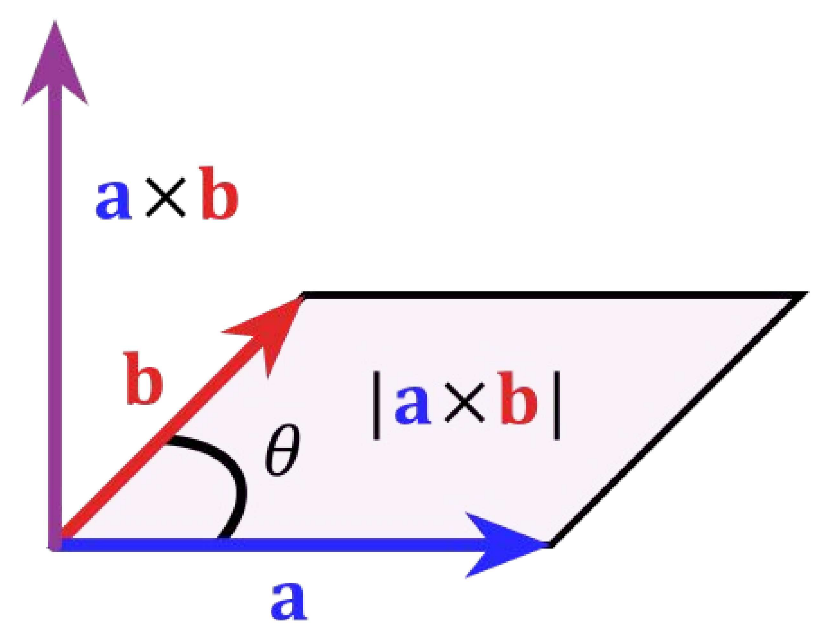
$$a\mathbf{n}_x + b\mathbf{n}_y + c\mathbf{n}_z + d = 0$$

Cross Product

$$\mathbf{a} \times \mathbf{b} = (\mathbf{a}_y\mathbf{b}_z - \mathbf{a}_z\mathbf{b}_y, \mathbf{a}_z\mathbf{b}_x - \mathbf{a}_x\mathbf{b}_z, \mathbf{a}_x\mathbf{b}_y - \mathbf{a}_y\mathbf{b}_x)$$

Cross Product

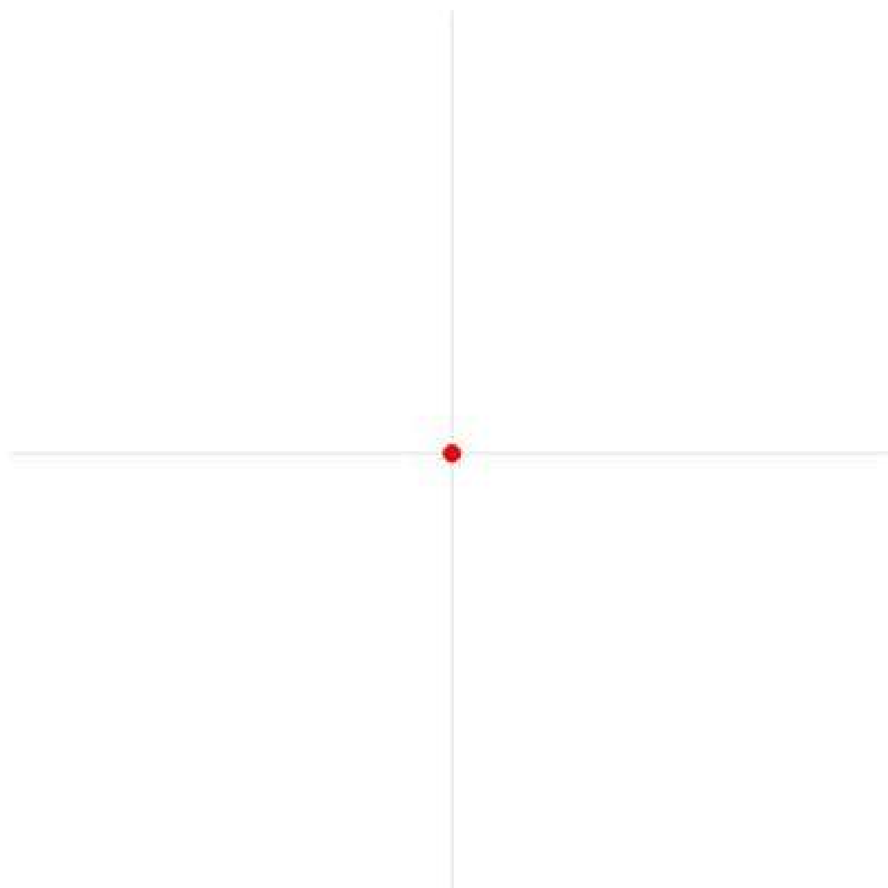
$$a \times b = |a||b| \sin\theta \hat{n}$$



Radians

A radian is the angle made at the center of a circle by an arc whose length is equal to the radius of the circle.

Radians



Questions?