On the impact of randomization on robustness in Machine Learning

A probabilistic point of view on adversarial examples

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Machine Intelligence and Learning Systems

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Context & motivations

Machine learning models are everywhere

- Machine learning models recently gave **outstanding results** (*e.g.* vision, NLP)
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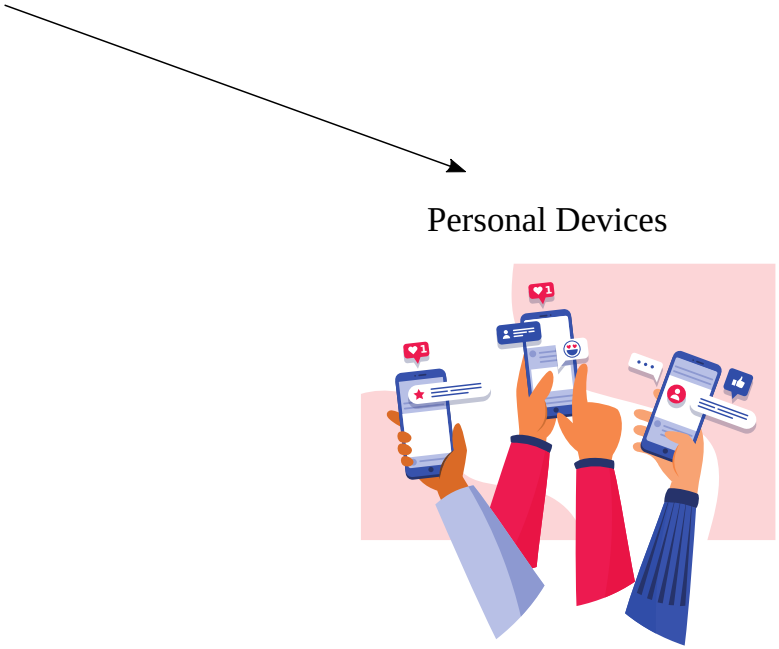

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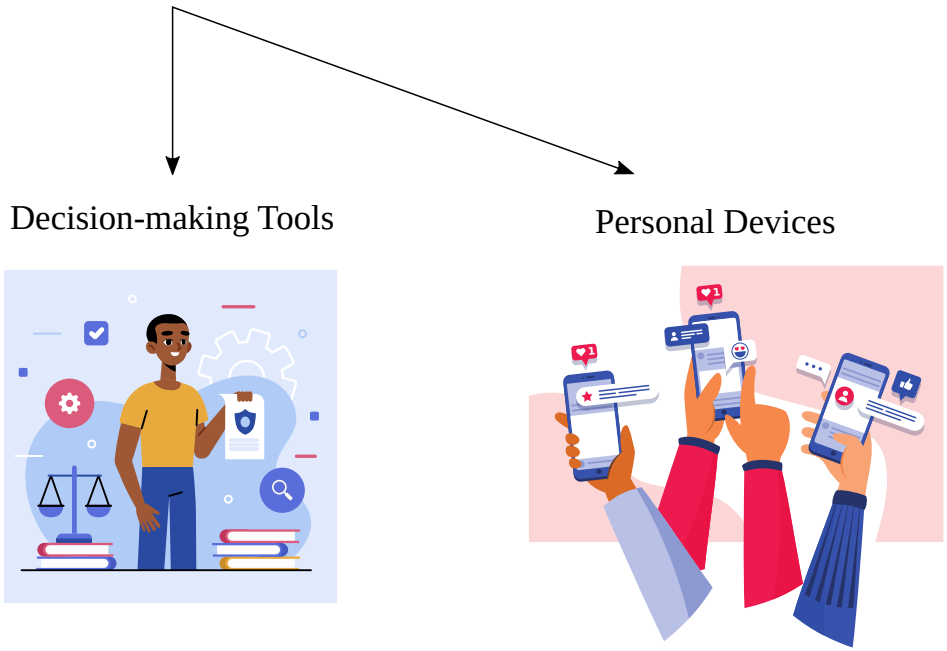

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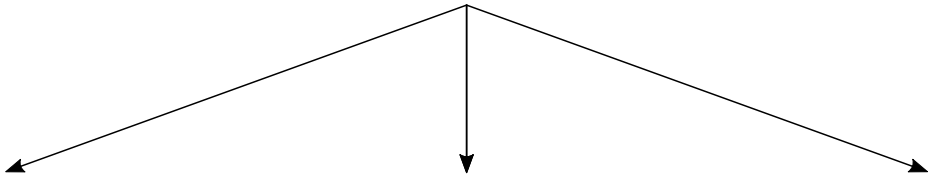
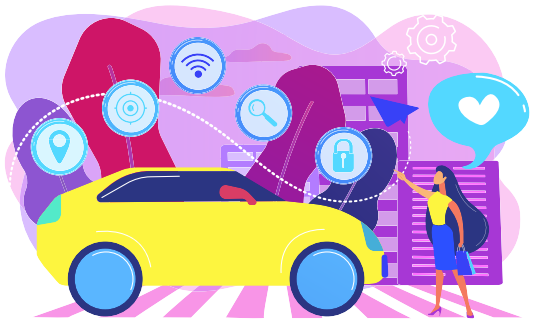


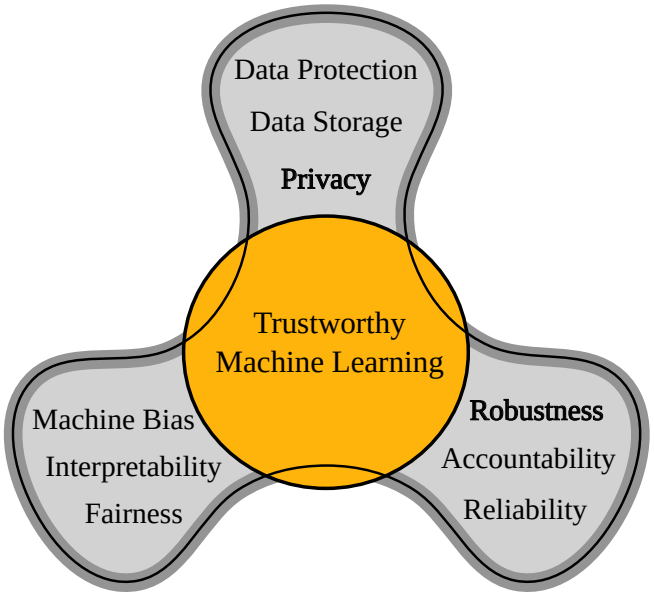
AI-driven Technologies



Decision-making Tools

Personal Devices

Trustworthy Machine Learning



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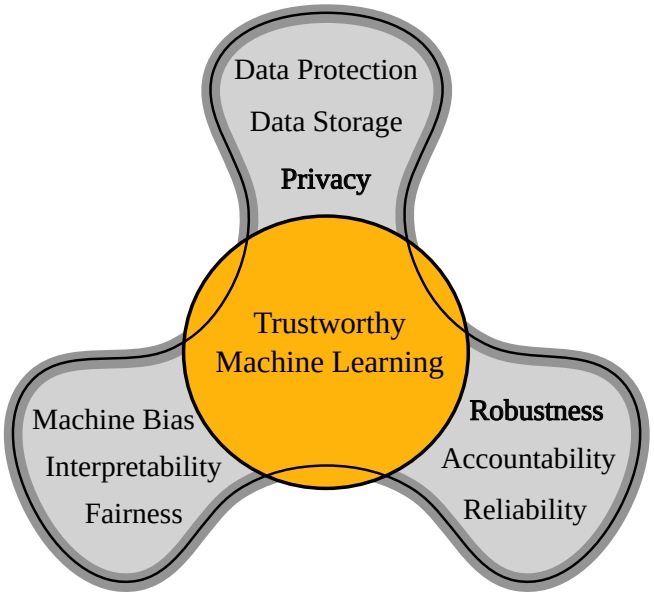

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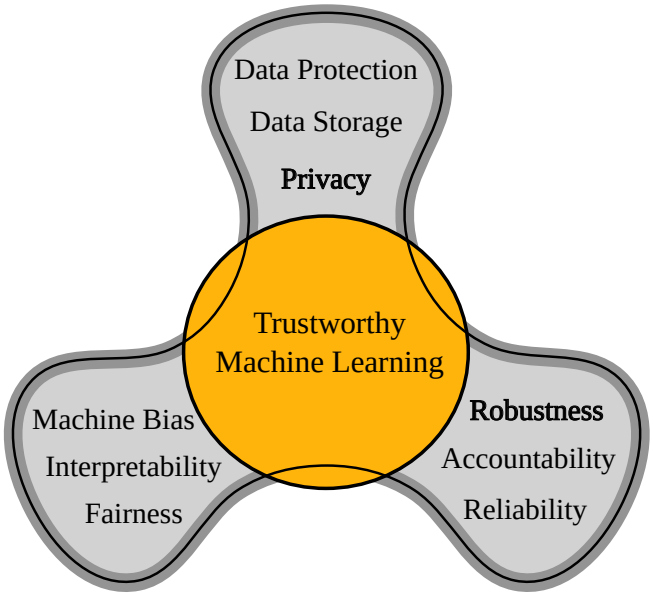

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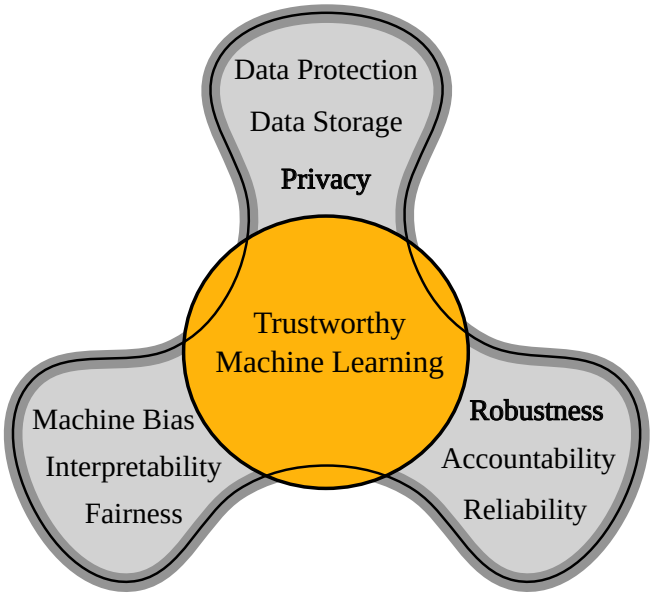

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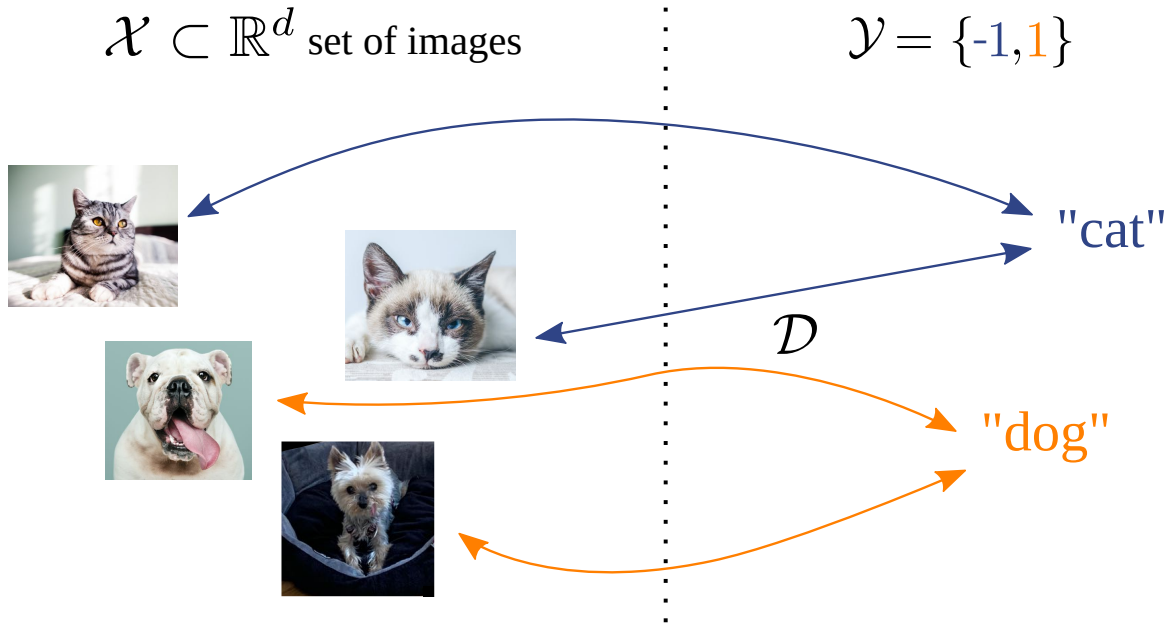
In this talk: How can we better understand the problem? Can we build machine learning models that are more robust to input manipulation?

Image classification and adversarial examples

Image classification



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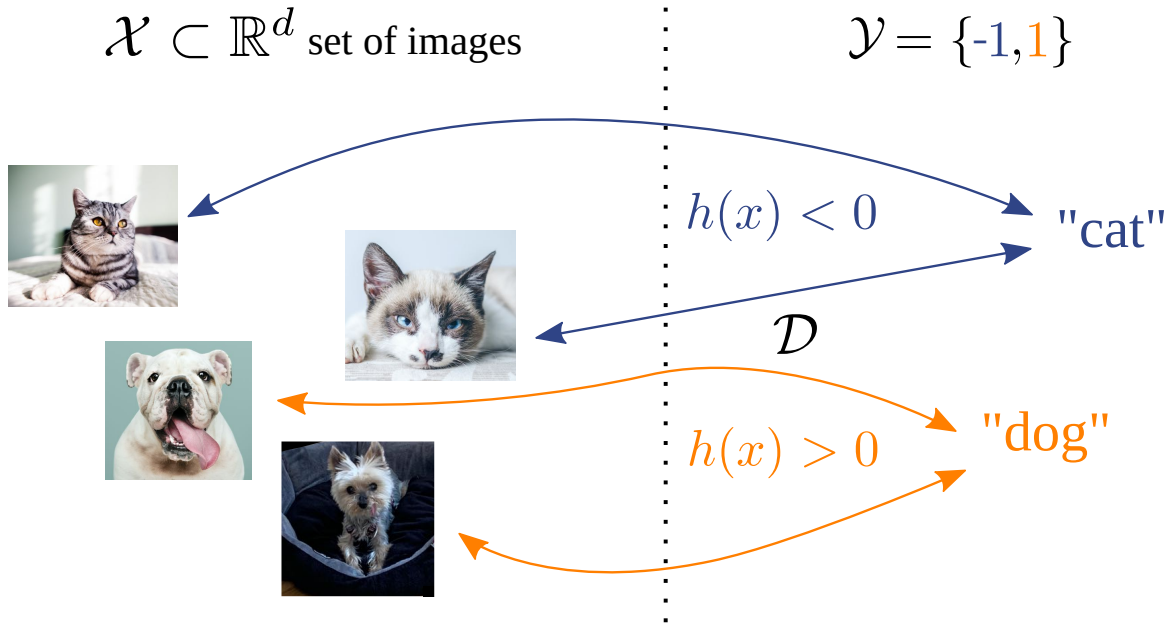

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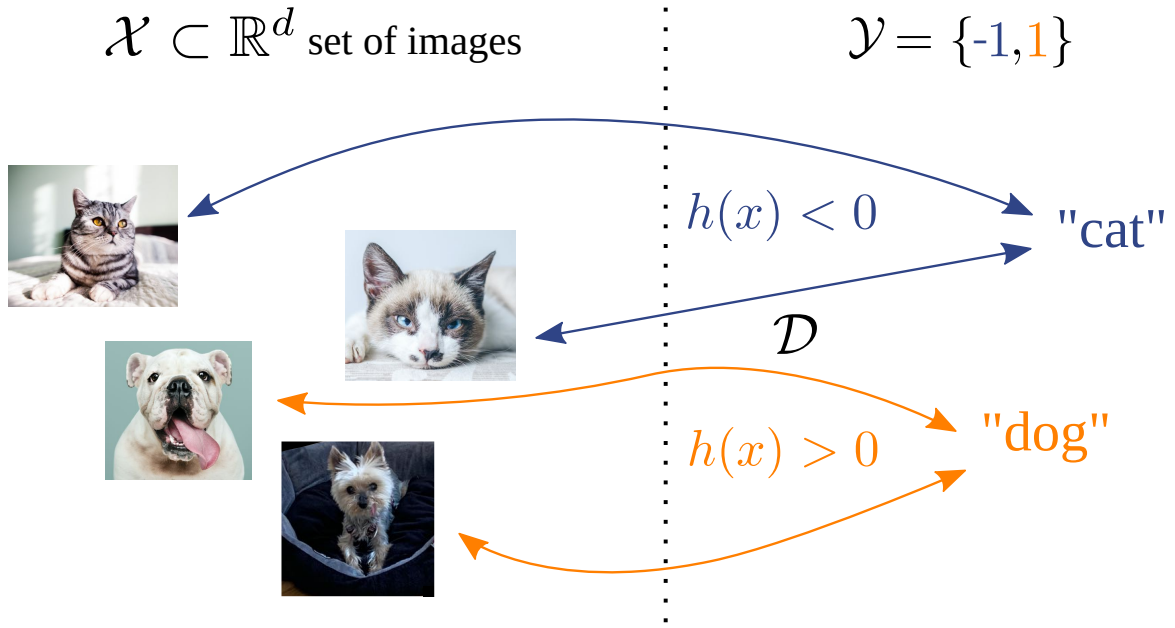

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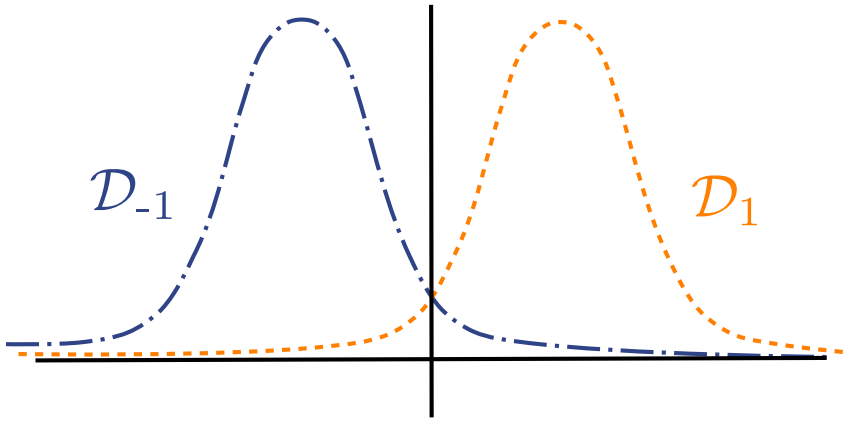
  1) Define a loss function $\ell : \mathbb{R} \times \mathcal{Y} \to \mathbb{R}^+$ and a hypothesis class $\mathcal{H}$

  2) Find $h \in \mathcal{H}$ to minimize the risk $\mathcal{R}(h) := \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\ell\left(h(x), y\right)\right]$

Image classification

In theory:

We have access to the distribution $\mathcal{D} = \mathbb{P}(y = \text{-}1)\,\mathcal{D}_{\text{-}1} + \mathbb{P}(y = 1)\,\mathcal{D}_1$



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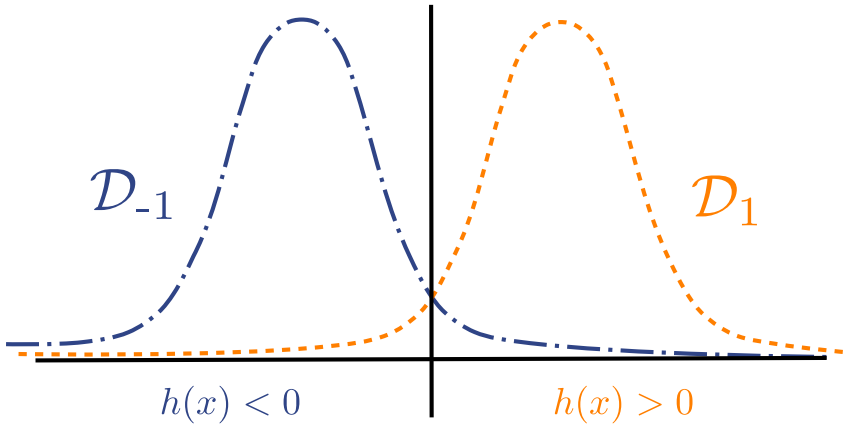

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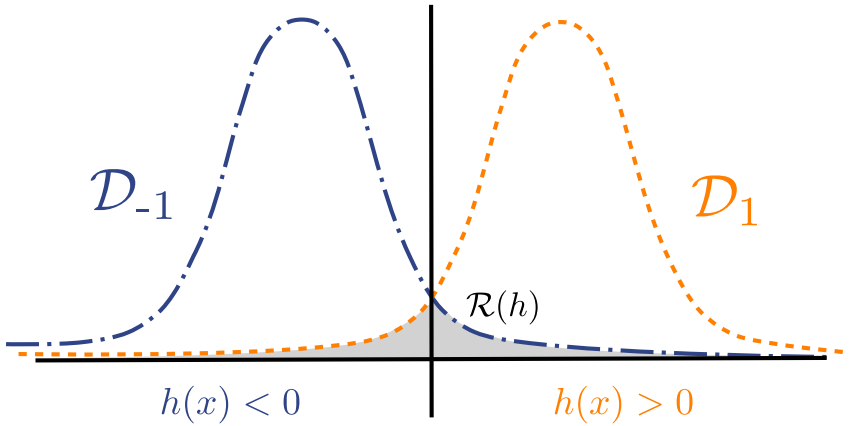

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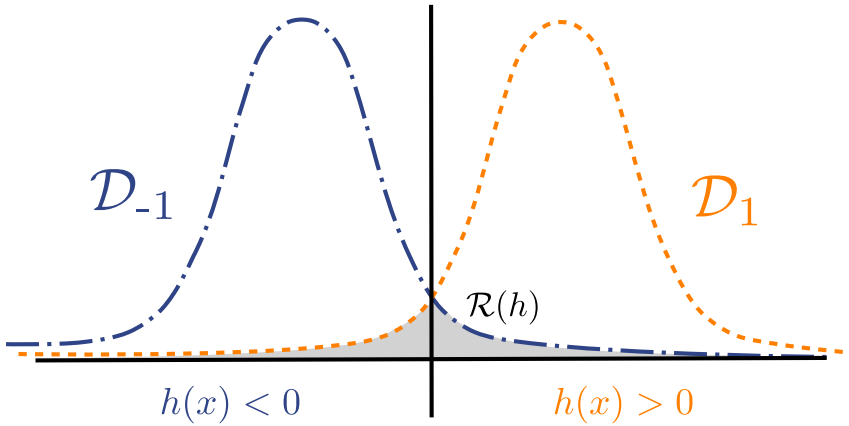
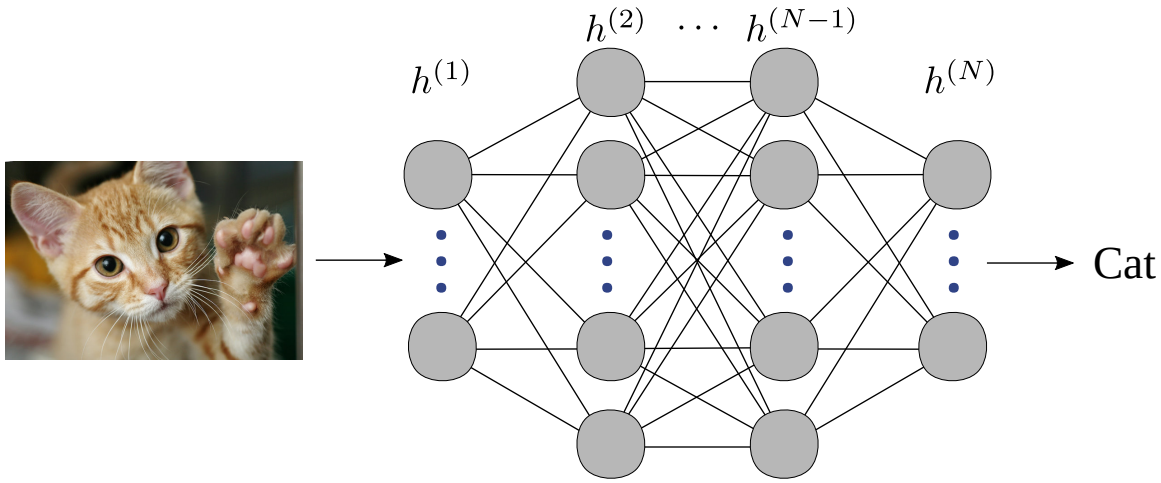


In practice:

- We only have access to a **training sample** $\{(x_1, y_1), ..., (x_n, y_n)\} \sim \mathcal{D}$
- Find $h \in \mathcal{H}$ the **empirical risk** instead $\mathcal{R}_{\text{n}}(h) := \frac{1}{n}\sum_{i=1}^{n} \ell(h(x_i), y_i)$
- The **more data**, the best $\mathcal{R}_{\text{n}}(h) \underset{n\to\infty}{\to} \mathcal{R}(h)$ (depending on $\mathcal{H}$)

Neural networks: the state-of-the-art for image classification

Hypothesis class of neural networks $\mathcal{H} := \left\{ x \mapsto h^{(N)} \circ h^{(N-1)} \circ \cdots \circ h^{(1)}(x) \right\}$



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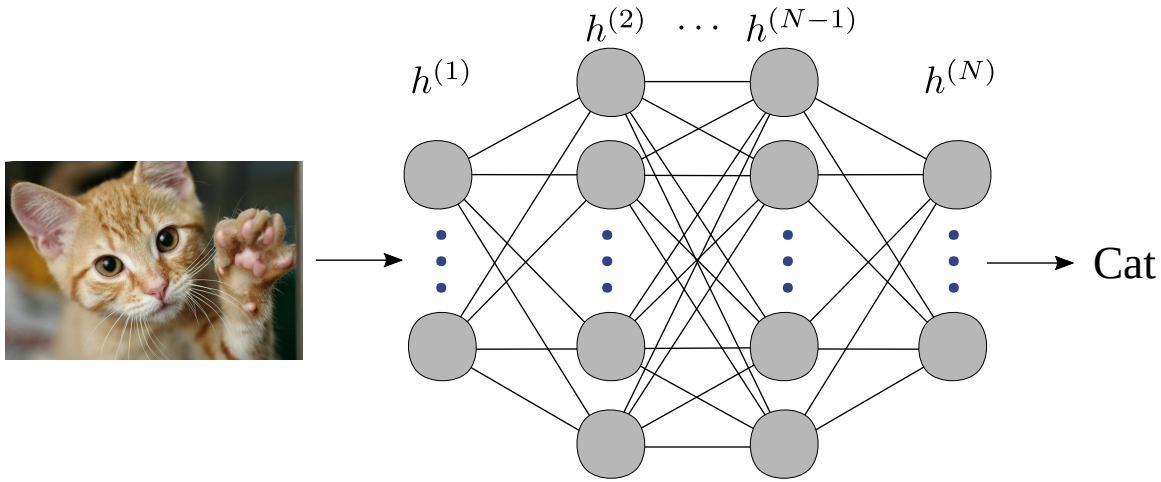
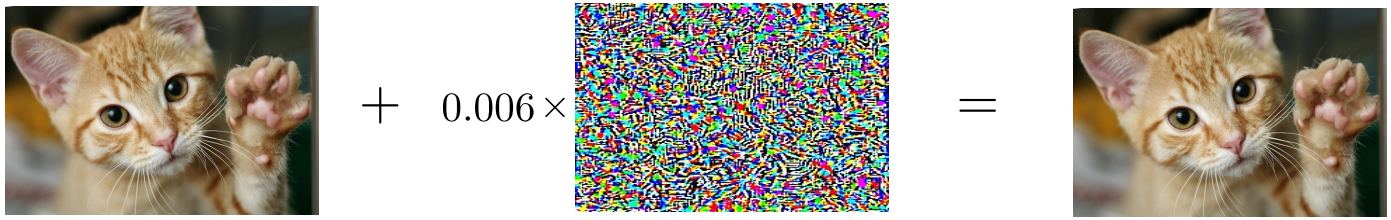


- **Deep convolutional neural networks** offer strong performances
- Still **lack theoretical guarantees** but widely used in **real-life applications**
- **Very vulnerable** to malicious data manipulations (*a.k.a.* adversarial attacks)

Adversarial examples for image classification

Adversarial attack (Biggio et al., 2013; Szegedy et al., 2014): small, imperceptible change of an image maliciously designed to fool the model

Clean example x | Perturbation τ | Adversarial example $x + \tau$



Label: "cat" | Label: "dog"

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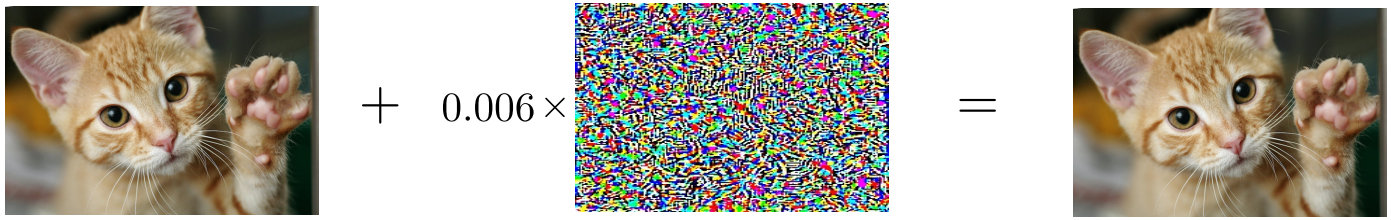

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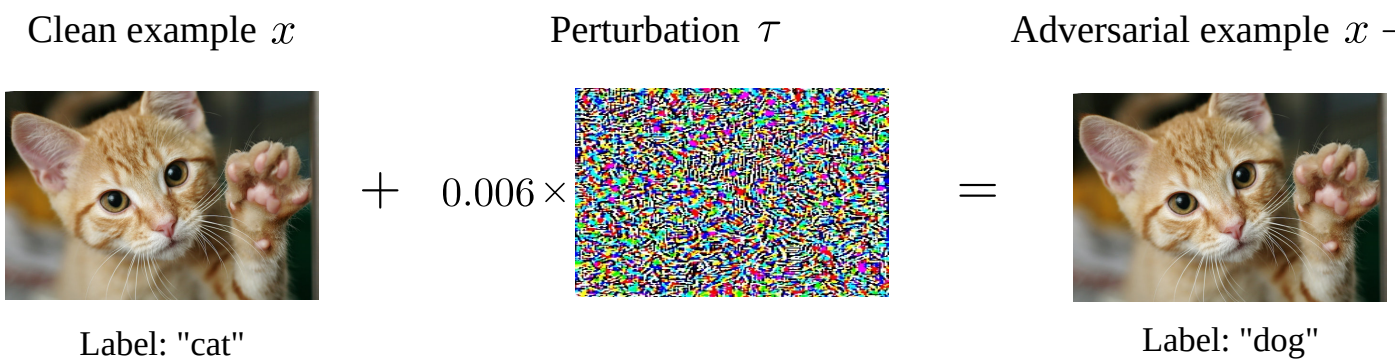

- Surrogate notion of imperceptibility is modeled with $||\tau||_p \leq \alpha^*$
- Numerous attack methods exist: attacks are **easy** to design for **most models**
- **Genuine security issues:** Face recognition in smartphones or autonomous cars

Adversary's point of view

Adversary's goal: find $\tau \in B(\alpha^*)$ to maximize $\ell(h(x+\tau), y)$

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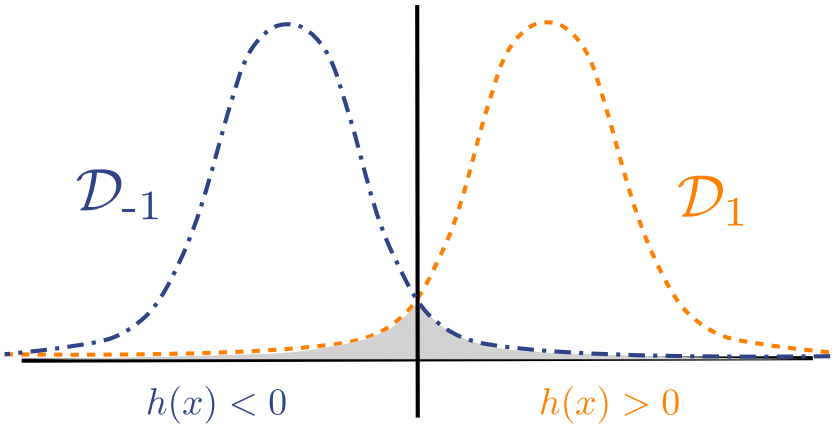

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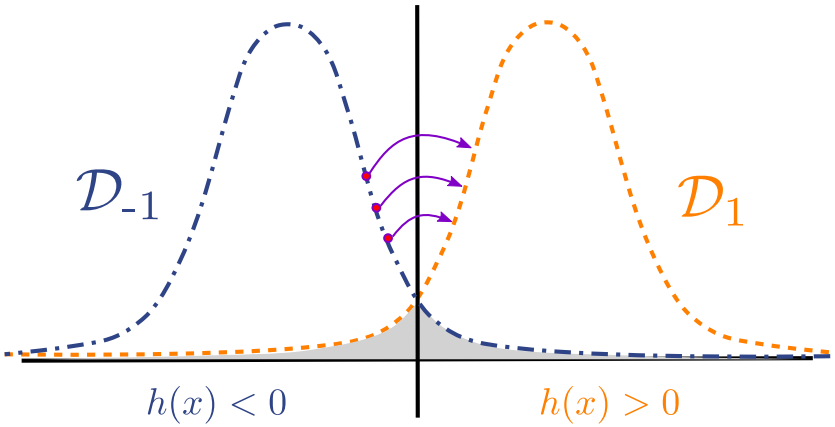

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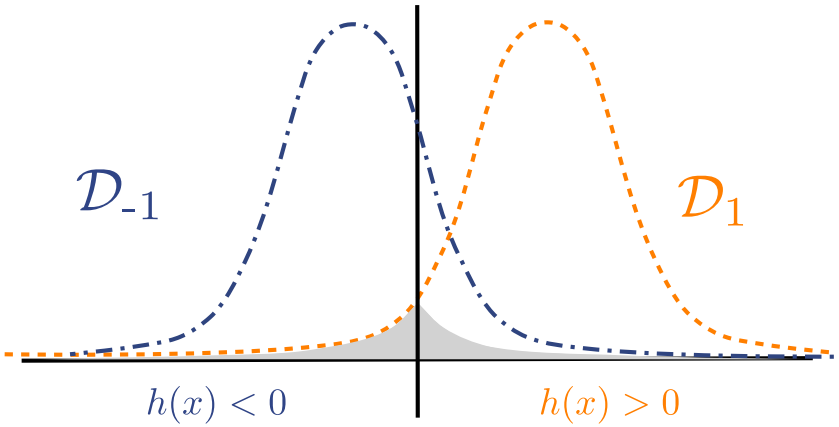

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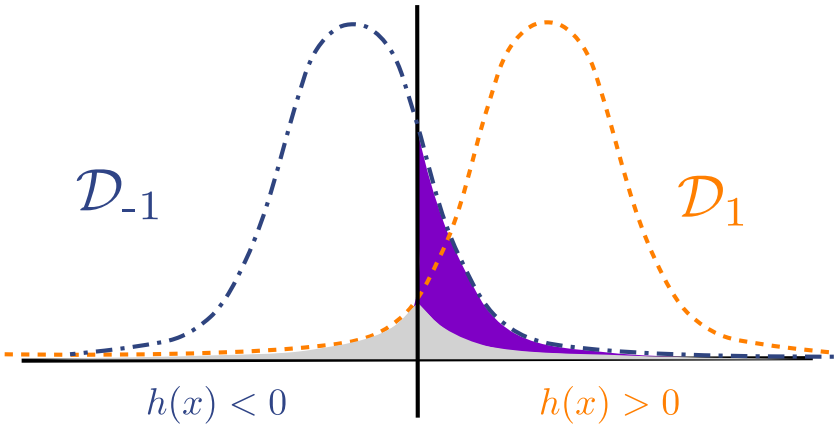

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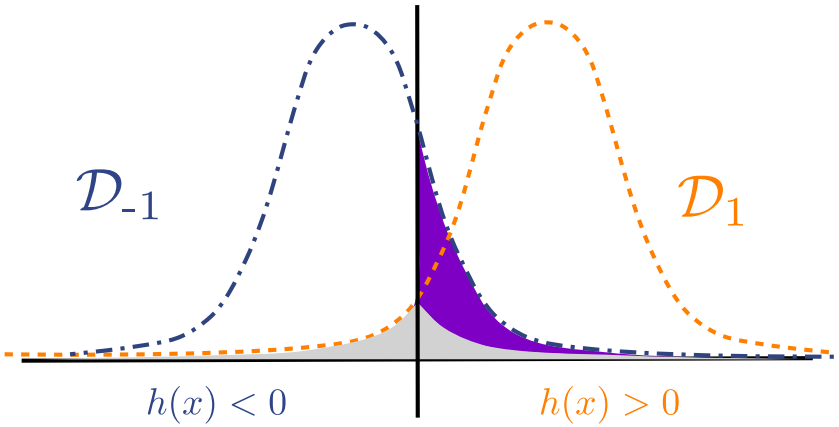

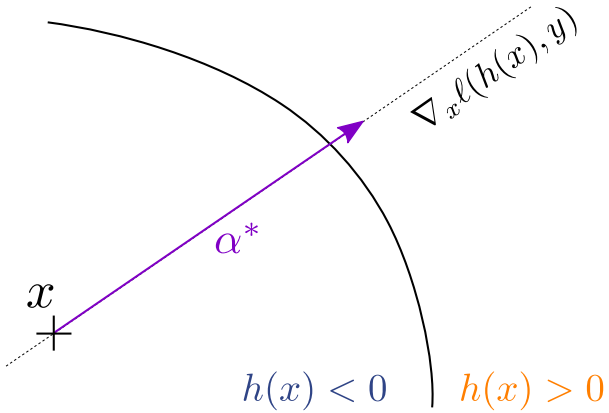

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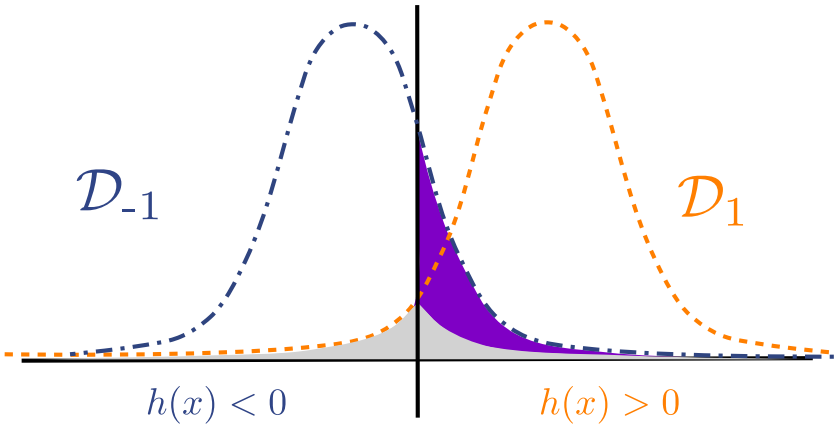

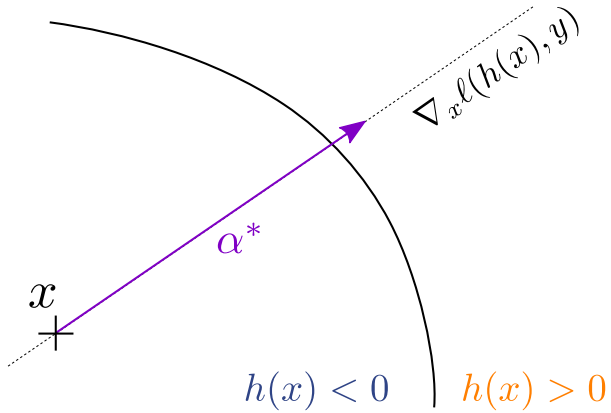

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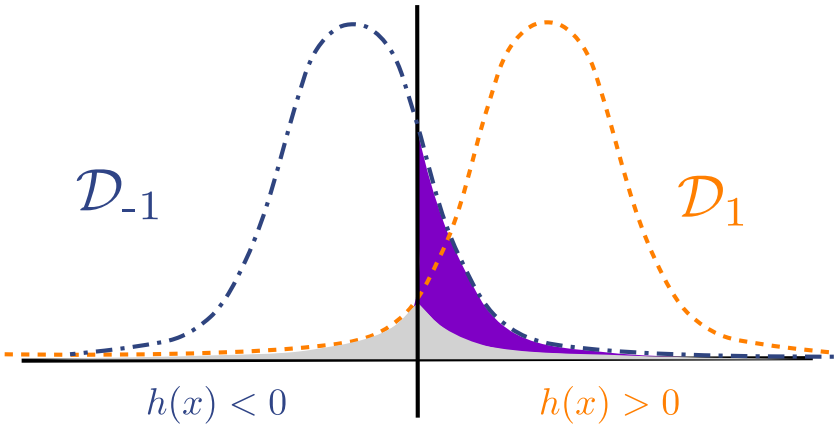

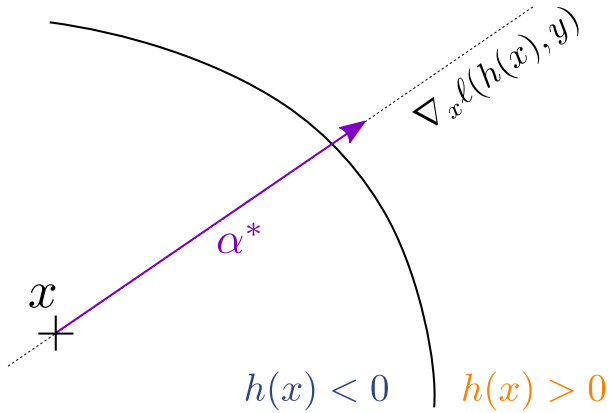

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- Attacking every point in the dataset makes the accuracy **drop to 0%**

State-of-the-art defense strategies

New goal: find $h \in \mathcal{H}$ minimizing the adversarial risk

$$\mathcal{R}_{\text{adv}}(h) := \mathop{\mathbb{E}}_{(x,y)\sim\mathcal{D}} \left[\sup_{\tau \in B(\alpha *)} \ell(h(x+\tau), y) \right]$$

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- **Randomized smoothing:** point-wise certification of the classification
  State-of-the-art **certified** defense for ℓ_2 attacks (Cohen et al.; Salman et al.,2019)

$$h_{\text{RS}}(x) = \mathop{\mathbb{E}}_{\delta\sim\mathcal{N}(0,\sigma^2\mathbb{I})} [h(x+\delta)]$$

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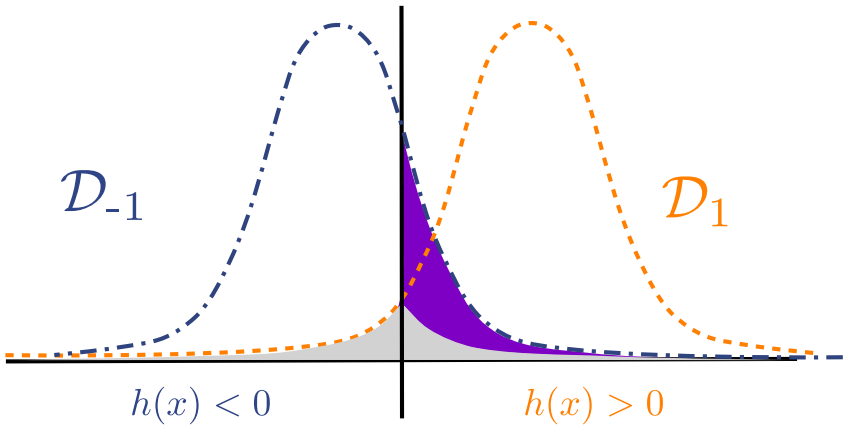

- $\mathcal{R}_{\text{adv}}(h) = \mathcal{R}(h) + \mathcal{R}_{\text{adv}>0}(h)$

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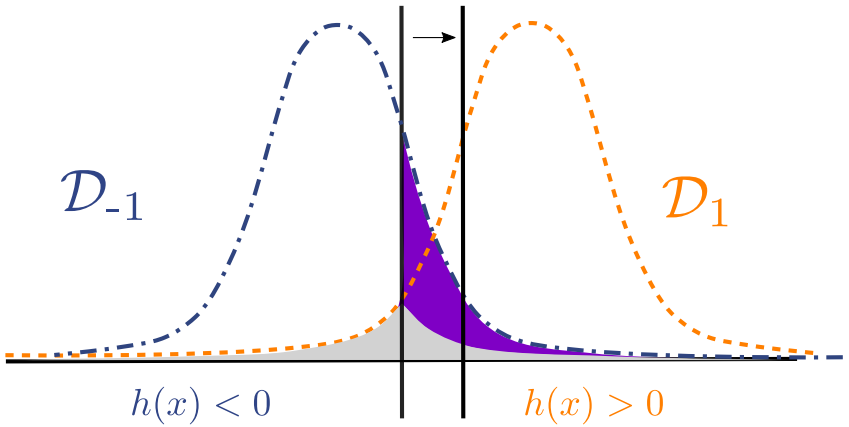

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- $\mathcal{R}_{\text{adv}>0}(h) \searrow$

On the hardness of adversarial defense

- Adversarial examples are **hard to mitigate** ($\approx 50\%$ on CIFAR10)
- Several works evaluated this hardness **more formally**
  - $\rightarrow$ Lower bound on the adversarial risk (Bhagoji et al.;Pydi and Jog, 2019)
  - $\rightarrow$ Sample complexity (Yin et al., 2019; Awasthi et al., 2020)
  - $\rightarrow$ Computational constraints (Bubeck et al., 2019)
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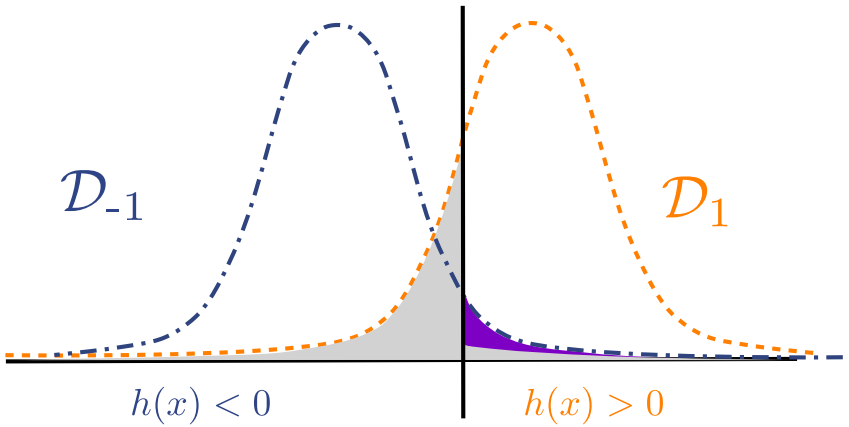

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Presented contributions

Main Question: Is there a class of hypotheses that could ensure both accuracy and robustness against adversarial attacks? $\rightarrow$ randomized classifiers

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- **Result 1:** non-existence of a Pure Nash Equilibrium in this game
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- **Result 2:** randomized classifiers are more robust than deterministic ones

Part 2: Demonstrating the robustness of randomized classifiers

- **Formalization:** define adversarial robustness for randomized classifier
- **Result 1:** bound the accuracy vs robustness trade-off for these classifiers
- **Result 2:** design classes of robust (randomized) neural networks

Part 1:
Justifying the use of randomized classifiers

Recall on Zero-sum games

Generic optimization problem:

$$\inf_{h} \sup_{\phi} \text{Score}(h, \phi)$$

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Nash Equilibrium: stable state (h_{NE}, ϕ_{NE}) s.t. $\begin{cases} h_{NE} \in \mathfrak{BR}(\phi_{NE}) \\ \phi_{NE} \in \mathfrak{BR}(h_{NE}) \end{cases}$

A closer look at adversarial risk

In the following we set:

$$\mathcal{H} := \{h : \mathcal{X} \mapsto \mathbb{R} \text{ continuous}\} \quad \text{and} \quad \ell_{0/1}(h(x), y) = \mathbb{1}\{\operatorname{sgn}(h(x)) \neq y\}$$

Decomposition of the adversarial risk:

$$\mathcal{R}_{\text{adv}}(h) := \mathop{\mathbb{E}}_{(x,y)\sim\mathcal{D}} \left[\sup_{\tau \in B(\alpha^*)} \ell_{0/1}(h(x+\tau), y) \right]$$

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Where $\mathcal{F} := \left\{ \phi : \mathcal{X} \to \mathcal{X} \mid \phi \text{ measurable, and } \sup_{x\in\mathcal{X}} \|\phi(x) - x\|_p \leq \alpha^* \right\}$

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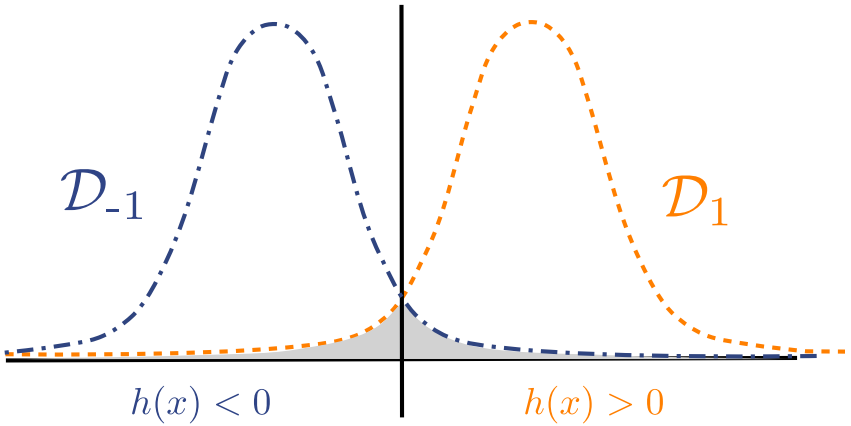

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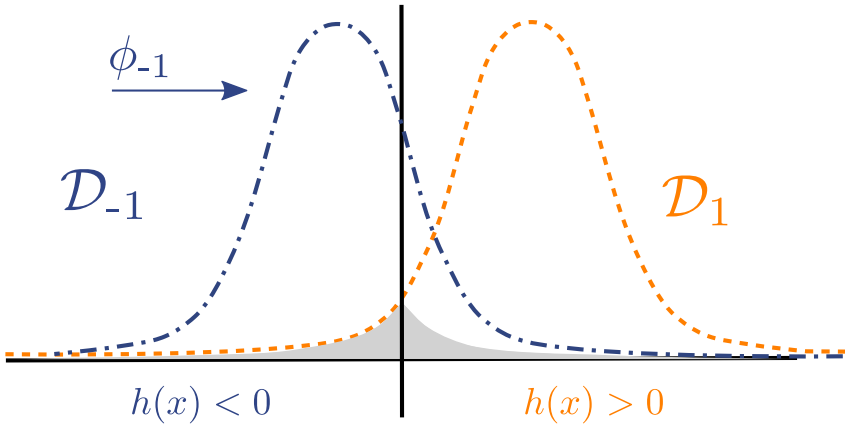

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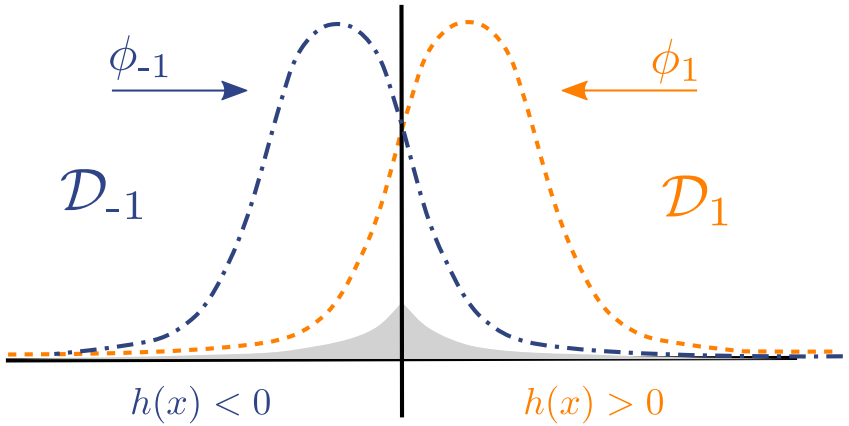

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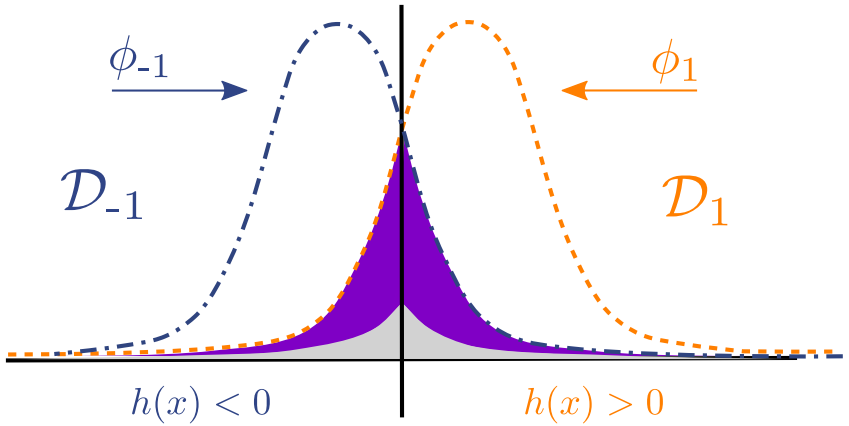

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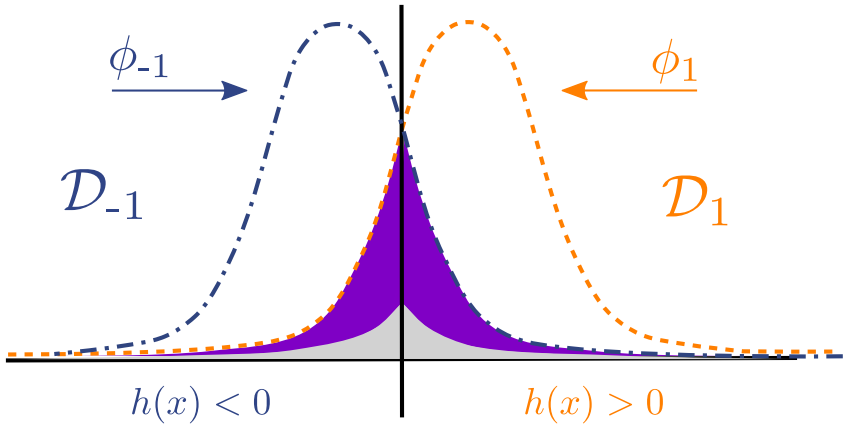

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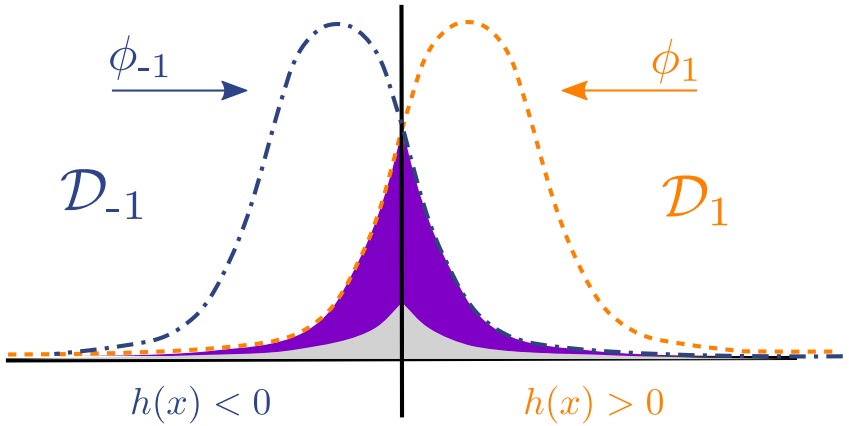

- Trivial Nash equilibrium
- Inevitably high adversarial risk

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- The adversary attacks all points (no budget constraint)

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Typical penalty: penalize the expected number of queries

$$\Omega(\phi) := \mathop{\mathbb{E}}_{(x,y) \sim \mathcal{D}} \left[\mathbb{1} \left[x \neq \phi_y(x) \right] \right]$$

Non-existence of a Pure Nash Equilibrium

Theorem (Non-existence of PNE)

Let us consider the above zero-sum game with penalty Ω and $\lambda > 0$.

If $\mathcal{D}_1$ and $\mathcal{D}_{\text{-}1}$ have full support, then the game has no Pure Nash Equilibrium.

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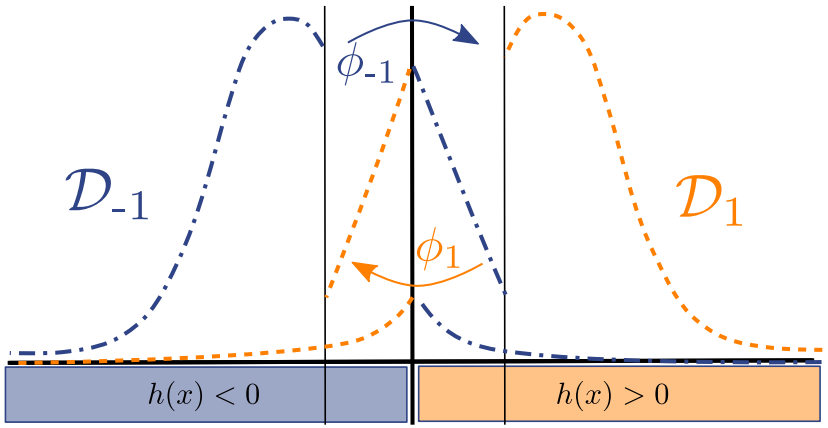

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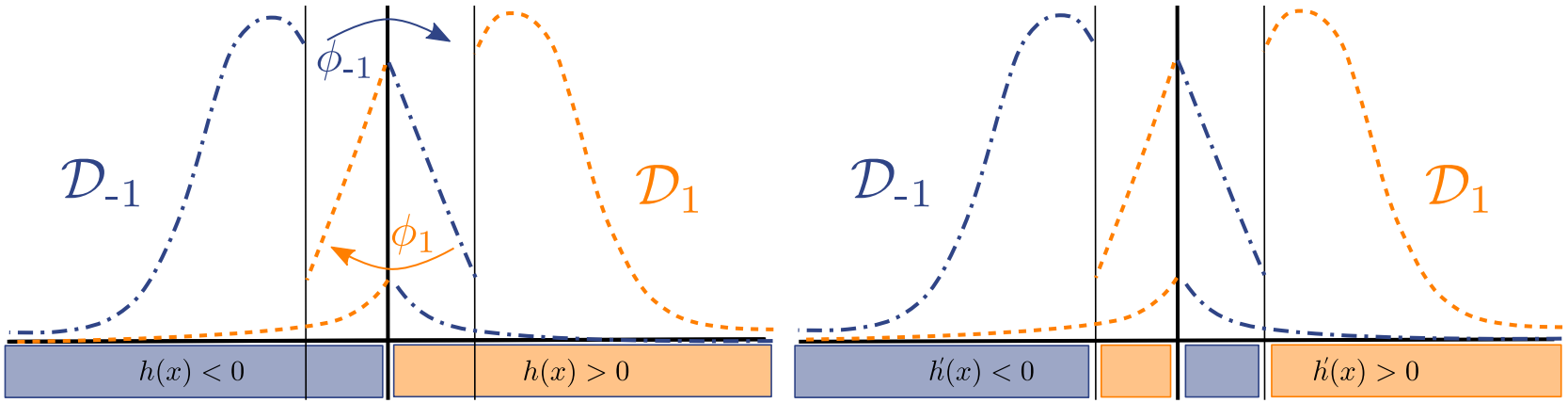

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→ *Study what happens if **only the defender** uses randomized strategies*

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Randomized classifier:

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Adaptation of the score:

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- If $1 > q > \max(\lambda, 1 - \lambda)$ the result holds

From theorem to algorithm

Algorithm: Boosted adversarial training

Inputs: D the training set and $q \in (0, 1)$

> Train h on D with adversarial training

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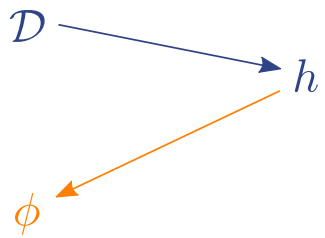

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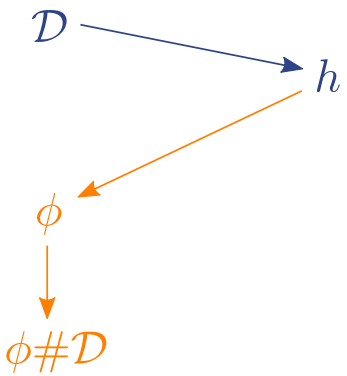

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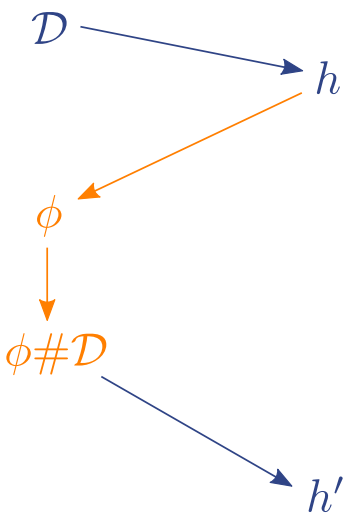

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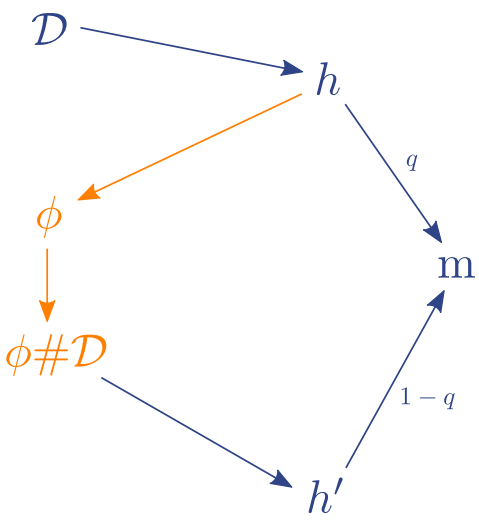

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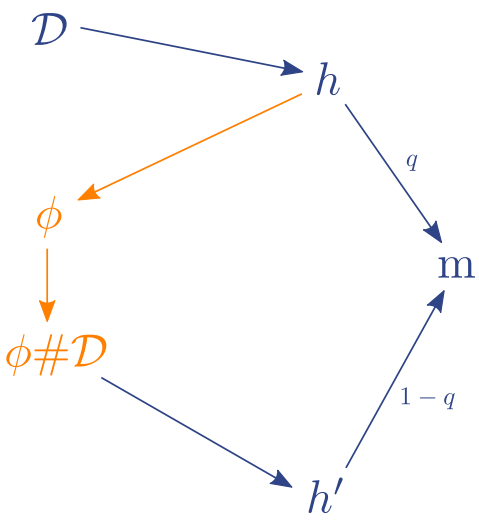

Dataset	Method	Accuracy	l_∞-PGD
CIFAR10	Undefended	0.88	0.00
	Adversarial training	0.83	0.42
	Boosted adversarial training	0.80	**0.55**

Summary and next step

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- These results give a **formal motivation** for studying randomized classifiers

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$$\Delta_\Sigma := \{\mathrm{m} : x \mapsto h(x+z) \mid h : \mathcal{X} \to \mathbb{R} \text{ continuous, and } z \sim \mathcal{N}(0, \Sigma)\}$$

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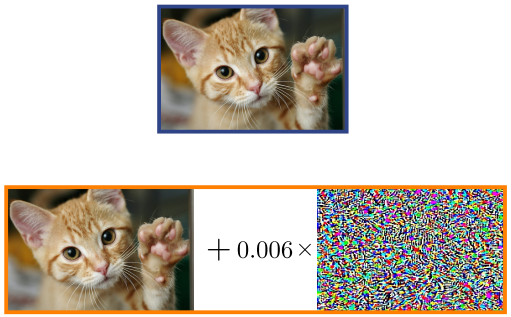
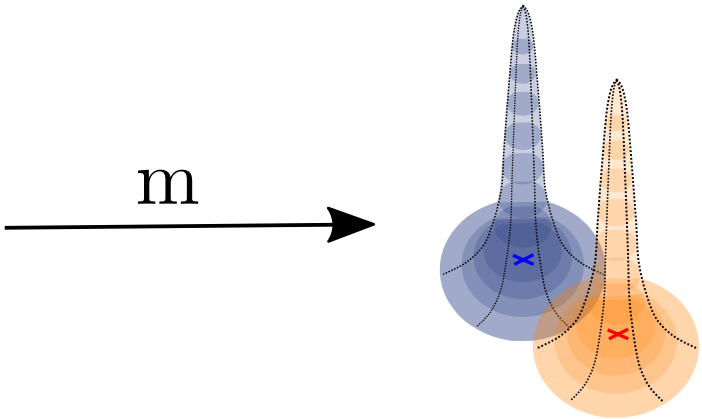
Remark: Gaussian noise injection has been proved to regularize the training phase
Here we use it at **test time** to get robustness to data manipulation

Part 2:
Demonstrating the robustness of $\Delta_{\sum}$

Randomized robustness: a Lipschitz condition





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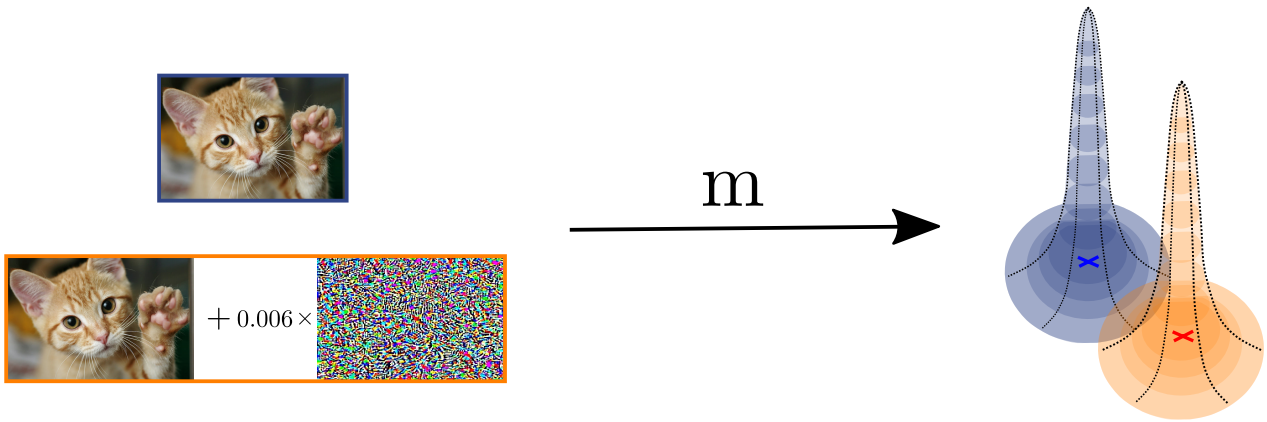

We define robustness for randomized classifiers with a **stability condition**

$$\forall \tau \in B(\alpha^*), \quad D\left(\mathrm{m}(x+\tau), \mathrm{m}(x)\right) \leq \epsilon$$

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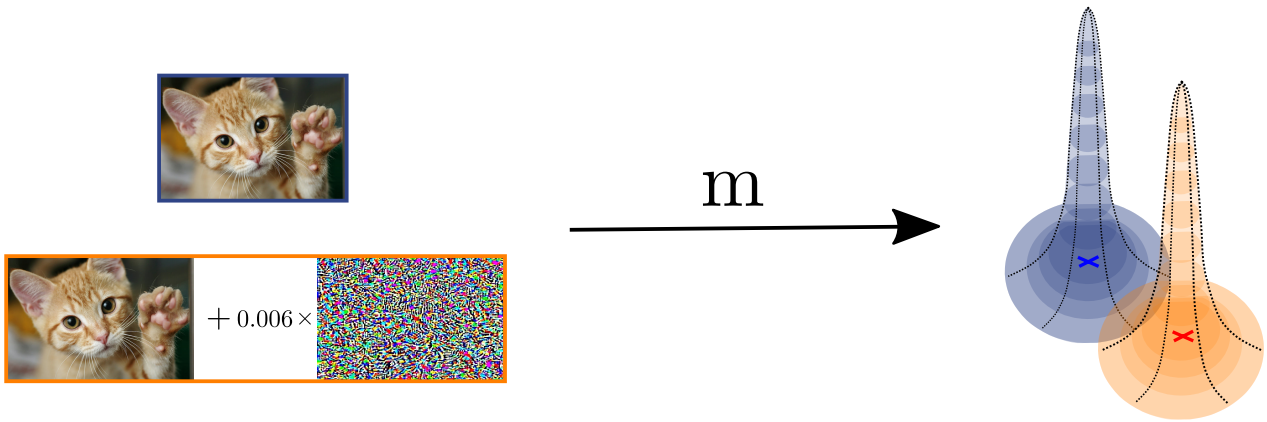

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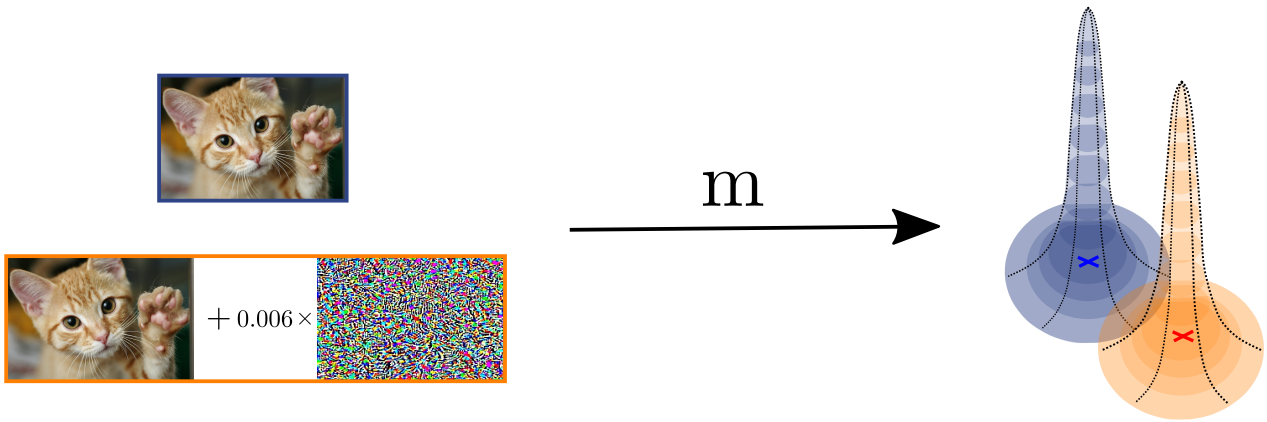

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Noise injection offers stability

Proposition (local Lipschitzness)

Let $\mathrm{m} \in \Delta_\Sigma$, then for any $x \in \mathcal{X}$, the following holds

$$\|\tau\|_{\Sigma^{-1}} \leq \alpha^* \implies D_\beta\left(\mathrm{m}(x), \mathrm{m}(x+\tau)\right) \leq \frac{(\alpha^*)^2 \beta}{2}.$$

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Idea of the proof:

- Simple calculus $D_\beta\left(\mathcal{N}(x, \Sigma), \mathcal{N}(x+\tau, \Sigma)\right) \leq \frac{\beta}{2}\|\tau\|^2_{\Sigma^{-1}}$

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- Data-processing inequality (Beaudry and Renner, 2012; Dwork and Roth, 2013)

$$D_\beta\left(h\#\mathcal{N}(x,\Sigma), h\#\mathcal{N}(x+\tau,\Sigma)\right) \leq D_\beta\left(\mathcal{N}(x,\Sigma), \mathcal{N}(x+\tau,\Sigma)\right)$$

Bounding the adversarial gap

Theorem (Bounding the accuracy vs robustness trade-off)

Let $\Sigma = \sigma^2 \times \mathbb{I}$ and $\mathrm{m} \in \Delta_\Sigma$. Then the following holds

$$\mathcal{R}_{\mathrm{adv}}(\mathrm{m}) - \mathcal{R}(\mathrm{m}) \leq 1 - e^{-\frac{(\alpha^*)^2}{2\sigma}} \mathop{\mathbb{E}}_{x \sim \mathcal{D}_{|\mathcal{X}}} \left[e^{-H(\mathrm{m}(x))} \right].$$

Where H is the Shannon entropy $H(P) = -\mathbb{E}_{z \sim P} \left[\log(p(z)) \right]$.

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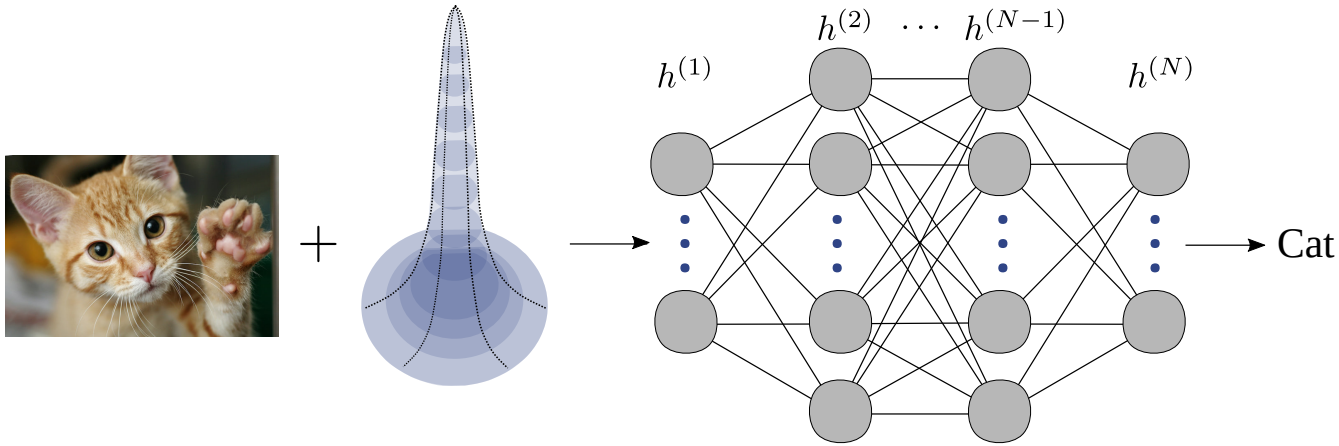
Remark: we get similar results with $\mathcal{R}_{\mathrm{adv}}(\mathrm{m}) - \mathcal{R}(\mathrm{m}) \leq 2F_{\mathcal{N}(0,1)}\left(\frac{\alpha^*}{2\sigma}\right) - 1$

Consequences and applications

Simple noise injection methods to build robust randomized neural networks



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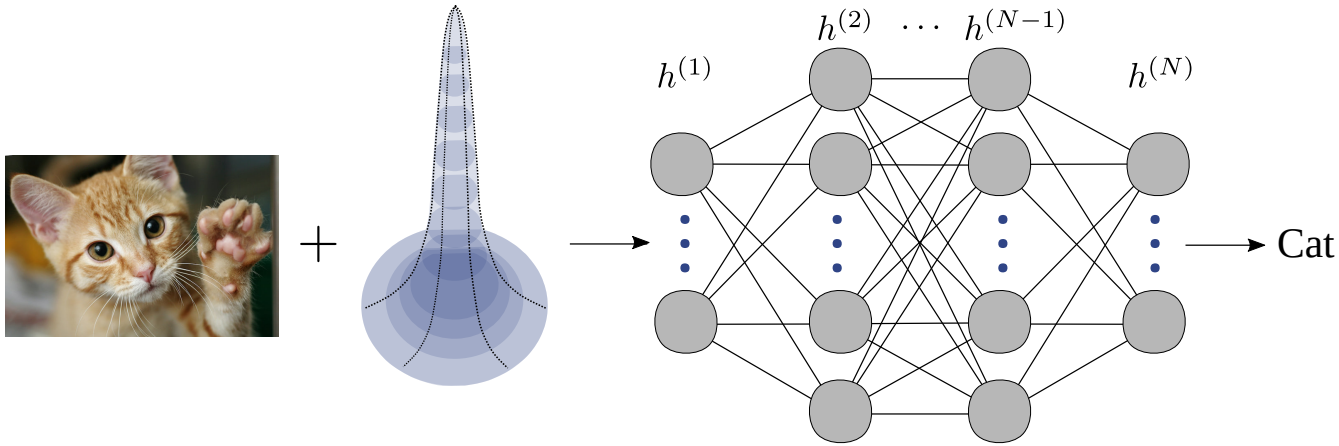

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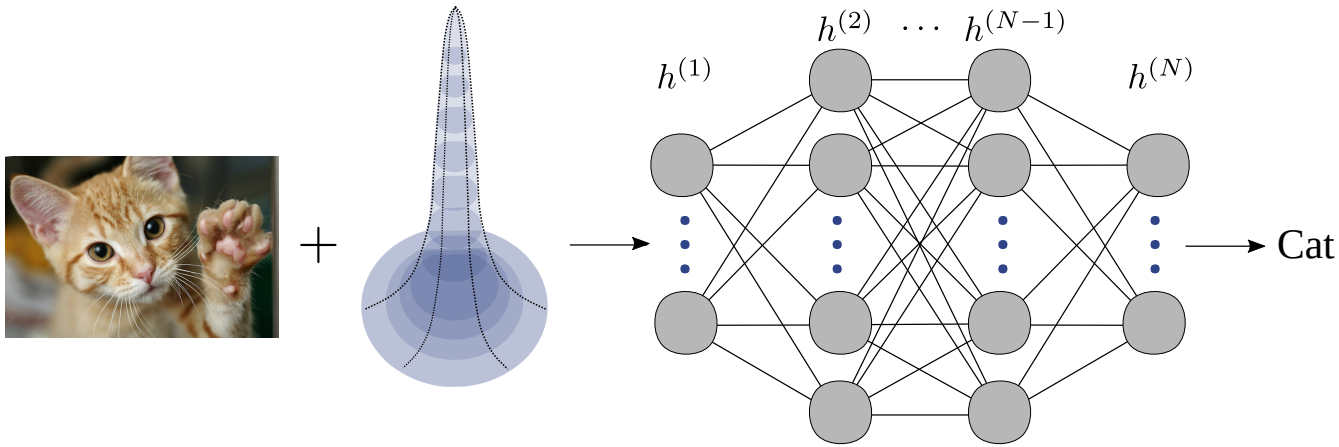

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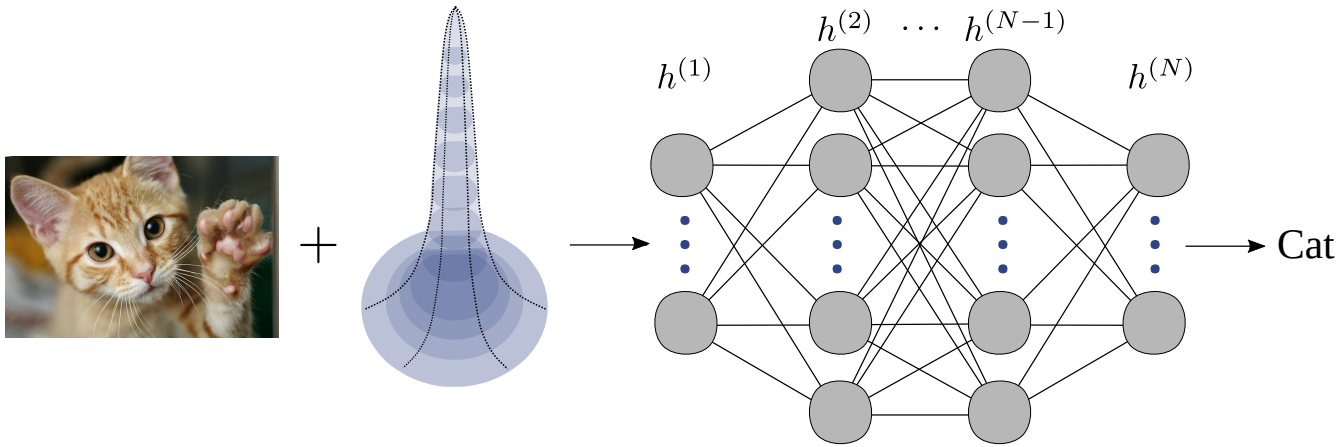

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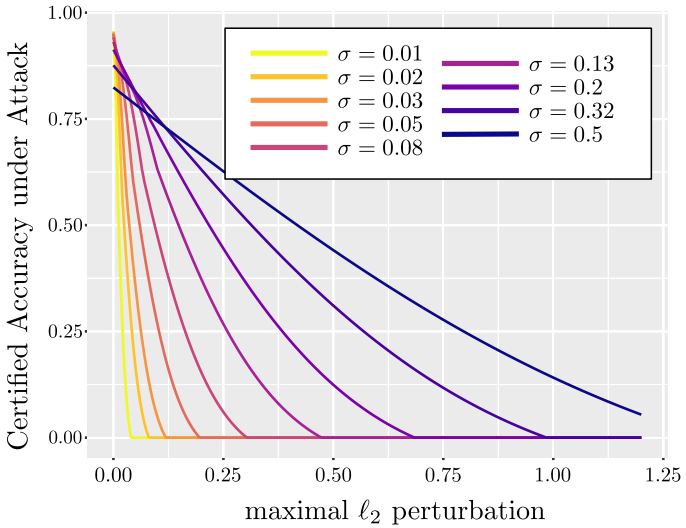
- Similar results for exponential family noises (Laplace, Weibull, etc.)
- Results work for K-class classification (application to benchmark datasets)

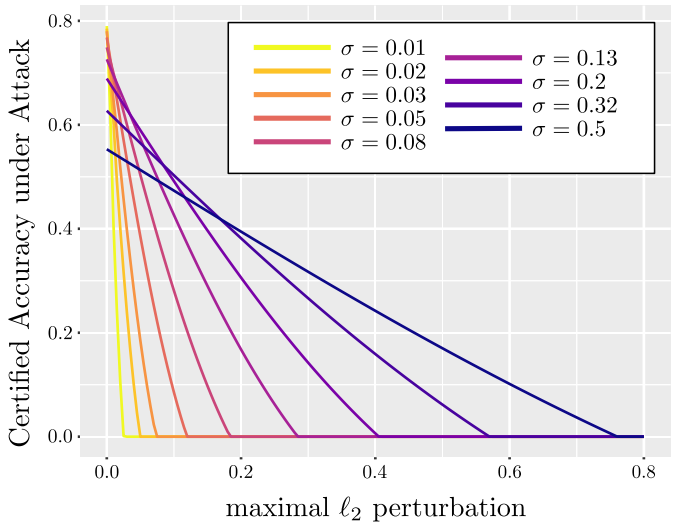
Application to benchmark datasets

Some results for WideResNets 28 (state-of-the-art model + Gaussian noise)

CIFAR-10



CIFAR-100



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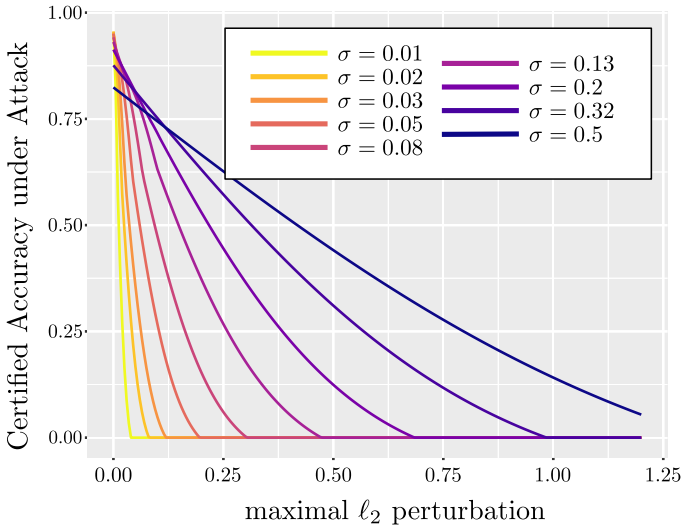

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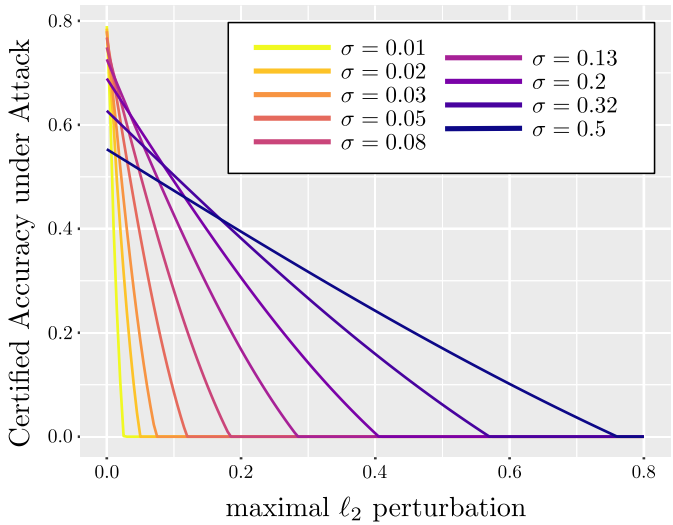

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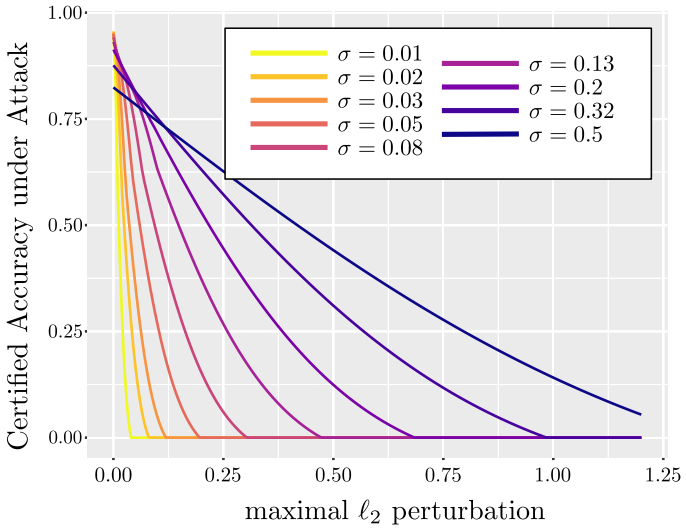

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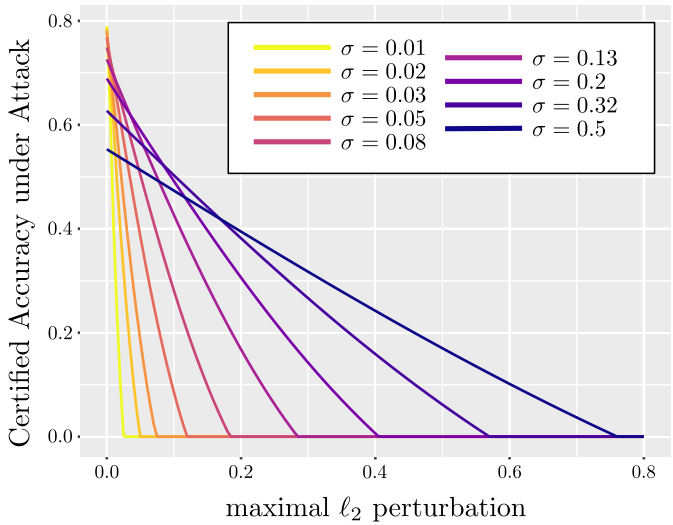

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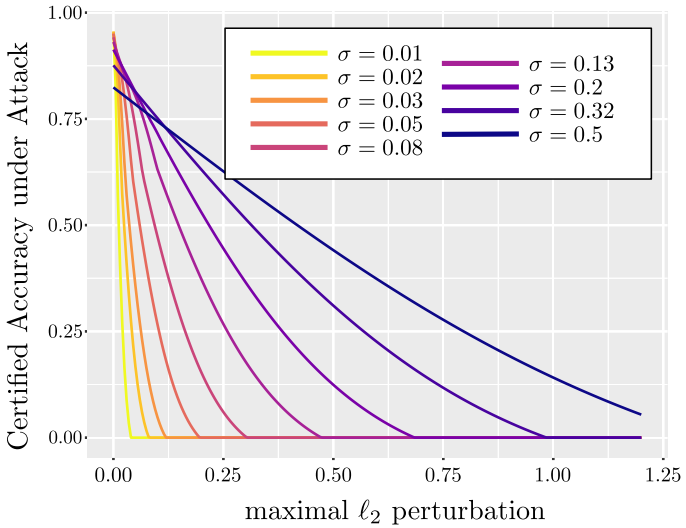

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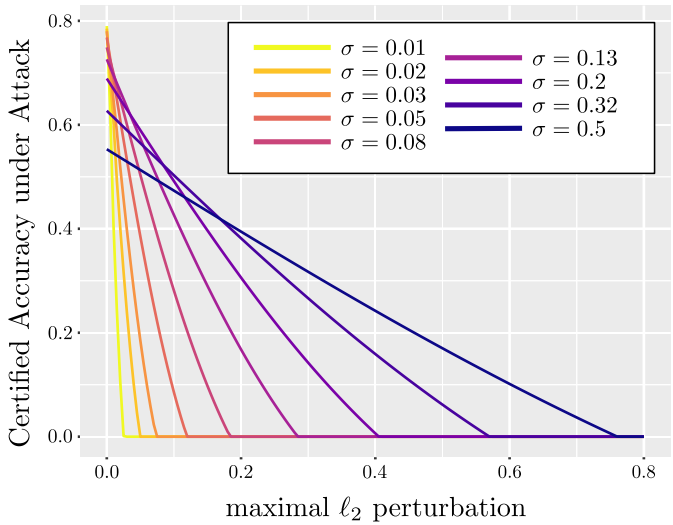

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- For **further improvements:** specify classifier and study the impact of the noise

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> Analysis based on D_β and D_{TV}
> Extend the class of divergences

Some broader perspectives

- Revisiting the adversarial game with **different objectives**

$$\begin{cases} \sup\limits_{\phi \in \mathcal{F}} & \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\ell_{\text{adv}} \left(h \left(\phi(x) \right), y \right) \right], \quad \text{for a given hypothesis } h \\ \inf\limits_{h \in \mathcal{H}} & \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\ell_{\text{def}} \left(h \left(\phi(x) \right), y \right) \right], \quad \text{for a given adversary } \phi \end{cases}$$

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- Revisiting adversarial learning theory with regard to double descent and interpolation regimes (Zhang et al., 2017; Belkin et al., 2019)

- Transfer existing results from image to sound processing. Adversarial examples also work well on speech-to-text (Carlini and Wagner, 2018)

Published works

On robustness to adversarial examples:

Randomization matters. how to defend against strong adversarial attacks
(Pinot et al., 2020) ICML 2020

Advocating for Multiple Defense Strategies against Adversarial Examples
(Araujo et al., 2020) Workshop at ECML 2020

Theoretical evidence for robustness through randomization (Journal TBA)
(Pinot et al., 2019a) NeurIPS 2019

On privacy preserving data analysis:

SPEED: Secure, PrivatE, and Efficient Deep learning (under review)
(Sébert et al., 2020) Journal track ECML 2021

Unified view on differential privacy and robustness to adversarial examples
(Pinot et al., 2019b) Workshop at ECML 2019

Graph-based Clustering under Differential Privacy
(Pinot et al., 2018) UAI 2018

We also participated to: Teaching, scientific outreach, coding, etc.

Questions

f-divergences (f convex): $D_f(P,Q) := \mathbb{E}_{z\sim Q}\left[f\left(p(z)/q(z)\right)\right]$

- Total variation $f(t) = |t-1|$, Kullback Leibler $f(t) = |t\log t|$
- Pro: Data-processing inequality / Con: accuracy vs robustness trade-off

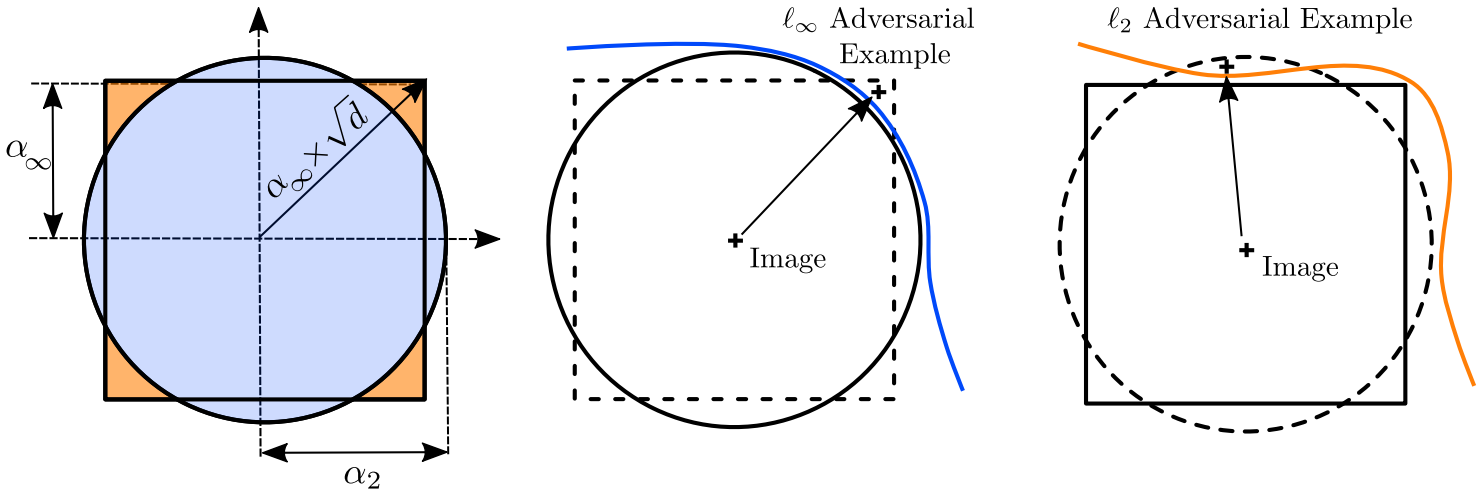
Integral probability metrics: $\gamma_{\mathcal{G}}(P,Q) := \sup_{g\in\mathcal{G}} |\mathbb{E}_{z\sim Q}\left[g(z)\right] - \mathbb{E}_{z\sim P}\left[g(z)\right]|$

- Total variation $\mathcal{G} = \{g \mid ||g||_\infty \leq 1\}$, Wasserstein $\mathcal{G} = \{g \mid ||g||_L \leq 1\}$
- Links with f-divergences (Sriperumbudur et al., 2009)

Bregman divergences (U convex): $D_U(P,Q) := U(p) - U(q) - \langle \nabla U(q), p-q\rangle$

- Kullback Leibler $U(p) = \sum p_i \log(p_i)$, Itakura–Saito $U(p) = -\sum \log(p_i)$
- Asymptotic equivalence with f-divergences (Pardo and Vajda, 2003)

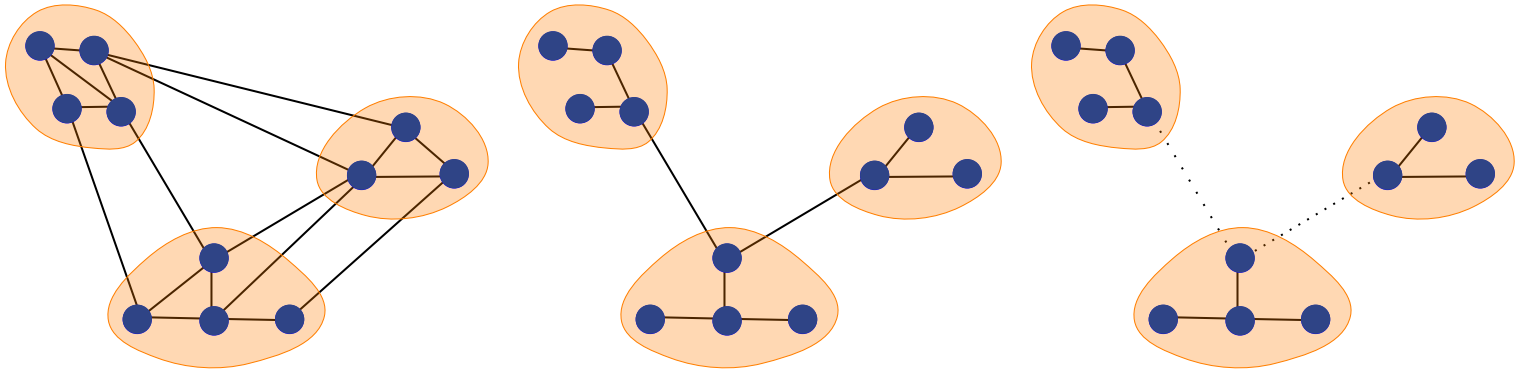
Selection of the norm in high dimension



- The volume of the intersection is negligible when d is sufficiently large
- Results obtained for an ℓ_p norm **do not generalize** to other norms

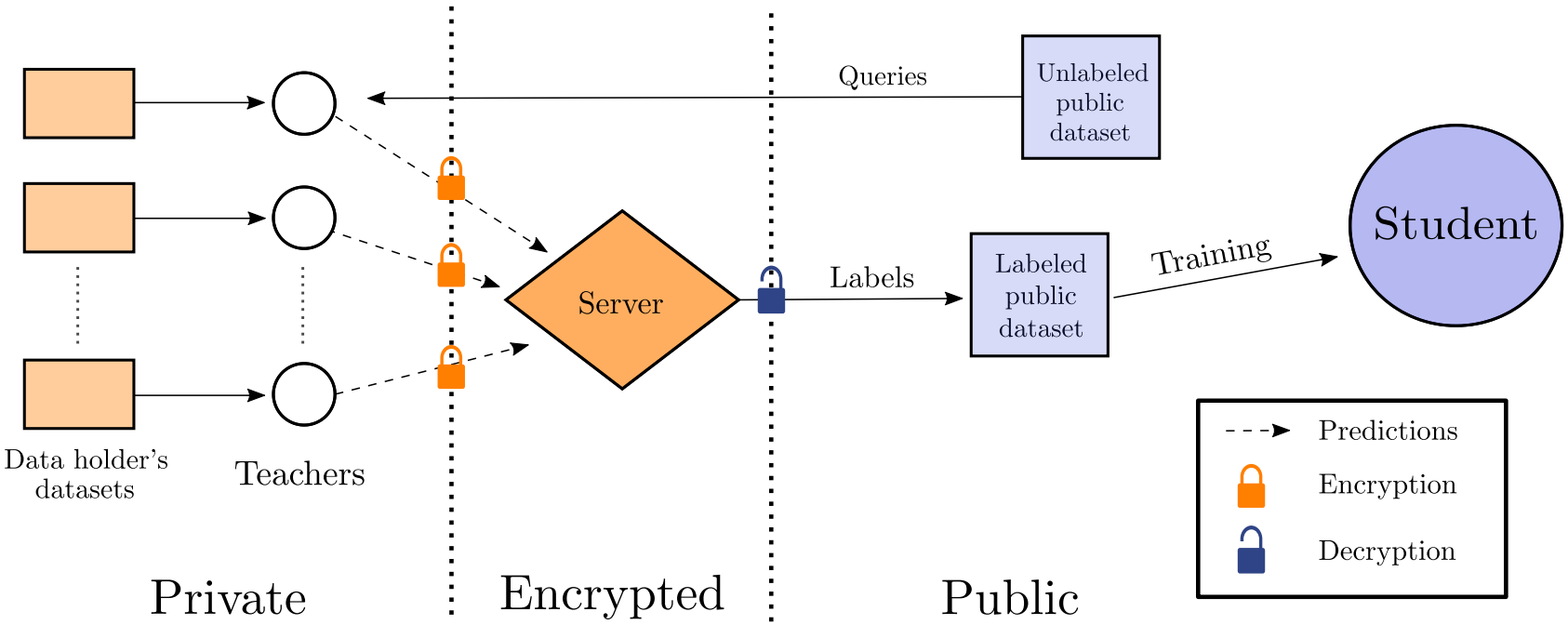
$\rightarrow$ Mixing defense strategies (Tramèr and Boneh, 2019; Maini et al., 2020)

Graph clustering with differential privacy constraints



- Start from a Tree-based clustering algorithm (Schaeffer, 2007)
- Demonstrate that the clustering method provably works (Pinot et al., 2018)
- Change Step 1 with a **privacy preserving** Tree algorithm (Pinot, 2017)
- Use the **data-processing inequality** to get results for Step 2

Secure and private deep learning with encrypted aggregation operator



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