Quantum properties

- **Discretization** - Energy is discrete in atoms and molecules. This means that only some values of energy are allowed. These allowed values of energy are also called energy levels. Discretization allows quantum objects to be used for computing. Their energy levels can be used as 0 and 1.

- **Superposition** - Quantum objects can be in a combination of multiple possible states. For example, in an atom, electrons are in a superposition of many possible positions.

- **Interference** - The possible states of quantum objects can add up or cancel out.
    - Ex: noise cancelling headphones, which produce sound waves that cancel out external noise through interference.

- **Entanglement** - Two quantum objects are entangled if the state of one object depends on the state of another. If you know the state of one quantum object, you know the state of the other. Entanglement is unaffected by distance. Entangled quantum particles remain entangled even if they are separated by millions of miles.

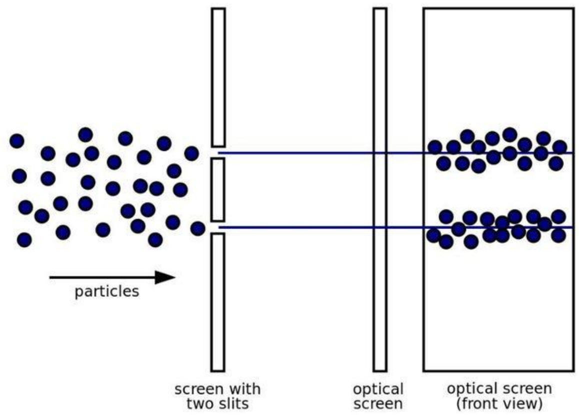
- **Measurement** - The results of measurements on quantum objects can be random. Phenomena like superposition, interference, and entanglement and the reason and make it extremely difficult to predict the exact outcome of the measurement. In addition, the state of the quantum object being measured can change as a result of the measurement.

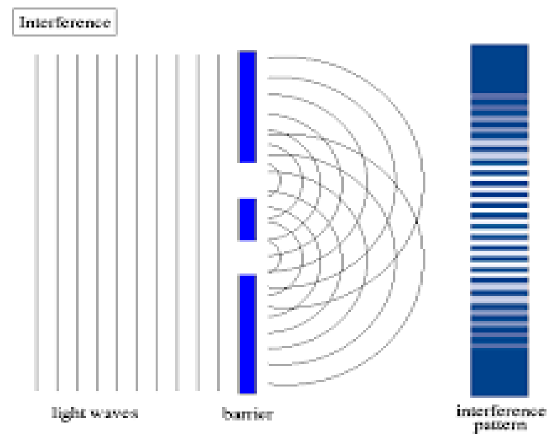
What makes Quantum properties possible?

Wave properties: Waves, such as light and water waves, travel with a unique velocity. Further, waves interact with each other to form complex patterns in a process called interference. Sometimes waves can add onto each other to create a bigger wave (constructive interference), and sometimes waves can cancel each other out (destructive interference)

Particle properties: Particles have mass, a definite, discrete location, and also travel with a certain velocity. Anything that can be measured consistently in a particle is called a particle property.

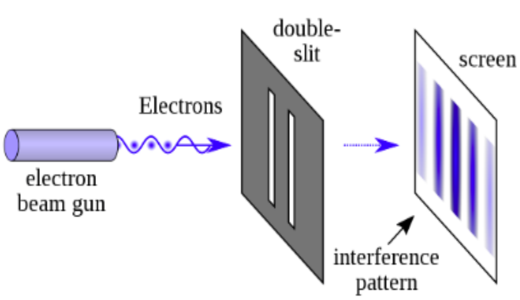
Double-slit experiment: The double-slit experiment helps visualize the differences between waves and particles. In this experiment, the wave or particle is aimed at two slits, behind which is a plain wall. Particles pass through either the left or the right slit, and create two lines of discrete spots on the wall. Waves passing through the slits interfere with each other and create a pattern of bright and dark lines on the wall.





<u>Explanation:</u>

Wave-particle duality: Quantum objects show both wave-like and particle-like properties. When quantum objects, such as photons and electrons, are used in a double slit experiment, they create discrete spots (like particles) but the spots are arranged in an interference pattern (like waves). Because of wave-particle duality, we can think of qubits as both waves and particles.



Superposition with waves: Superposition stems from wave-particle duality. Using the wave nature of qubits, we can represent the two states of the qubit (0 and 1) with two waves. To create a superposition state, we can combine these waves.

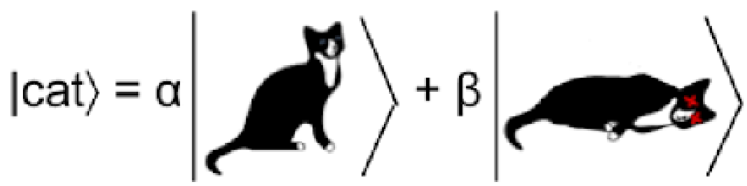
Interference with waves: Interference also stems from wave-particle duality, and can be described as the addition or subtraction of the waves representing qubit states. Both superposition and interference involve overlap between waves.

Discretization with waves: The discreteness of the quantum world comes into play when quantum objects are confined. Confinement of waves forces them to only take certain shapes or energies. In the quantum world, confinement can exist naturally or artificially.

- Ex. in a trapped ion qubit, where the negatively charged electron is confined by the positively charged nucleus.
- Ex. such as through electric circuits in superconducting qubits.

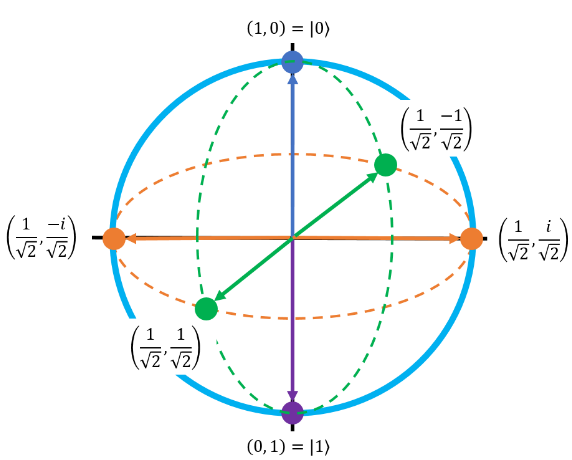
Representing Qubits

Ket notation: The ket notation is used to represent the state of qubits. Putting a “0” or a “1” inside a ket shows that it represents a quantum state. Implicitly, the state is a vector but more on that later.



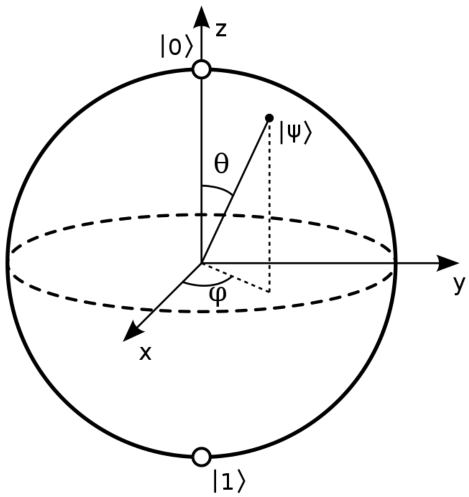
Bloch sphere: The Bloch sphere is a way to visually represent qubit states. Any individual qubit state can be represented on the Bloch sphere. However, entangled states are a notable exception.

- The |0> state is located at the top of the Bloch sphere, and the |1> state at the bottom.
- Any other state on the Bloch sphere represents a superposition of |0> and |1>. A superposition can be equal, meaning that |0> and |1> contribute equally to the state, or unequal, meaning that either |0> contributes more or |1> does. If the state is closer to |0>, it has a greater contribution from |0>. If it is closer to |1>, it has a greater contribution from |1>.



Quantum Gates: Quantum gates manipulate or change the state of qubits. Gates are how we create superposition, interference, and entanglement. The operation of gates on qubits can be visualized as rotations on the Bloch sphere for a single qubit. Quantum Gates are a generalization of classical logic gates.

To visualize these rotations, we need to associate a coordinate system with the Bloch sphere. Here is the conventional coordinate system:



The X gate: The X gate can be visualized as a 180 degree rotation about the X axis.

- | 0 > →| 1 > or | 1 > → | 0 >

The H gate: The H gate creates superposition. It is a uniquely quantum gate. Here, |+> and |-> represent two superposition states.

- | 0 > →| + > or | 1 > → | - >

The Z gate: The Z gate performs a 180 degree rotation about the Z axis.

- | + > →| - > or | - >→| + >

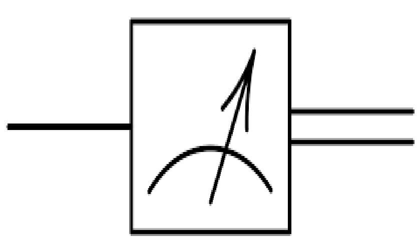
The | 1 > and | 0 > states are unaffected by the Z gate since it points in the vertical direction as well. However, the Z gate gives the |1> state a negative sign. This however, does not change where the state lies on the Bloch sphere.

- | 1 > → - | 1 >

How to find out what state the qubit was in?

The Quantum Circuit model captures the 3 major parts of any quantum circuit: the qubit states, gates, and measurements.

a. Measurement is the final step of any circuit. It is how we extract information about the state of our qubits in the actual circuit. Without measurements, we would never know what state our qubits were in and we would not get the results of our computations.



When we make measurements on a circuit, we get one of two answers, either |0> or |1>. Some circuits have a definite answer to this question. For other circuits there isn't a definite answer to this question. When dealing with qubits in superposition, the measurement we get is a random result. Sometimes we will get |0>, and sometimes we will get |1>.

b. After the measurement, the state of the qubit changes to the state that it was measured in. Therefore, if the result of the measurement was |1>, the qubit will be in state |1> after the measurement. Measurement can change the state of the qubit. This change is also known as collapse.

c. We can visualize this collapse by thinking about the wave representation of qubit states. If the qubit is in a superposition state before measurement, its wave will collapse to the wave representing the |0> state or the |1> state after measurement.

How to find state of qubit with superposition

- We can run the circuit and measure the state of the qubit many, many times. Repeated measurements on an identically prepared qubit increase our knowledge about the qubit by revealing the statistical distribution it emerges from.

- Although we cannot always predict what the exact result of an individual measurement will be, we can always predict the probability of different possible results of a measurement, however doing so is not always computationally efficient.

Measuring with Different Bases

In the Z basis, these two possible answers are |0> and |1>. In the X basis, the two possible answers are |+> and |->. In other bases, the two answers would be two other states. Luckily, as all vectors in space can be expressed with a complete spanning basis, no one particular basis is superior to the others. Physicists choose the Z basis for historic reasons, mainly because projections along the Z-axis in spherical coordinates are especially convenient.

The randomness of a quantum measurement is determined partially by both the state being measured, and the basis that the measurement is performed in.

Math of Quantum Measurements

Born rule: The probability of measuring a qubit in the |0> state is given by the square of the contribution of |0> to that qubit's normalized state vector. Similarly, the probability of measuring a qubit in the |1> state is given by the square of the contribution of |1> to that qubit's normalized state vector.