19AIE201 "Introduction to Robotics"

Project Report

Estimation of Forward Kinematics of BAXTER robot

Bachelor of Technology in
Artificial Intelligence & Engineering

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Abstract

The Baxter research robot offers potential application areas in both human-robot interaction and industry. Baxter robots are employed in a variety of sectors, are thru to make roles such as grasping, lifting, and positioning a thing. In this paper, we will study the Baxter robot and forward kinematics of its parts under robotics principles. our report explains how to find forward kinematics. To find forward kinematics we will find the values for the table of DH parameters that is link labelling, link twist, link offset, joint angle. We have found forward kinematics for the 7dof robot in this report which will get 28 tables of DH. The notions of forward kinematics and DH parameters are critical for understanding end-effector location in relation to the world frame (will give the knowledge in the kinematics of Baxter robot.)

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Chapter 1

1 Introduction

1.1 Background Study

1.1.1 What is Robot??

The robot is an autonomously operated system that replaces human labour, even if it does not look like people or conducts functions in a humanlike way. Robotics, as a technological area, is concerned with the design, construction, and operation of robots.

The modern name robot stems from the Czech word robota, which was originally used in Karel Capek’s play R.U.R. (1920). In the drama, the robots were built by humans who had been cruelly exploited by factory owners until they revolted and wiped-out humanity. Like the monster in Mary Shelley's Frankenstein (1818), It was unknown if they were biological or mechanical, but the mechanical possibility inspired decades of innovators to create electrical humanoids.

As of now we have got the idea of what a robot is now we will go to Baxter robot.

1.1.2 What is Baxter Robot??

As in the previous section we got to know about robots but in robots, In industries, there are many different types of robots, including industrial robots produced by Rethink Robotics, which was established by Rodney Brooks. It is known as the Baxter Robot.

Figure 1: Baxter Robot

The ideas and techniques utilized in the Baxter robot to accomplish some operations, such as lifting, holding, and moving an object with the robot's gripper, and then abandoning the thing at a certain spot, are shown in this picture

Now we will see what forward kinematics is, why because we will see in Motivation section

1.1.3 What is Forward Kinematics

Kinematics is a fundamental and well-known topic in robotics that investigates the relationship between a robot's joint coordinates and its spatial arrangement. Many problems, such as positioning a gripper in space, creating a mechanism to transport a tool from point A to point B, or predicting Kinematics may be used to determine whether or not a robot's motion will collide with barriers. Kinematics is solely concerned with the robot's coordinates' immediate values and ignores their movement under pressures and torques. The kinematics problem may be simple for some robots, such as mobile robots with essentially rigid bodies, but it requires substantial research for others with multiple joints, such as humanoid robots and parallel mechanisms.

The method of computing the frames of a robot's links given a configuration and the robot's kinematic structure as input is known as forward kinematics. The forward kinematics of a robot can be computed in microseconds in a software library for tasks like motion prediction, collision detection, and rendering, or they can be theoretically determined in closed form for further investigation during mechanism design.

Literature Survey

Robot is a machine that senses, thinks and automatically operating machine that helps in human efforts or replaces human effort. In robot there are so many types of Robots in this paper we will work on **Baxter robot**, Baxter is an industrial

robot with a wide range of capabilities. It can respond to changes in the surroundings thanks to its cameras and force-sensing actuators, and a user may program a new task by merely waving its arms around. The work on this paper is focusing on forward kinematics of a Baxter robot by using the DH parameters which are Link Length, Link Twist, Link Offset, Link Angle. **Link length** is the Perpendicular distance between joint variable and axis perpendicular to it,
Link twist is the angle between joint variable and axis perpendicular to it
Link offset is the distance along the joint variable from the joint to axis perpendicular to it, **Link angle** is the angle between joints that it. which we use to find the Length and angle of the joint and link labeling, joint variables, Kinematic Chains of a robot. After finding the values we will get the dh parameter table for Each joint means we will get 28 tables. With these terminologies we can estimate the forward kinematics of a 7-dof robot arm.

Motivation

The safety of human operators has been a major concern since the first robots were deployed into industrial contexts. Initially, the approach was to establish a clear boundary between humans and robots in the workplace: traditionally, robots have been caged and surrounded by various safety peripherals to protect the operators from harm. With the emergence of cooperative robots, or cobots, and the concept of HRI (Human Robot Interaction), this has begun to change. The cobots are meant to collaborate with humans and are used to complete activities in places where a normal industrial robot would be inconvenient. Due to its unique approach to more co-located work and a more adaptive system, the Baxter robot is an attractive research platform. Aside from that, it promises to be cost-effective, which makes it appealing to medium-sized businesses. As a result, the Alberta Centre for Advanced MNT Products (a camp) has purchased two Baxter research robots because the robot's use cases fit into their business model of advancing Alberta industry through new methodologies. Acampo works with the University of Calgary to research the possibilities of robotics by supplying one robot to the university. The primary goal of this thesis is to investigate the Baxter robot and gain a better understanding of its forward kinematics which will be calculated in microseconds in a software library for tasks and study the DH (the Denavit–Hartenberg) parameters.

Chapter 2

Work & Methodology

Overview

The overview of this chapter is to write and explain about to calculation of forward kinematics by using DH parameter. We have specifically done on left hand of Baxter robot, as its right arm is almost same. We find the DH parameter by using the free body diagram of the left arm of Baxter and calculating solution is straightforward. It is based on substituting each line of the D-H Parameters we have found in the equation.

so the convention we use to make life a bit easier for us and everyone else is called the denavit hartenberg convention so first of all a general transformation matrix has six variables three in the rotation component which is at the top left and three in the position or displacement vector this denavit hartenberg approach or DH is what most people call it collapses these six variables to only four link parameters but this only works if we follow a certain process or procedure for how we set up these XYZ coordinate frames at each link and the four parameters for each link are a subscript I which is the link length alpha subscript I which is the link twist D subscript I is the link offset and theta subscript I is the joint angle of joint I so the first two parameters are generally fixed for a particular link even if the robot is moving around these parameters are pretty much fixed they would only change if you took apart your robot and maybe made the arm half as long or twisted it and changed it in some permanent way like this the last two parameters can change so for a prismatic link which extends or contracts these which D would change and for a revolute joint which rotates the theta parameter would change.

In-depth study

Baxter Forward Pose Kinematics

Next we read the DH parameters of Baxter's arm and each arm of the Baxter robot has a 7-dof meaning seven members, so the arm in turn has a number of seven angles to indicate the rotation of these joints. As the lengths of these arms do not move and the angles of the members are also known and their limit. Therefore, position calculations and gripper directions will be easy to find. The main method of interaction is to use the DH parameter where each member is divided by four parameters in the DH parameter table. These DH parameters are repeated together to find the end point (gripper) and direction according to a single integrated integration frame. The area calculation and direction of the result (gripper) from the angle values of the joints to be used as an inclusion in the conversion equation in kinematics is called the advanced kinematics solution. Using this method will provide only one solution for the result area as the angle values of the joints are set to one location or values. Another way to find these seven-member angelic values can be retrieved and detected with an ROS tool called tf (short for conversion). Another way to get the angle of the seven members of each arm is to reach the topic of shared position.

$$\left[{}^{i-1}_{\ i}T\right] = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & \left[{}^{i-1}_{\ i}R\right] & & \{{}^{i-1}P_i\} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Denavit-Hartenberg-related methods are commonly used in many kinematic problems. It works great for serial manipulator problems. No matter how difficult the DH method has been used to improve the kinematic model of the robot due to its flexibility and acceptance to any number of members and serial manipulator links.

Baxter Robot Denavit - Hardenberg (DH) Parameters

Denavit-Hartenberg (DH)

Although it is possible to transfer all research into this project using an independent framework attached to each link, it is helpful to plan for the selection of these frameworks. The most widely used assembly to determine the limits of robotic claims is Denavit-Hartenberg, or D-H. In this commitment, the homogeneous transformation of Ai is seen as the introduction of four simple transformations.

DH Matrix

- $R_{z,\theta_i}\mathrm{Trans}_{z,d_i}\ \mathrm{Trans}_{x,a_i}\ R_{x,\alpha_i}$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_1} \\ s_{\theta_i} & c_{\theta_1}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Dividing DH parameters into 6 variables to 4 parameters of the link, which are introduced following a specific set of connection frames.
  $\left.\begin{array}{l} -a_i \text{ is link length of link } i \\ -\alpha_i \text{ is link twist of link } i \end{array}\right\}$ These don't change unless you reconfigure your robot
  $-d_i$ is link offset of link $i \leftarrow$ prismatic variable
  $-\theta_i$ is joint angle of joint $i \leftarrow$ revolute variable

- Rules for Assigning Frames
  Rule 1: z_{i-1} is axis of actuation of joint i.
  The axis of revolution for the revolute joint
  Axis of translation of prismatic joint
  Rule 2: The x_i axis is set and therefore perpendicular and intersects z_{i-1}.
  Rule 3: Derive y_i from x_i and z_i.

Rule 1: Base Frame

- z_0 is the axis of actuation of joint 1.
- x_0 and y_0 are set as convenient, provided they make right-handed set.
- Usually set to a useful reference such as orientation of work cell or table.

Rule 1: Tool Frame

- Where rules allow
  $-z_n$ is the approach direction of the tool
  $-y_n$ is the slide direction of the gripper
  $-x_n$ is the normal direction to other axes

Rule 2: Case 1
z_{i-1} and z_i are not coplanar

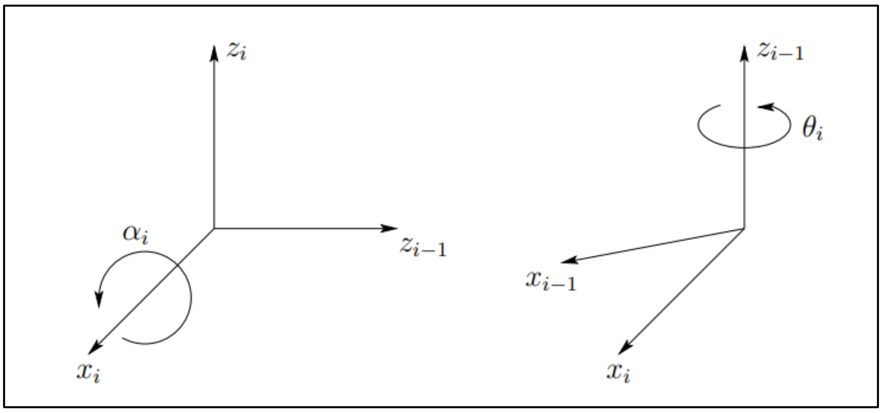
- There is only one line possible for x_i, which is the shortest line from z_{i-1} to z_i.
  $-o_i$ is at intersection of x_i and z_i.

Rule 2: Case 2
z_{i-1} and z_i are parallel

- There are an infinite number of opportunities for x_i from z_{i-1} to z_i.
- Usually easiest to choose an x_i that passes through o_{i-1} (so that $d_i = 0$).
- o_i is at intersection of x_i and z_i.
  $-\alpha_i = 0$ always for this case.

Rule 2: Case 3

- z_{i-1} intersects z_i
  $-x_i$ is normal to the plane of z_{i-1} and z_i.
- Positive direction of x_i is arbitrary.
- o_i naturally sits at intersection of z_{i-1} and z_i but can be anywhere on z_i.
  $-a_i = 0$ always for this case.



Finding DH parameters

- a_i distance from z_{i-1} to z_i is calculated according to x_i..
- angle α_i is the angle from z_{i-1} to z_i is calculated approximately x_i.
- d_i distance from x_{i-1} to x_i calculated in accordance with z_{i-1}.
- the θ_i angle from x_{i-1} to x_i is calculated approximately z_{i-1}.

Table 1. Seven-dof Left Arm R Joints Naming Convention

Joint Name	Joint Motion
S_0	shoulder roll
S_1	shoulder pitch
E_0	elbow roll
E_1	elbow pitch
W_0	wrist roll
W_1	wrist pitch
W_2	wrist roll

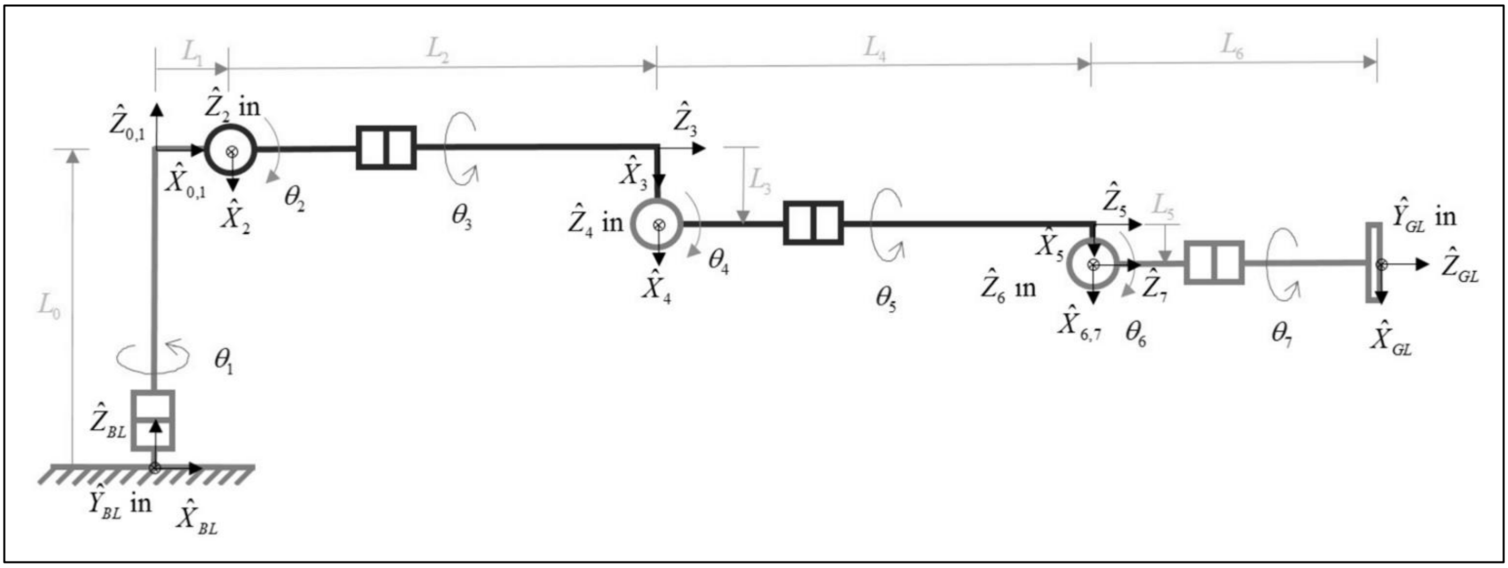


Table 2. Seven-dof Left Arm DH Parameters Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_j
1	0	0	0	θ_1
2	$-90°$	L_1	0	$\theta_2+90°$
3	$90°$	0	L_2	θ_3
4	$-90°$	L_3	0	θ_4
5	$90°$	0	L_4	θ_5
6	$-90°$	L_5	0	θ_6
7	$90°$	0	0	θ_7

Table 3. Seven-dof Right Arm DH

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-90°$	L_1	0	$\theta_2+90°$
3	$90°$	0	L_2	θ_3
4	$-90°$	L_3	0	θ_4
5	$90°$	0	L_4	θ_5
6	$-90°$	L_5	0	θ_6
7	$90°$	0	0	θ_7

Table 4. Seven-dof Left- and Right-Arm Joint Limits

Joint Name	**Joint Variable**	θ_i**min**	θ_i**max**	θ_i **range**
S_0	θ_1	$+51°$	$-141°$	$192°$
S_1	θ_2	$+60°$	$-123°$	$183°$
E_0	θ_3	$+173°$	$-173°$	$346°$
E_1	θ_4	$+150°$	$-3°$	$153°$
W_0	θ_5	$+175°$	$-175°$	$350°$
W_1	θ_6	$+120°$	$-90°$	$210°$
W_2	θ_7	$+175°$	$-175°$	$350°$

Table 5. Seven-dof Left- and Right-Arm Link- and Offset-Lengths

Length	**Value (mm)**
L_0	270.35
L_1	69.00
L_2	364.35
L_3	69.00
L_4	374.29
L_5	10.00
L_6	368.30

$$\left[{}^{W_o}_{BL}T\right] = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & L \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -h \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left[{}^{W_o}_{BL}T\right] = \begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -L \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -h \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 6. $\{B\}$ to $\{Wo\}$ Lengths

Length	**Value** (mm)
L	278
h	64
H	1104

Table 7. Baxter 'Neutral' Joint Angles

Joint Name	**Joint Variable**	$\boldsymbol{\theta_i}$
S_0	θ_1	$0°$
S_1	θ_2	$-31°$
E_0	θ_3	$0°$
E_1	θ_4	$43°$
W_0	θ_5	$0°$
W_1	θ_6	$72°$
W_2	θ_7	$0°$

Seven-dof Left Arm FPK Expressions

The problem statement for the Forward Pose Kinematic of the seven - dof (degrees of freedom) for the left arm serial chain humanoid robot Baxter is written as

Given $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)$, calculate $[\,{}^{0}_{7}T]$ and $[\,{}^{W}_{G}T]$.

Given the joint variables as $(\theta_1 \ldots \theta_7)$ and we are going to find the transformation matrix for the end effector of robot with the help of the ***{W}*** which is called as World fixed orientation frame on the floor and here the ***{G}*** is the left-arm's end-effector gripper frame.

$$[\,{}^{0}_{1}T] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [\,{}^{1}_{2}T] = \begin{bmatrix} -s_2 & -c_2 & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -c_2 & s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [\,{}^{2}_{3}T] = \begin{bmatrix} c_3 & -S_3 & 0 & 0 \\ 0 & 0 & -1 & -L_2 \\ S_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\,{}^{3}_{4}T] = \begin{bmatrix} c_4 & -S_4 & 0 & L_3 \\ 0 & 0 & 1 & 0 \\ -S_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [\,{}^{4}_{5}T] = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & -L_4 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [\,{}^{5}_{6}T] = \begin{bmatrix} c_6 & -S_6 & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ -S_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\,{}^{6}_{7}T] = \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By Replacing each row of the DH parameters with the help of using the table 2 into the equation for $[\,{}^{i-1}_{i}T]$ to gather these 7 homogeneous transformation matrices which might be functions of the joint angles of the seven-dof left arm. These are acquired with the aid of substituting the rows inside the DH parameters that's table 2

Now we have 7 matrices, and we are going to substitute these seven homogeneous transformation matrices into the given following homogeneous transform equation to achieve the active joints of the Forward Pose Kinematics result.

$$[\,{}^{0}_{7}T] = [\,{}^{0}_{1}T(\theta_1)][\,{}^{1}_{2}T(\theta_2)][\,{}^{2}_{3}T(\theta_3)][\,{}^{3}_{4}T(\theta_4)][\,{}^{4}_{5}T(\theta_5)][\,{}^{5}_{6}T(\theta_6)][\,{}^{6}_{7}T(\theta_7)]$$

Note: Here the left-arm active-joint Forward Pose Kinematic solution aka matrices will be grouped into 3 components, one by the 2 - dof shoulder, two by the 2 - dof elbow, and third with 3 - dof wrist joints.

$$[\,{}^{0}_{7}T(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7)] = [\,{}^{0}_{2}T(\theta_1,\theta_2)][\,{}^{2}_{4}T(\theta_3,\theta_4)][\,{}^{4}_{7}T(\theta_5,\theta_6,\theta_7)]$$

Where each of the component Is written with respect to the Transformation matrix of the Baxter Robot as *[*$[T_{\text{shoulder}}]$*]* of first two joint variables [theta1,2] and *[*T_{elbow}*]* of next joint variables and *[*T_{wrist}*]* with last final joint variables.

$$[T_{\text{shoulder}}] = [\,{}^{0}_{2}T(\theta_1,\theta_2)] = \begin{bmatrix} -c_1s_2 & -c_1c_2 & -s_1 & L_1c_1 \\ -s_1s_2 & -s_1c_2 & c_1 & L_1s_1 \\ -c_2 & s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_{\text{elbow}}] = [\,{}^{2}_{4}T(\theta_3,\theta_4)] = \begin{bmatrix} c_3c_4 & -c_3s_4 & -s_3 & L_3c_3 \\ s_4 & c_4 & 0 & -L_2 \\ s_3c_4 & -S_3s_4 & c_3 & L_3s_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_{\text{wrist}}] = [\,{}^{4}_{7}T(\theta_5,\theta_6,\theta_7)] = \begin{bmatrix} -s_5s_7 + c_5c_6c_7 & -s_5c_7 - c_5c_6s_7 & c_5s_6 & L_5c_5 \\ s_6c_7 & -s_6s_7 & -c_6 & -L_4 \\ c_5s_7 + s_5c_6c_7 & c_5c_7 - s_5c_6s_7 & s_5s_6 & L_5s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We are combining the homogenous transformation matrices of 2 - dof *[*T_{shoulder}*]* and 2 - dof *[*T_{elbow}*]* by the matrix multiplication:

$$[\,{}^{0}_{7}T(\theta_1,\theta_2,\theta_3,\theta_1,\theta_5,\theta_6,\theta_7)] = [\,{}^{0}_{1}T(\theta_1,\theta_2,\theta_3,\theta_1)][\,{}^{4}_{7}T(\theta_5,\theta_6,\theta_7)]$$

where:

$$[T_{\text{stouuldecrelbow}}] = [\,{}^{0}_{2}T(\theta_1,\theta_2)][\,{}^{2}_{4}T(\theta_3,\theta_4)] = [\,{}^{0}_{4}T(\theta_1,\theta_2,\theta_3,\theta_4)]$$

$$= \begin{bmatrix} -(s_1s_3 + c_1s_2c_3)c_4 - c_1c_2s_4 & -c_1c_2c_4 + (s_1s_3 + c_1s_2c_3)s_4 & -s_1c_3 + c_1s_2s_3 & (L_1 + L_2c_2)c_1 - L_3(s_1s_3 + c_1s_2c_3) \\ (c_1s_3 - s_1s_2c_3)c_4 - s_1c_2s_4 & -s_1c_2c_4 - (c_1s_3 - s_1s_2c_3)s_4 & c_1c_3 + s_1s_2s_3 & (L_1 + L_2c_2)s_1 + L_3(c_1s_3 - s_1s_2c_3) \\ s_2s_4 - c_2c_3c_4 & s_2c_4 + c_2c_3s_4 & c_2s_3 & -L_2s_2 - L_3c_2c_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The final analytical Forward Pose Kinematics expressions for the active Baxter joints are given below as:

$$[\,{}^{0}_{7}T(\theta_1,\cdots,\theta_7)] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & {}^{0}x_7 \\ r_{21} & r_{22} & r_{23} & {}^{0}y_7 \\ r_{31} & r_{32} & r_{33} & {}^{0}z_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} a &= s_1s_3 + c_1s_2c_3 \\ b &= s_1c_3 - c_1s_2s_3 \\ d &= c_1s_3 - s_1s_2c_3 \\ f &= c_1c_3 + s_1s_2s_3 \\ g &= s_2s_4 - c_2c_3c_4 \\ h &= s_2c_4 + c_2c_3s_4 \end{aligned}$$

Note: *As equations are too long, few repeated expressions are being taken as above variable.*

Below are the calculations of orthonormal rotation matrix elements of the result matrix:

$$\begin{aligned} r_{11} = {} & -(s_1s_3 + c_1s_2c_3)c_4 - c_1c_2s_4(-s_8s_7 + c_5c_6c_7) - c_1c_2c_4 + (s_1s_3 + c_1c_2c_3)s_4(s_6c_7) - s_1c_3 \\ & + c_1s_2c_3(c_5s_7 + s_5c_6c_7) \end{aligned}$$

$$= ac_4 + c_1c_2s_4 + (s_5s_7) + (ac_4 + c_1c_2s_4)(c_5c_6c_7) + (as_4 - c_1c_2c_4)s_6c_7 + b(c_5s_7 + s_5c_6c_7)$$

$$\boldsymbol{r_{11} = \big((ac_7 + c_1c_2s_4)s_5 - bc_5\big)s_7 + \{(ac_4 + c_1c_2s_4)s_6 - (bs_5 + (ac_4 + c_1c_2s_4)c_5)c_6]c_7}$$

$$\begin{aligned} r_{12} = {} & [-(s_1s_3 + c_1s_2c_3)c_4 - c_1c_2s_4][-s_5c_7 - c_5c_6s_7] + [-c_1c_2c_4 + (s_1s_3 + c_1s_2c_3)s_4][-s_6s_7] \\ & + [-s_1c_3 + c_1s_2s_3][c_5c_7 - s_5c_6s_7] \end{aligned}$$

$$= ac_4s_5c_7 + c_1c_2s_4s_5c_7 + ac_4s_7 + ac_4c_5c_6c_7s_7 - \mathrm{a}s_4s_6s_7 - bc_5c_7 + bs_5c_6s_7 + c_1c_2s_4c_5c_6c_7$$

$$\boldsymbol{r_{12} = (ac_4 + c_1c_2s_4)s_5 - bc_5]c_7 + [((as_4 - c_1c_2c_4)s_6 - (bs_5 + (ac_4 + c_1c_2s_4)c_5]c_6]s_7}$$

$$r_{13} = [-(a)c_4 - c_1c_2s_4]c_5s_6 + [-c_1c_2c_4 + as_4](-c_6) + [-s_1s_5 + c_1s_2s_3]s_5s_6$$

$$= -ac_4c_5s_6 - c_1c_2s_4c_5s_6 + c_1c_2c_4c_6 - as_4c_6 - bs_5s_6$$

$$\boldsymbol{r_{13} = -(as_4 - c_1c_2c_4)c_6 - [bs_5 + (ac_4 + c_1c_2s_4)c_5]s_6}$$

$$\begin{aligned} \mathrm{r}_{21} = {} & [(c_1s_3 - s_1s_2c_3)c_4 - s_1c_2s_4][-s_5s_7 + c_6c_5c_7][-s_1c_2c_4 - (c_1s_3 - s_1s_2c_3)s_4][s_6c_7] \\ & + [c_1c_3 + s_1s_2s_3][c_5s_7 + s_5c_6c_7] \end{aligned}$$

$$= -(dc_4 - s_1c_2s_4)s_5s_7 + (dc_4 - s_1c_2s_4)(c_5c_6c_7) + [-s_1c_2c_4 - ds_4][s_6c_7] + [fc_5c_7] + fs_5c_6c_7$$

$$\boldsymbol{\mathrm{r}_{21} = -\big((ds_4 + s_1c_2c_4)s_6 - (fs_5 + (dc_4 - s_1c_2s_4)c_5)c_6\big)c_7 - \big((dc_4 - s_1c_2s_4)s_5 - fc_5\big)s_7}$$

$$r_{22} = \quad [dc_4 - s_1c_2s_4][-s_5c_7 - c_5c_6s_7] + [-s_1c_2c_4 - ds_4][-s_Cs_7] + \; [f][c_5c_7 - s_5c_6s_7]$$

$$= \quad -dc_4s_5c_7 - dc_4c_5c_6s_7 + s_1c_2s_4s_5c_7 + s_1s_4c_2c_5c_6c_7 + ds_4s_6s_7 + s_1c_2c_4s_6s_7$$

$$\mathbf{r_{22}} = \quad -[(\boldsymbol{dc_4} - \boldsymbol{s_1c_2s_4}) - \boldsymbol{fc_5}]\boldsymbol{c_7} - [(\boldsymbol{ds_4} + \boldsymbol{s_1c_2c_4})\boldsymbol{s_6} - (\boldsymbol{fs_5} + (\boldsymbol{dc_4} - \boldsymbol{s_1c_2s_4})\boldsymbol{c_5})\boldsymbol{c_6}]\boldsymbol{s_7}$$

$$r_{23} \quad = [(c_1s_3 - s_1s_2c_3)c_4 - s_1c_2s_4]c_5s_6 + [-s_1c_2c_4 - (c_1s_3 - s_1s_2c_3)d_4](-c_6) + [c_1c_3 + s_1s_2s_3]s_5s_6$$

$$= dc_4c_5s_6 - s_1c_2s_4c_5s_6 + [s_1c_2c_4c_6] + [ds_4c_6]fs_5s_6$$

$$\boldsymbol{r_{23}} = [(\boldsymbol{ds_4} + \boldsymbol{s_1c_2c_4})\boldsymbol{c_6}] + [\boldsymbol{fs_5} + (\boldsymbol{dc_4} - \boldsymbol{s_1s_2s_4})\boldsymbol{c_5}]\boldsymbol{s_6}$$

$$r_{31} = [s_2s_4 - c_2c_3c_4][-s_5s_7 + c_5c_6c_7] + [s_2c_4 + c_2c_3s_4][s_6c_7] + c_2s_3[c_5s_7 + s_5c_6c_7]$$

$$= -gs_5s_7 + gc_5c_6c_7 + hs_6c_7 + c_2s_3[c_5s_7] + c_2s_3[s_6c_6c_7]$$

$$\boldsymbol{r_{31}} = (\boldsymbol{hs_6} + (\boldsymbol{gc_5} + \boldsymbol{c_2s_3s_5})\boldsymbol{c_6})\boldsymbol{c_7} - (\boldsymbol{gs_5} - \boldsymbol{c_2s_3c_5})\boldsymbol{s_7}$$

$$r_{32} = [g_7[-s_5c_7 - c_5c_6s_7] + [h] \cdot [-s_6s_7] + c_2s_3[c_5s_7 - s_5c_6s_7]$$

$$= -gs_5c_7 - gc_5c_6s_7 - hs_6s_7[c_2s_3c_5c_7] - c_2s_3s_5c_6s_7$$

$$\boldsymbol{r_{32}} = -[\boldsymbol{hs_6} + (\boldsymbol{gc_5} + \boldsymbol{c_2s_3s_5})\boldsymbol{c_6}]\boldsymbol{s_7} - [\boldsymbol{gs_5} - \boldsymbol{c_2s_3c_5}]\boldsymbol{c_7}$$

$$\begin{aligned} r_{33} \quad &= [g]c_5s_6 + [h](-c_6) + c_2s_3s_5s_6 \\ &= gc_5s_6 - hc_6 + c_2s_3s_5s_6 \\ \boldsymbol{r_{33}} \; &= (\boldsymbol{gc_5} + \boldsymbol{c_2s_3s_5})\boldsymbol{s_6} - \boldsymbol{hc_6} \end{aligned}$$

Applying another level of substitutions:

$$\begin{aligned} A &= as_4 - c_1c_2c_4 \\ B &= ac_4 + c_1c_2s_4 \\ D &= ds_4 + s_1c_2c_4 \\ F &= dc_4 - s_1c_2s_4 \\ G &= gs_5 - c_2s_3c_5 \\ H &= gc_5 + c_2s_3s_5 \end{aligned}$$

the same orthonormal rotation matrix elements are:

$$\begin{aligned} r_{11} &= (As_6 - (bs_5 + Bc_5)c_6)c_7 + (Bs_5 - bc_5)s_7 \\ r_{12} &= (Bs_5 - bc_5)c_7 + (As_6 - (bs_5 + Bc_5)c_6)s_7 \\ r_{13} &= -Ac_6 - (bs_5 + Bc_5)s_6 \\ r_{21} &= -(Ds_6 - (fs_5 + Fc_5)c_6)c_7 - (Fs_5 - fc_5)s_7 \\ r_{22} &= -(Fs_5 - fc_5)c_7 - (Ds_6 - (fs_5 + Fc_5)c_6)s_7 \\ r_{23} &= Dc_6 + (fs_5 + Fc_5)s_6 \\ r_{31} &= (hs_6 + Hc_6)c_7 - Gs_7 \\ r_{32} &= -Gc_7 - (hs_6 + Hc_6)s_7 \\ r_{33} &= -hc_6 + Hs_6 \end{aligned}$$

The translational terms for this result are:

$$
\begin{aligned}
{}^{0}x_7 &= L_1c_1 + L_2c_1c_2 - L_3(s_1s_3 + c_1s_2c_3) - L_4\big((s_1s_3 + c_1s_2c_3)s_4 - c_1c_2c_4\big) \\
&\quad - L_5\Big((s_1c_3 - c_1s_2s_3)s_5 + \big((s_1s_3 + c_1s_2c_3)c_4 + c_1c_2s_4\big)c_5\Big) \\
{}^{0}y_7 &= L_1s_1 + L_2s_1c_2 + L_3(c_1s_3 - s_1s_2c_3) + L_4\big((c_1s_3 - s_1s_2c_3)s_4 + s_1c_2c_4\big) \\
&\quad + L_5\Big((c_1c_3 + s_1s_2s_3)s_5 + \big((c_1s_3 - s_1s_2c_3)c_4 - s_1c_2s_4\big)c_5\Big) \\
{}^{0}z_7 &= -L_2s_2 - L_3c_2c_3 - L_4(s_2c_4 + c_2c_3s_4) + L_5\big((s_2s_4 - c_2c_3c_4)c_5 + c_2s_3s_5\big)
\end{aligned}
$$

substituting the $a - h$ terms defined above:

$$
\begin{aligned}
{}^{0}x_7 &= L_1c_1 + L_2c_1c_2 - L_3a - L_4(as_4 - c_1c_2c_4) - L_5(bs_5 + (ac_4 + c_1c_2s_4)c_5) \\
{}^{0}y_7 &= L_1s_1 + L_2s_1c_2 + L_3d + L_4(ds_4 + s_1c_2c_4) + L_5(fs_5 + (dc_4 - s_1c_2s_4)c_5) \\
{}^{0}z_7 &= -L_2s_2 - L_3c_2c_3 - L_4h + L_5(gc_5 + c_2s_3s_5)
\end{aligned}
$$

and substituting the $A - H$ terms defined above:

$$
\begin{aligned}
{}^{0}x_7 &= L_1c_1 + L_2c_1c_2 - L_3a - L_4A - L_5(bs_5 + Bc_5) \\
{}^{0}y_7 &= L_1s_1 + L_2s_1c_2 + L_3d + L_4D + L_5(fs_5 + Fc_5) \\
{}^{0}z_7 &= -L_2s_2 - L_3c_2c_3 - L_4h + L_5H
\end{aligned}
$$

$$
\{\,{}^{0}P_7\} = \{\,{}^{0}P_7(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)\} = \begin{Bmatrix} {}^{0}x_7 \\ {}^{0}y_7 \\ {}^{0}z_7 \end{Bmatrix}
$$

The above *x, y, z* represents the positions of end effector

$$
[\,{}^{BL}_{0}T] = [\,{}^{BR}_{0}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [\,{}^{7}_{GL}T] = [\,{}^{7}_{GR}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
[\,{}^{W}_{GL}T] = [\,{}^{W}_{BL}T][\,{}^{BL}_{0}T][\,{}^{0}_{7}T(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)][\,{}^{7}_{GL}T]
$$

Qualitative Discussion

In order to calculate the forward kinematics so work out these transformations you could just draw detailed 2d or 3d diagrams of your joints and links and just use geometry and your knowledge of geometry and trigonometry to work out what the forward kinematics are of course as your robot arms get complicated or they're arranged in complex configurations this gets very hard or impossible for most people so what we need and this is a general theme running through this whole subject is we need an approach which everyone can apply in the same way or in a very similar way with standard rules which makes the math as simple as possible so at least likely to make mistakes.

Chapter 3

Summary

To learn and approximating the forward kinematics of Baxter Robot i.e., the forward kinematics and study about the DH (Denavit–Hartenberg) parameters. So, we have studied some parts of Baxter robot in order to calculate the kinematics of those robot arms. We particularly focused on the DH parameters which is the best approach as of now to find the joint variables, angles, length etc.… which we used to make the homogenous transformation matrices. We used the DH parameters and the kinematic models with respect to Baxter's robot left arm for the angles and for joint motion conventions we used in the regard to the zero position/location. We have done this because for the right and light arm the dynamics equations, analytical terms, kinematics may be equal and specially those are with recognize to the respective base frames. Usually, Inside the forward Pose Kinematics hassle of a serial-chain robot is stated here we will be having the joint values, and we are going to calculate the location and orientation which is the pose of the end-effector body. For these serial-chain robots, the forward Kinematic problem system is straight-ahead. the set of steps to locate these is primarily based on the substituting the Denavit-Hartenberg (DH) Parameters into the equation which is the pose of body {i} and its miles with appreciate to the closest neighbor of its frame {i–1}.

Find and attach the axes (Z_0..., $Z_{\mathrm{n}-1}$). Then create a base frame. Set the origin somewhere in the Z_0-axis. X_0 and Y_0 axes are carefully selected to form the right frame. In i = 1,.. n - 1, then you get the root Oi where the normal values of Z_{i} and $Z_{\mathrm{i}-1}$ interfere with zi. If Z_{i} breaks $Z_{\mathrm{i}-1}$ get Oi from this member. If Z_{i} and $Z_{\mathrm{i}-1}$ go hand in hand, find Oi at any convenient location near Z_{i} . Start X_{i} with the normal range between $Z_{\mathrm{i}-1}$and Z_{i} pass O_{i} , or in the normal way to flight $Z_{\mathrm{i}-1}$- Z_{i} if $Z_{\mathrm{i}-1}$and Z_{i} met. Create Y_{i} to complete the frame on the right.

Set effect frame $O_{\mathrm{n}}X_{\mathrm{n}}Y_{\mathrm{n}}Z_{\mathrm{n}}$. Assuming the nth junction is round, set X_{n} = a to$Z_{\mathrm{n}-1}$. guidance. Set the source O_{n} correctly next to Z_{n}, preferably in the middle of the capture area or at the end of any tool trick. Place Y_{n} = s on the side of the holding set X_{n} and Y_{n} correctly to create the right frame.

Create a table of parameters a_{i} , d_{i} , α_{i} , θ_{i} . a_{i} To find this, you need to follow the steps below. a_i = distance from Oi to the intersection of axes X_i and Z (i1) along X_i di = di is variable if the joint i is prismatic. Otherwise, the distance from O_ (i1) to the intersection of axes X_i and Z_ (i1) along Z_ (i1). αi = the angle between zi – 1 and zi measured around xi .θi = θi is variable if the joint i is revolted If not the angle between xi - 1 and xi measured around zi – 1

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