Lecture 1

Jan 29, 2026

Robot Dynamic Models

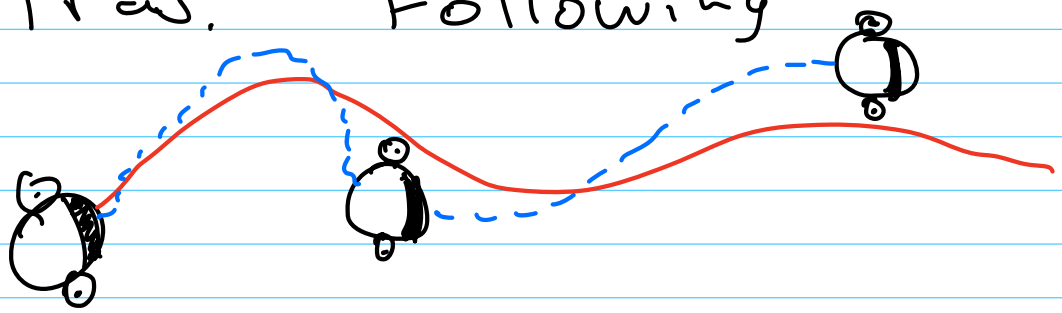
- Particle
- Unicycle

Tips for Angles

Safety!

Thanks to Prof. Schwager, Stanford, AA274A

Traj. Following



u_t - Control Inputs

Dynamics Model:

$$x_{t+1} = f(x_t, u_t)$$

We use Hamiltonian Dynamics,

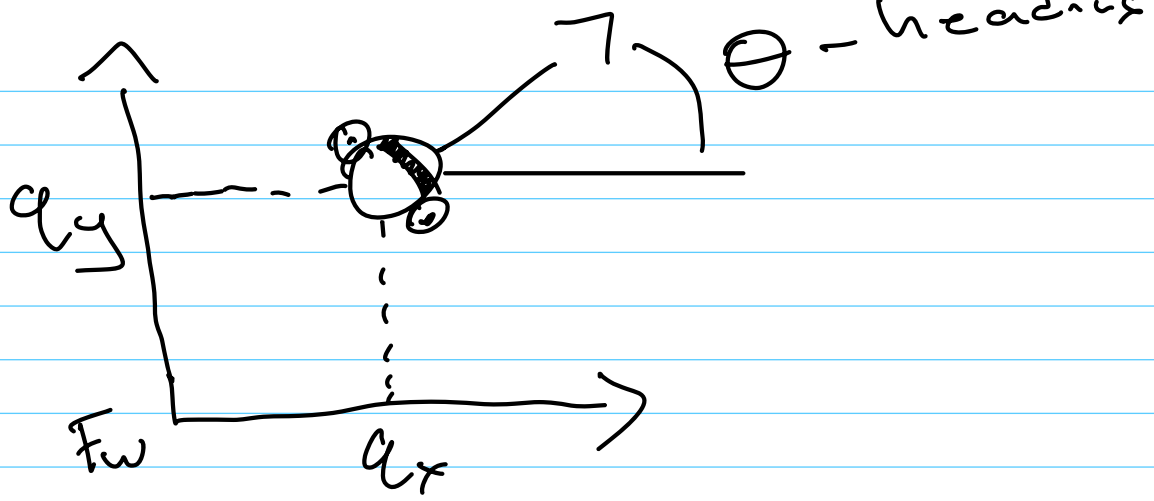
Configuration Variables:

q - Where is the robot, how is it posed?

$\dot{q}$ - generalized velocities - how fast config variables change in time

State: $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$

Unicycle Model



Configuration

$$\begin{bmatrix} q_x \\ q_y \\ \theta \end{bmatrix}$$

Constraint: (Non-holonomic)

$$\begin{bmatrix} \dot{q}_x \\ \dot{q}_y \end{bmatrix} = v\,d$$

v: speed, velocity, Magnitude (scalar)

d: unit vector, heading direction

$$d = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Kinematics

$$\begin{bmatrix} \dot{q}_x \\ \dot{q}_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix}$$

Control Input

$$u = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Model Error, can't instantaneously Influence velocity, but via forces.

Full dynamics Model

$$q = \begin{bmatrix} q_x \\ q_y \\ \theta \end{bmatrix} \qquad \dot{q} = \begin{bmatrix} v \\ w \end{bmatrix}$$

$$x = \begin{bmatrix} q_x \\ q_y \\ \theta \\ v \\ w \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{q}_x \\ \dot{q}_y \\ \dot{\theta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ w \\ a \\ \alpha \end{bmatrix}$$

Controls: $u = \begin{bmatrix} a \\ \alpha \end{bmatrix}$ — a: linear acceleration magnitude; α: angular acceleration

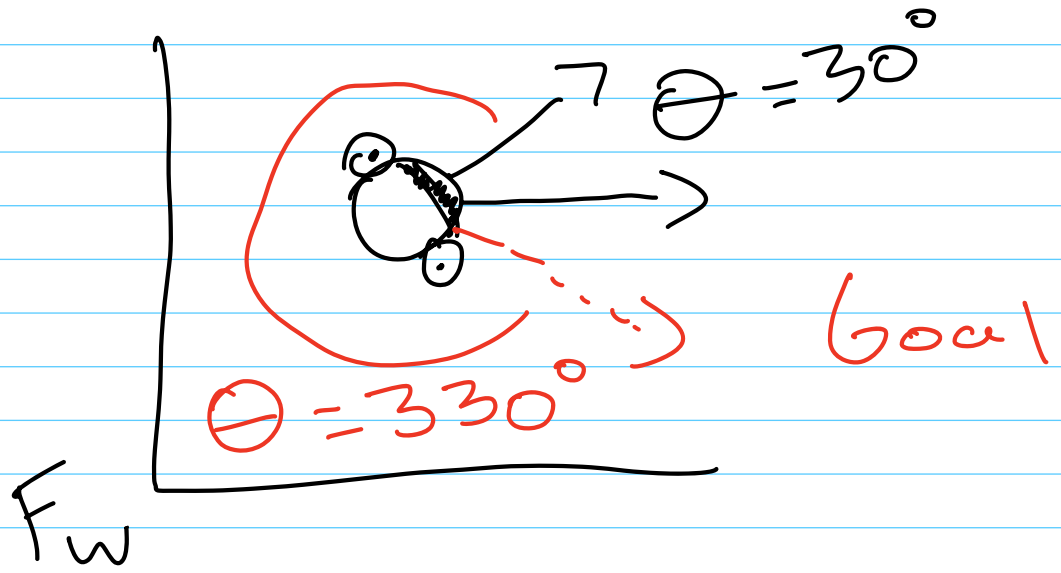
Compromise — Half-Kinematic Model

$$\begin{bmatrix} \dot{q}_x \\ \dot{q}_y \\ \dot{\theta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ w \\ a \end{bmatrix}$$

Control $\begin{bmatrix} a \\ w \end{bmatrix}$

Other robot models:
Planar Quadrotor
Bicycle

Tips For Angles



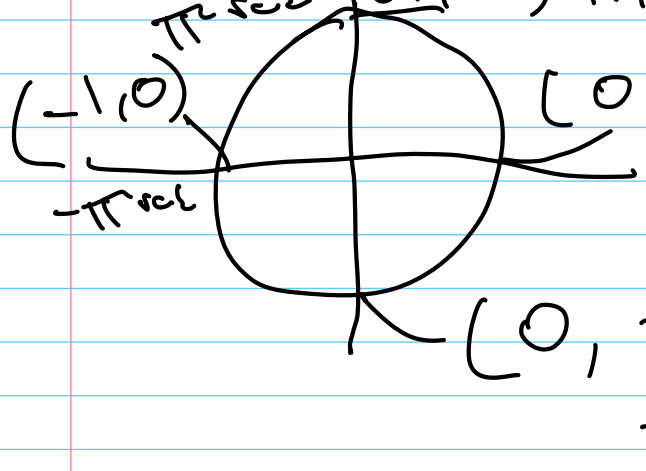
What is the heading error?

$330° - 30° = 300°$?

Or, $-60°$?

Heading Angles are discontinuous, need special handling!

We work in radians!



Bound Angle to Between $-\pi$, π!

```
function WrapAngle(angle)
    while angle > π do
        angle -= 2π
    end while
    while angle < -π do
        angle += 2π
    end while
    return angle
end function
```

Safety:

https://youtu.be/Wa70oKtmtLM?si=Yf2Rl19_zn98kGrZ

1: E-Stop!

2: No fingers near Joints!
Space around robot!

3: Watch Out for nearby People