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Optimization of module, shaft diameter and rolling bearing for spur gear through genetic algorithm

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ABSTRACT

In this study, the dimensional optimization of motion and force transmitting components of a gearbox is performed by genetic algorithm (GA). It is aimed to obtain the optimal dimensions for gearbox shaft, gear and the optimal rolling bearing. GA is a non-conventional useful search technique. In genetic algorithm, the best results can be obtained within the solution space which is formed by the design constraints. With the optimization of the gearbox components, the design with smallest volume which can carry the system load is obtained. The results obtained by GA optimization are compared to those obtained by analytical methods. These obtained results indicate that GA can be used reliably in machine element design problems.

1. Introduction

Conventional methods have been widely used in various mechanical design problems. They are deterministic in nature and use only a few geometric design variables due to their complexity and convergence problems (Chakraborthy, Kumar, Nair, & Tiwari, 2003). The main disadvantages of them are slow convergence along with local minima (or maxima) problems. When the number of design parameters increase, the complexity increases drastically. Many practical optimum design problems are characterized by mixed continuous-discrete variables, and discontinuous and non-convex design spaces. If the optimization problem involves the objective function and constraints that are not stated as explicit functions of the design variables or which are too complicated to manipulate, it is hard to solve by classical optimization methods. Therefore, some optimization methods such as genetic algorithm (GA) have been developed to solve complex optimization problems recently (Rao & Tiwari, 2007).

GA, which is one of the stochastic methods of optimization, has been commonly used for the optimal design machine systems. The main advantages of genetic algorithms (GAs) are an assured convergence without use of derivatives and functions with discrete and non-derivable variables (Marcelin, 2005). GA one of the optimization methods for solving complex optimization problems has been applied to many area including machine design. Rao and Tiwari (2007) applied GA to rolling element bearings design problem. Adeli and Cheng (1994) and Hasancebi and Erbatur (2000) applied GA in structural optimization. Marcelin (2001) applied GAs for optimum design of gears. Choi and Yoon (2001) used to optimize automotive wheel-bearing unit using GA. Their study has presented maximized system life of the wheel bearing. Periaux (2002) discussed in detail the application of GAs to aeronautics and turbomachinery. Chakraborthy et al. (2003) investigated design optimization problem for rolling element bearings with five design parameters using GAs. Marcelin (2004) proposed a different numerical approach based on genetic algorithms and some neural networks allowing optimization of gearbox.

In this study selection of optimum module, shaft diameter and rolling bearing for spur gear has been carried out using genetic algorithms.

Rest of this paper is organized as follows. Section 2 presents briefly description of GA. Section 3 presents formulation of problem. Section 4 presents results of optimization problem. In Section 5, conclusions of this paper are presented.

2. Genetic algorithm

GAs are global optimization methods based on the principles of natural selection and evolutionary theory (Goldberg, 1989; Holland, 1975). The algorithm is provided with a set of possible solutions (represented by *chromosomes*) termed a *population*. Solutions from one population are taken and used to form a new population. This is motivated by a hope that the new population will perform better than its predecessors. Solutions chosen to form new solutions (*offsprings*) are selected based on their *fitness* – the more suit-

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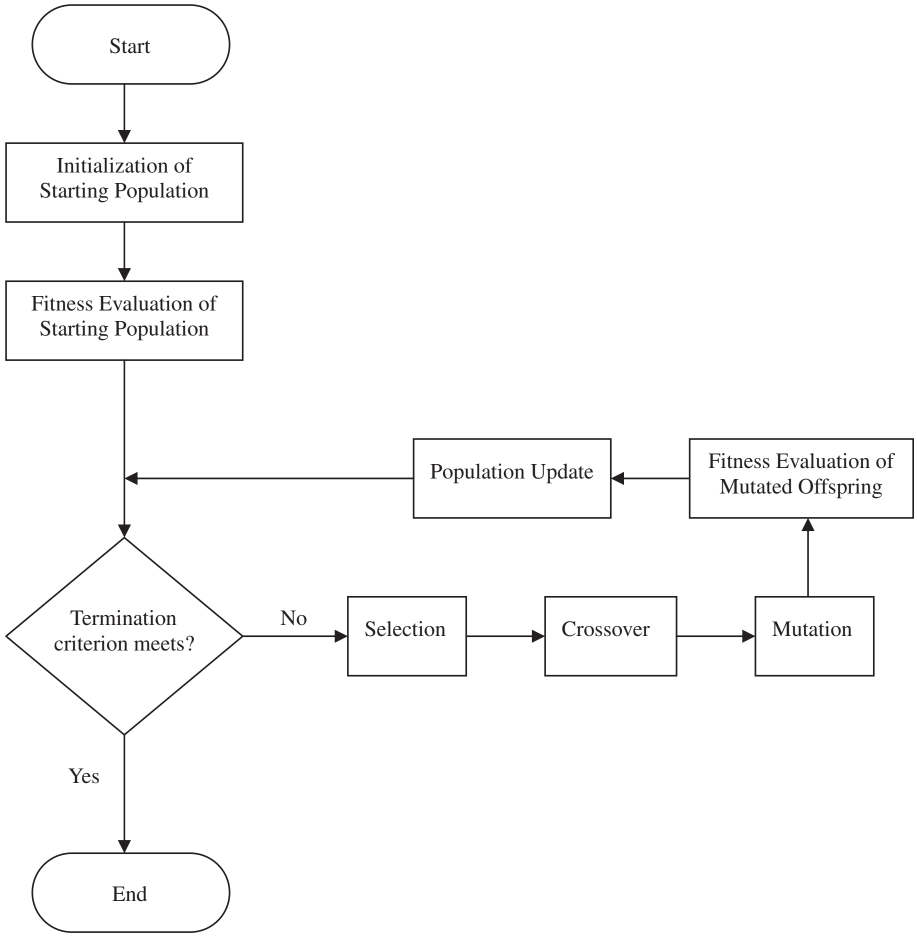

Fig. 1. Flowchart of GA for solving problem.

able they are, the better their chances of being reproduced. This process of selection is repeated till some predetermined condition based on, for instance, the number of populations or improvement of the best solution, is satisfied (Chakraborthy et al., 2003). Procedure for solving the discrete optimization problem mentioned using GA is illustrated in Fig. 1.

2.1. Genetic algorithm operators

Selection is the first genetic operator. Fitness function is evaluated for each individual in the population and at least two individuals with low fitness function are selected to form next generation. Crossover is the second genetic operator that allows producing offspring by recombining the chromosomes of two individuals (Table 1). Mutation is the third genetic operator allowing creating a new individual by changing 0 to 1 or changing 1 to 0. Each bit of each individual is possible subject to mutation (Table 2).

Table 1
Crossover operation.

Crossover	
Parents	
Parent 1	01110110 **00101001011111**
Parent 2	11100100 *10100011001010*
Offsprings	
Offspring 1	01110110*10100011001010*
Offspring 2	11100100**00101001011111**

Table 2
Mutation operation.

Mutation	
Individual gene before mutation	1110010**0**00101001011111
New individual gene after mutation	1110010**1**00101001011111

3. Model formulation

Optimization of module, shaft diameter and rolling bearing selection is carried out by using GA with help of a program developed on Matlab 7.0 platform. Optimum dimensions are obtained for the design of the gearbox. Factors (torque, material, width of tooth, tip speed and so on) having influence on gear module selection are given in Table 3.

3.1. Objective function

Objective function is expressed in Eq. (1) based on gear volume:

$$F_{\text{1-objective}} = F(m, Z, \Psi_m) = \left[\pi m \Psi m/4\left[(mZ)^2 - d_{\text{mil}}^2\right] + \pi m^3 \Psi mZ\right] \quad (1)$$

In Eq. (1), m represents module, Ψ_m, represents width ratio based on module, Z represents the number of teeth in the gear and d_{mil}, represents hole diameter of the gear and $d_{\text{mil}} = 10$ in the initial solution.

Table 3
Description of parameters and variables.

Parameters	Symbol	Unit	Value
Input power	P	Php	25
Gear revolution number	n	rev/min	600
Transmission ratio	i	–	600/375
Shaft torque	M	N mm	298,420
Ratio of clutch	ε	–	1.25
Circular speed	V	m/s	At searching space, changeable as to objective function minimization
Dynamic load factor	k_d	–	
Form factor	k_f	–	
Gear and shaft material resistance	σ_{gr}, σ_{sr}	N/mm^2	700, 450
Gear and shaft material life resistance	σ_D	N/mm^2	385
Notch coefficient	k_c	–	1.5
Safety coefficient	S	–	2
Elasticity module	E	N/mm^2	2.1×10^5
Hardness of material	HB	N/mm^2	2100
Safety surface pressure	P_{sef}	N/mm^2	525
Gear tangent force	F_t	N	5305.2
Gear radial force	F_r	N	1930.9
Shaft bending momentum	M_{max}	Nmm	168,550
Gearbox main dimensions	$b * h * l$	mm	$300 * 400 * 400$
Gearbox wall thickness	t	mm	10
Gear side distance	Y_1	mm	10
Shaft distance	l_1, l_2	mm	Variable for diameter of shaft
Confidence factor	a_1	–	1.00
Heat factor	f_t	–	0.90
Work duration	L_H	h	10,000
Bearing load	R_{Ad}, R_{Ay}	N	904, 2482
Bearing combine load	$F_{bearing}$	N	2643.7
Bearing load coefficient	X, Y	–	$X = 1$, $Y = 0$
Dynamic load capacity	C	N	Changeable dependent on bearing volume

The objective function for minimizing shaft volume is expressed in Eq. (2)

$$F_{2\text{-objective}} = F(d_{mil}, l_1, l_2) = \pi d_{mil}^2/4(2b + l_1 + B + l_2) \quad (2)$$

Here, d_{mil} represents shaft diameter, b represents shaft bush length (b = 15 mm), B represents gear width and l_1, l_2 represent shaft lengths, respectively (Fig. 2).

Eq. (3) expresses rolling bearing selection

$$F_{3\text{-objective}} = F(d, D, b_r) = [\pi b_r(D^2 - d^2)]/4 \quad (3)$$

Here, d represents bearing shaft diameter, D represents bearing external diameter and b_r represents bearing width.

3.2. Variables

The upper and lower limits of variables which appear in the objective function are given in Table 4 for gear elements.

3.3. Constraints

Constraints for tooth root fracture and deformation of the surface which are necessary for optimum module selection are formulated in Eqs. (4) and (5), respectively (Halowenken, 1993)

$$g_1 = [(2SMk_dk_f)/(Z\Psi_m\varepsilon m^3)] - \sigma_{gr}/k_c \leq 0 \quad (4)$$

$$g_2 = [(2SMk_dE(i+1))/(Z^2\Psi_m\varepsilon m^3 i)] - P_{sef}^2 \leq 0 \quad (5)$$

For spur gear design, minimum shaft diameter of the spur gear which can bear the torque where the system encounters is calculated based on the optimum shaft diameter using the constraints by Eqs. (6)–(8) based on variable load, shear stress and design (Halowenken, 1993)

$$g_3 = [(16S/\pi)[(M/(\sigma_{sr})]^2 + (M_{max}/(\sigma_D))^2]^{1/2} - d_{mil}^3 \leq 0 \quad (6)$$

$$g_4 = [(R_{Ad}^2 + R_{Ay}^2)^{1/2}]/[(\pi d_{mil}^2)/4] - [(\sigma_{sr}/S)0,5] \leq 0 \quad (7)$$

$$g_5 = d_{mil} - [l_1 + l_2] \leq 0 \quad (8)$$

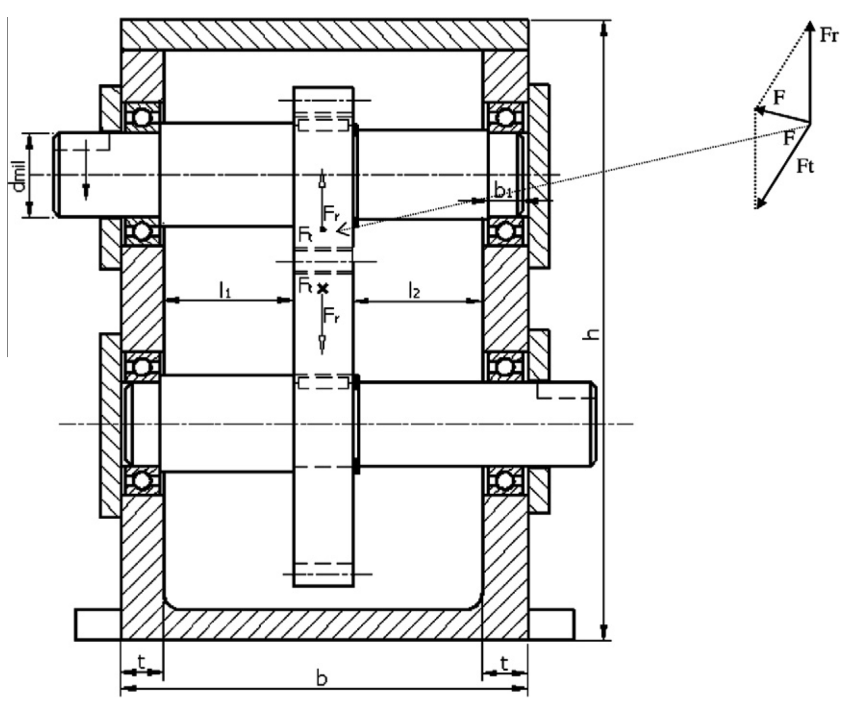

Fig. 2. Gearbox elements.

Table 4
Design variables.

	Design variables	Interval
m	Module, (mm)	$1 \leqslant m \leqslant 20$
Z	Number of teeth	$13 \leqslant Z \leqslant 30$
Ψ_m	Width ratio	$18 \leqslant \Psi_m \leqslant 23$
d_{mil}	Shaft diameter (mm)	$10 \leqslant d_{mil} \leqslant 100$
l_1	Shaft length (mm)	$5 \leqslant l_1 \leqslant 25$
l_2	Shaft length (mm)	$5 \leqslant l_2 \leqslant 25$
b_r	Bearing Width (mm)	$8 \leqslant B \leqslant 74$
d	Bearing shaft diameter (mm)	$10 \leqslant d \leqslant 300$
D	Bearing external diameter (mm)	$26 \leqslant D \leqslant 460$

The rolling bearing design constraints of gearbox are represented in Eqs. (9)–(12), respectively. Eq. (9) presents for bearing life, Eq. (10) represents constraint for life of the rolling bearing while Eq. (11) represents constraint for relation between bearing width and bearing external diameter. Eq. (12) represents constraint for relation between bearing shaft diameter and bearing external diameter

$$g_6 = [(60L_H n)/10^6] - [(a_1 C f_t)/(XF_{bearing} + YF_a)]^3 \leq 0 \tag{9}$$

$$g_7 = [(a_1 C f_t)/(XF_{bearing} + YF_a)]^3 - [(60L_H n)/10^6] \leq 0 \tag{10}$$

$$g_8 = b_r - (10^{-4}D^2 + 0.0927D + 7.4426) \leq 0 \tag{11}$$

$$g_9 = (1.4834d + 7.5351) - D \leq 0 \tag{12}$$

Dimensions of the gearbox used in transmitting of spur gear moment and teeth forces are illustrated in Fig. 2.

Table 5
Genetic algorithm parameters.

Genetic algorithm parameters		
Module selection parameters	Chromosome length	22
	Population size	41
	Number of generation	150
	Crossover probability	1.0
	Mutation rate	0.0001
Shaft diameter selection parameters	Chromosome length	36
	Population size	311
	Number of generation	100
	Crossover probability	1.0
	Mutation rate	0.0001
Rolling bearing selection parameters	Chromosome length	25
	Population size	63
	Number of generation	150
	Crossover probability	1.0
	Mutation rate	0.0001

3.4. Description of variables and genetic algorithm parameters

Amount of iteration and variables in a function are coded in bit patterns (Tables 5 and 6). Generally binary coded chromosomes having l's and 0's were used. Genetic operators could be used due to this coding. Eq. (13) was used to carry out coding (Gen & Cheng, 1997; Jang, Sun, & Mizutani, 1997)

$$2^l \geq \frac{x(i)_{max} - x(i)_{min}}{\varepsilon} + 1 \tag{13}$$

Here, l represents chromosome for all variables, $x(i)_{min}$ represents lower boundary of variable, $x(i)_{max}$ represents upper boundary of variable and ε represents variable step.

When the length of chromosome is determined, the chromosome is produced by the concatenation of l's and 0's. Genetic operators were used through binary code which is made up of the chromosome. Eq. (14) is used to decode after the algorithm is ended

$$(i) = x(i)_{min} + \frac{x(i)_{max} - x(i)_{min}}{2^l - 1} d(i) \tag{14}$$

In order to find out population size, Eq. (14) is suggested. l represented the chromosome length in the Eq. (14)

$$\text{Population size} = 1.65 * 2^{0.21*l} \tag{15}$$

The population size is equal to 41 for optimal gear module selection where l is equal to 22 in Eq. (15).

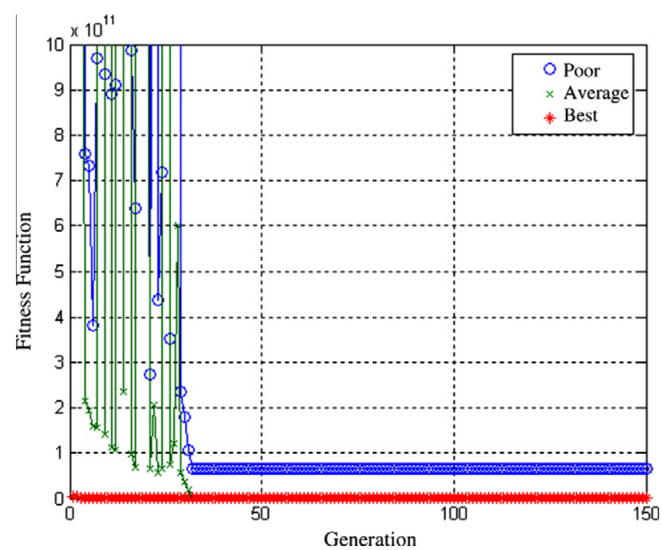

Fig. 3. Optimum fitness function.

Table 6
Coding of variables.

Module, m (ε = 0.1)	Number of teeth, Z (ε = 1)	Width ratio, Ψ_m (ε = 0.01)
01110 *(5 bit)*	11000101 *(8 bit)* 01110 + 11000101 + 001011111 0111011000101001011111 *(22 bit)*	001011111 *(9 bit)*
Shaft diameter, d_{mil} (ε = 0.01)	Shaft length, l_1 (ε = 0.01)	Shaft length, l_2 (ε = 0.01)
01101000101010 *(14 bit)*	00011100010 *(11 bit)* 01101000101010 + 00011100010 + 00011100010 011010001010100001110001000011100010 *(36 bit)*	00011100010 *(11 bit)*
Bearing width, b_r (ε = 1)	Bearing shaft diameter, d (ε = 1)	Bearing external diameter, D (ε = 1)
0101110 *(7 bit)*	001010111 *(9 bit)* 0101110 + 001010111 + 000100110 0101110001010111000100110 *(25 bit)*	000100110 *(9 bit)*

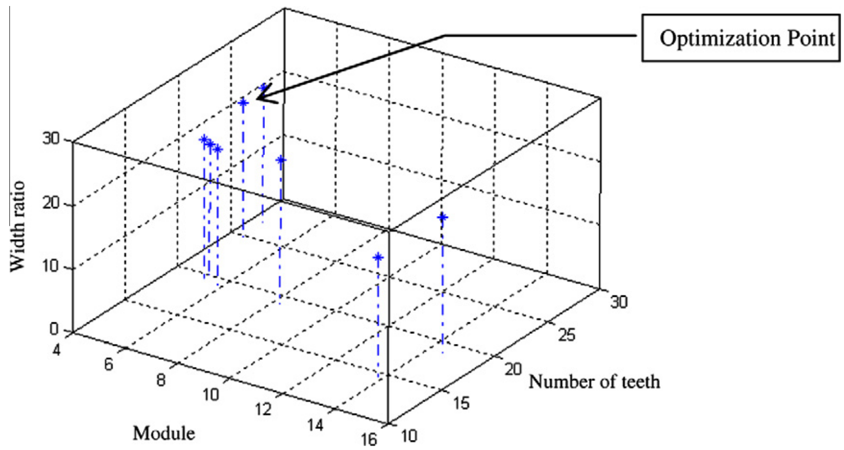

Fig. 4. Optimization results for module, number of teeth and width ratio.

The objective function was constrained by strength constraints in the solution space. These constraints enforced genetic search within the feasible region. Therefore, fitness function can be written as

$$F_{\text{fitness}} = F \pm (F_{\text{objective}}(m, Z, \Psi_m, l_1, l_2 d_{\text{mil}}, b_r, D, d) + P), \quad P = \sum_{j=1}^{KS} rj(\max[0, g_j])^2 \tag{16}$$

The fitness function is needed to be very much greater than the objective function as the fitness function is not equal to any negative value. In Eq. (16), P is penalty function, KS number of constraint and r is penalty coefficient.

Table 7
Optimization results of gearbox.

Function variables	Genetic algorithm
Number of teeth, Z	25
Module, m	4.5
Width ratio, Ψ_m	21.02
Circular speed, V (m/s)	3.53
Dynamic load factor, K_d	1.01
Form factor, K_f	2.74
Gear volume (mm^3)	1083327.189
Gearbox of width, b (mm)	139.1
Gearbox height, h (mm)	322.5
Gearbox breadth, l (mm)	210

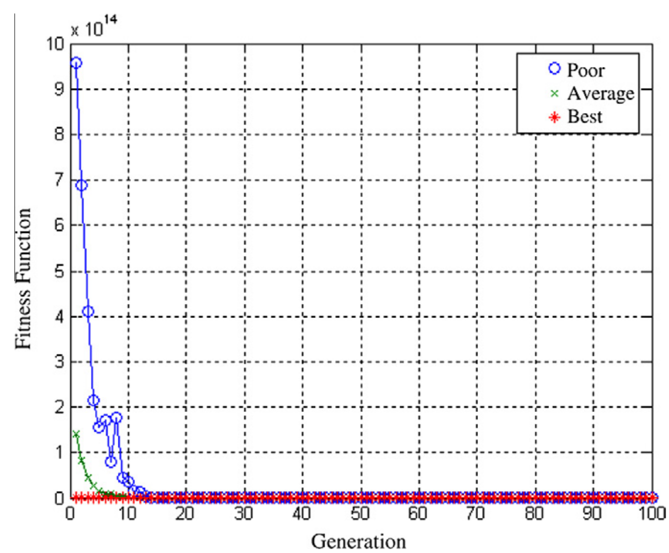

Fig. 5. Optimum fitness function for shaft diameter.

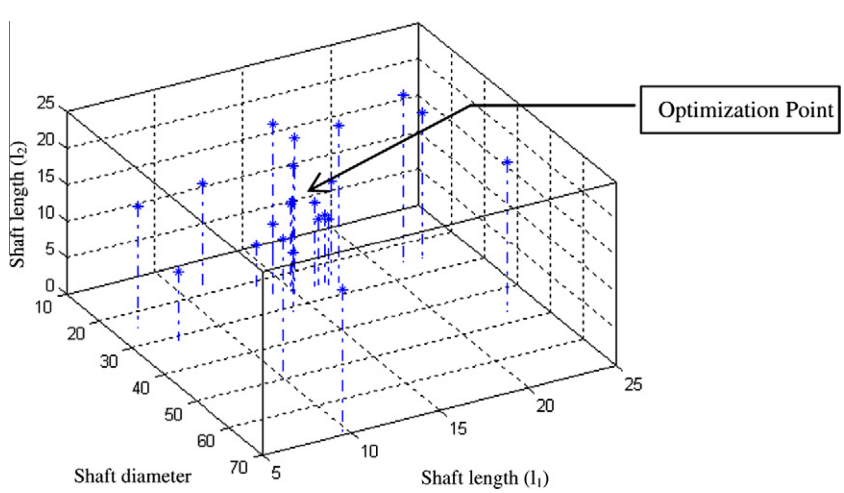

Fig. 6. Optimization results for shaft diameter, shaft length (l_1) and shaft length (l_2).

Table 8
Optimization results of shaft diameter for gearbox.

Function variables	Genetic algorithm
Shaft diameter, d_{mil} (mm)	25.30
Shaft length, l_1 (mm)	16.46
Shaft length, l_2 (mm)	8.85
Optimum shaft volume for gearbox (mm^3)	75359.124

Table 9
Optimization results of deep groove ball bearing.

Function variables	Genetic algorithm	SKF standard dimension
Bearing external diameter, D (mm)	59	72
Bearing shaft diameter, d (mm)	34	35
Bearing width, b_r (mm)	13	17
Gearbox optimum bearing volume (mm^3)	23,739 (C = 25,500 N, SKF 6007)	

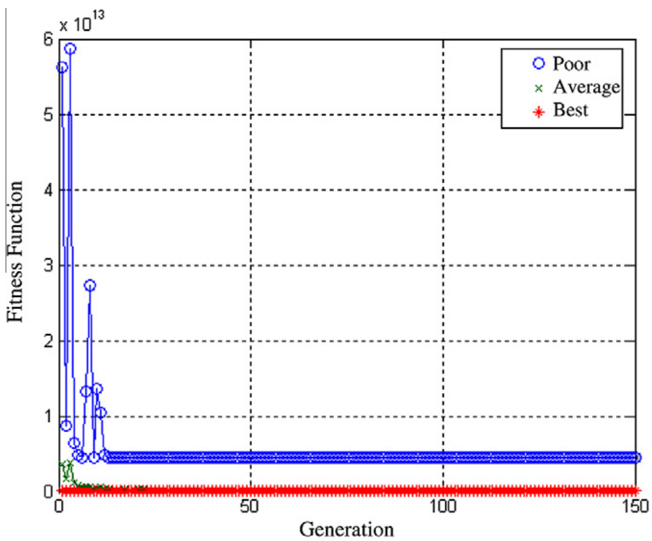

Fig. 7. Optimum fitness function for bearing selection.

4. Results of genetic algorithm

4.1. Results for optimization of gear volume

In order to end algorithm, all chromosomes in the population were converged to the best chromosome. Optimum fitness function and gear variables are illustrated in Figs. 3 and 4, respectively.

In Fig. 3, the best, average and poor curves are illustrated in the search space. The fitness function is optimized when the average curve is converged to the best curve after thirty-fifth generation. Optimum variables are obtained in Table 7.

4.2. Results for optimization of shaft volume

The optimum fitness function and variables of shaft diameter are illustrated in Figs. 5 and 6, respectively. In Fig. 5, the best, average and poor curves are illustrated in the search space. The fitness function is optimized when the average curve is converged to the best curve after tenth generation. The optimization results are obtained in Table 8.

4.3. Results for optimization of deep groove ball bearing volume

The optimum fitness function and variables of shaft diameter are illustrated in Figs. 7 and 8, respectively. In Fig. 7, the best, average and poor curves are illustrated in the search space. The fitness function is optimized when the average curve is converged to the best curve after 23rd generation. Optimization results for deep groove ball bearing volume are obtained in Table 9.

5. Analytical method

To facilitate the evaluation of genetic algorithm, a program was been developed on Borland Delphi 6.0 platform. Solution space was constructed by all combinations of variables in the program. Selection parameters belong to gear and the obtained results are illustrated in Fig. 9. Solution space scanning interval in this window was selected in a way that it was same with the genetic algorithm variable increasing intervals. The results obtained by the analytic method (AM) and GA for gear volume are given in Table 10.

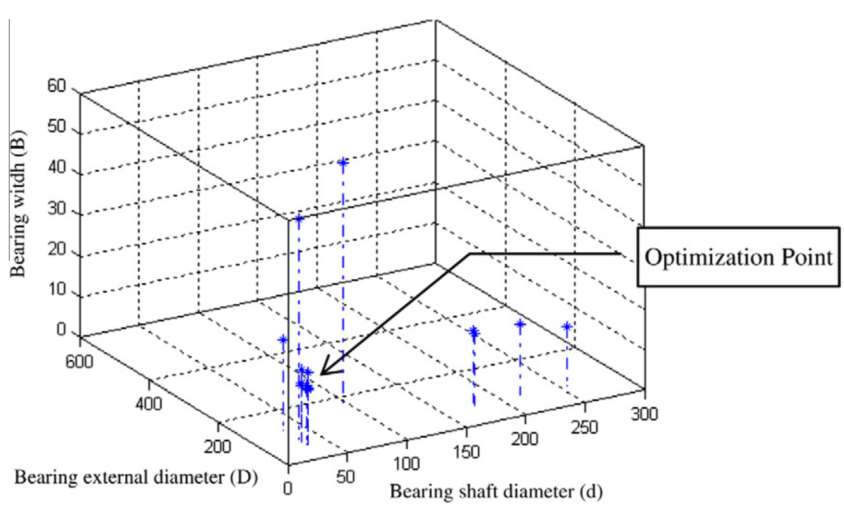

Fig. 8. Optimization results for bearing shaft diameter, bearing external diameter and bearing witdh.

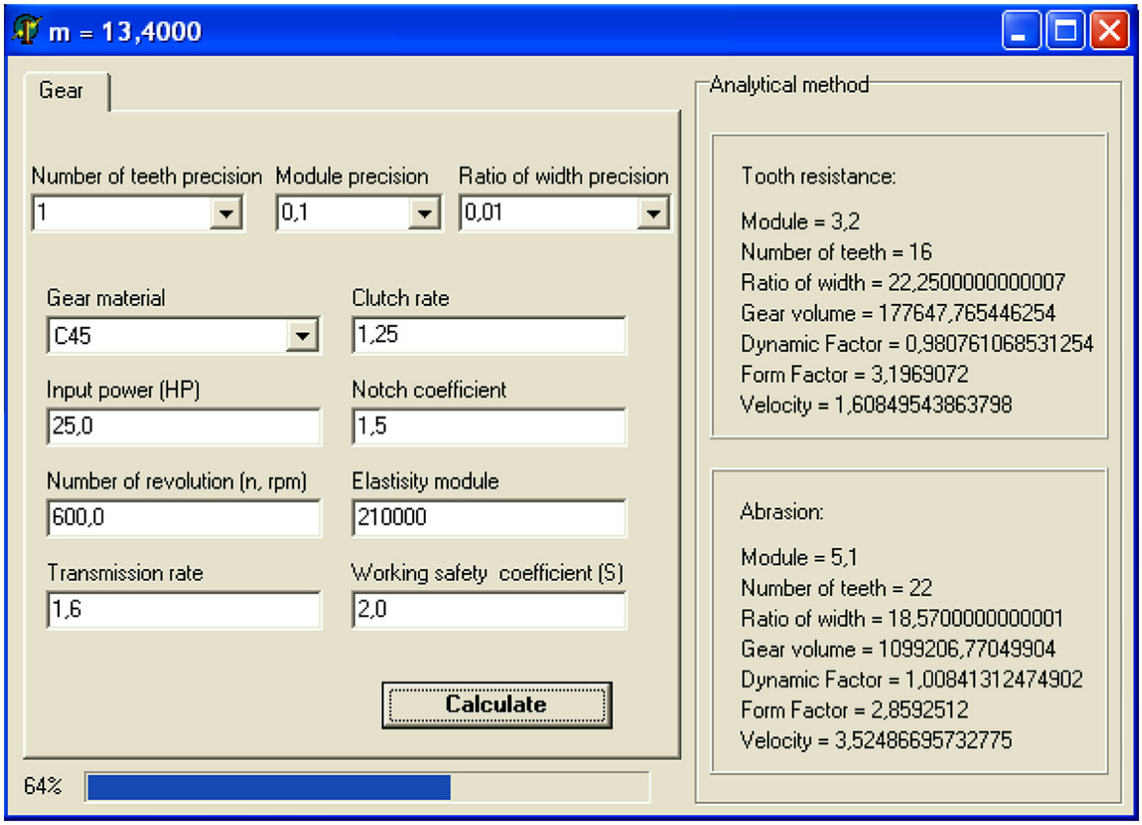

Fig. 9. Results obtained by analytical method for gearbox elements.

Table 10
Comparison of results.

Gear variables	GA	AM
Module, m (mm)	4.5	5.1
Number of teeth, Z	25	22
Width ratio, Ψ_m (mm)	21.02	18.57
Gear volume (dm^3)	1.083	1.099

6. Conclusions

- Minimization of gear, shaft and rolling bearing volumes was accomplished using GA. Gear volume was found to be equal to 1099 dm^3 using AM. Gear volume was equal to 1083 dm^3 using GA. These results showed that GA is a better method than AM to obtain minimum gear volume.
- Module was found to be equal to 4.5 mm and 5.1 mm through GA approach and AM, respectively. The number of teeth was found to be equal to 25 and 22 through GA approach and AM, respectively. Width ratio was found to be equal to 21.02 and 18.57 through GA approach and AM, respectively. The gear volume obtained by GA was 1.47% lower than the gear volume obtained by AM .The aim of the optimization was minimization of the gear volume. Therefore, GA provided better results than AM.
- In the light of this study, it was clearly seen that GA is more efficient approach for the optimization of complex systems such as the gearbox made up of various elements. GA is capable of providing better result in short time when compared with other optimization methods.

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