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American National Standard

Tooth Thickness Specification and Measurement

ANSI/AGMA 2002-B88

Tooth Thickness Specification and Measurement
ANSI/AGMA 2002-B88
(Revision of AGMA 231.52-1975)

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ABSTRACT

This Standard establishes the procedures for determining tooth thickness measurements of external and internal cylindrical involute gearing. It includes equations and calculation procedures for the commonly used measuring methods. A specific tooth thickness measurement limit can be established from the design thickness or from another tooth thickness measurement. The procedures can be entered with an established design tooth thickness, or with actual tooth thickness measurements. The effect of tooth geometric quality variations on tooth thickness measurements is discussed. Backlash information is provided in an appendix.

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FOREWORD

[This foreword, footnotes, and appendices, if any, are provided for informational purposes only and should not be construed as part of ANSI/AGMA 2002–B88, *Tooth Thickness Specification and Measurement.*]

This Standard presents calculation procedures for determining tooth thickness measurements of external and internal cylindrical involute gearing. It supersedes AGMA 231.52, *Inspection – Pin Measurement Tables for Involute Spur Gears.*

This Standard has been prepared to consolidate previously published AGMA tooth thickness information, to add more information on internal and helical gears and to add details on more measurement methods.

Previous AGMA publications have presented this information in tabular form, calculated for 1 DP and standard tooth proportions, with adjustment factors for nonstandard conditions. This Standard is arranged for direct calculation of the desired results, to eliminate the intermediate calculation steps and interpolation previously required.

The study of tooth thickness and backlash problems has been a major interest of gear technicians throughout the history of the industry. In the last fifty years, many clarifications and contributions have been made by men such as Buckingham, Candee, Leming, Vogel, and Wildhaber. Their work is consolidated here, without further attribution, and the work of more recent contributors is added where it improves the presentation.

The appendices provide further information on reasonable allowances for backlash and tooth thickness deviation, sample calculations, and information on four uncommon methods of measurement specified on some gear drawings.

The treatment of the effects of tooth profile, pitch, lead, and runout deviations on tooth thickness measurement is new in this Standard.

The information on backlash control is new in an AGMA Standard. It is based on AGMA Paper P239.14, *Assured Backlash Control – The ABC System.*[1]

The first draft of this revision was made in February 1984.

This version was approved by the AGMA membership on October 9, 1988 and as an American National Standard on October 17, 1988.

Suggestions for the improvement of this Standard will be welcome. They should be sent to the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia, 22314.

ERRATA July, 1992

The following editorial corrections have been made to ANSI/AGMA 2002–B88, *Tooth Thickness Specification and Measurement*, (originally printed October 1988). These changes, discovered after publication, have been made in the second standard printing, as shown below:

PAGE	ITEM	CHANGE
10	Fig 3–1	The position of minimum and maximum backlash is shown on the specified circle, also 1/2 specified tolerance and 1/2 specification bands labeled correctly.
26	Fig 3–1	The angle Ψ_b and the assumed form diameter, $D_o - 4a$, indicated correctly.
29	Eq 8.2	The right hand bracket should be at the end, with the full equation reading, $$f_3 = \text{arc inv}\left[\frac{P_{nd}(t_1 + t_2) - p}{N_1 + N_2} + \text{inv} f_c\right] \quad \text{(Eq 8.2)}$$
32	Table A–1	The last value in the table , for 64 inch center, should read 0.058.

ERRATA June, 1995 (Additional correction made in this printing).

PAGE	ITEM	CHANGE
29	Eq 8.2	Changed to transverse plane. $$\phi_3 = \text{arc inv}\left[\frac{P_d(t_1 + t_2) - \pi}{N_1 + N_2} + \text{inv}\phi_s\right] \quad \text{(Eq. 8.2)}$$

[1] Numbers in brackets refer to the bibliography.

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1. Scope

This Standard establishes the calculation procedures for determining tooth thickness measurements of external and internal cylindrical involute gearing.

The information is intended for use by the gear specifier or manufacturer in establishing values for tooth thickness measurement limits.

> CAUTION: It is important that tooth thickness measurement limits be reasonable for the specified quality class of the gears, to permit economical manufacture. This Standard provides guidance in the selection of reasonable tooth thickness measurement limits.

The designed tooth thickness is established from engineering considerations. It is determined by gear geometry, gear tooth strength, and backlash. The methods for establishing designed tooth thickness for a given application are beyond the scope of this Standard.

This Standard assumes the designed tooth thickness is known in cases where the values for various measuring techniques are to be established.

It includes equations and procedures for the following measuring methods:

(1) Chordal
(2) Pins (wires, rolls and balls)
(3) Span
(4) Composite Action Test

This Standard also establishes methods of determining tooth thickness of a gear based upon measurement limits by means of pins, span, chordal thickness or composite action test. These methods are often used to convert a tooth thickness specified by one method, such as *over pins* to another more convenient method, such as *span over X teeth*.

> CAUTION: The effect of tooth geometry variations on tooth thickness measurements made by different measuring methods may be significant. This must be considered if close control of backlash is required. When this is necessary the tooth thickness should be measured by the method specified on the drawing. Refer to 3.8 for additional discussion of the problem.

Examples included are for coarse pitch gears. The same mathematical principles apply to gear teeth of all sizes. For information on fine pitch gears, see AGMA 370.01, *Design Manual for Fine Pitch Gears*.

This Standard does not contain tolerances on tooth thickness. See AGMA 2000–A88, *Gear Classification and Inspection Handbook – Tolerances and Measuring Methods for Unassembled Spur and Helical Gears (Including Metric Equivalents)*, for tolerances.

AGMA 115.01, *Reference Information – Basic Gear Geometry* is a source for the derivations and detailed explanations of the geometrical relationships used here.

AGMA 112.05, *Gear Nomenclature (Geometry) Terms, Definitions, Symbols and Abbreviations* is a source of definitions of common gear terms as used in this Standard.

2. Symbols, Terminology and Definitions

2.1 Symbols and Terminology. Symbols and terminology used in this Standard are shown in Table 2–1 and Table 2–2.

> NOTE: The symbols, terminology, and definitions used in this Standard may differ from other AGMA standards. The user should not assume that familiar symbols can be used without a careful study of these definitions.

SI (Metric) units of measure are shown in parentheses in Table 2–1, Table 2–2 and in the text. Where equations require a different format or constant for use with SI units, a second expression is shown after the first, indented, in smaller type, and with "M" included in the equation number.

Example:

$$p_x = \frac{\pi}{P_{nd} \sin \psi_s} \qquad \text{(Eq 4.4)}$$

$$p_x = \frac{\pi m_n}{\sin \psi_s} \qquad \text{(Eq 4.4M)}$$

2.2 Definitions. The terms used, wherever applicable, conform to the following standards:

ANSI Y10.3 – 1968, *Letter Symbols for Quantities Used in Mechanics of Solids*

AGMA 112.05, *Gear Nomenclature, Terms, Definitions, Symbols, and Abbreviations*

AGMA 600.01, *Standard for Metric Usage*

Table 2-1
Alphabetical Table of Symbols and Terms, by Symbols

Symbol	Terms	Units	Where First Used
a	Addendum	in (mm)	Eq 5.1
a_c	Chordal Addendum	in (mm)	Eq 5.7
Δa_c	Correction to Chordal Addendum	in (mm)	5.3.1
B	Backlash (Transverse Operating)	in (mm)	4.3
B_{min}	Minimum Transverse Backlash	in (mm)	3.1.4
B_f	Normal Backlash (feeler gage)	in (mm)	4.3
B_t	Circular Transverse Backlash	in (mm)	Eq 4.7
C	Tightest Center Distance	in (mm)	3.1.4
C_{max}	Maximum Center Distance	in (mm)	Eq 8.3
C_{min}	Minimum Center Distance	in (mm)	Eq 8.5
D	Specified Diameter	in (mm)	4.1
D'	Operating Pitch Diameter	in (mm)	3.7
D_b	Base Circle	in (mm)	3.7
D_{b1}	Base Circle Diameter of Test Gear	in (mm)	8.4.1
D_{b2}	Base Circle Diameter of Master Gear	in (mm)	8.4.1
D_i	Tip Diameter of Internal Gear	in (mm)	6.4
D_o	Outside Diameter	in (mm)	6.4
$D_{o\,max}$	Maximum Outside Diameter	in (mm)	5.3
D_s	Standard (Generating) Pitch Diameter	in (mm)	Eq 4.6
D_W	Contact Diameter for Best Pin Size	in (mm)	Eq 6.1
$D_{2\,W}$	Diameter over/between Two Pins	in (mm)	6.5
F	Facewidth	in (mm)	7.3
L	Lead	in (mm)	4.1
L_{best}	Best Length of Base Tangent	in (mm)	Eq 7.7
L_{max}	Maximum Length of Base Tangent	in (mm)	Eq 7.3
L_{min}	Minimum Length of Base Tangent	in (mm)	Eq 7.1
M	Span Measurement	in (mm)	Eq 7.10
M_m	Span Measurement, Modified for Tooth Variation	in (mm)	Eq 7.13
m_n	Normal Module	mm	Eq 3.6M
N	Number of Teeth in Gear	--	3.1.4
N_1	Number of Teeth in Test Gear	--	8.4.1
N_2	Number of Teeth in Master Gear	--	8.4.1
n	Number of Teeth in Pinion	--	3.1.4
P_{nd}	Normal Diametral Pitch	in^{-1}	4.1
p'	Operating Transverse Circular Pitch	in (mm)	Eq 3.2
p_x	Axial Pitch	in (mm)	4.1
p_b	Transverse Base Pitch	in (mm)	6.5.1

Table 2-1 (cont)
Alphabetical Table of Symbols and Terms, by Symbols

Symbol	Terms	Units	Where First Used
p_N	Normal Base Pitch	in (mm)	7.3
R_{max}	Maximum Measuring Radius	in (mm)	5.3
R_m	Master Gear Test Radius	in (mm)	8.4.1
R_{1W}	Radius over/to One Pin	in (mm)	Eq 6.11
$R_{T\max}$	Maximum Test Radius (work gear)	in (mm)	Eq 8.4
$R_{T\min}$	Minimum Test Radius (work gear)	in (mm)	Eq 8.6
S	Number of Teeth to be Spanned	--	7.1
S_{best}	Best Number of Teeth to be Spanned	--	Eq 7.8
S_{max}	Maximum Number of Teeth to be Spanned	--	Eq 7.4
S_{min}	Minimum Number of Teeth to be Spanned	--	Eq 7.2
s_W	Transverse Space Width at Best Pin Contact Diameter	in (mm)	Eq 6.4
t	Circular Tooth Thickness	in (mm)	2.2
t_1	Transverse Tooth Thickness of the Test Gear at ϕ_c	in (mm)	8.4.1
t_2	Transverse Tooth Thickness of the Master Gear at ϕ_c	in (mm)	8.4.1
t_b	Transverse Base Tooth Thickness	in (mm)	Eq 4.11
t_{b1}	Transverse Base Tooth Thickness of Test Gear	in (mm)	8.4.1
t_{b2}	Transverse Base Tooth Thickness of Master Gear	in (mm)	8.4.1
t_{bm}	Transverse Base Tooth Thickness, Modified for Runout and Pitch Variation	in (mm)	Eq 7.12
t_m	Measured Transverse Normal Chordal Tooth Thickness	in (mm)	Eq 5.10
t_{max}	Maximum Transverse Tooth Thickness at Operating Pitch Diameter	in (mm)	3.7
$t_{G\max}$	Maximum Transverse Tooth Thickness of Gear	in (mm)	Eq 3.1
$t_{P\max}$	Maximum Transverse Tooth Thickness of Pinion	in (mm)	Eq 3.1
t_{min}	Minimum Specified Transverse Tooth Thickness	in (mm)	Eq 3.3
t_n	Normal Tooth Thickness	in (mm)	Eq 4.1
t_{nb}	Normal Base Tooth Thickness	in (mm)	Eq 7.6
t_{nR}	Normal Tooth Thickness at R_{max}	in (mm)	Eq 5.3
t_R	Transverse Tooth Thickness at R_{max}	in (mm)	Eq 5.5
t_t	Transverse Tooth Thickness, Circular	in (mm)	4.1
t_{ts}	Maximum Transverse Generated Tooth Thickness	in (mm)	Eq 4.8
t_W	Transverse Tooth Thickness at Best Pin Contact Diameter	in (mm)	Eq 6.3
t_T	Transverse Tooth Thickness Tolerance	in (mm)	3.7
V_{apk}	Accumulated Pitch Variation, Sector of k Pitches	in (mm)	7.3
V_{cq}	Total Composite Variation	in (mm)	3.7
V_r	Radial Runout of Reference Diameter	in (mm)	5.3
V_{rT}	Radial Runout Tolerance	in (mm)	6.5.5

Table 2-1 (cont)
Alphabetical Table of Symbols and Terms, by Symbols

Symbol	Terms	Units	Where First Used
$V_{r\,W}$	Correction to Pin Measurement for Runout	in (mm)	Eq 6.20
W	Pin Diameter in the Calculation	in (mm)	6.5.1
W_{best}	Best Pin Size	in (mm)	Eq 6.6
W'_{best}	Best Pin Size–Transverse Plane	in (mm)	Eq 6.5
ϕ	Transverse Pressure Angle	--	6.1
ϕ'	Transverse Operating Pressure Angle	--	Eq 3.4
ϕ_c	Normal Profile Angle of the Equivalent Standard Rack Cutter	--	3.7
ϕ_m	Transverse Pressure Angle at Measuring Diameter	--	7.3
ϕ_s	Transverse Generating Pressure Angle	--	4.5
ϕ_W	Transverse Pressure Angle at Best Pin Diameter	--	Eq 6.2
ϕ_2	Pressure Angle at Center of Pin	--	6.5.1
ϕ_3	Operating Transverse Pressure Angle in Tight Mesh	--	Eq 8.1
ψ	Helix Angle at a Specified Diameter	--	Eq 4.1
ψ_b	Base Helix Angle	--	Eq 3.7
ψ_R	Helix Angle at Measuring Radius R	--	5.3
ψ_s	Helix Angle at Standard Pitch Diameter	--	Eq 4.5
τ	Normal Angular Thickness	--	5.3.1

The following definitions are specifically used in this Standard. The user should be familiar with these definitions and symbols before applying this information.

Backlash, B**.** Backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the operating pitch circles (see Fig 2-1). As actually indicated by measuring devices, backlash may be determined variously in the transverse, normal, or axial planes, and either in the direction of the pitch circles, or on the line of action. Such measurements should be converted to corresponding values in the transverse plane at the operating pitch circle for general comparisons. If not otherwise identified, values for backlash refer to transverse operating backlash.

Backlash, Minimum, B_{min}**.** Minimum backlash is the minimum transverse backlash at the operating pitch circle allowable when the gear tooth with the greatest allowable functional tooth thickness is in mesh with the pinion tooth having its greatest allowable functional tooth thickness, at the tightest allowable center distance, under static conditions.

Standard Pitch Circle. A circle defined by the number of teeth and a specified module or circular pitch. (Reference AGMA 112.05)

Tooth Thickness. Tooth Thickness is the thickness of a gear tooth at a specified diameter. Unless otherwise defined it is taken as the transverse circular tooth thickness (see Fig 2-2).

Tooth Thickness, Chordal, Normal. The normal chordal tooth thickness is the length of the chord subtending a tooth thickness arc in the normal plane.

Tooth Thickness, Circular. The circular tooth thickness is the length of arc between two sides of a gear tooth, on a specified diameter.

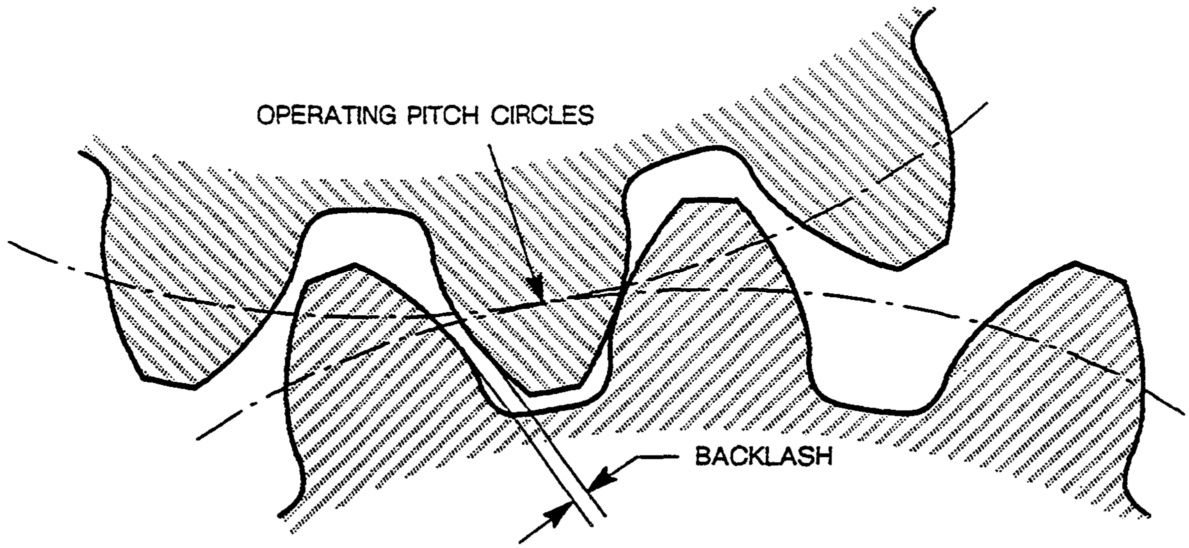


Fig 2-1 Backlash

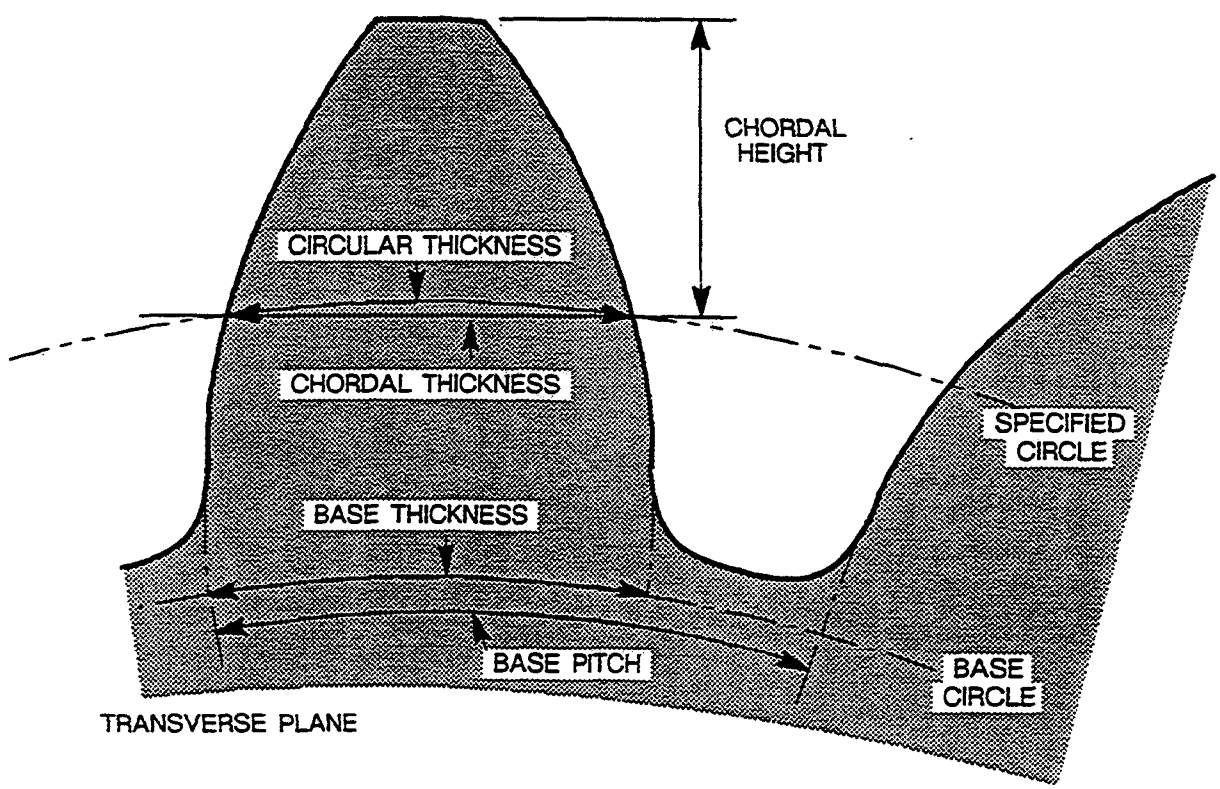


Fig 2-2 Circular Tooth Thickness

Table 2-2
Alphabetical Table of Terms and Symbols, by Terms

Terms	Symbol	Units	Where First Used
Addendum	a	in (mm)	Eq 5.1
Backlash (Transverse Operating)	B	in (mm)	4.3
Backlash, Normal (feeler gage)	B_f	in (mm)	4.3
Backlash, Transverse, Circular	B_t	in (mm)	Eq 4.7
Backlash, Transverse, Minimum	B_{min}	in (mm)	3.1.4
Base Circle	D_b	in (mm)	3.7
Base Circle Diameter of Master Gear	D_{b2}	in (mm)	8.4.1
Base Circle Diameter of Test Gear	D_{b1}	in (mm)	8.4.1
Base Tangent, Best Length of	L_{best}	in (mm)	Eq 7.7
Base Tangent, Maximum Length of	L_{max}	in (mm)	Eq 7.3
Base Tangent, Minimum Length of	L_{min}	in (mm)	Eq 7.1
Center Distance, Maximum	C_{max}	in (mm)	Eq 8.3
Center Distance, Minimum	C_{min}	in (mm)	Eq 8.5
Center Distance, Tightest	C	in (mm)	3.1.4
Chordal Addendum	a_c	in (mm)	Eq 5.7
Chordal Addendum Correction Factor	Δa_c	in (mm)	5.3.1
Diameter, Contact, for Best Pin Size	D_W	in (mm)	Eq 6.1
Diameter, Maximum Outside	$D_{o\,max}$	in (mm)	5.3
Diameter, Outside	D_o	in (mm)	6.4
Diameter over/between Two Pins	D_{2W}	in (mm)	6.5
Diameter, Specified	D	in (mm)	4.1
Diameter, Tip of Internal Gear	D_i	in (mm)	6.4
Face width	F	in (mm)	7.3
Helix Angle at Measuring Radius R_{max}	ψ_R	--	5.3
Helix Angle at Specified Diameter	ψ	--	Eq 4.1
Helix Angle at Standard Pitch Diameter	ψ_s	--	Eq 4.5
Helix Angle, Base	ψ_b	--	Eq 3.7
Lead	L	in (mm)	4.1
Normal Module	m_n	mm	Eq 3.6M
Number of Teeth in Gear	N	--	3.1.4
Number of Teeth in Master Gear	N_2	--	8.4.1
Number of Teeth in Pinion	n	--	3.1.4
Number of Teeth in Test Gear	N_1	--	8.4.1
Number of Teeth to be Spanned	S	--	7.1
Number of Teeth to be Spanned, Best	S_{best}	--	Eq 7.8
Number of Teeth to be Spanned, Maximum	S_{max}	--	Eq 7.4

Table 2-2 (cont)
Alphabetical Table of Terms and Symbols, by Terms

Terms	Symbol	Units	Where First Used
Number of Teeth to be Spanned, Minimum	S_{min}	--	Eq 7.2
Pin Diameter in the Calculation	W	in (mm)	6.5.1
Pin Measurement Correction for Runout	V_{rW}	in (mm)	Eq 6.20
Pin Size, Best	W_{best}	in (mm)	Eq 6.6
Pin Size, Best - Transverse Plane	W'_{best}	in (mm)	Eq 6.5
Pitch, Axial	p_x	in (mm)	4.1
Pitch, Base, Normal	p_N	in (mm)	7.3
Pitch, Base, Transverse	p_b	in (mm)	6.5.1
Pitch Diameter, Operating	D'	in (mm)	3.7
Pitch Diameter, Standard (Generating)	D_s	in (mm)	Eq 4.6
Pitch, Normal, Diametral	P_{nd}	in^{-1}	4.1
Pitch, Operating Transverse Circular	p'	in (mm)	Eq 3.2
Pitch Variation, Accumulated, Sector of *k* Pitches	V_{apk}	in (mm)	7.3
Pressure Angle at Center of Pin	ϕ_2	--	6.5.1
Pressure Angle, Transverse, Operating in Tight Mesh	ϕ_3	--	Eq 8.1
Pressure Angle, Transverse	ϕ	--	6.1
Pressure Angle, Transverse at Best Pin Diameter	ϕ_W	--	Eq 6.2
Pressure Angle, Transverse at Measuring Diameter	ϕ_m	--	7.3
Pressure Angle, Transverse Operating	ϕ'	--	Eq 3.4
Pressure Angle, Transverse Generating	ϕ_s	--	4.5
Profile Angle, Normal, of the Equivalent Standard Rack Cutter	ϕ_c	--	3.7
Radial Runout of Reference Diameter	V_r	in (mm)	5.3
Radial Runout Tolerance	V_{rT}	in (mm)	6.5.5
Radius, Maximum Measuring	R_{max}	in (mm)	5.3
Radius over/to One Pin	R_{1W}	in (mm)	Eq 6.11
Space Width, Transverse at Best Pin Contact Diameter	s_W	in (mm)	Eq 6.4
Span Measurement	M	in (mm)	Eq 7.10
Span Measurement, Modified for Tooth Variation	M_m	in (mm)	Eq 7.13
Test Radius, Master Gear	R_m	in (mm)	8.4.1
Test Radius, Maximum (work gear)	R_{Tmax}	in (mm)	Eq 8.4
Test Radius, Minimum (work gear)	R_{Tmin}	in (mm)	Eq 8.6
Transverse Tooth Thickness, Angular	τ	degrees	5.3.1
Tooth Thickness, Transverse Base, of Master Gear	t_{b2}	in (mm)	8.4.1
Tooth Thickness, Transverse Base, Modified for Runout and Pitch Variation	t_{bm}	in (mm)	Eq 7.12
Tooth Thickness, Transverse Base, of Test Gear	t_{b1}	in (mm)	8.4.1

Table 2-2 (cont)
Alphabetical Table of Terms and Symbols, by Terms

Terms	Symbol	Units	Where First Used
Tooth Thickness, Base, Transverse	t_b	in (mm)	Eq 4.11
Tooth Thickness, Circular	t	in (mm)	2.2
Tooth Thickness, Normal	t_n	in(mm)	Eq 4.1
Tooth Thickness, Normal, at R_{max}	t_{nR}	in (mm)	Eq 5.3
Tooth Thickness, Normal Base	t_{nb}	in (mm)	Eq 7.6
Tooth Thickness, Normal Chordal Measured	t_m	in (mm)	Eq 5.10
Tooth Thickness, Transverse, of the Test Gear at ϕ_c	t_1	in (mm)	8.4.1
Tooth Thickness, Transverse, of the Master Gear at ϕ_c	t_2	in (mm)	8.4.1
Tooth Thickness, Transverse, Specified Minimum	t_{min}	in (mm)	Eq 3.3
Tooth Thickness, Transverse, Tolerance	t_T	in (mm)	3.7
Tooth Thickness, Transverse	t_t	in (mm)	4.1
Tooth Thickness, Transverse, at Best Pin Contact Diameter	t_W	in (mm)	Eq 6.3
Tooth Thickness, Transverse, at R_{max}	t_R	in (mm)	Eq 5.5
Tooth Thickness, Transverse Maximum, of Gear	$t_{G\max}$	in (mm)	Eq 3.1
Tooth Thickness, Transverse Maximum Generated	t_{ts}	in (mm)	Eq 4.8
Tooth Thickness, Transverse Maximum, at Operating Pitch Diameter	t_{max}	in (mm)	3.7
Tooth Thickness, Transverse Maximum, of Pinion	$t_{P\max}$	in (mm)	Eq 3.1
Total Composite Variation	V_{cq}	in (mm)	3.7

Tooth Thickness, Design. Design tooth thickness is the thickness established from engineering consideration of strength, deflection, mounting and backlash upon the theoretical tooth thickness.

Tooth Thickness, Effective. The effective tooth thickness is the apparent circular thickness at the operating pitch diameter with a mate, established by the mounting (See 3.1.3).

Tooth Thickness, Functional. The tooth thickness as determined by meshing with a specified gear on a calibrated composite action test fixture.

Tooth Thickness, Measured. The measured tooth thickness is the actual value of circular tooth thickness calculated from a specific measurement over pins, a span or tooth caliper measurement.

Tooth Thickness, Normal, t_n. The circular tooth thickness in a normal plane.

Tooth Thickness, Tolerance, t_T. The permissible amount of tooth thickness variation.

Tooth Thickness, Transverse, t_t. The circular tooth thickness in a transverse plane.

Tooth Thickness, Variation. The variation from a specified value of normal circular tooth thickness.

Total Accumulated Pitch Variation, V_{apk}. Total accumulated pitch variation is equal to the algebraic difference between the maximum and minimum values obtained from the summation of successive values of pitch variation, V_p, and is the same as total index variation.

Total Composite Variation, V_{cq}. The total change in center distance while the gear being tested is rotated one complete revolution during double flank composite action test.

3. Application

3.1 Tooth Thickness Concepts. Various concepts dealing with tooth thickness are discussed within this Standard.

(1) Design Tooth Thickness
(2) Measured Tooth Thickness
(3) Effective Tooth Thickness

3.1.1 Design Tooth Thickness. Design tooth thickness is usually established from engineering considerations of gear geometry, gear tooth strength, mounting and consideration of backlash. The methods for establishing design tooth thickness for given applications are beyond the scope of this Standard.

This Standard assumes the design tooth thickness is known and the values for various measuring techniques are to be established.

3.1.2 Measured Tooth Thickness. The measured tooth thickness is used to evaluate the size of an entire tooth or all of the teeth on a given gear. It can be based on a few measurements between two points or two very short contact lines. The nature and the location of these contacts is determined by the type of measurement (pins, span, or tooth caliper). It is customary to assume that the entire gear is characterized by the measured data from as few as one or two measurements.

Depending upon the method of measurement, variations in tooth alignment, profile, and pitch will affect the measured values to varying degrees. The effects of these variations on the measured values may either be additive or may cancel one another, depending on the magnitude of the variation where the measurements are made.

There is no way to separate these variations from the measurement for tooth thickness. If a given gear is measured by each of these methods somewhat different results will be observed. These differences are due to the different tooth variations that enter each measurement. The differences are usually ignored, but, when results are critical, or backlash is closely controlled, it is necessary to specify the measurement method to be used.

3.1.2.1 Functional Tooth Thickness. The functional tooth thickness is that family of tooth thickness values obtained on a composite action test (double flank) by means of a calibrated master gear. It is a measurement which encompasses the effects of element variations in profile, pitch, tooth alignment, etc., (similar to the concept of maximum material condition). Section 8 explains this measurement method.

3.1.3 Effective Tooth Thickness. In most designs it is desirable to establish the maximum effective thickness equal to the maximum design thickness. That is the basis of this Standard.

The effective tooth thickness of a gear will be different than the measured tooth thickness by an amount equal to all the combined effects of the tooth element variation, and mounting, similar to functional tooth thickness. It is the final envelope condition which encompasses all the effects which must be considered to determine the maximum material condition (see Fig 3–1). As in the case of measured tooth thickness, the effects of the tooth element variations may be additive or may cancel each other at various angular positions within a given mesh. It is not possible to segregate the individual tooth element variations from the effective tooth thickness.

3.1.4 Maximum Tooth Thickness, t_{max}. The maximum tooth thickness of a gear, measured on the transverse plane is the thickness it would have if meshed at the tightest center distance and minimum backlash with a perfect, maximum tooth thickness, mating gear.

The maximum effective tooth thickness is the thickness of the thickest tooth, with reference to the mounting surfaces, at the operating pitch diameter with its mating gear. In this Standard, maximum tooth thickness and maximum effective tooth thickness are taken as numerically identical.

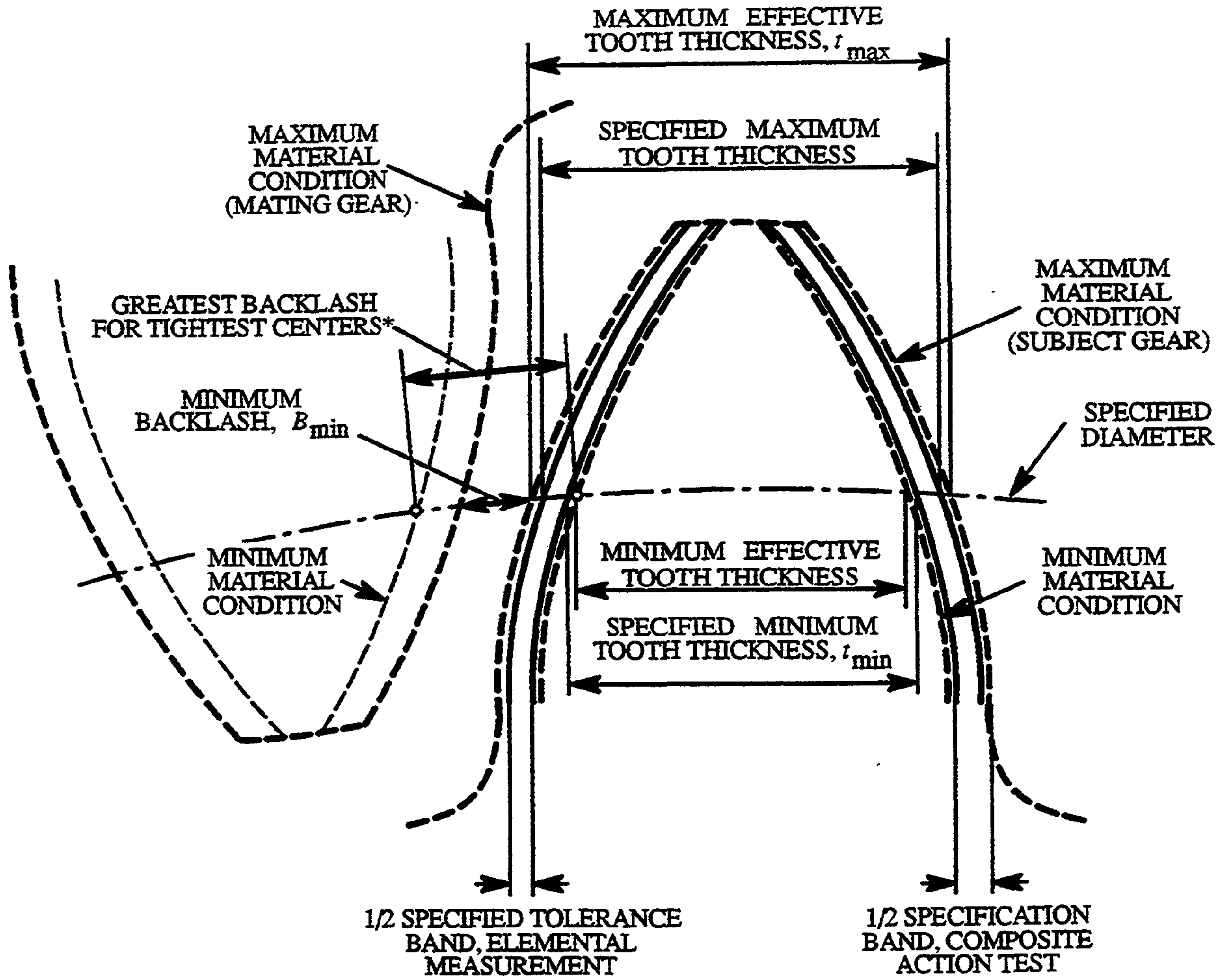
The selection of tooth thickness is up to the designer, but, the following relationship must be satisfied:

$$t_{G\max} = p' - B_{\min} - t_{P\max} \qquad \text{(Eq 3.1)}$$

where

$t_{G\max}$ = maximum transverse tooth thickness of gear, at operating pitch radius, in (mm)

$B_{\min}$ = minimum backlash, transverse, in (mm)



* THIS FIGURE IS DRAWN AT THE POSITION OF TIGHTEST CENTER DISTANCE; IF CENTER DISTANCE IS INCREASED BACKLASH WILL INCREASE.

Fig 3–1 Tooth Thickness Transverse Plane

$t_{P\max}$ = maximum transverse tooth thickness of pinion, at operating pitch radius, in (mm)

p' = transverse circular pitch at operating (tightest) center distance, in (mm)

$$p' = 2\pi\left(\frac{C}{N+n}\right) \qquad \text{(Eq 3.2)}$$

where

N = number of teeth in the gear

n = number of teeth in the pinion

C = tightest center distance, in (mm) (minimum center distance for external gears or the maximum center distance for internal gears)

For gears of standard proportions, operating at standard center distance, it is common to reduce the tooth thickness of each member by one half the backlash allowance, but it is not a requirement. As long as Eq 3.1 is satisfied, the set will have the specified minimum backlash.

For gear sets with nonstandard proportions, or operating at nonstandard center distances, the designer has a wide range of choices for $t_{\max}$. The usual approach is to select a center distance, then to vary the addendum (tip) diameters of the gears until an approximate balance of strength rating is achieved. An attempt is made to keep the cutting depths of both members equal, assuming that they are to be cut with the same tool. The design is then rechecked for tip land width, contact ratio, limit diameter *, root clearance, and rating before it is finalized.

3.2 Backlash. The amount of backlash which is appropriate for different sizes and classes of gears is discussed in Appendix A.

An individual gear does not have backlash. It has only a tooth thickness. Backlash of a meshed set of gears is governed by the center distance at which the gears are operated and the tooth thickness of each of the gears.

This Standard uses the term *minimum backlash* in a carefully defined way. Minimum backlash, $B_{\min}$, is the minimum transverse backlash allowable on the operating pitch circle when the gear tooth with the greatest allowable effective tooth thickness is in mesh with the pinion tooth having its greatest allowable effective tooth thickness, at the tightest allowable center distance, under static conditions. This is the traditional *backlash allowance* provided by the designer to allow for deflections, misalignments, bearing runouts, temperature effects, and any unknowns.

The tightest center distance is the minimum center distance for external gears or the maximum center distance for internal gears.

3.3 Mounting Surfaces. Mounting surfaces are the surfaces (usually two) which determine the axis of rotation and axial location (usually a plane perpendicular to the axis of rotation) of the finished gear in the gear assembly. These surfaces must be specified, because they are used as the reference surfaces (tooling points) for all backlash and effective tooth thickness measurements. If the mounting surfaces are finished after the teeth are cut or inspected, an auxiliary pair of reference surfaces (trueing registers or proof surfaces) should be specified for tooth inspection.

3.4 Reference Surfaces. The expressions *reference surface, reference diameter, reference plane,* and *reference axis* are used to denote surfaces, actual or hypothetical, which form the basis for the calculation or measurement under discussion. For example, chordal measurement uses the outside diameter as a reference surface. The definition of circular tooth thickness in 2.2 uses *specified diameter* in this way.

3.5 Total Composite Variation. Total composite variation, V_{cq}, is the variation in center distance when a test gear is rolled in tight mesh for a complete revolution with an appropriate master gear, in a variable center distance fixture. It is not a factor limiting maximum tooth thickness, but it has an important effect on operating backlash, tooth thickness measurement, and minimum tooth thickness specifications. Appendix A covers this subject in more detail. AGMA 2000–A88 provides tables of values for V_{cq} for gears of various sizes and quality numbers.

[*] Limit diameter is the diameter on a gear at which the line of action intersects the maximum addendum circle of the mating gear. This is sometimes referred to as the start or end of contact. See AGMA 112.05, *Gear Nomenclature Definitions of Terms with Symbols*.

3.6 Specifying Maximum Tooth Thickness. Because it is very difficult to measure tooth thickness directly, and the indirect methods include different effects of tooth variations, the specified tooth thickness measurement should be adjusted for each specific method of measurement. These differences are often ignored, particularly in coarse pitch gearing with large backlash allowance, but the effects are important in fine pitch gears and where backlash is tightly controlled. Table 3-1 shows the influence of tooth geometry variations on each measurement method.

Where variations in tooth geometry or reference surface geometry influence the tooth thickness measurement, the magnitude of the variations is taken as the maximum for that gear and quality class per AGMA 2000-A88 and the direction is taken so that maximum effective tooth thickness is not exceeded.

In each of the following sections, the maximum tooth thickness in the transverse plane at the minimum operating pitch diameter (maximum for internal) will be used as a basis for calculating the specified maximum value for each measurement method.

Tooth thickness is calculated in the transverse plane and specified as the appropriate value for each measurement method, usually in the normal plane.

3.7 Specifying Minimum Tooth Thickness. The specified minimum tooth thickness is equal to maximum tooth thickness, less an allowance for manufacturing variation. The manufacturing variation allowance should be a function of the manufacturing method and the actual gear quality (total composite variation and tooth thickness tolerance).

$$t_{min} = t_{max} - t_T - 2\,V_{cq} \tan \phi' \qquad \text{(Eq 3.3)}$$

where

t_{min} = minimum specified transverse tooth thickness, in (mm)

t_{max} = maximum transverse tooth thickness at operating pitch diameter, in (mm)

t_T = tooth thickness tolerance, in (mm) (taken from AGMA 2000-A88, and applied to the transverse plane)

V_{cq} = total composite variation

ϕ' = transverse operating pressure angle

$$\phi' = \arccos\left(\frac{D_b}{D'}\right) \qquad \text{(Eq 3.4)}$$

D' = operating pitch diameter, in (mm)

Table 3-1
Other Gear Variations Included in Tooth Thickness Measurement

Method of Measurement	Variation: Concentricity in reference to Mounting Surfaces	Outside Diameter	Out-of-Roundness	Profile	Pitch	Tooth Alignment
Chordal Thickness		X		X*		
over-two-pins			X	X*	X	
over-one-pin	X		X	X*	X	
Span				X*	X	X†
Test Radius with master gear	X		X	X	X	X

* If pin size or number of teeth spanned is selected to locate the measurement at one half the working depth from the addendum (tip) circle, the effect of profile deviation is minimized. This Standard uses this method.

† Helical gears only.

$$D' = 2C\left(\frac{N}{N+n}\right) \quad \text{(Eq 3.5)}$$

C = tightest center distance

D_b = base circle diameter, in (mm)

$$D_b = \left(\frac{N}{P_{nd}}\right)\frac{\cos\phi_c}{\cos\psi_b} \quad \text{(Eq 3.6)}$$

$$D_b = \frac{N\ m_n\ \cos\phi_c}{\cos\psi_b} \quad \text{(Eq 3.6M)}$$

where

P_{nd} = normal diametral pitch

m_n = normal module

ϕ_c = normal profile angle of the equivalent standard rack cutter, degrees [*]

ψ_b = base helix angle, degrees

$$\psi_b = \arcsin\left(\frac{\pi\ \cos\ \phi_c}{P_{nd}\ p_x}\right) \quad \text{(Eq 3.7)}$$

$$\psi_b = \arcsin\left(\frac{\pi\ m_n\ \cos\ \phi_c}{p_x}\right) \quad \text{(Eq 3.7M)}$$

For spur gears, $\cos\psi_b = 1$

The tooth thickness tolerance (allowance for tool wear or adjustment of the cutting machine) is a function of pitch and quality number. Values are tabulated in AGMA 2000-A88.

3.8 Measurement Method Effects. The effect of measurement method on the specified value of minimum tooth thickness is discussed in Sections 5 through 8 as it applies to each measurement method. The magnitude and direction of adjustments for variations in tooth or reference surface geometry are taken so that tooth thickness is decreased by inaccuracies inherent in each measurement method.

3.9 Selection of Tooth Thickness. Usually, maximum backlash does not affect the function or smoothness of transmission motion, and effective tooth thickness variation is not the main consideration in the selection of gear quality. Under these conditions, the selection of tooth thickness and measurement method is not critical and the most convenient method can be used.

In many applications, allowing a larger range of tooth thickness tolerance or operating backlash will not affect the performance or load capacity of gears, and may allow more economical manufacturing. A tight tooth thickness tolerance should not be used unless absolutely necessary, since it has a strong influence on manufacturing cost.

For any given value of minimum backlash, B_{min}, and tooth thickness tolerance, t_T, maximum backlash, B_{max}, increases as the quality level decreases and as the size increases, since total composite variation, V_{cq}, increases with size and lower quality.

In those cases where maximum backlash must be closely controlled, a careful study of these factors must be made and the gear quality class, center distance tolerance, and measurement methods must be carefully specified. It may be necessary to specify a higher quality class to hold maximum backlash within the desired limits.

A method to calculate maximum backlash from tolerances for center distance, tooth thickness, and total composite variation is included in Appendix A.

4. Gear Geometry Calculations

4.1 Circular Tooth Thickness. Circular tooth thickness may be specified in the plane of rotation, (transverse plane), t_t, or in the plane normal to the helix angle at the reference circle (normal plane), t_n.

Tooth thickness calculations are usually made in the transverse plane and tooth thickness measurements are made in the normal plane.

At any specified diameter at or above the base circle:

$$t_n = t_t\ \cos\psi \quad \text{(Eq 4.1)}$$

where

t_t = tooth thickness in transverse plane

t_n = tooth thickness in normal plane
For spur gears, $t_n = t_t$

ψ = helix angle at the specified diameter

[*] For complete discussion, see 9.01 of AGMA 112.05.

$$\psi = \arctan\left(\frac{\pi D}{N p_x}\right) \qquad \text{(Eq 4.2)}$$

where

D = the specified diameter, in (mm)

p_x = axial pitch [*], in (mm)

$$p_x = \frac{L}{N} \qquad \text{(Eq 4.3)}$$

L = lead of gear or machine guide

When the helix angle at the standard pitch diameter is given:

$$p_x = \frac{\pi}{P_{nd} \sin \psi_s} \qquad \text{(Eq 4.4)}$$

$$p_x = \frac{\pi m_n}{\sin \psi_s} \qquad \text{(Eq 4.4M)}$$

where

ψ_s = helix angle at standard pitch diameter

$$\psi_s = \arcsin\left(\frac{\pi}{p_x P_{nd}}\right) \qquad \text{(Eq 4.5)}$$

$$\psi_s = \arcsin\left(\frac{\pi m_n}{p_x}\right) \qquad \text{(Eq 4.5M)}$$

For spur gears, $\cos \psi_s = 1$

4.2 Standard Pitch Diameter. A standard pitch diameter (generating pitch diameter), D_s, is one calculated according to the standard pitch of the gear cutting tool. [**]

$$D_s = \frac{N}{P_{nd} \cos\psi_s} = \frac{N}{P_d} \qquad \text{(Eq 4.6)}$$

$$D_s = \frac{N m_n}{\cos\psi_s} \qquad \text{(Eq 4.6M)}$$

where

P_d = transverse diametral pitch

P_d = $P_{nd} \cos \psi_s$

4.3 Backlash Calculations. Backlash, B, in an assembled gear set is the clearance or play between the teeth of the meshing gears. It is the amount by which the width of the tooth space exceeds the tooth thickness of the engaging tooth on the operating pitch circles (see Fig 2–1). Backlash may be measured in the transverse plane, in the normal plane along the operating pitch cylinder or normal to the tooth surface in the plane of action, as measured by a feeler gage. In this Standard, Backlash is specified in the transverse plane.

$$B_t = \frac{B_f}{\cos\phi' \cos\psi_b} = \frac{B_f D'}{D_b \cos\psi_b} \qquad \text{(Eq 4.7)}$$

where

B_t = circular transverse backlash, in (mm)

B_f = backlash measured normal to tooth surface (feeler gage) in the plane of action, in (mm)

4.4 Effective Tooth Thickness Calculation. The maximum transverse tooth thickness at the operating pitch diameter determined in 3.1.4 is used as the basic dimension. If the gears operate at standard center distance, the effective tooth thickness is also the maximum material condition tooth thickness at the generating diameters (maximum generated tooth thickness).[***]

4.5 Maximum Generated Tooth Thickness. The maximum generated tooth thickness (tooth thickness at the standard pitch diameter), t_{ts}, is:

For external gears:

$$t_{ts} = D_s\left[\left(\frac{t_{max}}{D'}\right) + \text{inv}\,\phi' - \text{inv}\,\phi_s\right] \qquad \text{(Eq 4.8)}$$

where

t_{ts} = maximum transverse generating tooth thickness, in (mm)

ϕ_s = transverse generating pressure angle

$$\phi_s = \arcsin\left(\frac{\sin \phi_c}{\cos \psi_b}\right) \qquad \text{(Eq 4.9)}$$

For spur gears, $\phi_s = \phi_c$

For internal gears:

$$t_{ts} = D_s\left[\left(\frac{t_{max}}{D'}\right) - \text{inv}\,\phi' + \text{inv}\,\phi_s\right] \qquad \text{(Eq 4.10)}$$

[*] This calculation is based on standard gear hobbing practice, with P_{nd} and p_x given. For a detailed text on geometry see AGMA 115.01 (R1988), *Information Sheet – Basic Gear Geometry.*

[**] See 8.16 of AGMA 112.05 for more information.

[***] The discussion of short pitch cutters is beyond the scope of this Standard.

4.6 Base Tooth Thickness. The base tooth thickness, used in subsequent calculations, is:

For external gears:

$$t_b = D_b\left[\left(\frac{t_{ts}}{D_s}\right) + \mathrm{inv}\,\phi_s\right]$$

or (Eq 4.11)

$$t_b = D_b\left[\left(\frac{t_{max}}{D'}\right) + \mathrm{inv}\,\phi'\right]$$

where

t_b = transverse tooth thickness at base circle, in (mm)

For internal gears:

$$t_b = D_b\left[\left(\frac{t_{ts}}{D_s}\right) - \mathrm{inv}\,\phi_s\right]$$

or (Eq 4.12)

$$t_b = D_b\left[\left(\frac{t_{max}}{D'}\right) - \mathrm{inv}\,\phi'\right]$$

5. Chordal Tooth Thickness

5.1 Advantages of Chordal Tooth Thickness. The vernier gear tooth caliper, Fig 5–1, is a portable hand held instrument used to measure the thickness of external gear teeth. Its portability and its simplicity are its principal advantages.

5.2 Limitations of Chordal Tooth Thickness. The tooth caliper requires an experienced operator, because the anvils make contact with the tooth flank on their corners, rather than on the flats.

The instrument is hard to read consistently with a deviation less than 0.001 in (25 μm). Instruments are not available for very large or very small teeth.

For coarse pitches and small numbers of teeth, the addendum must be corrected and the chordal thickness must be calculated (see Fig 5–2).

The theoretical addendum, a, is affected by variations in the outside diameter, taper and runout of the blank since the outside diameter is used as a reference surface for the caliper.

The tooth thickness caliper cannot be used for internal gears.

5.3 Calculation of Chordal Tooth Thickness. The addendum bar setting is usually based on a standard addendum, even if the gear has a non-standard nominal outside diameter. This puts the point of measurement at approximately half the working depth, to minimize the effect of profile deviation.

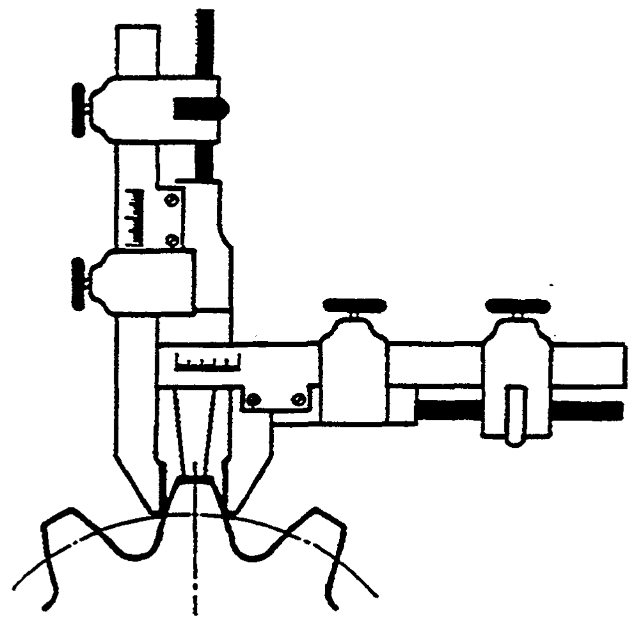


Fig 5–1 Chordal Tooth Thickness Measurement by Means of a Gear Tooth Caliper

The maximum expected reference radius, equal to half the maximum outside diameter plus half the allowable runout, is the basis for calculation. If the actual outside diameter and the runout of the outside diameter at the point of measurement are known, they should be used. If the runout of the outside diameter is not known, it may be assumed to be equal to the allowable runout of the teeth.

$$a = \frac{1}{P_{nd}} \text{ for full depth teeth} \qquad \text{(Eq 5.1)}$$

$$a = m_n \qquad \text{(Eq 5.1M)}$$

$$a = \frac{0.8}{P_{nd}} \text{ for stub teeth} \qquad \text{(Eq 5.2)}$$

$$a = 0.8\, m_n \qquad \text{(Eq 5.2M)}$$

where

a = addendum, in (mm)

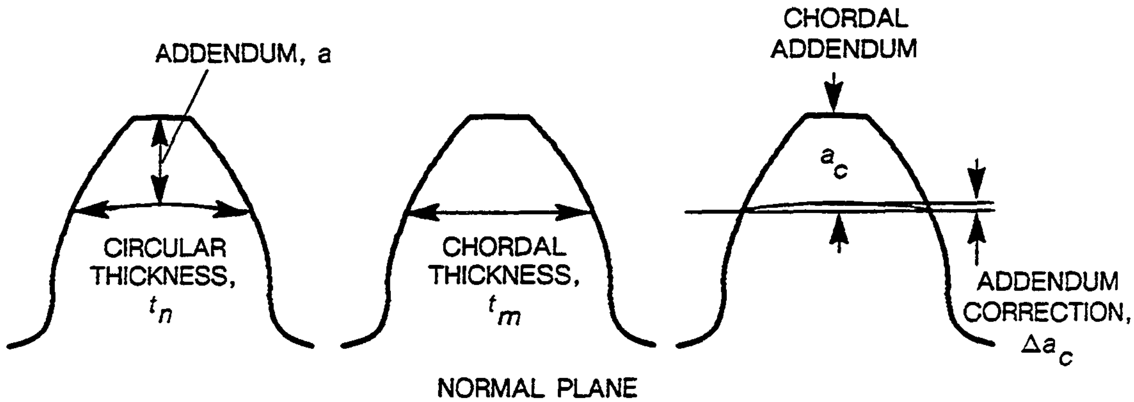


Fig 5-2 Addendum and Chordal Tooth Thickness Corrections

$$t_{nR} = t_R \cos \psi_R \qquad \text{(Eq 5.3)}$$

where

t_{nR} = maximum normal tooth thickness, in (mm)

ψ_R = helix angle at measuring radius, R_{max}

$$\psi_R = \arctan \left(\frac{2 \pi R_{max}}{N p_x} \right) \qquad \text{(Eq 5.4)}$$

For spur gears, $\cos \psi_R = 1$

t_R = transverse tooth thickness at R_{max} in (mm)

$$t_R = 2 R_{max} \left[\frac{t_b}{D_b} - \text{inv} \left(\arccos \left(\frac{D_b}{2 R_{max}} \right) \right) \right] \qquad \text{(Eq 5.5)}$$

R_{max} = maximum measuring radius, in (mm)

$$R_{max} = \left(\frac{D_{o\,max} + V_r}{2} \right) - a \qquad \text{(Eq 5.6)}$$

where

$D_{o\,max}$ = maximum outside diameter

V_r = radial runout of reference diameter, total indicator movement, (may be assumed to be equal to allowable runout of the gear teeth per AGMA 2000-A88), in (mm)

If the maximum runout of the outside diameter to the mounting diameter is specified, the specified value of runout should be used, instead of the assumed value.

5.3.1 Addendum Correction. To calculate the chordal addendum, a_c, a correction, Δa_c, must be made for the height of the chord spanned by the tooth caliper.

$$a_c = a + \Delta a_c \quad \text{or} \qquad \text{(Eq 5.7)}$$

$$a_c = \left(\frac{D_{o\,max}}{2} \right) - R_{max} \cos \left(\frac{\tau}{2} \right) \qquad \text{(Eq 5.8)}$$

where

τ = normal angular thickness

$$\frac{\tau}{2} = \left(\frac{t_R \,(\cos^2 \psi_R)}{2\; R_{max}} \right) \text{, radians} \qquad \text{(Eq 5.9)}$$

Since they have such a large influence on gear tooth thickness measurement, outside diameter size, outside diameter runout, and gear tooth runout must be carefully controlled when tooth thickness is controlled by tooth caliper measurement.

5.3.2 Chordal Correction. Since the tooth caliper measures on a straight chordal line, the chordal thickness measurement, t_m, is slightly less than the distance along the arc of the reference circle. Although this difference is frequently ignored, it is significant for coarse pitches and low numbers of teeth.

$$t_m = 2 R_{max} (\cos \psi_R) \sin \left(\frac{t_R}{2 R_{max}} \right) \qquad \text{(Eq 5.10)}$$

where

t_m = measured normal chordal tooth thickness, in (mm)

5.3.3 Specifying Chordal Tooth Thickness Measurement. If the outside diameter of the gear is under the maximum size, the tooth will appear to be thicker than it is. To avoid accepting gears which are thicker than t_{max}, the maximum chordal tooth thickness must be calculated from the maximum outside diameter. To avoid rejecting gears which are at the minimum tooth thickness, it is also necessary to calculate the minimum chordal tooth thickness from the maximum outside diameter. This procedure will allow some thin gears to be accepted, if their outside diameters are less than the maximum. If tight control of tooth thickness is required, the size and concentricity of the outside diameter must also be tightly controlled.

6. Measurement by Pins

6.1 Advantages of Measurement by Pins. Pins or balls afford a method of measuring tooth thickness of gears of any diameter within the capacity of available micrometers (see Fig 6–1). Measurements are not affected by outside diameter deviation or by runout of the outside diameter.

Measurements over one pin or ball, from the mounting diameter, show the effects of runout in the gear teeth and approximate the measurement of functional tooth thickness.

The *amplifying effect* of pin or ball measurement; i.e., the fact that the measurement over a pin is a function of $t/\tan\phi$, makes it easy to detect small changes in tooth thickness.

This is a common method of tooth thickness inspection.

6.2 Limitations of Measurement by Pins. Measurements are affected by deviations in pitch and profile.

The following should be noted:

- Pins on spur gears form line contacts
- Balls on spur gears form point contacts
- Pins and balls form point contacts on helical gears.

Therefore, deflection, because of the limited contact, can cause variation in readings and will vary with gaging pressure.

Micrometers are often used to measure the

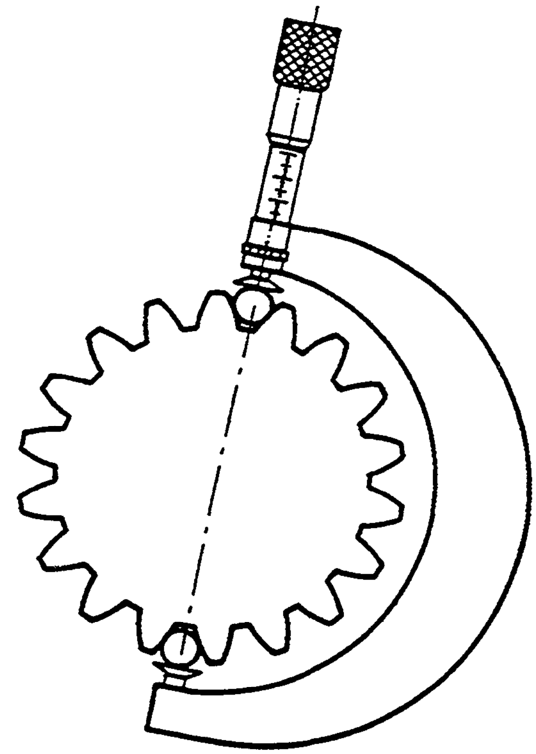


Fig 6–1 Tooth Thickness Measurement Over Pins

dimension over pins. Even though the micrometers may be graduated to 0.0001, the variation of the measurement among several operators may exceed 0.001.

Balls must be held exactly in the plane of rotation; a difficult task.

Internal helicals cannot be measured with pins and are usually measured with balls.

External helical gears with odd numbers of teeth should be measured with balls or with three pins between parallel planes. Both are difficult setups.

The following is quoted from *Analytical Mechanics of Gears*, by Earle Buckingham [2]

Measurements over rolls on helical gears are very difficult to make with any great degree of accuracy unless definite precautions are taken. In many cases, a pair of calibrated wedges, or rack teeth, make a much more reliable measurement for tooth thickness than do rolls. However rolls are often available when needed, while the special calibrated rack–tooth wedges may not be at hand. The measurement over rolls should

[2] Numbers in brackets refer to the bibliography.

be made between parallel flat surfaces and not with a micrometer alone. When the rolls are held in position on the gear by two parallels, the two rolls will be on opposite sides of the gear, or diametrically opposite to each other, whether the number of teeth in the gear is odd or even. With odd numbers of teeth, one roll may make contact near one edge of the gear while the other roll makes contact near the opposite edge of the face width. If an attempt is made to measure odd numbers of teeth over the rolls directly with a micrometer, one or both rolls will be tipped away from the correct plane of measurement, and any measured values so obtained are useless for any purpose.

Ball-point micrometers may be used, but here the two balls must be definitely aligned in respect to the face of the gear blank. For example, the gear blank may be laid flat on a surface plate, and the two ball points may be held against this same surface plate. Where balls are used, when odd numbers of teeth are involved, the calculation of the actual chordal measurement must include the offset condition or position in exactly the same way as the calculations are made for spur gears with odd numbers of teeth.

Large micrometers are required for large gears.

Measurements made over two pins or balls do not show functional tooth thickness.

Multiple readings taken around the gear should be averaged to find the mean. The mean value should be used in comparison of readings. The maximum reading, as previously stated, is probably closer to the functional tooth thickness, which is best measured by double flank testing.

6.3 Measurement Methods. It is important to use a measurement over pins [*] setup for which there is a suitable calculation method relating the measurement to the tooth thickness. For all types of spur and helical gears, there are measurement setups using pins or balls for which there are corresponding geometrically exact calculation methods.

For external spur gears with even numbers of teeth, the measurement is made across the high points of two properly sized pins placed in diametrically opposed tooth spaces. In the case of spur gears with odd numbers of teeth, the tooth spaces used are those nearest to diametrically opposed.

Measurement over pins can also be performed on medium and small external helical gears. When the gear has an even number of teeth, the measurement technique is similar to that on spur gears. Although the two pins are not parallel, it is possible to position the anvils on a conventional micrometer so as to measure at diametrically opposite points.

For helical gears with odd numbers of teeth, there are two techniques with geometrically exact calculation methods. One method uses three pins instead of two, but is limited to gears whose face widths are greater than their axial pitches. This method also requires the use of a micrometer or other measuring instrument with an anvil of size greater than the axial pitch. The third pin is placed alongside one of the others so that the pair will be diametrically opposite the single pin. The wide anvil is positioned to span the axial pitch distance between the two adjacent pins and the second anvil, which need not be as wide, is positioned to contact the single pin at a location halfway along the axial pitch distance. When properly positioned for measurement, all three pins will be in line contact with their respective parallel anvil surfaces. The measurement is twice the calculated radius over one pin.

The other method uses a single pin and is suitable for any external helical gear, whatever the face width and whether the number of teeth is odd or even. The measurement over the single pin is made relative to the gear center line or relative to the bore or other concentric cylindrical reference surface on the gear. This measurement, when combined with the radius of the reference surface corresponds to the calculated radius over one pin.

To approximate the maximum functional tooth thickness, repeated measurements must be made, and the largest used.

[*] "Pin" is used in this text for "pin, wire or ball". The calculations are made using a disc of infinitesimal thickness, representing either.

All *two pin* measurements can be made with the pins replaced by balls of the same size. In the case of helical gears with odd numbers of teeth, it is also possible to measure over two balls. This measurement is the same as that for spur gears with odd numbers of teeth. All measurements over balls have the special requirement that the two balls be located with their centers in a single plane perpendicular to the axis of the gear.

6.4 Pin or Ball Sizes. The ideal (best) size pin or ball would contact the tooth profiles at half their working depth. This minimizes the effect of profile deviation.

It would extend above the outside diameter of an external gear or below the inside diameter of an internal gear, and would not touch the root of the tooth space.

To calculate the best pin size for external gears, see Fig 6-2.

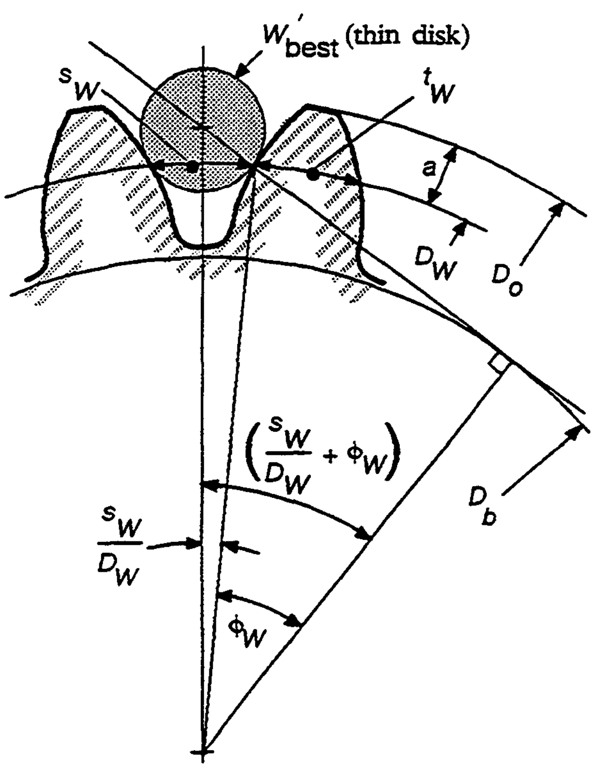


Fig 6-2 Best Pin Size, W'_{best} (External Gears)

This calculation is based on a disc of infinitesimal thickness in the transverse plane (see Fig 6-3).

The best disc diameter would contact the tooth profile at D_W.

For external gears:

$$D_W = D_o - 2a \tag{Eq 6.1}$$

The pressure angle at this diameter is:

$$\phi_W = \arccos\left(\frac{D_b}{D_W}\right) \tag{Eq 6.2}$$

The arc tooth thickness at this diameter is:

$$t_W = D_W\left[\left(\frac{t_b}{D_b}\right) - \operatorname{inv}\phi_W\right] \tag{Eq 6.3}$$

The arc space width at this diameter is:

$$s_W = \left(\frac{\pi D_W}{N}\right) - t_W \tag{Eq 6.4}$$

The best disc diameter is:

$$W'_{best} = \frac{D_W \sin\left(\frac{s_W}{D_W}\right)}{\cos\left[\left(\frac{s_W}{D_W}\right) + \phi_W\right]} \tag{Eq 6.5}$$

This value must be converted to the normal plane by rotation in the plane tangent to the base circle through the center of the disc (see Fig 6-3).

$$W_{best} = W'_{best} \cos\psi_b \tag{Eq 6.6}$$

where

- D_W = contact diameter for best pin size, in (mm)
- D_o = outside diameter, in (mm)
- ϕ_W = pressure angle at best pin diameter
- t_W = transverse tooth thickness at best pin diameter, in (mm)
- s_W = transverse space width at best pin diameter, in (mm)
- W'_{best} = best pin size – transverse plane, in (mm)
- W_{best} = best pin size, in (mm)

For internal gears (see Fig 6-4):

$$D_W = D_i + 2a \tag{Eq 6.7}$$

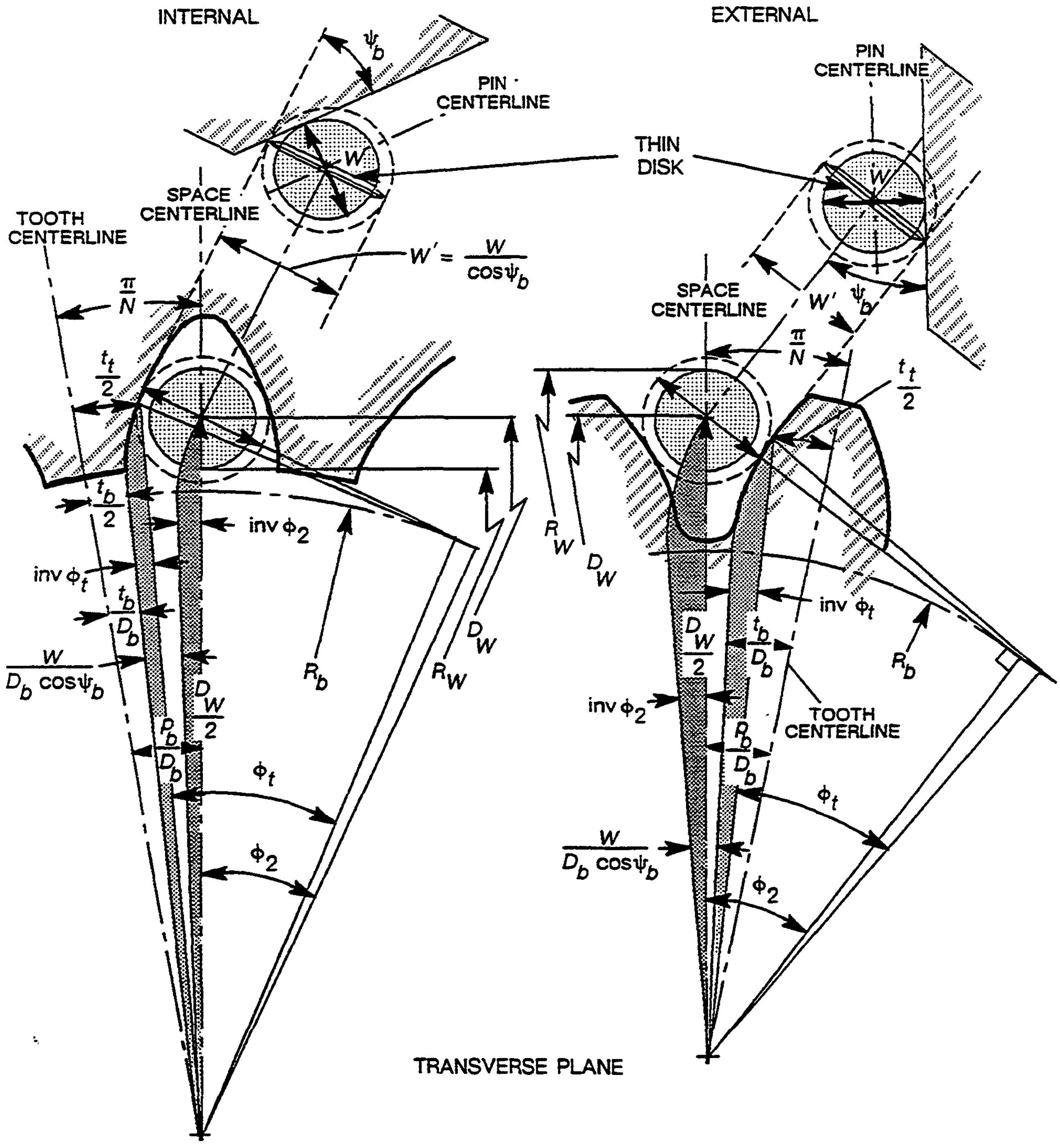


Fig 6-3 Pin Measurement Spur and Helical

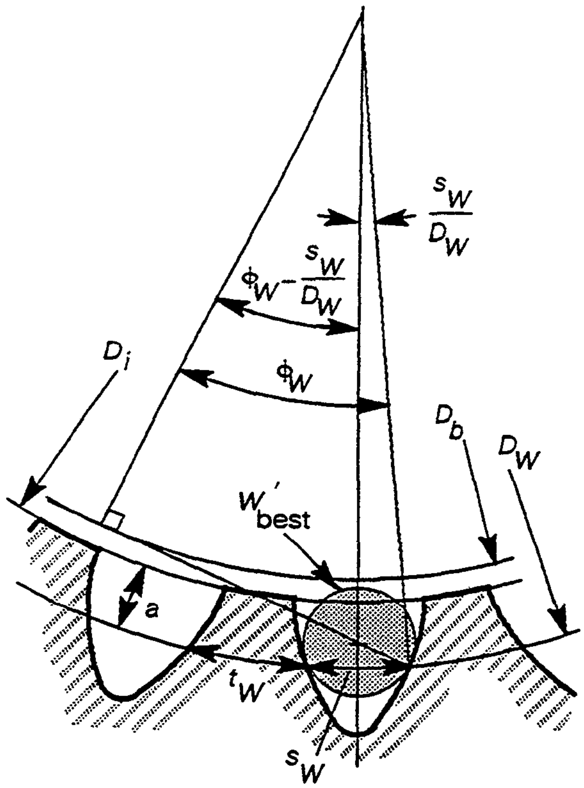


Fig 6–4 Best Pin Size (Internal Spur Gear)

where

D_i = tip diameter of internal gear

$$t_W = D_W\left[\left(\frac{t_b}{D_b}\right) + \text{inv}\ \phi_W\right] \qquad \text{(Eq 6.8)}$$

$$W'_{best} = \frac{D_W \sin\left(\frac{s_W}{D_W}\right)}{\cos\left[\phi_W - \left(\frac{s_W}{D_W}\right)\right]} \qquad \text{(Eq 6.9)}$$

For internal helical gears, W'_{best} must be converted to a ball diameter, W_{best}, using Eq 6.6. This is the theoretical best pin size; however, it may not be available.

The pin size specified should be the next largest selected from Table 6–1, or from a table of pins known to be available to the gear manufacturer. The size selected should be checked to be sure that it will not bottom in the root of the tooth space, and that it will protrude past the tip circle of the gear. This is done after the specified radius measurement over pins is calculated per Eq. 6.5, by comparing the pin measurement to the tip radius, and the pin measurement radius, plus or minus the pin diameter, to the root radius.

Dimensions over or between pins can be calculated for any pin diameter. Pin diameters have been standardized so that sets of pins for common pitches can be used. Table 6–1 lists some commonly used pin sizes. Further information on standard pin sizes can be found in manufacturer's catalogs. [3]

6.5 Calculation for Measurement by Pins. The final pin diameter selected is labeled W in the calculations.

6.5.1 Radius Over One Pin. Figure 6–3 shows the general case for external and internal gears.

For external gears:

$$\text{inv}\ \phi_2 = \frac{t_b}{D_b} + \frac{W}{D_b \cos\psi_b} - \frac{\pi}{N}$$

$$= \frac{t_b + \left(\frac{W}{\cos\psi_b}\right) - p_b}{D_b} \qquad \text{[*] (Eq 6.10)}$$

where

p_b = transverse base pitch

$$R_{1W} = \frac{D_b}{2\cos\phi_2} + \frac{W}{2} \qquad \text{(Eq 6.11)}$$

where

ϕ_2 = pressure angle at center of pin

R_{1W} = radius over or to one pin, in (mm)

W = pin diameter, in (mm)

Conversely

$$\phi_2 = \text{arc}\cos\left[\frac{D_b}{2R_{1W} - W}\right] \qquad \text{(Eq 6.12)}$$

$$t_b = D_b\left[\text{inv}\ \phi_2 + \frac{\pi}{N}\right] - \frac{W}{\cos\psi_b} \qquad \text{(Eq 6.13)}$$

[*] The inverse function is usually found by iteration.

Table 6–1
Standard Pin Diameters for Various Pitches in Inches

Diametral Pitch P_{nd}	For Standard External Gears $W = \frac{1.728}{P_{nd}}$	For Standard Internal Gears $W = \frac{1.680}{P_{nd}}$	For Long–Addendum Pinions $W = \frac{1.920}{P_{nd}}$
1	1.728	1.680	1.920
1 1/2	1.152	1.120	1.280
2	0.864	0.840	0.960
2 1/2	0.6912	0.672	0.768
3	0.576	0.560	0.640
4	0.432	0.420	0.480
5	0.3456	0.336	0.384
6	0.288	0.280	0.320
7	0.24686	0.240	0.27428
8	0.216	0.210	0.240
9	0.1920	0.18666	0.21333
10	0.1728	0.168	0.192
11	0.15709	0.16273	0.17454
12	0.144	0.140	0.160
14	0.12343	0.120	0.13714
16	0.108	0.105	0.120
18	0.096	0.09333	0.10667

Table 6–1M
Standard Pin Diameters in Millimeters

2	5.5	16
2.25	6	18
2.5	6.5	20
2.75	7	22
3	7.5	25
3.25	8	28
3.5	9	30
3.75	10	35
4	10.5	40
4.25	11	45
4.5	12	50
5	14	–
5.25	15	–

Abstracted from DIN 3977[4]

6.5.2 Radius To One Pin.

For internal gears:

$$\mathrm{inv}\,\phi_2 = \frac{\pi}{N} - \frac{t_b}{D_b} - \left(\frac{W}{D_b \cos\psi_b}\right)$$

$$= \frac{p_b - t_b - \left(\dfrac{W}{\cos\psi_b}\right)}{D_b} \qquad \text{(Eq 6.14)}$$

$$R_{1W} = \frac{D_b}{2\cos\phi_2} - \frac{W}{2} \qquad \text{(Eq 6.15)}$$

6.5.3 Dimension Over Two Pins.

For external gears with even numbers of teeth:

$$D_{2W} = 2\,R_{1W} \qquad \text{(Eq 6.16)}$$

For external gears with odd numbers of teeth:

$$D_{2W} = \frac{D_b}{\cos\phi_2}\cos\left(\frac{\pi}{2N}\right) + W \qquad \text{(Eq 6.17)}$$

where

D_{2W} = dimension over or between two pins, in (mm)

Equation 6.17 applies to helical gears with odd numbers of teeth when measured over balls. See 6.4 for further information on helical gears.

If D_{2W} is known from measurements:

$$\phi_2 = \arccos\left[\frac{D_b \cos\left(\dfrac{\pi}{2N}\right)}{D_{2W} - W}\right] \qquad \text{(Eq 6.18)}$$

6.5.4 Dimension Between Two Pins.

For internal gears with even numbers of teeth see Eq. 6.16.

For internal gears with odd numbers of teeth:

$$D_{2W} = \frac{D_b}{\cos\phi_2}\cos\left(\frac{\pi}{2N}\right) - W \qquad \text{(Eq 6.19)}$$

6.5.5 Correction for Tooth Deviations. If the pins make contact near the mid-point of the active profile, deviation effects are minimized.

The effect of allowable pitch deviation is much smaller than allowable runout, so it can be ignored, except with very low numbers of teeth and other unusual cases.

If pin measurements are made as a radius to one pin from the mounting diameter, the effects of runout are included and no correction is necessary. If the measurements are made with two pins, the effects of runout should be calculated and D_{2W} adjusted accordingly.

The amount of correction is:

$$V_{rW} = \frac{V_{rT}}{2} \qquad \text{(Eq 6.20)}$$

where

V_{rW} = correction to pin measurement for runout, in (mm)

V_{rT} = allowable runout of gear teeth, from AGMA 2000–A88, in (mm)

The direction of the correction reduces the allowable tooth thickness (see 3.1).

7. Span Measurement

7.1 Advantages of Span Measurement. This method utilizes a vernier caliper or a disc micrometer to measure the distance, M, over a number of teeth, S, along a line tangent to the base cylinder.

For external gears, the distance measured is the sum of (S–1) normal base pitches, plus the normal thickness of one tooth at the base cylinder.

For internal spur gears the measurement is made between teeth, and the distance measured is (S+1) normal base pitches minus one normal base tooth thickness.

Measurements are not affected by outside diameter deviations or by runout of the outside diameter (see Figs 7–1 and 7–2).

This method is particularly useful for large gears, because it does not require auxiliary balls or pins and a smaller micrometer or caliper can be used. The measurement can sometimes be made without stopping the gear cutting machine.

No unusual skill is required to make the measurement, which is similar to measuring a diameter.

7.2 Limitations of Span Measurement. Span measurement cannot be applied when a combination of high helix angle and narrow face width prevent the caliper from spanning a sufficient number of teeth. This can be overcome to some extent by making measurements on the machine when gears are stacked in cutting, or by using a disc micrometer.

Readings are influenced by deviations in base pitch, accumulated pitch over S teeth, tooth profile, and lead. The effects of profile deviation are greatly reduced if the measurement is made at half the working height of the teeth.

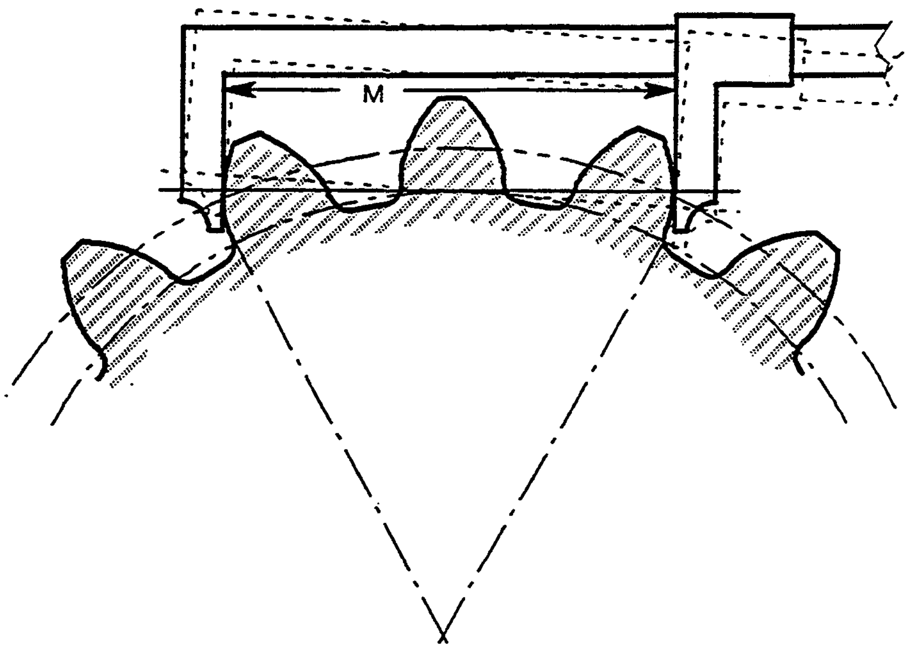


Fig 7-1 Span Measurement of Tooth Thickness

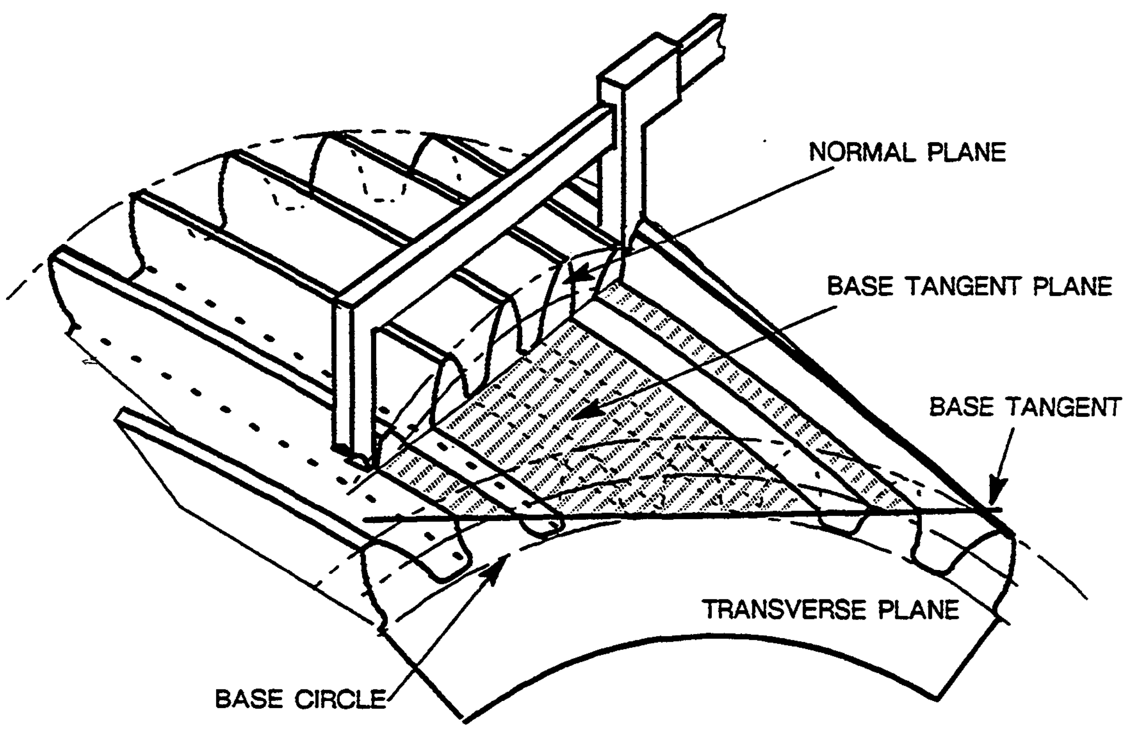


Fig 7-2 Span Measurement of Helical Gears

Readings are erroneous if attempted on a portion of the tooth flank which has been modified from true involute form.

Span measurement does not show the effect of runout, so it does not measure functional tooth thickness.

Span measurement cannot be used for internal helical gears or for pitches which are too fine for the anvils of the measuring instrument to enter the tooth space.

It does not have the *amplifying effect* of pin measurement.

7.3 Calculation of Span Measurement. Number of teeth to be spanned for external gears, S, can be a range.

The range of S is limited by the size of the plane which is tangent to the base cylinder, bounded by the outside diameter and the face width of the gear (see Fig 7–3). S is also limited by the limit diameter of the gear.

The following calculation also limits the contact between the measuring instrument and the gear so that no contact occurs within $0.125/P_{nd}$ ($0.125\ m_n$), of the outside diameter or $0.25/P_{nd}$ ($0.25\ m_n$), of the ends of the teeth.

$$L_{min} = \sqrt{(D_o - 4a)^2 - D_b^2} \qquad \text{(Eq 7.1)}$$

$$S_{min} = \text{integer portion of} \left[\frac{\left(\frac{L_{min}}{\cos \psi_b}\right) - t_{nb}}{p_N} + 2 \right] \qquad \text{(Eq 7.2)}$$

$$S_{min} \geqq 2$$

$$L_{max} = \sqrt{\left(D_o - \frac{1}{4 P_{nd}}\right)^2 - D_b^2} \qquad \text{(Eq 7.3)}$$

$$L_{max} = \sqrt{\left(D_o - \frac{m_n}{4}\right)^2 - D_b^2} \qquad \text{(Eq 7.3M)}$$

$$S_{max} = \text{integer portion of} \left[\frac{\left(\frac{L_{max}}{\cos \psi_b}\right) - t_{nb}}{p_N} + 1 \right] \qquad \text{(Eq 7.4)}$$

or, if helical, integer portion of

$$\left[\frac{\frac{\left[F - \left(\frac{1}{2 P_{nd}}\right) \cos \psi_b\right]}{\sin \psi_b} - t_{nb}}{p_N} + 1 \right] \qquad \text{(Eq 7.5)}$$

$$\left[\frac{\frac{\left[F - \left(\frac{m_n}{2}\right) \cos \psi_b\right]}{\sin \psi_b} - t_{nb}}{p_N} + 1 \right] \qquad \text{(Eq 7.5M)}$$

whichever is least.

where

S_{min} = minimum number of teeth to be spanned

S_{max} = maximum number of teeth to be spanned

F = face width, in (mm)

L_{max} = maximum length of base tangent plane, in (mm)

L_{min} = minimum length of base tangent plane, in (mm)

p_N = normal base pitch, in (mm)

S = number of teeth to be spanned

t_{nb} = normal base tooth thickness, in (mm)

$$t_{nb} = t_b \cos \psi_b \qquad \text{(Eq 7.6)}$$

If S_{max} is not greater than or equal to S_{min}, which must be greater than or equal to 2, the face width is too narrow for span measurement at this helix angle.

The best span measurement, S_{best}, is made where the base tangent plane intersects the teeth at approximately half their working height. When rounding S_{best} (Eq. 7.7) to the nearest integer; rounding up will place the caliper contact above half the working height, and rounding down will place the caliper contact below half the working height. For gears with extra tip relief or extra fillet clearance rounding may be favored one way or the other.

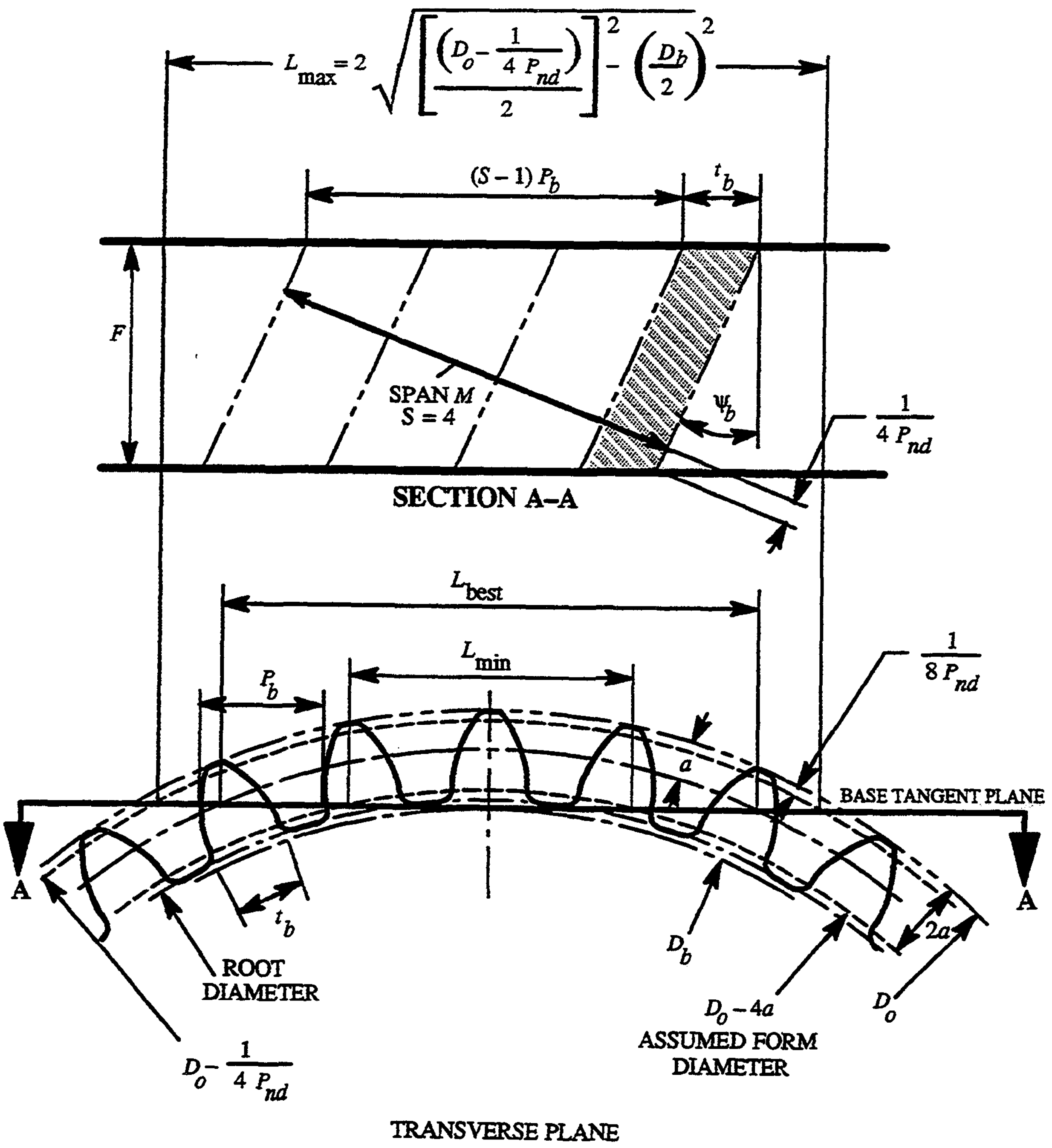


Fig 7–3 Limits of Span Measurement in Base Tangent Plane

$$L_{best} = \sqrt{(D_o - 2a)^2 - D_b^2} \qquad \text{(Eq 7.7)}$$

$$S_{best} = \text{rounded value of} \left[\frac{\frac{L_{best}}{\cos \psi_b} - t_{nb}}{p_N} + 1 \right] \qquad \text{(Eq 7.8)}$$

where

L_{best} = best length of base tangent

S_{best} = best number of teeth to be spanned

$$S_{min} \leqq S_{best} \leqq S_{max} \qquad \text{(Eq 7.9)}$$

Calculate span:

$$M = [(S_{best} - 1) p_b + t_b] \cos \psi_b \qquad \text{(Eq 7.10)}$$

where

M = span measurement, in (mm)

$$p_b = \frac{\pi D_b}{N} = \frac{p_N}{\cos \psi_b} \qquad \text{(Eq 7.11)}$$

This value of M is theoretical. It should be corrected for the effect of tooth geometry deviations and runout by decreasing the base tooth thickness. The value of allowable tolerances for each gear size and quality number is obtained from AGMA 2000–A88.

$$t_{bm} = t_b - V_{rT} \tan \phi_m - V_{apk} \cos \phi_m \qquad \text{(Eq 7.12)}$$

$$M_m = [(S_{best} - 1) p_b + t_{bm}] \cos \psi_b \qquad \text{(Eq 7.13)}$$

where

V_{rT} = Radial Runout Tolerance of gear teeth, from AGMA 2000–A88, in (mm)

t_{bm} = transverse base tooth thickness, modified for runout and pitch deviation, in (mm)

V_{apk} = accumulated pitch variation, sector of k pitches[*], see Appendix E of AGMA 2000–A88, in (mm)

M_m = span measurement, modified for tooth deviations, in (mm)

ϕ_m = transverse pressure angle at measuring diameter

For external gears

$$\phi_m \approx \arccos \left[\frac{D_b}{D_o - 2a} \right] \text{ approximately} \qquad \text{(Eq 7.14)}$$

For internal gears

$$\phi_m \approx \arccos \left[\frac{D_b}{D_i + 2a} \right] \text{ approximately} \qquad \text{(Eq 7.15)}$$

The effects of lead and profile deviations have been ignored since they are usually much smaller than runout and pitch.

Number of teeth to be spanned for internal spur gears (see Fig 7–4).

$$L_{max} = \sqrt{(D_i + 4a)^2 - D_b^2} \qquad \text{(Eq 7.16)}$$

The limit diameter is approximated by $D_i + 4a$

$$L_{min} = \sqrt{D_i^2 - D_b^2} \qquad \text{(Eq 7.17)}$$

$$L_{best} = \sqrt{(D_i + 2a)^2 - D_b^2} \qquad \text{(Eq 7.18)}$$

$$S_{max} = \text{integer portion of} \left[\left(\frac{L_{max} + t_b}{p_b} \right) - 1 \right] \qquad \text{(Eq 7.19)}$$

$$S_{min} = \text{integer portion of} \left[\left(\frac{L_{min} + t_b}{p_b} \right) - 1 \right] \qquad \text{(Eq 7.20)}$$

$$S_{best} = \text{rounded value of} \left[\left(\frac{L_{best} + t_b}{p_b} \right) - 1 \right] \qquad \text{(Eq 7.21)}$$

$$1 \leqq S_{min} \leqq S_{best} \leqq S_{max} \qquad \text{(Eq 7.22)}$$

Calculate span:

$$M = (S_{best} + 1) p_b - t_b \qquad \text{(Eq 7.23)}$$

To correct this theoretical span for tooth geometry deviations, t_{bm} must be calculated per Eq 7.12.

$$M_m = (S_{best} + 1) p_b - t_{bm} \qquad \text{(Eq 7.24)}$$

[*] Values of V_{apk} are not given in AGMA 2000–A88, and are a designers option (See Appendix A).

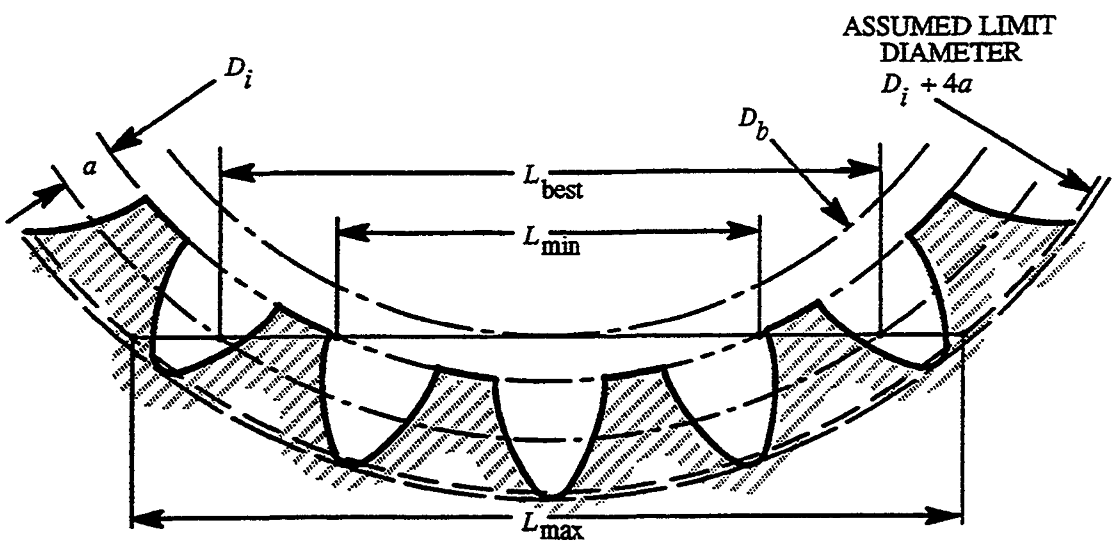


Fig 7-4 Limits of Span Measurement for Internal Gear

8. Composite Action Test Measurement

8.1 Advantages of Composite Action Test Measurement. This method measures functional tooth thickness, since it includes the effects of all tooth deviations. See Fig 8-1, Appendix A and AGMA 2000-A88 for a detailed description.

Where the size of the work permits and the tooling can be justified a composite action test, test radius measurement, is the best method to inspect tooth thickness.

Composite action test measurement inspects every tooth of the work gear in one operation. This is much faster than making multiple measurements with the other methods.

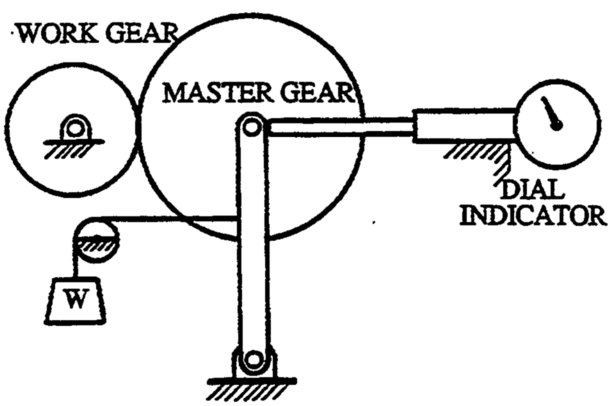


Fig 8-1 Schematic of Composite Action Test Measurement

8.2 Limitations of Composite Action Test Measurement. This method is limited to medium and smaller gears, since testing machines capable of more than twenty inch center distance are rarely available. In special circumstances testing can be accomplished in place on the cutting machine.

Small lot producers encounter significant tooling costs in using the test radius method, since special mounting fixtures and master gears are often required. Careful design to use existing tooling can save some of this expense.

Special attention must be paid to mounting surfaces, to assure that the test performed is representative of the gear as it will be installed.

Special machines or attachments are required for internal gears.

Test machines must be carefully calibrated, particularly for fine pitch and high quality gears. Refer to AGMA 2000 for detailed calibration instructions.

8.3 Master Gears. Master gears suitable for checking most spur gears are available in sizes and tooth proportions standardized by their manufacturers (see AGMA 2000-A88). The tooth thickness of these master gears is made equal or close to one half of the circular pitch at the standard pitch diameter.

The proportions of the master gear must be checked for proper meshing with the work gear to be sure that contact takes place near to the tip and true involute form diameters, without interference.

Master gears are usually marked with a *test radius* which is the radius at which they would mesh with a standard mating gear having a tooth thickness at D_S of $(\pi D_S/2N)$.

Special master gears are often required for spur gears with nonstandard proportions.

Helical gears usually require special master gears.

Master gears must be made very accurately since any deviation in the master gear is added, in the test results, to the deviations in the work gear. Accuracy requirements for master gears are included in AGMA 2000-A88.

8.4 Calculation for Composite Action Test Measurement. The following method applies to external gears.

8.4.1 Maximum Test Radius. The maximum test radius is based on the maximum effective tooth thickness as defined in 3.1.4. The calculation method assumes that the errors in the master gear are too small to affect the test results. This requires a very accurate master gear, if precision gears are to be measured.

If two gears are in tight mesh, the sum of their tooth thicknesses on their operating pitch circles is equal to the circular pitch on that circle. Also, the operating pitch diameters of the two gears must be in proportion to the numbers of teeth. These relationships, with the fundamental tooth thickness equations, yield a series of simultaneous equations, from which the operating transverse pressure angle can be found.

$$\phi_3 = \text{arc inv}\left[\frac{t_{b1} + t_{b2} - p_b}{D_{b1} + D_{b2}}\right] \qquad \text{(Eq.8.1)}$$

where

t_{b1} = maximum transverse base tooth thickness of test gear, in (mm)

t_{b2} = transverse base tooth thickness of master gear, in (mm)

D_{b1} = base circle diameter of test gear, in (mm)

D_{b2} = base circle diameter of master gear, in (mm)

ϕ_3 = transverse operating pressure angle in tight mesh

$$\phi_3 = \text{arc inv}\left[\frac{P_d\left(t_1 + t_2\right) - \pi}{N_1 + N_2} + \text{inv}\,\phi_s\right] \qquad \text{(Eq.8.2)}$$

where

P_d =transverse standard diametral pitch

t_1 = maximum transverse tooth thickness of the test gear at ϕ_s, in (mm)

t_2 = transverse tooth thickness of the master gear at ϕ_s, in (mm)

N_1 = number of teeth in the test gear

N_2 = number of teeth in the master gear.

All measurements are in the transverse plane

$$C_{max} = \frac{D_{b1}}{2\cos\phi_3}\,\frac{\left(N_1 + N_2\right)}{N_1} \qquad \text{(Eq.8.3)}$$

where

C_{max} = maximum center distance, in (mm)

The maximum test radius, $R_{T\,max}$, is:

$$R_{T\,max} = C_{max} - R_m \qquad \text{(Eq.8.4)}$$

where

R_m = master gear test radius, in (mm)

8.4.2 Minimum Test Radius. Figure 8-2 illustrates a typical composite action test chart. The "trace for maximum gear" represents a gear which has a tooth at the maximum effective thickness, t_{max}. The tolerance band for composite action test or test center distance must allow the full deviation of the total composite tolerance plus the tooth thickness tolerance. Both components vary with the test gear size and quality. AGMA 2000-A88 provides appropriate values.

$$C_{min} = C_{max} - V_{cq} - \frac{t_T}{2\tan\phi_3}[*] \qquad \text{(Eq.8.5)}$$

where

C_{min} = minimum center distance

$$R_{T\,min} = C_{min} - R_m \qquad \text{(Eq.8.6)}$$

[*] The use of ϕ_3 for the minimum pressure angle is an approximation. If greater accuracy is required, recalculate, using Eq. 8.1 and C_{min}, iterating for a final value.

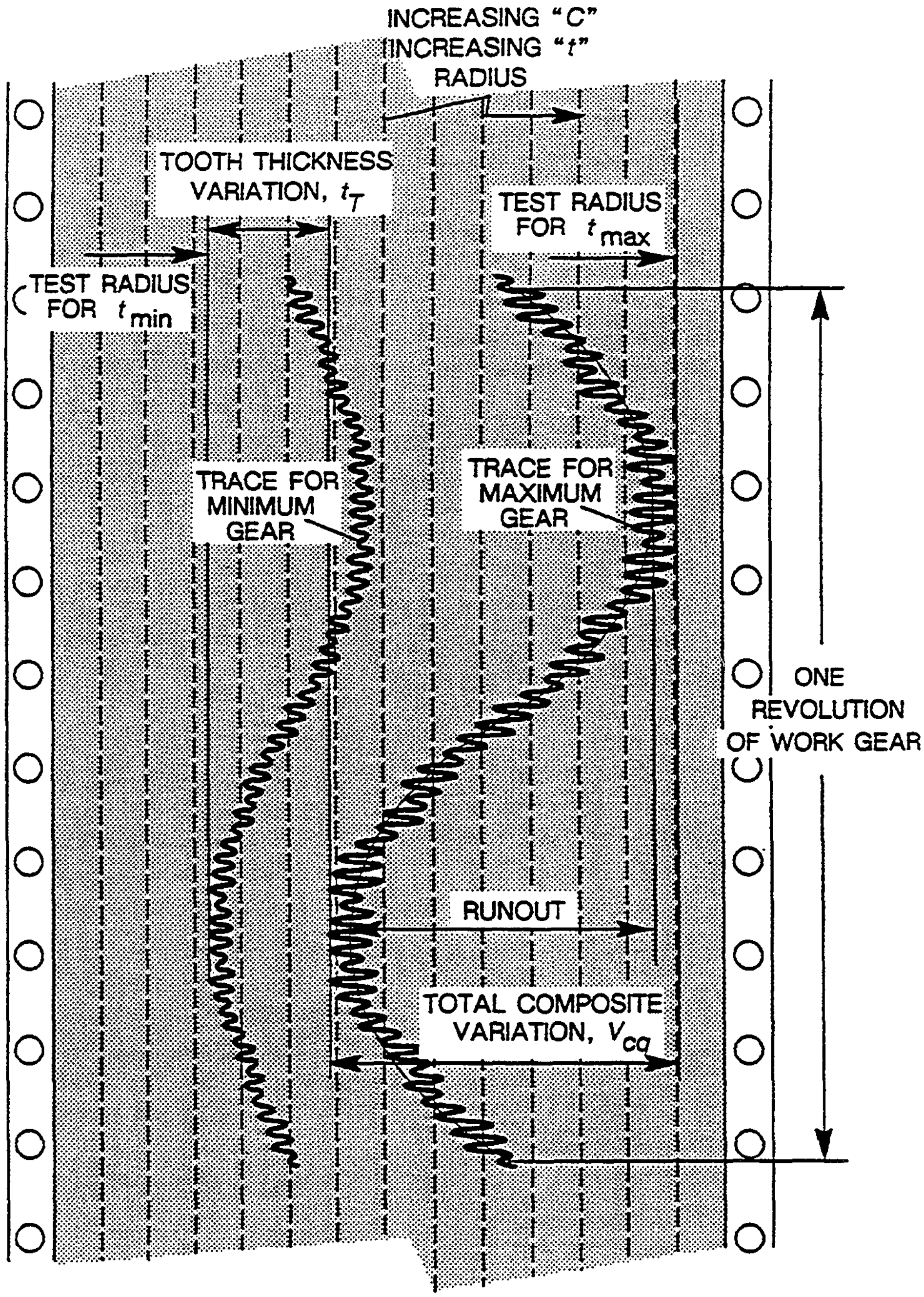


Fig 8–2 Composite Action Test Measurement of Tooth Thickness

Appendix A
Backlash and Tooth Thickness Tolerance [1]

[This Appendix is not a part of ANSI/AGMA 2002–B88, *Tooth Thickness Specification and Measurement,* but is included for information purposes only.]

A1. Purpose. This Appendix provides a rational method to select gear tooth thickness tolerances and minimum backlash. It also provides a method to calculate maximum expected backlash, in a gear mesh, using minimum backlash, tooth thickness tolerances, center distance tolerance and gear tooth quality tolerances. Suggested values for minimum backlash are included.

A2. Backlash. Backlash, B, in an assembled gear set is the clearance between the teeth of the meshing gears. It is the amount by which the width of the tooth space exceeds the tooth thickness of the engaging tooth on the operating pitch circles (See Fig A–1). Backlash may be measured in the normal plane or along the line of action, but it is calculated and specified in the transverse plane or in the plane of action.

An individual gear does not have backlash, it has only a tooth thickness. Backlash in a mesh is governed by the center distance at which the gears are operated and the effective tooth thickness of each of the gears.

Some backlash should be present in all gear meshes. It is required to assure that the nondriving sides of the teeth do not make contact. Backlash in a given mesh varies during operation as a result of changes in speed, temperature, load, etc. Adequate backlash should be present during static conditions, when it can be measured, to assure sufficient backlash under load at the most adverse operating condition.

The amount of backlash required depends on the size of the gears, their quality, mounting and the application.

A3. Maximum Tooth Thickness. Maximum tooth thickness of a gear is determined, in accordance with Eq 3.1, as if the gear were in mesh with a perfect mating gear at the minimum center distance, allowing for the desired minimum backlash. Tooth thickness variations reduce the maximum tooth thickness from the maximum value, and increase backlash.

For gears intended to operate at standard center distance, with standard tip diameters, the theoretical or basic tooth thickness is customarily equal to one half the circular pitch on the standard pitch circle. Unless otherwise specified, the actual maximum tooth thickness on an unassembled gear will usually be less than the theoretical value, since the manufacturer usually makes a reduction in tooth thickness to allow for backlash.

A4. Minimum Backlash. Minimum backlash, B_{min}, is the minimum backlash allowable when the gear tooth with the greatest allowable effective tooth thickness is in mesh with a mating tooth having its greatest allowable effective tooth thickness at the tightest allowable center distance, under static conditions. This is the traditional "backlash allowance" provided by the designer to provide for:

(1) deflections of housings, shafts and bearings

(2) misalignments of gear axes due to housing deviations and bearing clearances

(3) skew of gear axes due to housing deviations and bearing clearances

(4) mounting errors such as shaft eccentricity

(5) bearing runouts

(6) temperature effects (a function of temperature differences between housing and gear elements, center distance and material difference)

(7) centrifugal growth of rotating elements

(8) other factors, such as allowance for contamination of lubricant and swelling of nonmetallic gear materials

The European treatment of backlash allowance is described in DIN 3967 [5].

The value of minimum backlash can be small, if the factors listed above are controlled. Each factor can be evaluated, by analyzing the tolerances, and a minimum requirement calculated. Judgment and experience are required to assess the minimum expected requirement, since the worst case tolerances are not likely to coincide.

[1] Numbers in brackets refer to the bibliography.

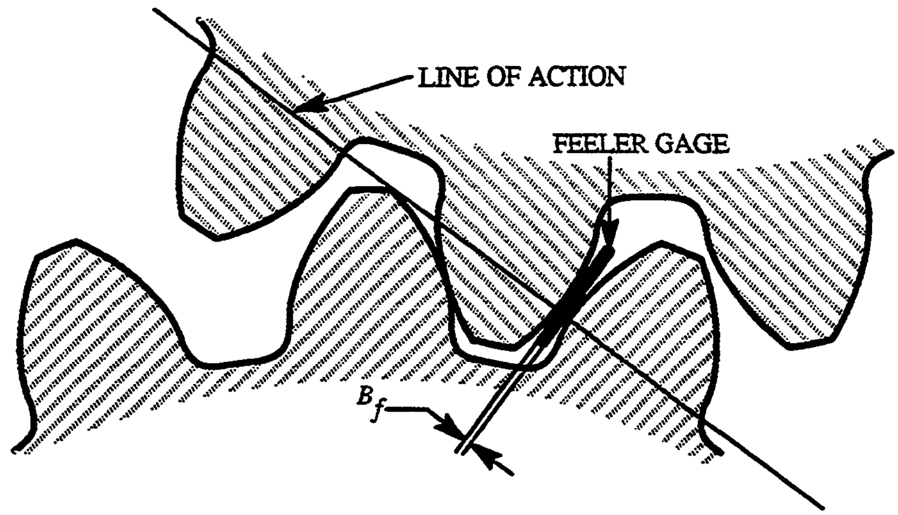


Fig A–1 Feeler Gage Backlash Measurement (Normal Plane)

Table A–1 shows conservative values of minimum backlash for ferrous gears in ferrous housings, operating at pitchline speeds less than 3000 fpm (15 m/s), with typical commercial manufacturing tolerances for housings, shafts and bearings.

The values found in Table A–1 may be calculated from Eq A.1.

$$B_{min} = 0.0024 + 0.0005\,C + \frac{0.03}{P_{nd}} \qquad \text{(Eq A.1)}$$

$$B_{min} = 0.06 + 0.0005C + 0.03\ m_n \qquad \text{(Eq A.1M)}$$

NOTE: C must be an absolute value.

Table A–1
Minimum Backlash, B_{min}, for Coarse Pitch Gears (inch values)

P_{nd}	Minimum Center Distance, *C*, in					
	2	4	8	16	32	64
18	0.005	0.006	—	—	—	—
12	0.006	0.007	0.009	—	—	—
8	0.007	0.008	0.010	0.014	—	—
5	—	0.010	0.012	0.016	—	—
3	—	0.014	0.016	0.020	0.028	—
2	—	—	0.021	0.025	0.033	—
1 1/4	—	—	—	0.034	0.042	0.058

Table A–1(M)
Minimum Backlash, B_{min}, for Coarse Pitch Gears (millimeter values)

m_n	Minimum Center Distance, *C*, mm					
	50	100	200	400	800	1,600
1.5	0.13	0.16	—	—	—	—
2	0.14	0.17	0.22	—	—	—
3	0.18	0.20	0.25	0.35	—	—
5	—	0.26	0.31	0.41	—	—
8	—	0.35	0.40	0.50	0.70	—
12	—	—	0.52	0.62	0.82	—
18	—	—	—	0.80	1.00	1.40

A5. Tooth Thickness Tolerance.

A5.1 Basic Tolerance Calculation. t_{max} is calculated in accordance with Eq 3.1. The sum of $t_{G\,max}$ and $t_{P\,max}$ is controlled by the center distance, *C*, and the minimum backlash, B_{min}. The values for tooth thickness are normally chosen by the designer, considering such things as balancing bending strength, sliding velocity, and cutting depths. The values used in the numerical examples are chosen for some arbitrary objective.

t_{min} is calculated by subtracting a tooth thickness tolerance selected from the table 6.3 in AGMA 2000–A88 (Eq 5.13, 5.14 or 5.15), and an allowance for tooth thickness variation as a function of total composite tolerance for the appropriate quality number, from t_{max}.

$$t_{min} = t_{max} - t_T - 2\,V_{cq} \tan \phi' \qquad \text{(Eq 3.3)}$$

The allowance for total composite variation is sometimes ignored, but the practice should be discouraged, since it may lead to interference in the assembled unit, if the parts are near the maximum allowable variation. An example calculation sequence of t_{min} and t_{max} for an AGMA Quality Q9–B gearset, as listed in Table A–2, is given in Table A–3[*].

Table A–2
Typical Input Data
(Dimensions in inches)

Data Item	Pinion	Both	Gear
n,N	34		197
P_{nd}		6	
ϕ_c		20	
p_x		3.01529	
Q_n		Q9–B	
F		6.030	
t_{Pmax}	0.3600		
C_{min}		19.801	
C_{max}		19.806	
D_o	$\frac{6.426}{6.421}$		$\frac{33.842}{33.837}$

Table A–3
Preliminary Calculations for
AGMA Quality Q9–B

Data Item	Pinion	Gear	Notes
ψ_b	9.391 27°	–.–	Eq 3.7
D_b	5.397 26	31.272 38	Eq 3.6
D'	5.828 87	33.773 13	Eq 3.5
ϕ'	22.187 28°	–.–	Eq 3.4
p'	0.538 59	–.–	Eq 3.2
B_{min}	0.017 30	–.–	Eq A.1
t_{Gmax}	–.–	0.161 29	Eq 3.1
t_T	0.003 2	0.003 2	AGMA 2000
V_{cq}	0.003 8	0.005 3	AGMA 2000
t_{min}	0.353 70	0.153 77	Eq 3.3
p_b	0.498 71	–.–	Eq 7.11

If this gearset were used in a high speed drive, manufactured to AGMA Quality Q12–C, the data might be modified as shown in Table A–4[*].

Table A–4
Preliminary Calculation for
AGMA Quality Q12–C

Data Item	Pinion	Gear	Notes
Q_n	Q12–C		
B_{min}	0.010		Designer's option
t_{max}	0.363 0	0.165 59	Eq 3.1
t_T	0.001 6	0.001 6	AGMA 2000
V_{cq}	0.001 4	0.001 9	AGMA 2000
t_{min}	0.360 26	0.162 44	Eq 3.3

The values of t_{max} and t_{min} calculated above are the values which would be measured by a composite action test. The values which would be used for other inspection methods are calculated in Table A–5[*].

A5.2 Specifications For Tooth Thickness Measurement. The maximum tooth thickness specified for any inspection method should be reduced to be sure that the effects of runout and other tooth cutting variations on the inspection results do not increase the maximum effective tooth thickness. The minimum specified tooth thickness should also be reduced, so that the tooth thickness tolerance, selected from AGMA 2000–A88, is available for economical gear manufacture, and is not used up by the other tolerances implied by the quality class.

A6. Maximum Backlash. The maximum backlash in a gear set is the sum of B_{min}, tooth thickness tolerance, the effect of center distance variation, and the effects of gear tooth geometry variation. The theoretical maximum backlash occurs when two perfect gears, made to the minimum tooth thickness specification, are meshed at the loosest allowable center distance. The loosest center distance is the maximum for external gears or the minimum for internals.

* Note: For the purpose of the mathematical example the values have been calculated to 16 significant figures and the results rounded to five decimal places. Normal practice would be to round to four decimal places.

Table A-5
Example Calculations for Tooth Thickness Measurements (Dimensions in inches)

Data Item	Pinion	Gear	Notes
Preliminary Data from Tables A-2 and A-3			
D'	5.828 87	33.773 13	
t_{max}	0.360 0	0.161 29	
t_{min}	0.353 70	0.153 77	
Q_n	Q9-B	Q9-B	
D_b	5.397 26	31.272 38	
$t_{b\,max}$	0.444 49	0.793 32	Eq 4.11
$t_{b\,min}$	0.438 65	0.786 35	Eq 4.11
Chordal Measurement:			
a	0.166 67	0.166 67	Eq 5.1
V_{rT}	0.002 7	0.004 0	AGMA 2000-A88
R_{max}	3.047 68	16.756 33	Eq 5.6
$t_{R\,max}$	0.248 96	0.262 95	Eq 5.5
ψ_R	0.184 66 rad	0.175 42 rad	Eq 5.4
$t_{nR\,max}$	0.244 72	0.258 92	Eq 5.3
a_c	0.167 69	0.165 15	Eq 5.8
$t_{m\,max}$	0.244 66	0.258 91	Eq 5.10
$t_{R\,min}$	0.242 37	0.255 49	Eq 5.5
$t_{m\,min}$	0.238 19	0.251 56	Eq 5.10
Pin Measurement:			
W	0.384	0.288	Section 6.4 and Table 6-1
$R_{1W\,max}$	3.352 09	16.953 43	Eq 6.11
$R_{1W\,min}$	3.346 47	16.943 93	Eq 6.11
$D_{2W\,max}$ (even)	6.704 18	- -	Eq 6.16
$D_{2W\,max}$ (odd)	- -	33.905 80	Eq 6.17
$D_{2W\,min}$ (theoretical)	6.692 95	33.886 78	- -
V_{rW} correction for runout	0.001 35	0.002 00	Eq 6.20
corrected $D_{2W\,max}$	6.702 83	33.903 80	6.5.5
corrected $D_{2W\,min}$	6.691 60	33.884 78	6.5.5
Span Measurement:			
S_{min}	5	23	Eq 7.2
S_{max}	7	25	Eq 7.4
S_{best}	6	24	Eq 7.8
M_{max}	2.898 64	12.099 19	Eq 7.10
M_{min}	2.892 88	12.092 32	Eq 7.10
$\cos\phi_m$	0.885 9	0.933 3	Eq 7.14
V_{rT}	0.002 7	0.004 0	AGMA 2000-A88
V_{apk}	0.001 7	0.004 0	[*]

[*] Values of V_{apk} are not in AGMA 2000-A88, approximate by; $V_{apk} = V_p \left(1 + \frac{S}{4}\right)$, and $V_{apk} \le V_{rT}$

Table A-5 (cont)
Example Calculations for Tooth Thickness Measurements (Dimensions in inches)

Data Item	Pinion	Gear	Notes
Span Measurement: (cont)			
$t_{bm\ \max}$	0.441 57	0.788 05	Eq 7.12
$t_{bm\ \min}$	0.435 73	0.781 08	Eq 7.12
$M_{m\ \max}$	2.895 76	12.093 99	Eq 7.13
$M_{m\ \min}$	2.890 00	12.087 12	Eq 7.13
Composite Action Test:			**Master**
N_2			24 TEETH
t_{b2}			0.309 61
D_{b2}			3.809 83
ψ_s			0.174 53 rad
D_s			4.061 70
R_m			2.030 80
ϕ_3	0.425 76 rad	0.365 69 rad	Eq 8.1
$C_{\max}$	5.054 81	18.783 10	Eq 8.3
$R_{T\max}$	3.024 01	16.752 30	Eq 8.4
$C_{\min}$	5.047 48	18.773 62	Eq 8.5
$R_{T\min}$	3.016 68	16.742 82	Eq 8.6

NOTE: The example is calculated as if the same master gear were used for both parts. This would not be true in practice, since the parts are of opposite hands.

The maximum theoretical backlash will also occur when two teeth, made to the minimum effective tooth thickness, coincide while operating at the loosest center distance. Neither occurrence is likely in practice.

$$B_{\max} = p' - t_{G\min} - t_{P\min} + \left[C_{\max} - C_{\min}\right] 2 \tan\phi' \qquad \text{(Eq. A.2)}$$

where

$t_{G\min}$ = minimum transverse tooth thickness of gear

$t_{P\min}$ = minimum transverse tooth thickness of pinion

$C_{\max}$ = maximum center distance

$C_{\min}$ = minimum center distance

The maximum expected backlash is a function of $B_{\max}$ and the statistical distribution of the individual elements of tooth and center distance variation. Any tooth variations due to manufacturing will decrease the maximum expected backlash. Experience and judgment are required to estimate reasonable values.

If maximum backlash must be controlled, a careful study of each element of maximum backlash must be made and a quality class selected which will limit tooth variations as necessary.

Example: Using the values for the example gearsets of Tables A-2 to A-4.

$B_{\max}$ = 0.034 4 in, for the Q9-B set

$B_{\max}$ = 0.019 2 in, for the Q12-C set

This example emphasizes the importance of quality number, if backlash is to be limited.

When maximum backlash of an assembled unit, particularly a unit with multiple stages, is used as an acceptance criterion, the maximum acceptable value must be carefully chosen to allow reasonable manufacturing tolerances for each part in the assembly.

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Appendix B
Alternate Methods of Tooth Thickness Measurement

[This Appendix is not a part of AGMA 2002–B88, *Tooth Thickness Specification and Measurement*, but is included for information purposes only.]

B1. Purpose. This Appendix provides information on the Measuring Block, Tooth Comparator, Coordinate Measuring Machine (CMM) and other alternate tooth thickness measurement methods, including standard block dimensions and a calculation method.

B2. Measuring Blocks.

B2.1 Advantages of Measuring Blocks. Gear measuring blocks can be used on external spur and helical gears. The blocks rest firmly on the lines of action of helical gears without rocking, which is an advantage over pins.

Measurements made over blocks are not affected by deviations in blank geometry.

Measurements over blocks have a similar *amplifying factor* to measurements over pins.

B2.2 Limitations of Measuring Blocks. Measuring blocks cannot be used on internal teeth.

They are more expensive than pins, and are seldom specified on new drawings.

Block measurements are independent of the mounting diameter of the part, so they require an allowance for eccentricity.

B2.3 Measuring Block Sets. These blocks are in effect theoretical rack teeth of standard proportions, made to the exact tooth thickness and normal pressure angle of the standard rack. Each set consists of three blocks, two males and one female, and is constructed for a specific pitch and pressure angle. Two male blocks are used on gears with an even number of teeth, and the combination of one male and one female for gears with an odd number of teeth. Standard proportions for these blocks are shown in figures Figs B–1 and B–2. [6]

B2.4 Calculation for Measuring Blocks.

$$R_{B1} = \frac{D_s}{2} + \frac{1.6}{P_{nd}} + \left(\frac{t_{ns} - \frac{\pi}{2 P_{nd}}}{2 \tan \phi_c} \right) \qquad \text{(Eq B.1)}$$

$$R_{B2} = \frac{D_s}{2} + \frac{3.0}{P_{nd}} + \left(\frac{t_{ns} - \frac{\pi}{2 P_{nd}}}{2 \tan \phi_c} \right) \qquad \text{(Eq B.2)}$$

for even number of teeth

$$D_B = 2 R_{B1} \qquad \text{(Eq B.3)}$$

for odd number of teeth

$$D_B = R_{B1} + R_{B2} \qquad \text{(Eq B.4)}$$

where

t_{nr}	=	normal tooth thickness of standard rack at the reference line, in
	=	$\pi / 2 P_{nd}$
t_{ns}	=	normal generating tooth thickness of the gear, in
R_{B1}	=	Radius over male block, in
R_{B2}	=	Radius over female block, in
D_B	=	Measuring Dimension, in

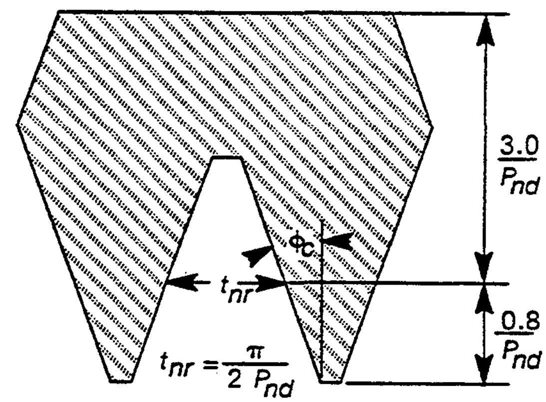

Fig B–1 Female Block

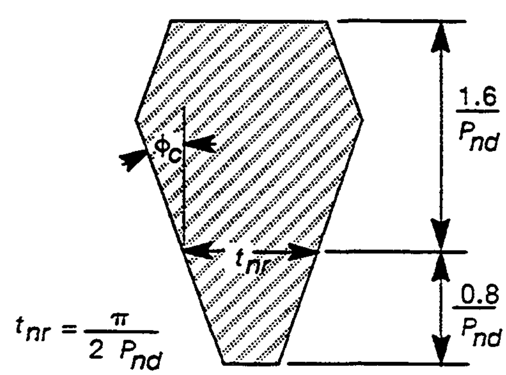

Fig B–2 Male Block

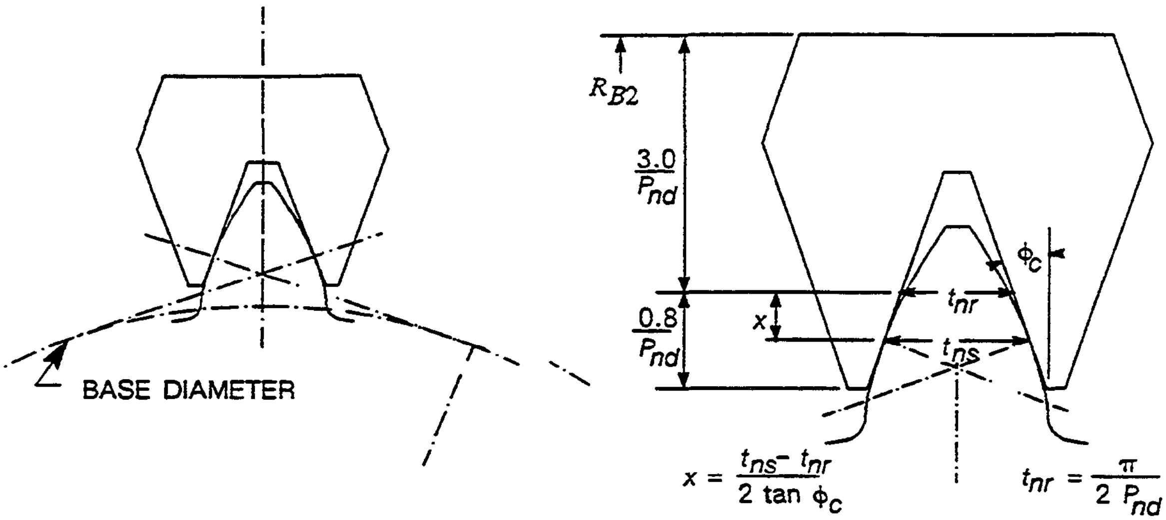

Fig B–3 Measuring Block Engagement, Spur Gear

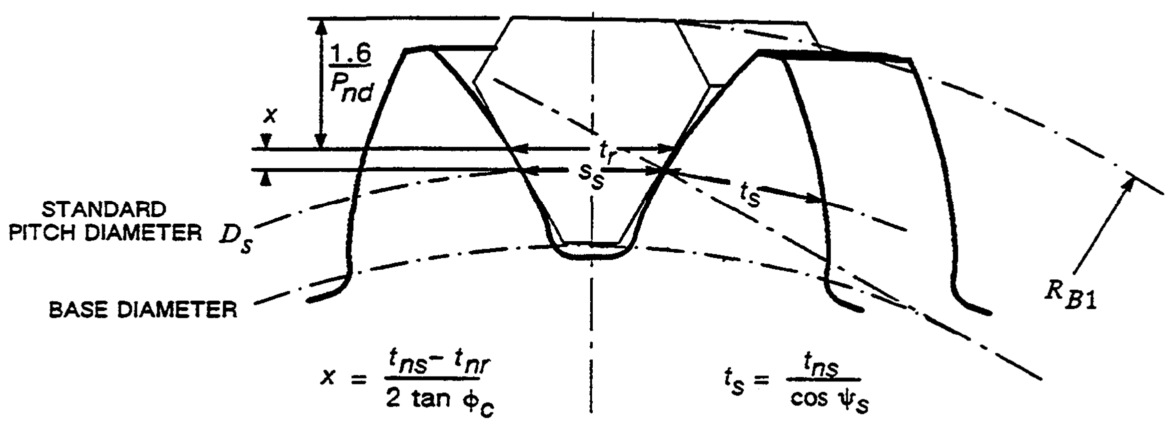

Fig B–4 Measuring Block Engagement, Helical Gear

B.2.5 Correction for Tooth Deviations. The effect of allowable pitch deviation is much smaller than allowable runout, so it can be ignored, except with very low numbers of teeth and other unusual cases.

If block measurements are made as a radius to one block from the mounting diameter, the effects of runout are included and no correction is necessary. If the measurements are made with two blocks, the effects of runout should be calculated and D_B adjusted accordingly.

The amount of correction is:

$$V_{rB} = \frac{V_{rT}}{2} \qquad \text{(Eq B.5)}$$

The direction of the correction reduces the allowable tooth thickness (see 3.1).

where

V_{rB} = correction to block measurement for runout, in (mm)

V_{rT} = allowable runout of gear teeth, from AGMA 2000–A88, in (mm)

B3. CNC Gear Tooth Thickness Measurement.

B3.1 Alternate Methods. This method of gear tooth thickness measurement is based on using a CNC Gear Measuring Instrument equipped with a high resolution rotary table and measuring stylus which can be moved to a known position.

B3.1.1 The instrument stylus should have a small tip radius.

B3.1.2 The instrument tip contact is to be known relative to the center of the rotary table.

B3.2 **General Method of Measurement.**

B3.2.1 The gear to be measured is mounted concentric to the rotary table.

B3.2.2 The probe is moved to the measuring radius, R, into the space adjacent to the tooth to be measured.

B3.2.3 The gear is rotated until the tooth flank contacts the probe and is at radial position on the probe.

B3.2.4 The rotary position is recorded in radians, θ_0 .

B3.2.5 The probe is moved into the next space for the opposite tooth flank measurement and positioned to the measuring radius.

B3.2.6 The gear is rotated back until the opposite tooth flank (from step B3.2.3) is contacted with the probe and at the radial position.

B3.2.7 The new rotary position for the opposite flank is recorded in radians, θ_1.

B3.2.8 The tooth thickness can now be computed from the measuring radius, and difference in angular positions in steps B3.2.4 and B3.2.7 according the following equation:

$$T = R(\theta_0 - \theta_1) - \text{tip diameter} \quad \text{(Eq B.6)}$$

= Circular tooth thickness at measuring Radius R

θ = Angular position in radians

B4. Optical Comparator.

An optical comparator is best suited for fine pitch gears (see Fig B-5).

B4.1 Comparator layout of the space from the basic rack for gear to be measured is made to a suitable scale depending on capacity of the instrument and gear pitch. A scale of at least 20 to 1 is recommended.

B4.2 Position layout and gear to be measured so that the centerline of the rack space and the centerline of the gear tooth are coincident, and pitch line of rack is at a distance equal to the pitch radius from the centerline of the gear.

B4.3 The projection of the gear tooth must fall within the tolerance lines shown. Outside diameter can also be checked by tolerance lines in root of rack.

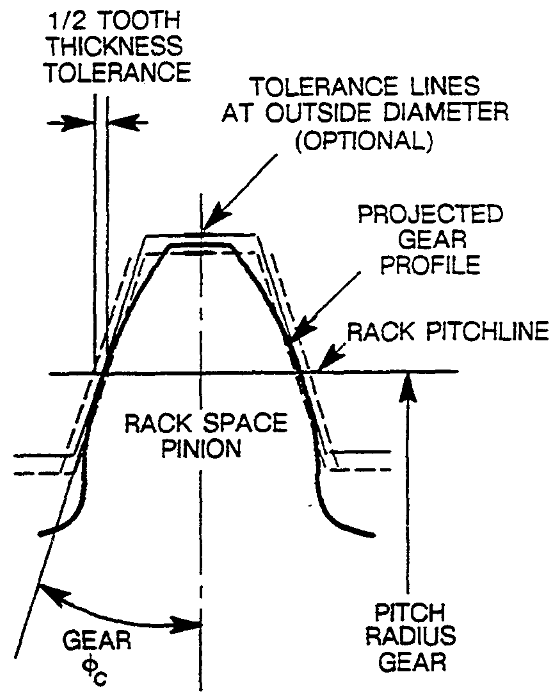

Fig B-5 Optical Comparator Measurement

B5. Tooth Comparator.

B5.1 Advantages of Tooth Comparator. Gear tooth comparator measurements can be made on large external gears without the use of large micrometers. If the outside diameter is accurately known, the method has the same ease of measurement and amplifying factor as measurement over pins or balls. It is not limited by helix angle or face width.

B5.2 Limitations of Tooth Comparator. Gear tooth comparator measurements are affected by all deviations, such as runout, taper, undersize, and oversize in the reference outside diameter of the gear.

This method is seldom specified for new gear designs, but is seen on many older drawings.

The requirement for an accurate outside diameter to be used as a reference surface, separate comparators for each pressure angle, and precision setting blocks for each pitch and pressure angle can outweigh the advantages.

B5.3 Comparators. The gear tooth comparator compares the thickness of a standard rack with the sample gear tooth. The principle can be understood from Fig B-6. The anvils of the comparator are equivalent to the sides of the generating rack tooth and have the same profile angle. The dial indicator is set to zero with the anvils against a standard setting block (having an addendum of $1/P_{nd}$ and a tooth thickness of $\pi/2P_{nd}$). The instrument is then placed on the gear tooth to be measured. The anvils contact the tooth flanks such that the centerline of the tooth at the generating pitch circle is the measurement point and the indicator reads the difference between the actual addendum and the standard addendum.[7] If the outside diameter is known, the tooth thickness can be calculated. Tooth thickness is specified as *thick* (minus reading) or *thin* (plus reading), directly as read from the instrument indicator. See Fig B-7.

B5.4 Calculation. The anvils of the instrument always make contact with the tooth at the standard pitch diameter, D_s. When the instrument is set, the indicator reads zero when the addendum, measured from D_s, is $1/P_{nd}$ and the tooth thickness at D_s is $\pi/2P_{nd}$.

The theoretical indicator deflection, Δ, is:

$$\Delta = \frac{\left(\frac{\pi}{2P_{nd}} - t_{ns}\right)}{2\tan\phi_c} \qquad \text{(Eq B.7)}$$

To correct for actual outside diameter and to account for runout:

$$\Delta h = \Delta - (D_s + 2a - D_{o\max}) + \frac{V_r T}{2} \qquad \text{(Eq B.8)}$$

When tooth thickness is controlled by comparator measurement, outside diameter size and runout and gear tooth runout must be carefully controlled, because they have such a large influence on gear tooth thickness measurements.

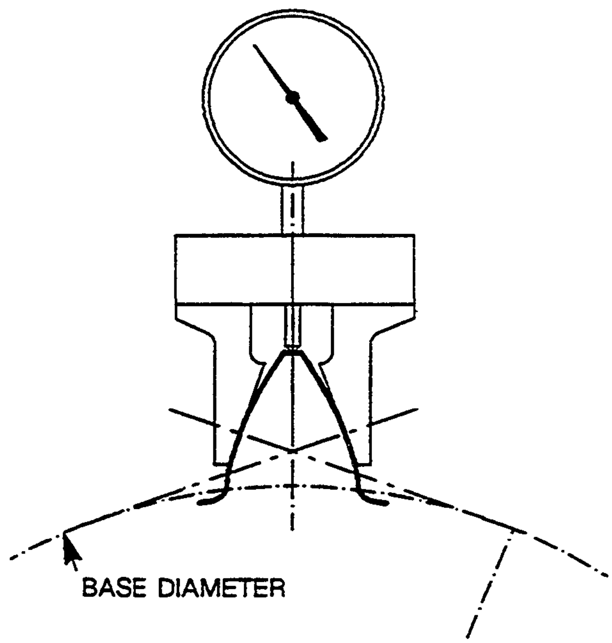

Fig B-6 Measurement of Tooth Thickness by Means of a Gear Tooth Comparator

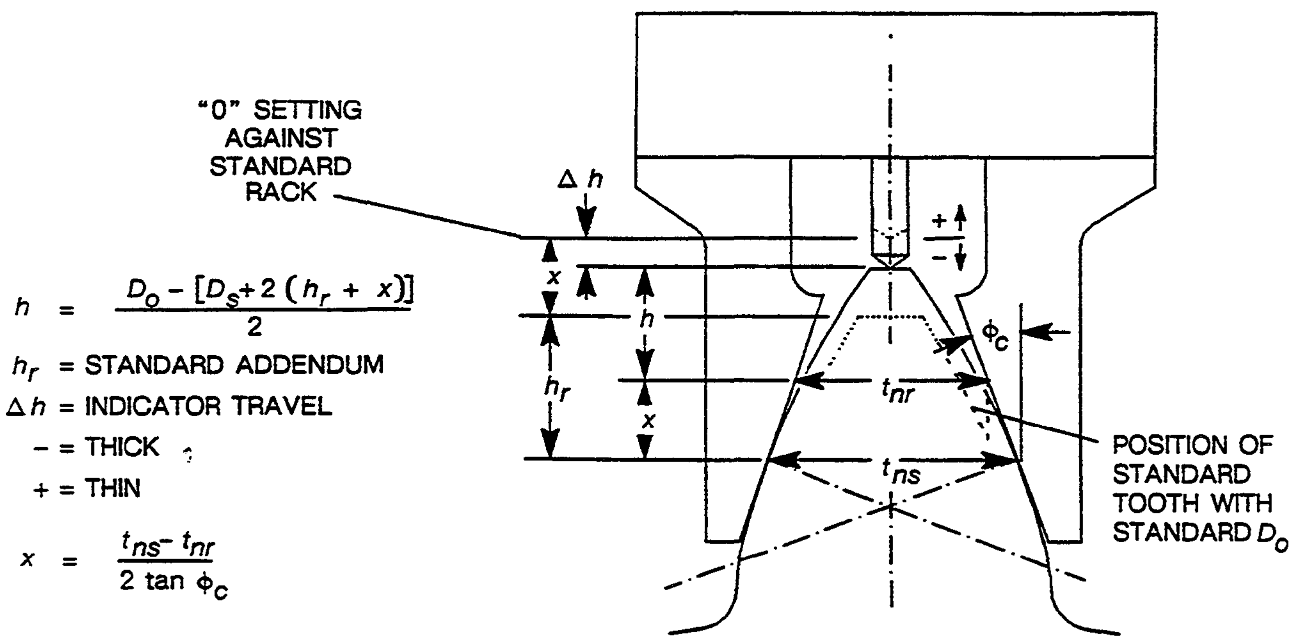

Fig B-7 Comparator Measurement Variations

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