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Geometry / Helical Gears

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Description: Review of geometry for helical gears and helical gear meshes.

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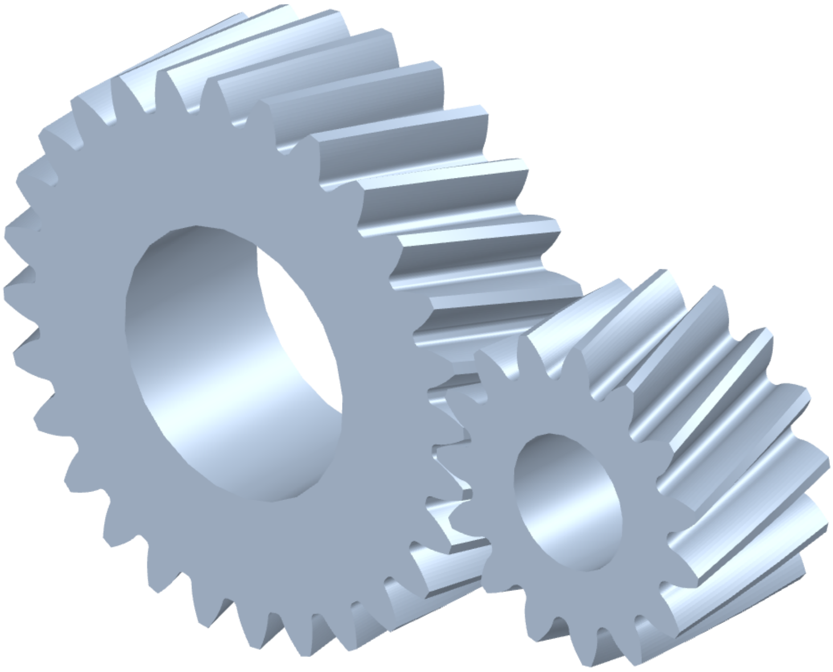
Introduction

The geometry of helical gears and gear meshes is reviewed here. Helical gears have similarities with spur gears, but fundamental differences do exist. Helical and spur gears similarities include:

- Mechanical power is transferred between rotating parallel bodies
- Gear tooth geometry uses the involute curve to transmit motion
- Plane of action is along the path tangent to the involute base circles

The visualization of helical gears clearly shows the key geometric difference between it and a spur gear.



Helical gear pair modeled in Gears App (https://drivetrainhub.com/gears)

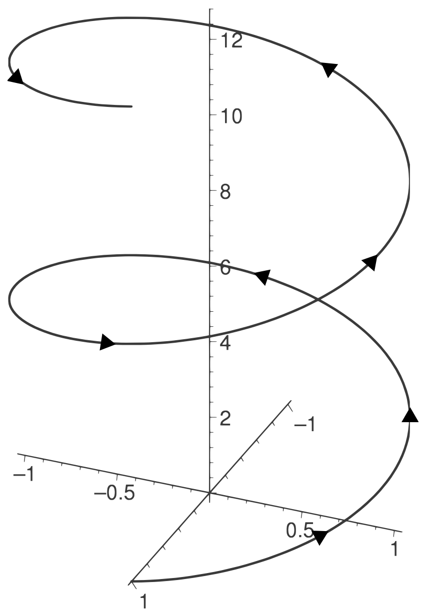
A helical gear is *similar* to a spur gear with an applied twist along its axial axis. Conceptually, this description is sufficient, but the mathematics of helical gear geometry must be considered in more detail to accurately define it. This notebook will review the geometric attributes and their significance to the operation of helical gear pairs.

The geometry of a helical gear is directly dependent on the manufacturing tooling and processes used to make the gear.

To understand the tooling and manufacturing processes used to make a helical gear, see the notebooks on gear tooling. This notebook focuses on the resulting geometry of manufactured gears and gear pairs, not on how they are made.

Helix

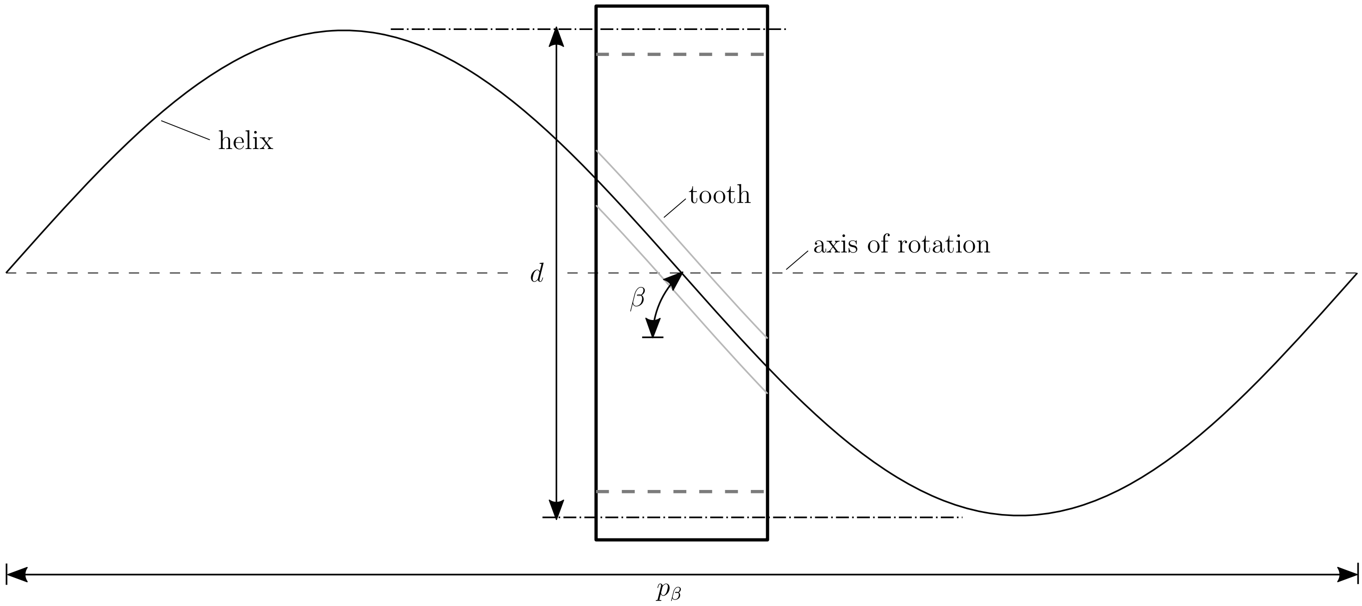
To define the geometry of a helical gear tooth, we shall first define the *helix*. A helix is a shape commonly associated with components such as coil springs, bolt threads, and gear hobs. A helix contains certain characteristics that make it well suited for many applications, including the manufacturing and operation of mechanical gears.



Helix curve (right-handed)

Image credit: [RobHar (//commons.wikimedia.org/wiki/User:RobHar)](//commons.wikimedia.org/wiki/User:RobHar)

For helical gears, a reference helix is defined by a *helix angle*, β, at the theoretical pitch diameter. The sign of the helix angle determines the hand of the helix, with positive (+) being right-hand. The diagram below illustrates the helix angle, not to be confused with the helix lead angle.



Helix angle of gear tooth

If we imagine unwrapping the helix curve along its helix angle, we can derive an expression for the helix pitch length as a function of helix angle and theoretical pitch diameter:

$$p_\beta = \frac{\pi d}{\tan \beta}$$

and by acknowledging the helix pitch length as a constant regardless of diameter, the helix angle at an arbitrary diameter, d_y, is:

$$\beta_y = \tan^{-1}\left(\frac{d_y \tan \beta}{d}\right)$$

For the geometry of a helical gear, the helix angle varies as a function of radius.

This is observed in a helical gear tooth since the involute profile spans over a range of diameters. This results in the helix angle at the base diameter being less than at the tip diameter. This is particularly important when considering contact on the tooth surface and the resulting forces of a helical gear pair.

A helix curve can be expressed in Cartesian coordinates with a set of parametric equations:

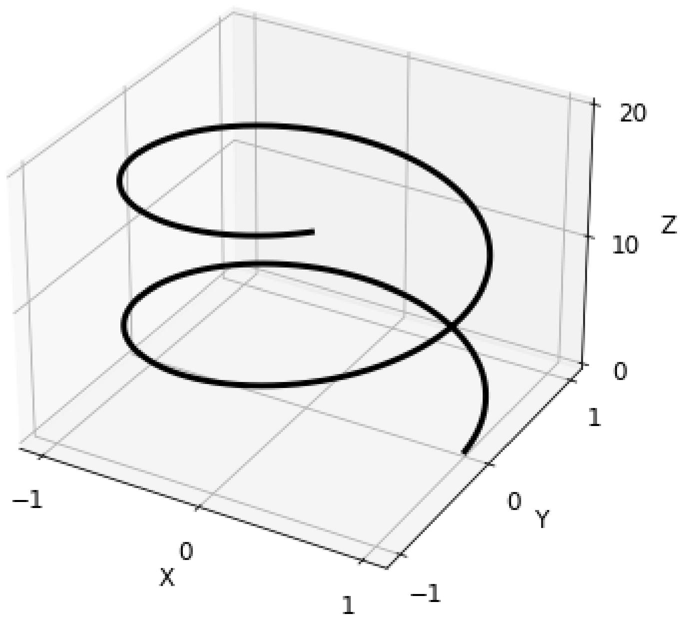
$$x(t) = r \cos t$$

$$y(t) = r \sin t$$

$$z(t) = ct$$

where the helix pitch, i.e. the z distance of one helix loop, equals $2\pi c$.

r	β	b	p_β
1	30	20	10.88

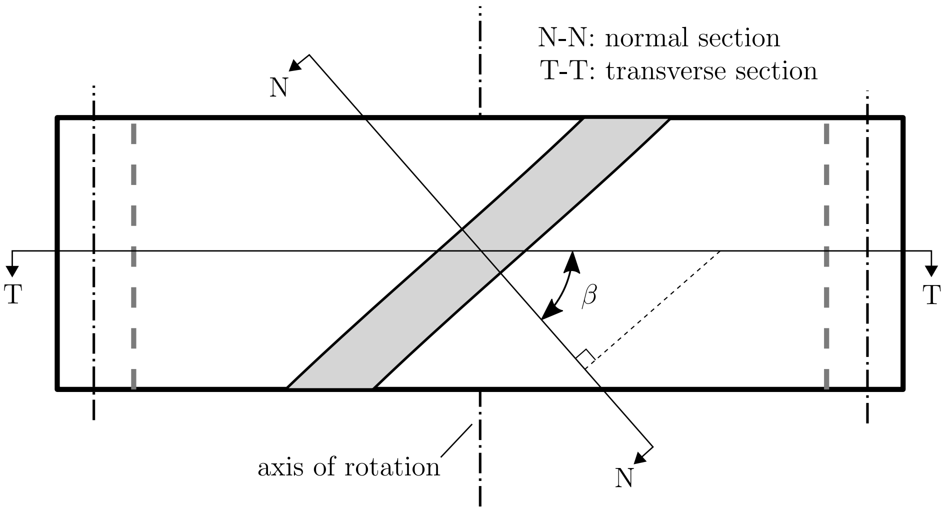


Gear Teeth

This section reviews the geometric properties of helical gear teeth. A working example is provided at the end of the section.

To understand the geometry of a helical gear, it is appropriate to consider both the *transverse* and *normal* planes. These distinct planes exist due to the helical shape of the gear teeth. In the case of spur gears, these planes are coincident since $\beta = 0^\circ$.

- **Transverse plane**: Plane normal to the axis of rotation, i.e. it is the plane of gear rotation.
- **Normal plane**: Plane normal to the gear tooth surface, oriented by the helix angle relative to the transverse plane.



Transverse and normal planes

Lengths in the transverse and normal planes are related by helix angle according to:

$$\cos\beta = \frac{\text{normal length}}{\text{transverse length}}$$

Nomenclature

This table provides a set of input parameters commonly used to define the geometry of helical gear teeth.

Symbol	Description
m_n	Normal module
α_n	Normal pressure angle
z	Number of teeth
β	Helix angle
b	Facewidth
d_a	Tip diameter
d_f	Root diameter
s_n	Normal tooth thickness
ρ_f	Tool tip radius

It is also common to use addendum, dedendum, and profile shift coefficients of the basic rack or cutting tool instead of tip diameter, root diameter, and tooth thickness. The former are better associated with the manufacturing process, while the latter are in reference to the manufactured geometry of a helical gear.

Notice tool tip radius is included in this set of input parameters. It may seem out of place, but it is necessary to define since it relates directly to the manufacturing tooling responsible for the helical gear root geometry. To understand this further, refer to the notebooks on gear tooling.

The next table provides a set of parameters commonly calculated for the geometry of helical gear teeth. Notice that both normal and transverse components exist for many parameters. Certain parameters inherently exist only in the transverse plane, such as diameters and roll angles.

Symbol	Description
m_t	Transverse module
α_t	Transverse pressure angle
d	Theoretical pitch diameter
d_b	Base diameter
d_F	Root fillet boundary diameter
β_b	Base helix angle
p_β	Helix pitch length
p_{bn}	Normal base pitch
p_{bt}	Transverse base pitch
p_a	Axial pitch
P_n	Normal diametral pitch
P_t	Transverse diametral pitch
x	Profile shift
s_n	Normal tooth thickness
s_t	Transverse tooth thickness
s_{na}	Normal tooth tip thickness
s_{ta}	Transverse tooth tip thickness
ψ	Tooth thickness half angle
θ	Roll angle
h_a	Basic rack addendum or tool dedendum
h_f	Basic rack dedendum or tool addendum
ρ_f	Basic rack root radius or tool tip radius

Notice the inverse relationship used for a basic rack and rack cutting tool, such as a hob. Any symbols combined with a superscript asterisk refer to a *coefficient*, a term normalized by the module. For example, x^* is the profile shift coefficient.

Basic Parameters

The basic parameters defining the geometry of helical gear teeth are normal module, normal pressure angle, number of teeth, and helix angle. Symbolically, each is denoted as m_n, α_n, z, and β, respectively. Different basic parameters could be used, but these are the most common.

Module

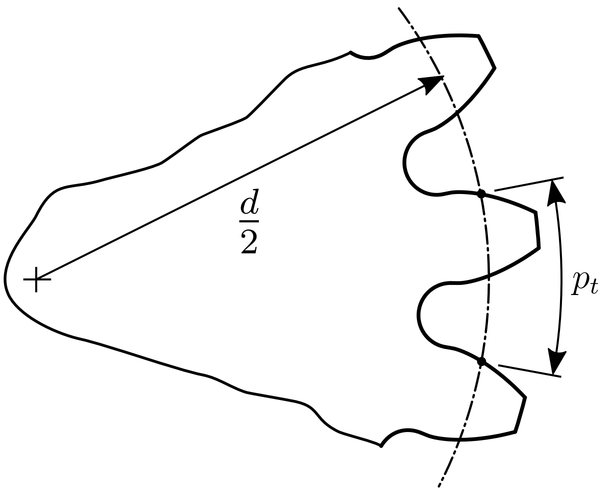
Module is a defining parameter of gear tooth size, with units of mm. Module values exist in both the normal and transerve planes. Geometrically, it is easiest to understand the transverse module, expressed as millimeters of theoretical pitch diameter per tooth:

$$m_t = \frac{d}{z}$$

As a function of basic parameters, the transverse module is calculated as:

$$m_t = \frac{m_n}{\cos\beta}$$

By examining a transverse section of a helical gear, we can derive the transverse pitch and its relationship to module.



Transverse pitch and theoretical pitch diameter

where the transverse pitch equals:

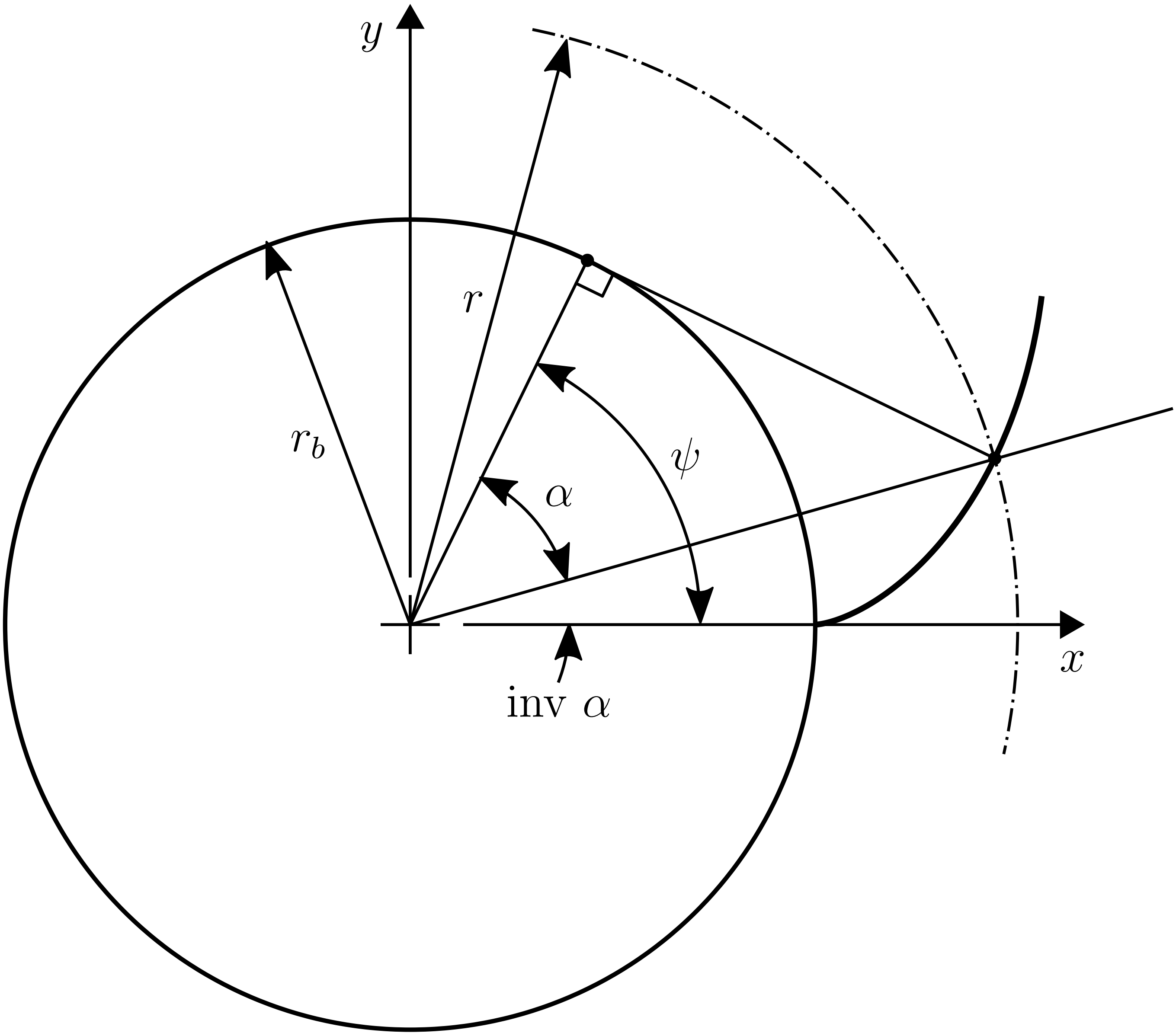
$$p_t = \frac{\pi d}{z} = \pi m_t$$

Diametral pitch, P_d, is the module equivalent parameter used with the system of imperial units, with units of in^{-1}:

$$P_d = \frac{25.4}{m}$$

Pressure Angle

Since helical gears are a form of cylindrical involute gears, the pressure angle at the theoretical pitch diameter is used as a defining parameter. Geometrically, it is easiest to understand the transverse pressure angle as illustrated below.



Involute curve pressure angle in transverse plane

The transverse pressure angle can be calculated as a function of the normal pressure angle and helix angle:

$$\alpha_t = \tan^{-1}\left(\frac{\tan\alpha_n}{\cos\beta}\right)$$

> The pressure angles explained here are sometimes called *reference* values, since they refer to the pressure angles associated with the theoretical pitch diameter. Recall, involute pressure angle changes as a function of diameter. In a later section, the *working* pressure angle of a gear pair is explained.

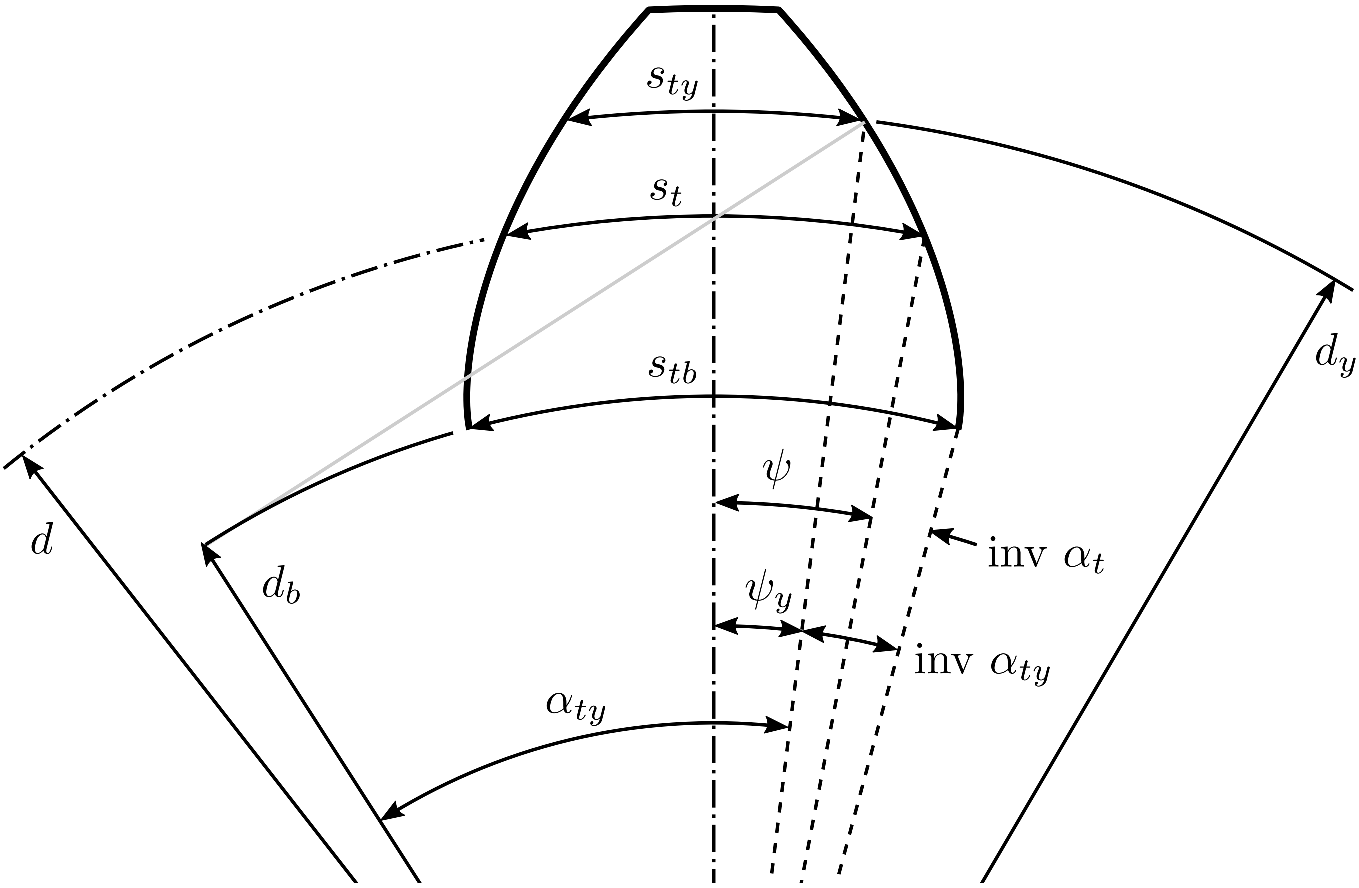
From the previous figure, a useful equation for transverse pressure angle at an arbitrary diameter, d_y, is derived as:

$$\cos \alpha_{ty} = \frac{d_b}{d_y}$$

TODO guidelines on selecting pressure angles

Tooth Thickness

Tooth thickness of a helical gear is defined in the transverse and normal planes. Transverse tooth thickness is defined as the arc length of tooth material at the theoretical pitch diameter. Tooth thickness is important to tooth bending strength and is primarily a function of the gear module and profile shift. For more about profile shift, see the blue info box below.



Transverse tooth thickness diagram

Tooth thickness and profile shift are related by this expression:

$$s_t = \frac{m_n}{\cos\beta}\left(\frac{\pi}{2} + 2x^* \tan\alpha_n\right)$$

From the diagram, it is easy to see that transverse tooth thickness at an arbitrary diameter, d_y, is:

$$s_{ty} = d_y \psi_y$$

where the corresponding tooth thickness half angle, ψ_y, is observed as:

$$\psi_y = \psi + \text{inv }\alpha_t - \text{inv }\alpha_{ty}$$

and since s_t is known and $\psi = s_t/d$, a usable expression is obtained by substitution of the equations above:

$$s_{ty} = d_y\left(\frac{\pi}{2z} + \frac{2x^* \tan\alpha_n}{z} + \text{inv }\alpha_t - \text{inv }\alpha_{ty}\right)$$

Profile shift is in reference to common manufacturing processes used to make helical gears.

Physically, profile shift refers to the distance between a gear's theoretical pitch diameter and the cutting tool datum line. Without consideration for the manufacturing process, it is difficult to assign physical meaning to profile shift, in which case considering the gear tooth thickness directly is more intuitive.

Tooth tip thickness

The tooth tip thickness is of particular interest since it can fracture if too thin. Gear designers that are maximizing contact ratio will need to apply a constraint for minimum allowable tooth tip thickness. Without constraining it, the tooth tip thickness will trend towards zero, a condition known as *tooth peaking*.

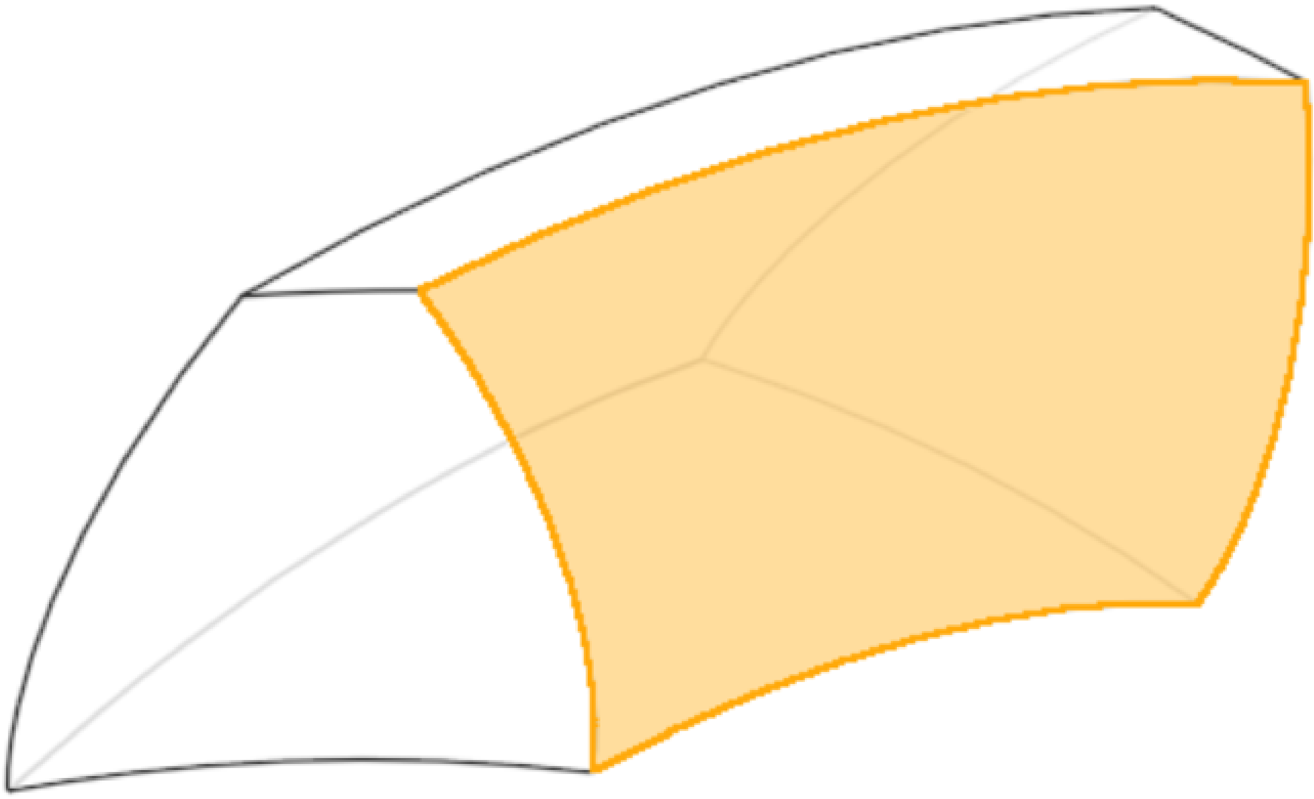
$$s_{na} = s_{ta} \cos \beta_a$$

The recommended minimum value for the normal tooth tip thickness ranges from $0.25 m_n$ to $0.4 m_n$.

The true physical limit of tooth tip thickness is dependent on several parameters of a gear mesh pertaining to the tooth strength and tooth tip loads.

Tooth Flank

The helical gear tooth flank is a 3-dimensional surface that can be defined numerically by computing an involute curve for each transverse section along the gear facewidth. Each successive involute is the same as the previous, but with a rotational transform applied to account for the helix hand and angle.



Helical gear tooth flank

The involute profile of a helical gear tooth is in the transverse plane.

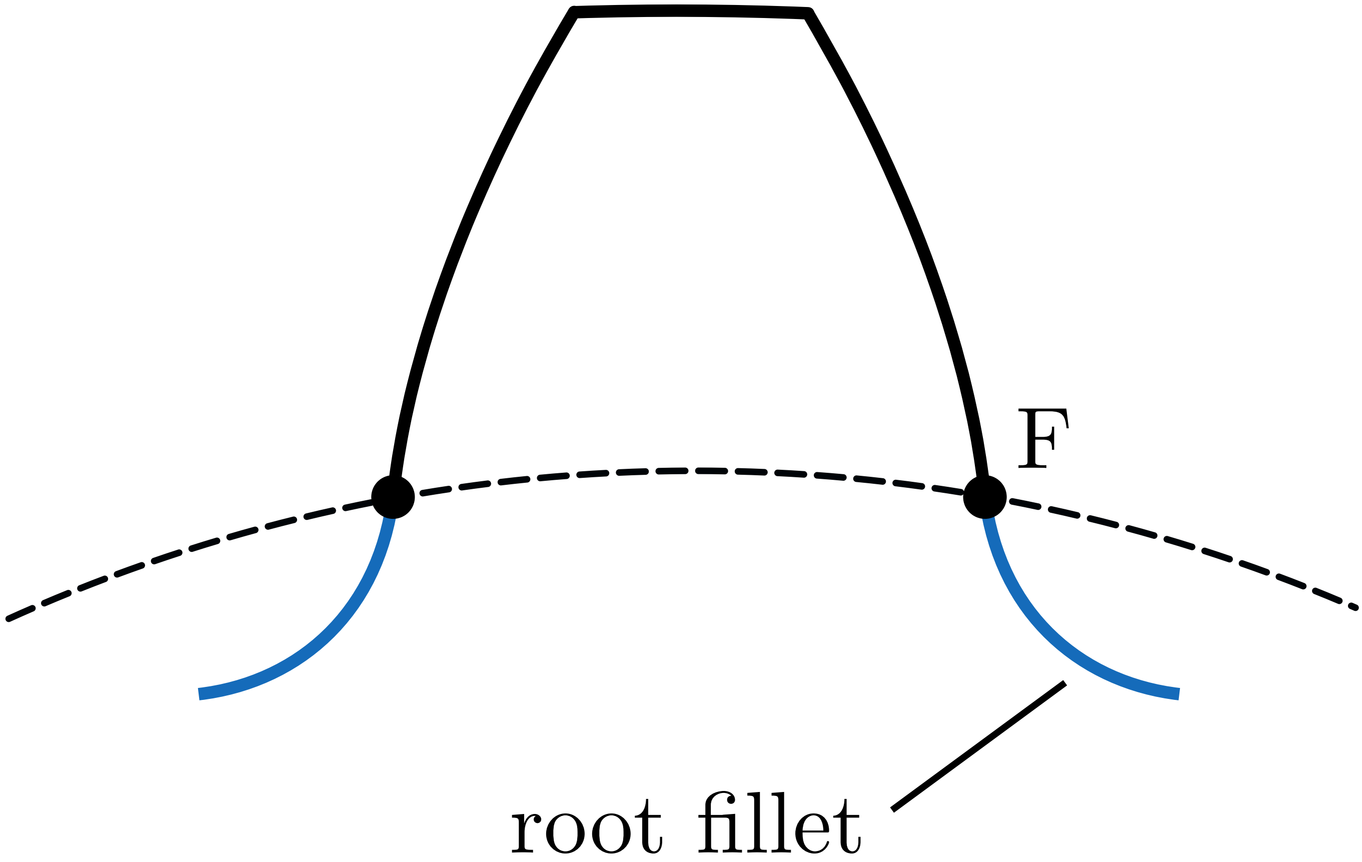
This is an important and intentional result of the manufacturing process when cutting helical gears since the transverse plane is the plane of rotation. Without this result, a pair of meshing helical gears would not possess the benefits associated with cylindrical involute gearing, such as conjugate action.

Microgeometry modifications are commonly applied to the tooth flanks of helical gears with medium-to-high quality.

Different methods exist for applying these flank surface modifications, such as gear grinding, but it is outside the scope of this notebook to review them.

Root Fillet

The root fillet is an important, but complex, feature of any cylindrical involute gear. The gear tooth bending strength is highly dependent on the root geometry.



Root fillet and boundary point

The root fillet is not a simple curve that can be described by a set of parametric equations. The reason for its complexity is because it depends on the manufacturing tool geometry and cutting process. To learn more about this, refer to our notebooks on gear tooling.

Form Diameter

The form diameter refers to the diameter at the boundary point, F, of the involute and root profiles. In the simplest case, the form diameter boundary point is the point of tangency between the involute and root profiles. For a rack generation cutting tool, such as a hob, the tool cut point at boundary point F is the tool tip point tangent to the rack profile. For a better understanding, refer to the notebooks on gear tooling.

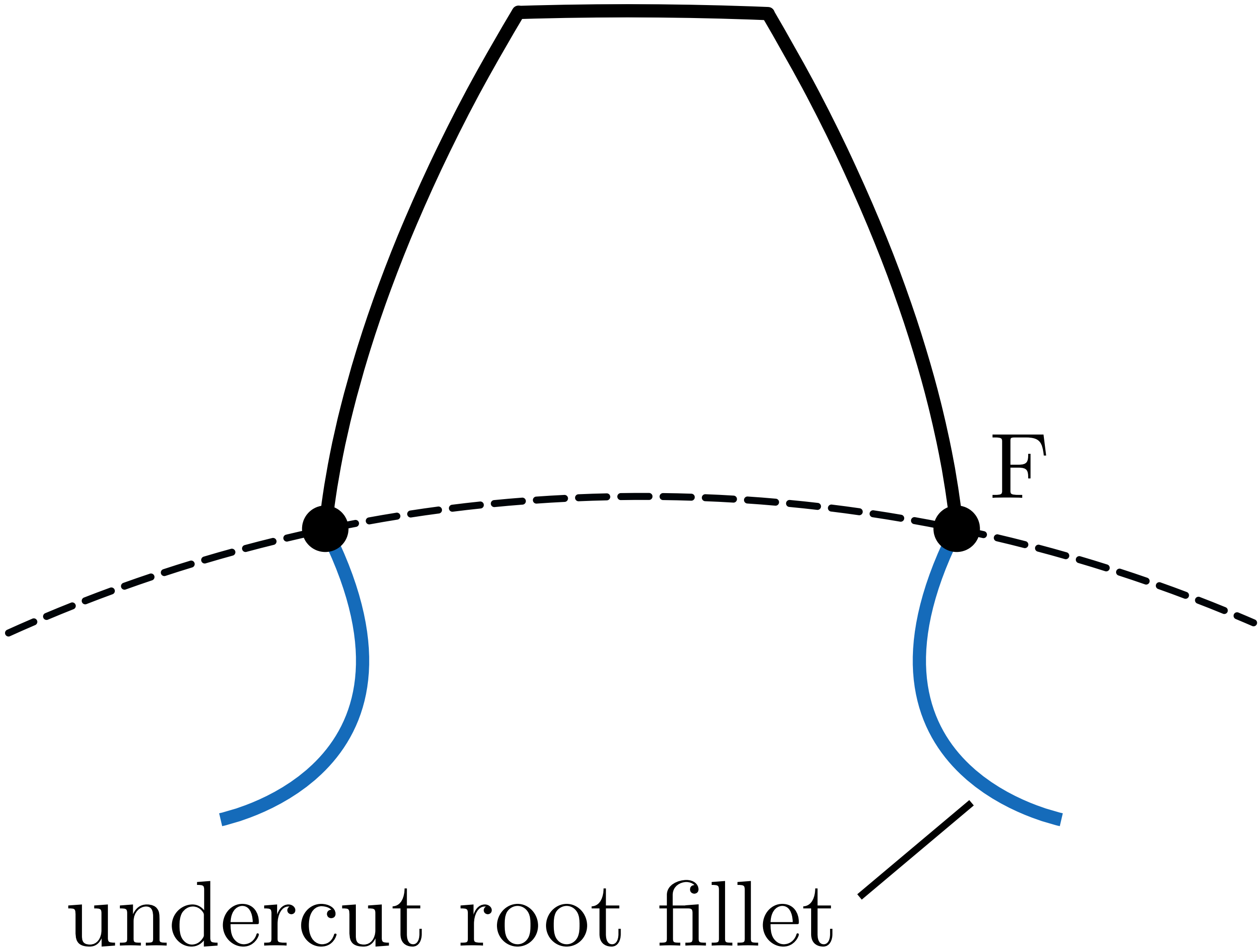
From a diagram of a rack tool generating the root fillet, an expression for transverse pressure angle at the boundary point can be derived as:

$$\tan\alpha_{tF} = \tan\alpha_t - \frac{h_f - \rho_f + \rho_f \sin\alpha_n - x}{r_b \sin\alpha_t}$$

which can be used to calculate form diameter. This equation is invalid if the gear tooth root is undercut.

Undercut

Undercut of a helical gear tooth root is a consequence of the cutting tool removing a portion of the involute profile instead of being tangent to the involute. The path of the cutting tool tip in plane rolling action with the gear is what generates the undercut root shape.



Undercut root fillet and boundary point

Undercut can be related back to certain geometric properties of the gear. Undercut caused by a rack generation cutting process can be avoided with positive profile shift, thus the following condition is formulated to avoid undercut:

$$x \geq h_a^* - \frac{z \sin^2 \alpha_t}{2 \cos \beta}$$

Similarly, a condition for minimum number of teeth to avoid undercut is:

$$z \geq \frac{2 \cos \beta}{\sin^2 \alpha_t}(h_a^* - x^*)$$

More complicated cases of undercut are possible with protuberance cutting tools, but are outside the scope of this notebook. To learn more about the influence of protuberance, see the notebooks on gear tooling.

Gear designers should avoid undercut of *finished* teeth since it reduces tooth bending strength.

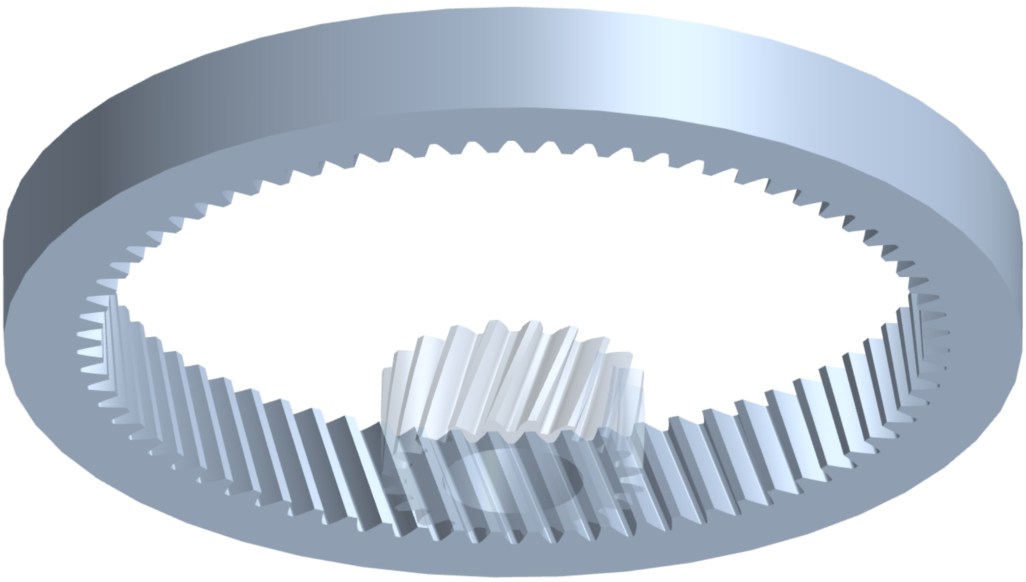
Depending on the application requirements, it may not always be possible to avoid undercut, e.g. if a large single-stage gear ratio is required. In such cases, other failure modes, such as gear scuffing, may become a greater concern due to high pitch line velocities.

Undercut is intentionally added to the *pre-finished* teeth of many medium-to-high quality gears that undergo a finishing process.

A *protuberance* cutting tool can be used to add undercut to gears that would otherwise not have it. Pre-finished undercut is beneficial to the finishing process because it provides *finishing stock* material to be removed and provides an exit path for the finishing tool without interfering with the root. Upon removal of the finishing stock, the finished involute profile is tangent to the root fillet.

Internal

Thus far, we have focused on helical gears of the *external* type, meaning the gear teeth are external to the gear blank (body). In certain gear trains or applications, helical gears of the *internal* type may be required or better suited. Below illustrates an internal helical gear with 70 teeth meshing with an external helical gear of 20 teeth, both having a right-hand helix.



Internal helical gear mesh in Gears App (https://drivetrainhub.com/gears)

Despite visual and operational differences, external and internal helical gears are geometrically similar in the use of involute tooth profiles. This is important to theoretically achieving conjugate action when an internal gear meshes with an external gear.

Key differences of internal gears:

- Root diameter is greater than tip diameter
- Rack generation manufacturing methods cannot be used
- Unintended undercut is not a concern

Internal gears are most commonly used in **planetary gear systems**, where they are typically referred to as **ring gears**. It is also possible to use them in simpler gear trains, such as a slewing gear.

Example | Helical Gear Geometry

GIVEN
Input parameters to define a helical gear.

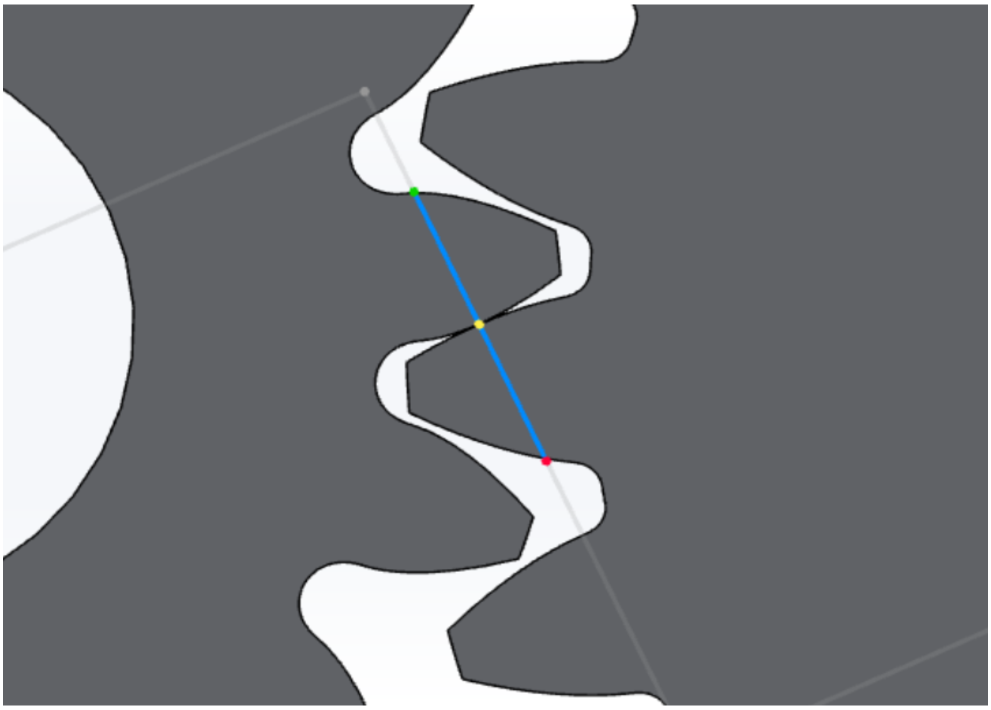
FIND
Detailed geometric parameters of the helical gear teeth.

SOLUTION
See below.

Description	Symbol	Value	Units
Normal module	m_n	1	mm
Normal pressure angle	α_n	20	deg
Helix angle	β	15	deg
Number of teeth	z	17	-
Addendum coefficient	h_a^*	1	-
Dedendum coefficient	h_f^*	1.25	-
Tool tip radius coefficient	ρ_f^*	0.38	-
Profile shift coefficient	x^*	0.2	-
Facewidth	b	10	mm
---	---	---	---
Transverse module	m_t	1.035276	mm
Transverse pressure angle	α_t	20.646896	deg
Theoretical pitch diameter	d	17.599695	mm
Base diameter	d_b	16.469288	mm
Tip diameter	d_a	19.999695	mm
Root diameter	d_f	15.499695	mm
Base helix angle	β_b	14.076095	deg
Helix pitch length	p_β	206.349093	mm
Transverse pitch	p_t	3.252416	mm
Normal pitch	p_n	3.141593	mm
Transverse base pitch	p_{bt}	3.043517	mm
Normal base pitch	p_n	9.274394	mm
Transverse diametral pitch	P_t	24.534516	in^{-1}
Normal diametral pitch	P_n	25.4	in^{-1}
Transverse tooth thickness	s_t	1.776932	mm
Normal tooth thickness	s_n	1.716384	mm
Transverse tooth tip thickness	s_{ta}	0.634641	mm
Normal tooth tip thickness	s_{na}	0.613016	mm
Tooth tip thickness coefficient	s_{na}^*	0.613016	-
Tooth thickness half angle	ψ	5.784799	deg
Tooth tip thickness half angle	ψ_a	1.81814	deg
Profile shift to avoid undercut	$x_{\min}$	-0.094104	-
Number of teeth to avoid undercut	$z_{\min}$	12.430259	-

Gear Mesh

This section reviews the properties of a helical gear mesh, both geometrically and kinematically. A working example is provided at the end of the section.



Gear mesh transverse view in Gears App (https://drivetrainhub.com/gears)

Nomenclature

This table provides a set of helical gear mesh parameters.

Symbol	Description
a	Center distance
a_0	Reference center distance
a_{j0}	Theoretical center distance
α_w	Working pressure angle
P	Pitch point
Y	Contact point
Σx	Sum of profile shifts
b	Effective facewidth
j_r	Radial backlash
j_{tt}	Circumferential backlash
j_{tn}	Profile backlash
j_{nn}	Normal backlash
ϵ_α	Transverse contact ratio
ϵ_β	Axial contact ratio
ϵ_γ	Total contact ratio

n_α	Transverse contact ratio remainder
n_β	Axial contact ratio remainder
l_D	Total length of contact lines

The next table provides a set of helical gear mesh parameters specific to each gear, despite that certain parameters must be equal for each gear.

Symbol	Description
d_w	Pitch diameter
p_t	Transverse pitch
p_n	Normal pitch
p_a	Axial pitch
p_{bt}	Transverse base pitch
p_{bn}	Normal base pitch
p_{ba}	Axial base pitch
c_a	Tip clearance
c_f	Bottom clearance
j_θ	Angular backlash
$\alpha_{t\mathrm{Y}}$	Transverse pressure angle at contact point
$\alpha_{t\mathrm{SAP}}$	Transverse pressure angle at SAP
$\alpha_{t\mathrm{EAP}}$	Transverse pressure angle at EAP
d_{SAP}	SAP diameter
d_{EAP}	EAP diameter
ψ_{SAP}	SAP roll angle
ψ_{EAP}	EAP roll angle
v_w	Pitch line velocity

where SAP and EAP are the start and end of active profile, respectively, as explained later.

For each gear in a gear pair, the subscripts $_1$ and $_2$ are used for the respective gear parameters. For example, d_{w1} is the pitch diameter of *gear 1*.

Compatibility

Two helical gears must have compatible geometry for their involute teeth to properly mesh. Fundamentally, each gear must have the same base pitch to be compatible. Since gear designers do not tend to think in terms of base pitch, we will consider the parameters responsible for the transverse and normal base pitch of a helical gear.

Base Pitch

From a previous figure illustrating the transverse pitch, the expression for transverse base pitch is:

$$p_{bt} = \frac{\pi d_b}{z}$$

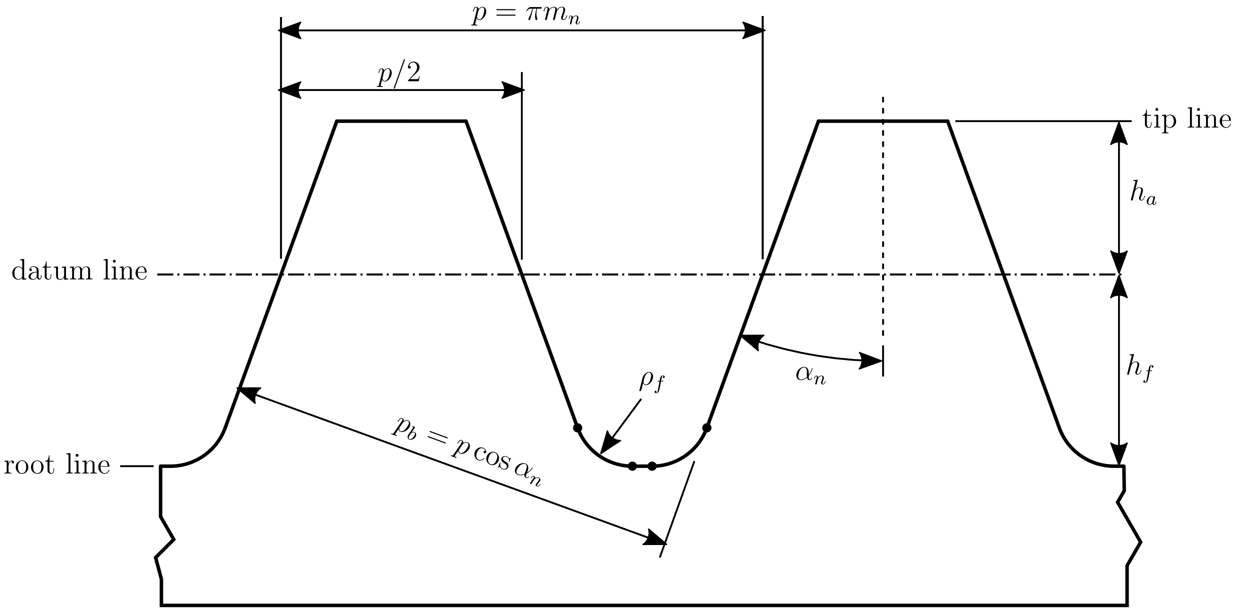
Expand and simplify the equation to obtain the transverse base pitch as a function of normal module, normal pressure angle, and helix angle:

$$p_{bt}\{m_n, \alpha_n, \beta\} = \frac{\pi}{z} d \cos \alpha_t = \frac{\pi m_n}{\cos \beta} \cos\left(\tan^{-1}\left(\frac{\tan \alpha_n}{\cos \beta}\right)\right)$$

The normal base pitch can be expressed as a function of normal module and normal pressure angle:

$$p_{bn}\{m_n, \alpha_n\} = \pi m_n \cos \alpha_n$$

which is easily observed from the basic rack geometry.



Basic rack profile illustrating base pitch

To learn more about the basic rack and its significance to gear geometry and manufacturing, refer to our notebooks on gear tooling.

Helix Angle

Helix angle was previously defined in the section on geometry of helical gear teeth, however an additional consideration is required for the helix angles of meshing gears. Namely, the sign ($\pm$) must be considered.

The following conventions are required for helical gears to mesh:

- An *external-external* mesh, i.e. two gears of external type, must have *equal and opposite sign* helix angles. For example, $\beta_1 = 15^\circ$, $\beta_2 = -15^\circ$.
- An *internal-external* mesh, i.e. an internal gear and external gear, must have *equal and same sign* helix angles. For example, $\beta_1 = 15^\circ$, $\beta_2 = 15^\circ$.

Note that *internal* gears can only mesh with *external* gears, not another internal gear.

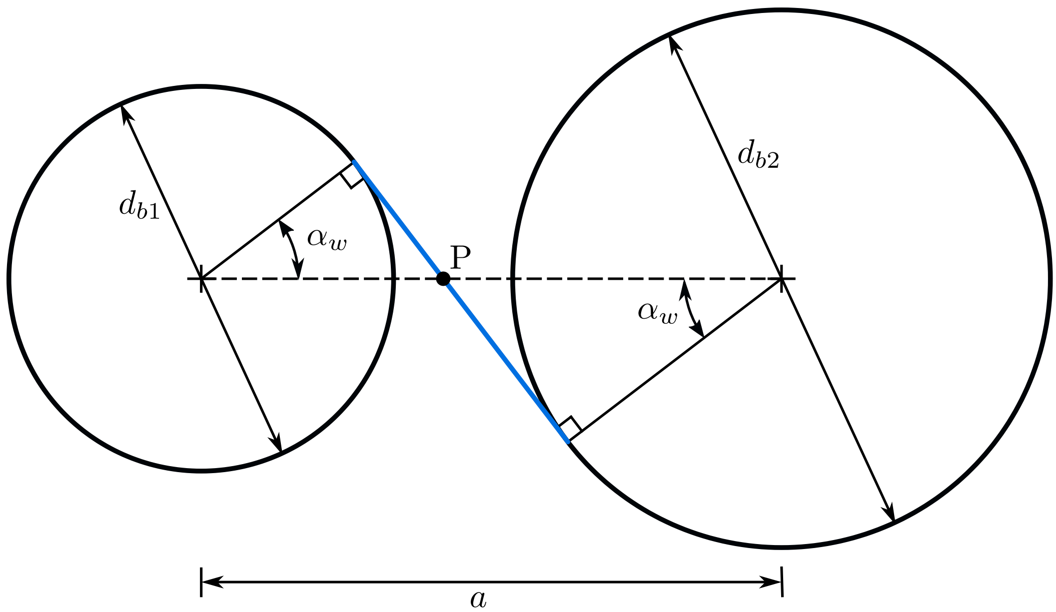
Helical gears are compatible to mesh if the normal module, normal pressure angle, and helix angle† are matching.

Notice that parameters pertaining to tip diameter, root diameter, and tooth thickness are excluded from the terms for *compatibility*. However, each of these parameters is important for a well designed gear mesh that is without interference.

†The helix angle sign convention depends on the gear mesh being external-external or internal-external type.

Pressure Angle

The pressure angle of main interest to a cylindrical involute gear mesh is the *working pressure angle*, α_w. It is the transverse pressure angle formed by the gear mesh *pressure line*, a line tangent to the base circles and commonly referred to as the *line of action*.



Working pressure angle, center distance, line of action, and pitch point

From this figure, working pressure angle is derived as:

$$\alpha_w = \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a}\right)$$

Notice that working pressure angle is not necessarily equal to the reference pressure angle, and the only *mesh* parameter determining so is the center distance. Working pressure angle can also be expressed as a function of profile shifts for the condition of zero backlash:

$$\mathrm{inv}\ \alpha_w = \mathrm{inv}\ \alpha_t + 2\tan\alpha_n \frac{x_1^* + x_2^*}{z_1 + z_2}$$

where inv is the involute function. Notice when the sum of profile shift coefficients equals zero, $\Sigma x^* = x_1^* + x_2^* = 0$, the working pressure angle equals the reference pressure angle (with zero backlash, otherwise not true).

If working pressure angle does not equal the reference pressure angle, then the theoretical pitch diameters do not intersect the pitch point.

The sum of profile shift coefficients can be used as a design metric to achieve performance characteristics. Here are rules of thumb:

- 0.00 - Large contact ratio for better noise and dynamic loading.
- 0.25 - Well-balanced for durability, efficiency, and dynamics.
- 0.50 - High load capacity and reduced sliding for efficiency.

Center Distance

The center distance of a helical gear pair is the radial distance between the rotational axis of each gear. Three types of center distances are classified:

1. Reference center distance
2. Theoretical center distance
3. Working center distance

Reference center distance

Center distance of a gear pair without profile shifts or backlash. Also known as *null center distance*. For this condition, working pressure angle is equal to the reference pressure angle, $\alpha_w = \alpha_t$, and therefore the reference center distance is calculated as:

$$a_0 = \frac{d_1 + d_2}{2} = \frac{m_n(z_1 + z_2)}{2\cos\beta}$$

Theoretical center distance

Center distance of a gear pair without backlash, but gears may have profile shift. If the sum of profile shifts is zero, $\Sigma x = 0$, the theoretical center distance equals the null center distance, and working pressure angle equals the transverse reference pressure angle.

$$a_{j0} = a_0 \frac{\cos\alpha_t}{\cos\alpha_w}$$

where the specified working pressure angle, α_w, must be for the condition of zero backlash.

Center distance

Actual center distance of an assembled gear pair. It can be calculated from the pitch diameters, which are simply related by the number of teeth, as explained in the kinematics section.

$$a = \frac{d_{w1} + d_{w2}}{2}$$

Influence of profile shifts when backlash is zero, i.e. when *actual* center distance equals the theoretical center distance:

- $\Sigma x = 0 \Rightarrow a = a_0 \text{ and } \alpha_w = \alpha_t$
- $\Sigma x < 0 \Rightarrow a < a_0 \text{ and } \alpha_w < \alpha_t$
- $\Sigma x > 0 \Rightarrow a > a_0 \text{ and } \alpha_w > \alpha_t$

Involute tooth profiles of helical gears allow for center distance variation without affecting the nominal transmission ratio or conjugate action.

Actual meshing behavior may be affected since tooth stiffness and conjugate relief of microgeometry modifications could change with center distance.

Clearances

Clearances between meshing helical gear teeth are important in accounting for manufacturing tolerances, operating deflections, thermal expansion, and lubricant film thickness. Clearances should not be more than necessary, while avoiding tooth interferences and having sufficient tooth tip thickness. Interferences will negatively affect tooth stresses, efficiency, and conjugate action, but excessive clearances could reduce tooth strength, increase undesired windup or endplay, and increase dynamic loads.

Tip and bottom clearance

Tip clearance is the distance between the tooth tip diameter and the mating tooth root diameter. It can be calculated as:

$$c_{a1,2} = a - \frac{d_{a1,2} - d_{f2,1}}{2}$$

Bottom clearance is the distance between the tooth root diameter and the mating tooth tip diameter. It can be calculated as:

$$c_{f1,2} = a - \frac{d_{a2,1} - d_{f1,2}}{2}$$

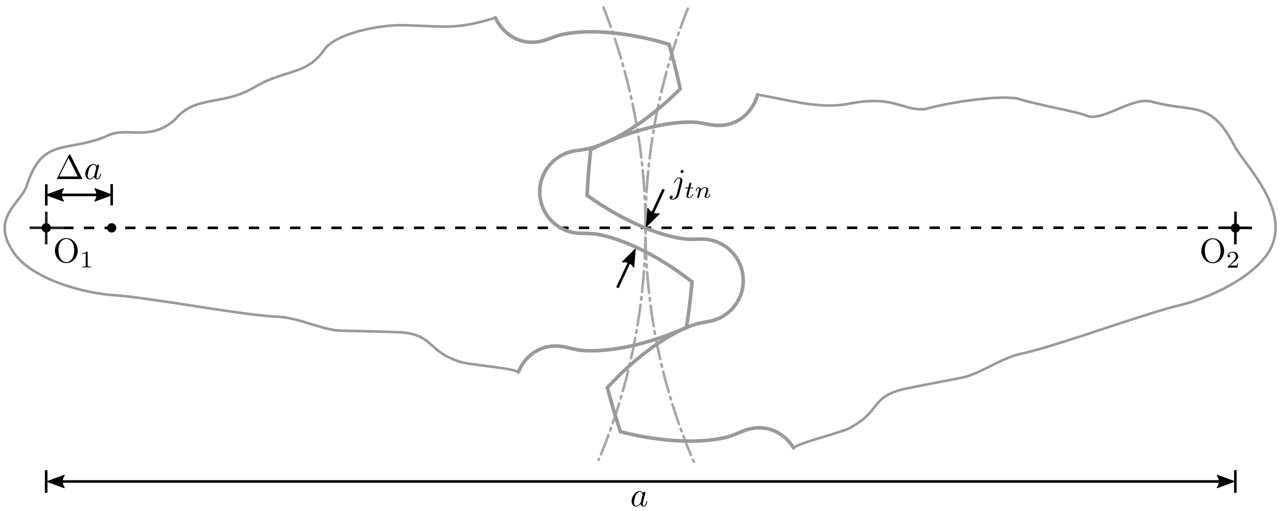
The recommended minimum value for both tip clearance and bottom clearance is $0.25m_n$.

Backlash

Backlash is the distance between the inactive tooth flanks when the active flanks are in contact. Different components of backlash are typically reported:

- **Radial backlash,** j_r: Linear length of backlash along the gear mesh center distance.
- **Circumferential backlash,** j_{tt}: Arc length of pitch circle in the backlash region.
- **Profile backlash,** j_{tn}: Linear length of backlash normal to the involute profile in the transverse plane.
- **Normal backlash,** j_{nn}: Linear length of backlash normal to the helicoid tooth surface in the normal plane.

Backlash is present in a helical gear pair when the center distance, a, is increased from the theoretical center distance, a_{j0}. This increase creates a gap between the inactive flanks of the meshing gear teeth. Backlash can also be interpreted as *tooth thinning* by reducing the sum of profile shifts from a theoretical value corresponding to zero backlash.



Center distance increase and backlash normal to involute profiles

The radial backlash, also known as *radial play*, can be computed as the difference of actual center distance and theoretical center distance:

$$j_r = \Delta a = a - a_{j0}$$

By resolving the backlash in the directions of interest, each component can be calculated accordingly:

$$j_{tt} = 2 j_r \tan \alpha_w$$

$$j_{tn} = j_{tt} \cos \alpha_w$$

$$j_{nn} = j_{tn} \cos \beta_b$$

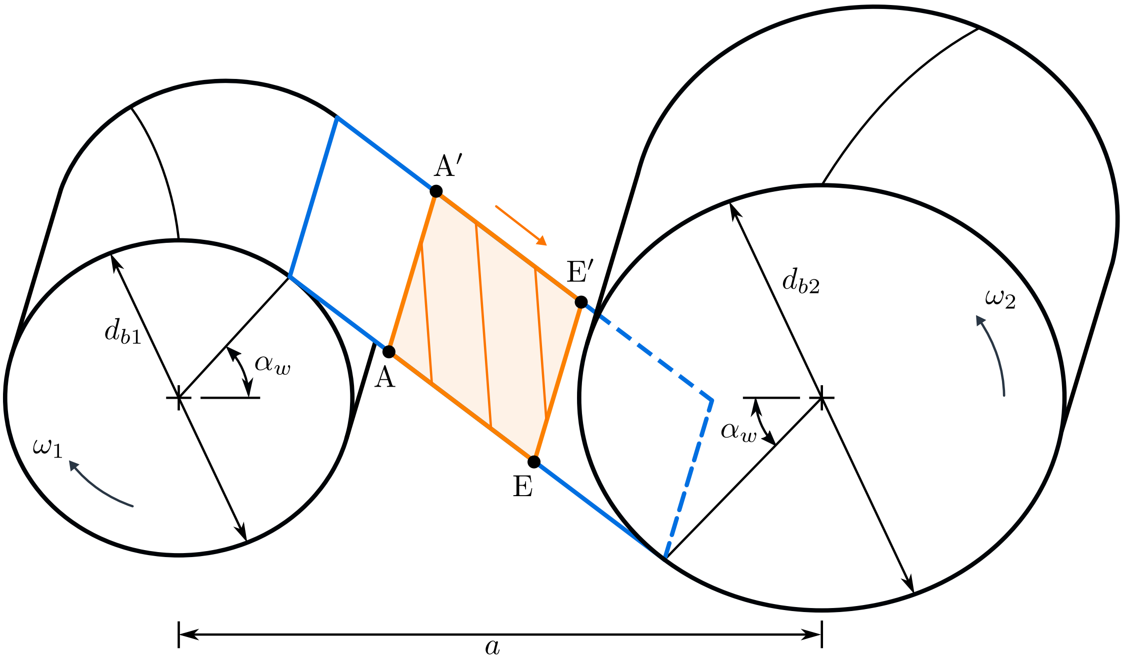
There is also a backlash value known as *angular backlash*. It is the angle of gear rotation in the backlash region, and can be expressed as:

$$j_\theta = \frac{2 j_{tt}}{d_w}$$

Since angular backlash depends on pitch diameter, it can differ for each gear in a mesh. Its values are related by the transmission ratio.

Plane of Action

The plane of action is coincident with the line of action in the transverse plane, and extends along the effective facewidth of a meshing gear pair. It is within this plane that flanks of helical gear teeth make contact as the gears roll through a mesh cycle.



Plane of action for helical gear pair

The figure above outlines the plane of action in blue, with the orange area being the active region of contact. The diagonal orange lines are lines of contact between successive pairs of meshing teeth. The following sections review the plane of action for contact geometry and kinematics of a helical gear pair.

Contact

Contact of helical gears is a complex subject and an area of active research. The focus here is on the geometric properties of contact for unloaded teeth in a helical gear pair. Loaded tooth contact analysis (LTCA) requires additional considerations, including tooth bending stiffness, tooth contact stiffness, tooth forces, contact pressure, tooth surface kinematics, microgeometry modifications, and lubrication.

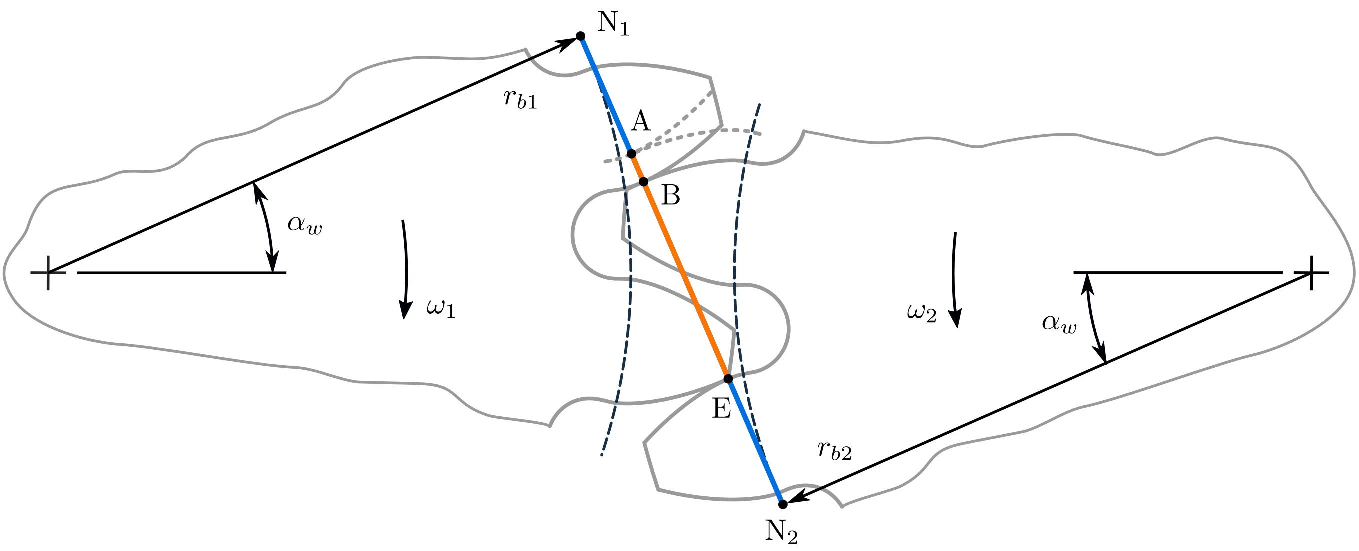
The following aspects of helical gear contact are studied here:

- Start and end of the active profile
- Contact path and contact lines
- Variation of contact lines
- Axial, transverse, and total contact ratios

Active Profile

During the mesh cycle of a helical gear pair, the involute tooth profiles have a starting point and ending point. The region of involute profile between these start and end points is termed the *active profile*. Intuitively, the start and end points are referred to as the *start of active profile* (SAP) and *end of active profile* (EAP). Diameters or roll angles are typically used to indicate the SAP and EAP, with SAP as the lower value by convention.

The active profile can be understood by visualizing the active portion of the line of action in a transverse section of a helical gear pair. This active segment may be called the *contact plane* or *plane of contact*.



Contact plane in the line of action

From the figure, line $\overline{\mathrm{AE}}$ is the active profile trace in the line of action, thus gear 1 SAP and EAP are at points A and E, respectively. Due to the convention of SAP always being the lower value, the SAP and EAP of gear 2 are at points E and A, respectively. An expression for transverse pressure angle at the point of contact can be derived as:

$$\alpha_{t\mathrm{Y}1} = \tan^{-1}\left(\tan\alpha_w - \frac{z_2}{z_1}(\tan\alpha_{t\mathrm{Y}2} - \tan\alpha_w)\right)$$

where Y indicates an arbitrary point of contact, thus $\alpha_{t\mathrm{Y}2} = \alpha_{ta2}$ can be used to calculate the SAP of gear 1 . The contact plane length is calculated as:

$$\overline{\mathrm{AE}} = r_{b1}\left(\tan\alpha_{ta1} - \tan\alpha_w\right) + r_{b2}\left(\tan\alpha_{ta2} - \tan\alpha_w\right)$$

It is worth noting that length $\overline{\mathrm{BE}}$ equals the transverse base pitch. Notice the importance of equal base pitch for two gears to achieve the desired engagement of gear teeth as they rotate. In actual manufactured helical gears, imperfect base pitch exists and contributes to uneven load share between gear teeth, which leads to higher-than-nominal loads on certain teeth and may cause amplitude modulated vibrations and noise.

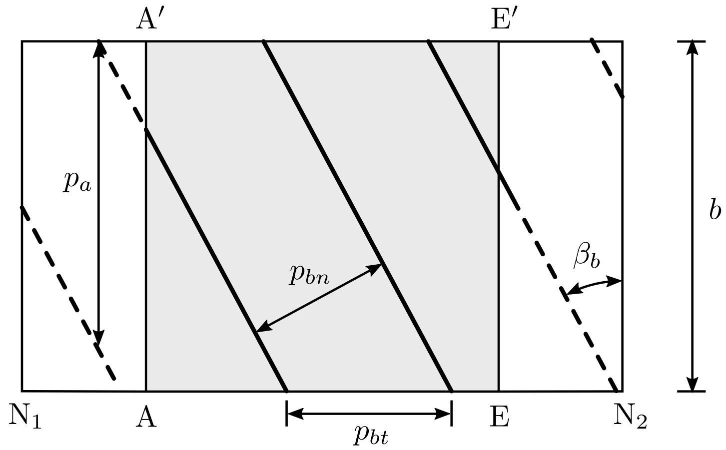
End of active profile does *not* always coincide with the tip diameter or tip chamfer diameter.

In cases of a gear tooth being undercut, the boundary point of the root fillet and involute can reduce the active profile region and cause loss of contact before the mating gear involute has reached its tip diameter.

Contact Path

A helical gear tooth surface is constructed from a circle involute swept along a helical path, creating an *involute helicoid*. The circle involute is a curve of continuously varying radius. Therefore, when two opposing helical gear tooth surfaces are brought into contact, the contact occurs along a line.

Since a helical gear mesh has multiple teeth in contact during a mesh cycle, multiple lines of contact exist. By examining the geometric properties of the contact lines in the plane of action, attributes of contact through a mesh cycle can be understood.



Contact plane with contact lines of meshing helical gear teeth

By following the convention of the previous figures for direction of rotation, tooth flanks begin contact at point A and end contact at point E'. As the contact lines move across the plane while the gears roll through mesh, the total length of the contact lengths is not necessarily constant. This is considered in more detail, but first we must define contact ratios.

Contact Ratio

Contact ratio is defined as the mean number of teeth in contact through a mesh cycle. For a helical gear pair, these contact ratio definitions are used:

- **Transverse contact ratio**: Mean number of teeth in contact in the transverse plane.
- **Axial contact ratio**: Mean number of teeth in contact in the axial plane. Also known as the *overlap ratio*.
- **Total contact ratio**: Mean number of teeth in contact in the plane of action.

The contact ratios can be derived from the relationship of contact path lengths and pitch lengths, as illustrated in the previous figure. Transverse contact ratio is the ratio of transverse contact path length to transverse base pitch, calculated as:

$$\epsilon_\alpha = \frac{\overline{\mathrm{AE}}}{p_{bt}} = \frac{z_1\left(\tan\alpha_{a1} - \tan\alpha_w\right) + z_2\left(\tan\alpha_{a2} - \tan\alpha_w\right)}{2\pi}$$

where it is assumed that tip diameter is the EAP. Axial contact ratio is the ratio of effective facewidth to axial pitch, calculated as:

$$\epsilon_\beta = \frac{b}{p_a} = \frac{b\sin\beta_b}{\pi m_n \cos\alpha_n} = \frac{b\sin\beta}{\pi m_n}$$

where it is obviously zero for spur gears and increases with helix angle. Lastly, total contact ratio is calculated as the sum of transverse and axial:

$$\epsilon_\gamma = \epsilon_\alpha + \epsilon_\beta$$

Contact Variation

As a helical gear pair rolls through a mesh cycle, the sum of contact line lengths can vary. This can be visualized with the previous figure by viewing the contact lines as moving across the plane of action during a mesh cycle. Remember, each contact line represents a pair of meshing teeth. The mean sum of contact line lengths can be expressed as:

$$l_{D\text{mean}} = \frac{b\epsilon_\alpha}{\cos\beta_b}$$

Axial contact ratio, ϵ_β, is responsible for the variation in the sum of contact line lengths.

This is most obvious for the case of zero overlap ratio, $\epsilon_\beta = 0$, where the total length of contact lines varies instantaneously by b due to the start or end of contact for a given tooth pair. For example, a spur gear pair with a contact ratio of 1.4 will have exactly 1 tooth pair in contact 60% of the time and 2 tooth pairs in contact 40% of the time, i.e. for steady state operation.

For helical gears, the overlap ratio is always non-zero, $\epsilon_\beta \neq 0$, which causes the total length of contact lines to vary more gradually through a mesh cycle. The higher the overlap ratio, the less variation in total length of contact lines relative to the mean value. The special case of an integer overlap ratio results in zero variation. Less variation is desireable for reducing dynamic tooth loads and noise.

The minimum value for the sum of contact line lengths is of particular interest because it is when the gear teeth experience maximum load. Its value depends on the sum of contact ratio remainders. For $n_\alpha + n_\beta \leq 1$, it is calculated as:

$$l_{D\text{min}} = l_{D\text{mean}}\left(1 - \frac{n_\alpha n_\beta}{\epsilon_\alpha \epsilon_\beta}\right)$$

and for $n_\alpha + n_\beta > 1$, it is calculated as:

$$l_{D\text{min}} = l_{D\text{mean}}\left[1 - \frac{(1-n_\alpha)(1-n_\beta)}{\epsilon_\alpha \epsilon_\beta}\right]$$

Kinematics

The kinematics of a helical gear pair are considered at two degrees of fidelity:

1. Kinematics of the gear bodies
2. Kinematics of the contacting gear teeth

The kinematics of a gear body can be expressed with a single degree of freedom (DOF), the angular velocity about its axis of rotation. This fidelity is sufficient unless analyzing the dynamics of a fully elastic system, in which case a 6-DOF mathematical model is recommended.

If the local z-axis is denoted as the axis of rotation for a helical gear, its angular velocity can be expressed as:

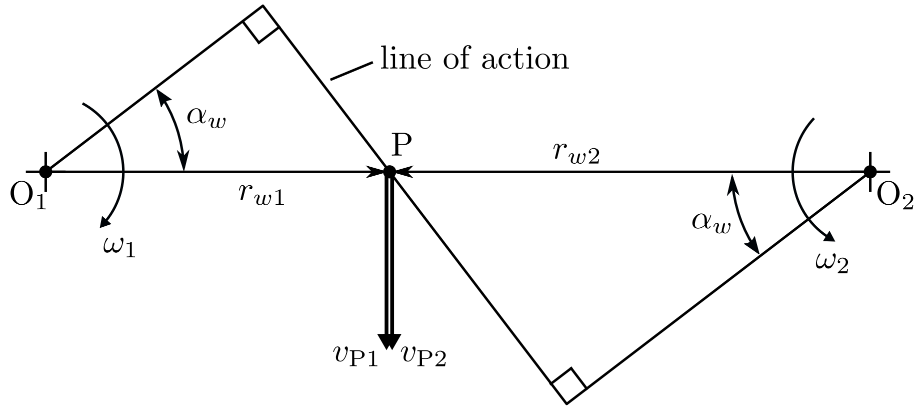
$$\vec{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega_z \end{bmatrix}$$

0:07 / 0:07

Helical gear pair animation in Gears App (https://drivetrainhub.com/gears)

Pitch Line Velocity

The pitch line velocity refers to the linear velocity at the gear mesh pitch point. Pitch line velocity is important to consider for gear lubrication and dynamics, which can influence stress, wear, and noise.



Pitch line speed of a helical gear pair

Pure rolling motion occurs at the pitch point of meshing helical gears and thus an equation for pitch line velocity is:

$$\vec{v}_w = \vec{r}_w \times \vec{\omega}$$

where $\vec{v}_w = \vec{v}_{P1,2}$ in the figure.

Transmission Ratio

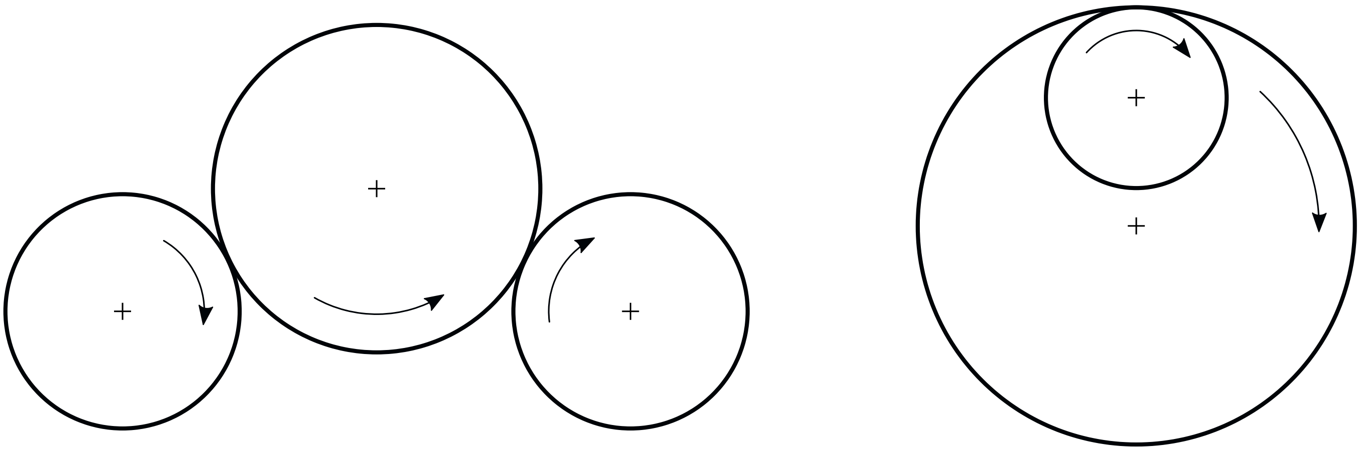
Nominally, a constant ratio of angular velocities exists for the gears in a helical gear mesh, known as the *transmission ratio* or *gear mesh ratio*. The ratio can be derived by considering the instantaneous linear velocity at the pitch point, where both gears are in pure roll.

$$\vec{r}_{w1} \times \vec{\omega}_1 = \vec{r}_{w2} \times \vec{\omega}_2$$

By solving this equation for the ratio of angular velocities, the gear mesh transmission ratio is defined as:

$$i = \frac{\omega_1}{\omega_2} = \frac{r_{w2}}{r_{w1}} = \frac{z_2}{z_1}$$

Direction of rotation depends on the gear mesh being external-external or internal-external type.



External Idler Gear Set

Internal Gear Mesh

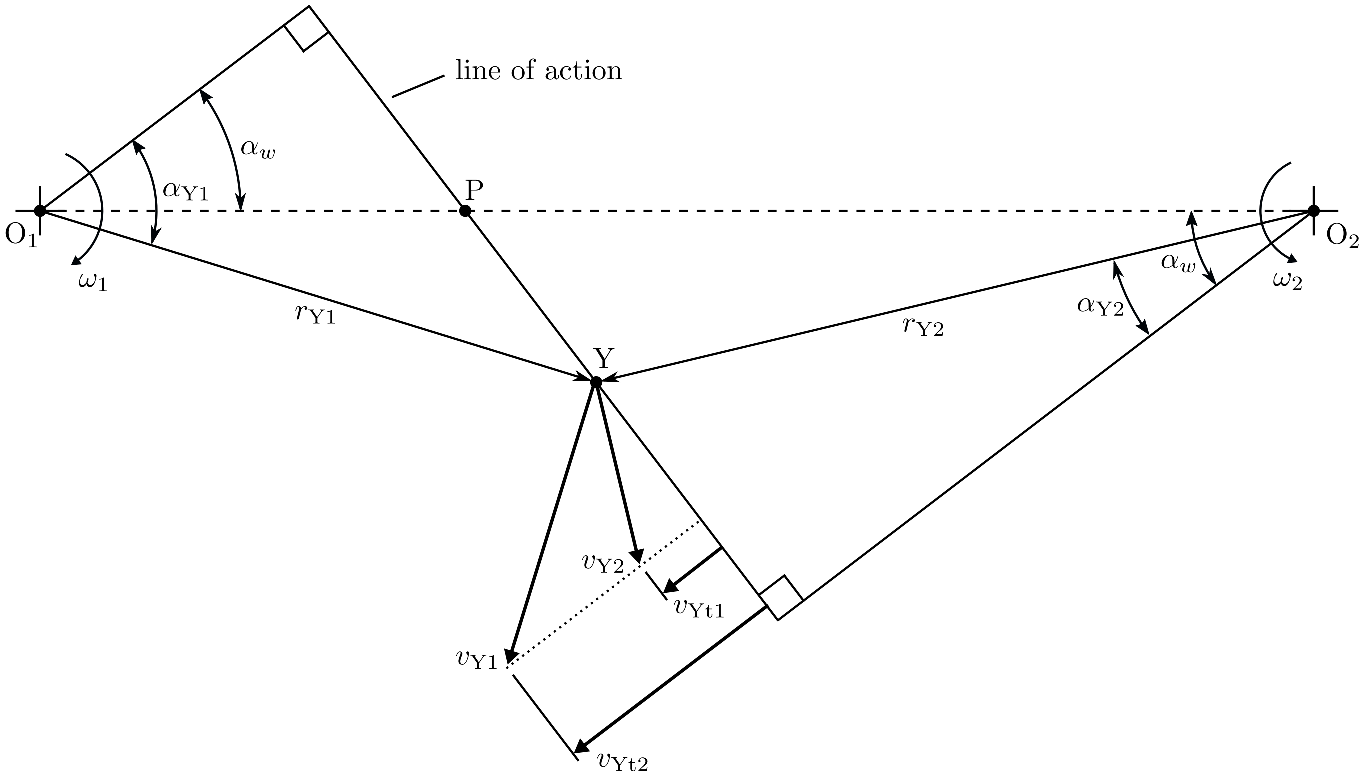
Rotational directions of helical gear meshes

Real helical gear meshes do not operate with a constant ratio of angular velocities at all times.

Actual velocities fluctuate about the transmission ratio defined here because true conjugate action is never achieved. This is due to manufactured gears being imperfect and gear tooth loads causing deflections of the involute profile. The transmission ratio, i, should be considered the *nominal* or *mean* ratio of angular velocities.

Sliding Velocity

While meshing helical gear teeth are in a state of pure roll at the pitch point, the tooth surfaces are sliding against each other in the approach and recess regions in the line of action. This sliding action is why cylindrical involute gearing is sometimes described as analogous to a cam-follower mechanism. Gear tooth sliding is important to gear mesh efficiency, wear, scuffing, and noise.



Contact point linear velocity vectors in the line of action

In the figure above, the linear velocities of contact point Y are denoted by v_{Y1} and v_{Y2} for gear 1 and gear 2, respectively.

$$v_{\mathrm{Y1}} = r_{\mathrm{Y1}}\omega_1$$

$$v_{\mathrm{Y2}} = r_{\mathrm{Y2}}\omega_2$$

The components of velocity normal to the line of action, and therefore tangential to the involute tooth profiles, are denoted by v_{Yt1} and v_{Yt2}.

$$v_{\mathrm{Yt1}} = r_{b1}\omega_1 \tan \alpha_{\mathrm{Y1}}$$

$$v_{\mathrm{Yt2}} = r_{b2}\omega_2 \tan \alpha_{\mathrm{Y2}}$$

Sliding action occurs when these tangential components of velocity are unequal, which is true everywhere except the pitch point, P. Sliding velocity is the relative tangential velocity of the tooth profiles, calculated as:

$$v_{\mathrm{R1}} = v_{\mathrm{Yt1}} - v_{\mathrm{Yt2}}$$

$$v_{\mathrm{R2}} = v_{\mathrm{Yt2}} - v_{\mathrm{Yt1}}$$

where v_{R1} is interpreted as the sliding velocity of the driving gear relative to the driven gear. Notice the equal and opposite relationship between the sliding velocities, $v_{\mathrm{R2}} = -v_{\mathrm{R1}}$. If sliding velocity is plotted as a function of mesh cycle, it can be observed that the direction of sliding reverses at the pitch point.

Specific Sliding

Specific sliding is the ratio of sliding velocity to the velocity of the contact point in the direction of sliding, i.e. tangential to the involute profile or normal to the line of action. It is a unitless metric dependent only on the gear mesh geometry, i.e. regardless of angular velocities, making it useful for gear design.

$$\vartheta_1 = \frac{v_{\mathrm{R1}}}{v_{\mathrm{Yt1}}}$$

$$\vartheta_2 = \frac{v_{\mathrm{R2}}}{v_{\mathrm{Yt2}}}$$

Sliding is of greatest concern at the SAP and EAP, when a pair of helical gear teeth are entering or exiting the mesh cycle.

Sliding velocity increases as the distance from the pitch point increases in the line of action. The points of contact furthest from the pitch point are the SAP and EAP. In cases of high profile shift, the pitch point may be at or below SAP and therefore sliding velocity does not reverse direction, but EAP sliding velocity may be excessive and lead to scuffing.

The kinematics for a system of rotationally coupled bodies, e.g. meshing helical gears, is formulated in detail in our notebooks on rotational mechanics.

Example | Helical Gear Mesh

GIVEN
Input parameters to define two parallel external helical gears meshing.

FIND
Geometry, contact, and kinematic properties of an external helical gear pair.

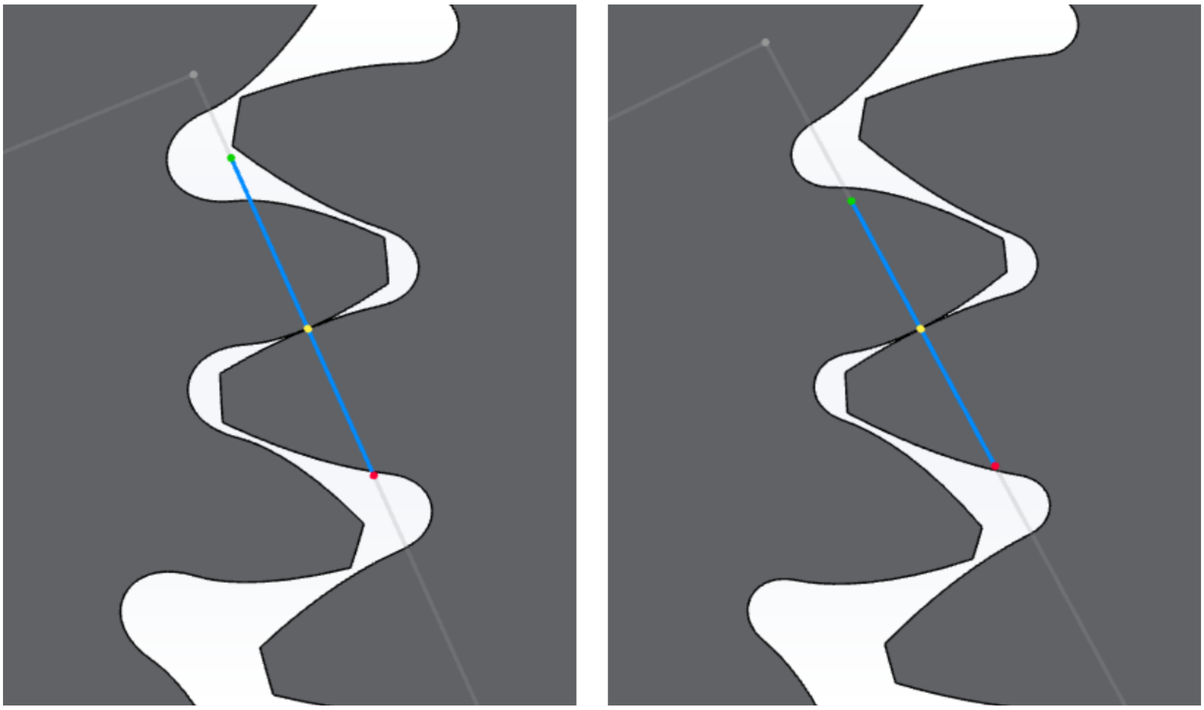
SOLUTION
See below.

Description	Symbol	Gear 1	Common	Gear 2	Units
Normal module	m_n	-	1	-	mm
Normal pressure angle	α_n	-	20	-	deg
Helix angle	β	-	15	-	deg
Base helix angle	β_b	-	14.076095	-	deg
Number of teeth	z	17	-	35	-
Addendum coefficient	h_a^*	1	-	1.0	-
Dedendum coefficient	h_f^*	1.25	-	1.25	-
Tool tip radius coefficient	ρ_f^*	0.38	-	0.38	-
Profile shift coefficient	x^*	0.2	-	-0.1	-
Theoretical pitch diameter	d	17.599695	-	36.234666	mm
Base diameter	d_b	16.469288	-	33.907359	mm
Root diameter	d_f	15.499695	-	33.534666	mm
Tip diameter	d_a	19.999695	-	38.034666	mm
Facewidth	b	10	-	9	mm
Effective facewidth	b	-	9	-	mm
Reference center distance	a_0	-	26.917181	-	mm
Theoretical center distance	a_{j0}	-	27.015921	-	mm
Actual center distance	a	-	27.5	-	mm
Working pressure angle	α_w	-	23.660563	-	deg
Transmission ratio	i	-	2.058824	-	-
Rotational speed	ω	100	-	48.571429	rpm
Pitch diameter	d_w	17.980769	-	37.019231	mm
Transverse pitch	p_t	-	3.252416	-	mm
Normal pitch	p_n	-	3.141593	-	mm
Axial pitch	p_a	-	12.138182	-	mm
Transverse base pitch	p_{bt}	-	3.043517	-	mm
Normal base pitch	p_{bn}	-	2.952131	-	mm
Axial base pitch	p_{ba}	-	12.138182	-	mm
Radial backlash	j_r	-	0.484079	-	mm
Circumferential backlash	j_{tt}	-	0.424197	-	mm
Profile backlash	j_{tn}	-	0.388539	-	mm
Normal backlash	j_{nn}	-	0.376873	-	mm
Angular backlash	j_θ	0.047183	-	0.022918	deg
Tip clearance	c_a	0.732819	-	0.732819	mm
Bottom clearance	c_f	0.732819	-	0.732819	mm
Transverse pressure angle at SAP	α_{tSAP}	16.379883	-	17.5533	deg

Description	Symbol	Gear 1	Common	Gear 2	Units
Transverse pressure angle at EAP	α_{tEAP}	34.565617	-	26.939471	deg
Roll angle at SAP	ψ_{SAP}	16.841207	-	18.123906	deg
Roll angle at EAP	ψ_{EAP}	39.474986	-	29.117456	deg
Diameter at SAP	d_{SAP}	17.166004	-	35.5633	mm
Diameter at EAP	d_{EAP}	19.999695	-	38.034666	mm
Transverse contact ratio	ϵ_{α}	-	1.068817	-	-
Axial contact ratio	ϵ_{β}	-	0.741462	-	-
Total contact ratio	ϵ_{γ}	-	1.810279	-	-
Contact plane length	$\overline{\mathrm{AE}}$	-	3.252964	-	mm
Mean total length of contact lines	$l_{D\mathrm{mean}}$	-	9.917132	-	mm
Minimum total length of contact lines	$l_{D\mathrm{min}}$	-	9.278603	-	mm
Variation of total length of contact lines	Δl_D	-	6.438643	-	$\%$
Pitch line velocity	v_w	-	188.294175	-	-
Sliding velocity at SAP	$v_{R,\mathrm{SAP}}$	-18.476363	-	-32.134436	$\mathrm{m/s}$
Sliding velocity at EAP	$v_{R,\mathrm{EAP}}$	32.134436	-	18.476363	$\mathrm{m/s}$
Specific sliding at SAP	ϑ_{SAP}	-0.728941	-	-1.178062	-
Specific sliding at EAP	ϑ_{EAP}	0.540876	-	0.421611	-

Gear Design

This section reviews the design of helical gears for geometric packaging. A working example is provided at the end of the section.



Comparing gear mesh profile shifts in Gears App (https://drivetrainhub.com/gears)

Transverse Plane

In the transverse plane, i.e. the plane of rotation, the key items of geometric design for spur and helical gears are:

1. Gear ratio
2. Center distance
3. Sum of profile shifts

The gear tooth forms are determined as a function of these items. Tooth depth may also be considered as a function of the desired bottom clearance. The tooth form design must ultimately relate back to the tooling and work piece used to manufacture the gear, namely the profile shift, addendum, and dedendum coefficients. Additionally, tool tip radius and its influence on root strength and the active profile may be studied.

As you can see, full consideration of the detailed tooth forms can cause the design space to become very large. Usually, a designer has several constraints that can be imposed to help reduce the design space, allowing for quicker exploration of viable design candidates.

In the working example provided at the end of this section, the design space is limited based on a required center distance, target gear ratio, and sum of profile shifts. With these constraints, a human can reasonably explore and interpret the design space. Without these constraints, it is advised that optimization algorithms be used with computational methods to efficiently explore the very large design space.

Axial Dimension

Adding to the design space are the parameters associated with the axial dimension: facewidth, b, and helix angle, β. Recall from a prior section that the axial contact ratio is solely responsible for the variation in total length of contact lines, Δl_D, through a mesh cycle. Nominally, this variation will be zero to minimize the gear mesh dynamic loads and noise, calculated as:

$$\beta = \arcsin \frac{k\pi m_n}{b}$$

where k must be an integer to achieve $\Delta l_D = 0$.

Unfortunately, the use of helical teeth is not entirely beneficial. Disadvantages of helical gears compared to spur gears include:

1. Axial forces occur at the gear mesh and must be reacted by bearing supports.
2. Normal module is reduced for a given package size, but the normal force increases.
3. Sliding between gear flanks occurs over a larger surface area, reducing efficiency.

Unless dynamic loads and noise are of little concern, the benefits of helical gearing outweigh its disadvantages.

The axial contact ratio is *not* affected by center distance or tooth depth, a beneficial trait when designing the helix angle to minimize dynamic loading.

Example | Helical Gear Design

GIVEN

1. Target gear ratio, with +/- range.
2. Required center distance.
3. Required pressure angle.
4. Required helix angle.

FIND
Helical gear mesh designs that satisfy the input constraints.

SOLUTION
See below.

Description	Symbol	Value	Units
Target ratio	i_o	1.75	-
Target ratio deviation	Δi	1.00	%
Actual center distance	a	46.35	mm
Sum of profile shift coeff.	Σx^*	-0.40	-
Helix angle	β	0.00	$^\circ$

SOLUTION IS UNDER DEVELOPMENT

References

1. Gears and Gear Drives, 1st Edition. Damir Jelaska (https://www.wiley.com/en-us/Gears+and+Gear+Drives-p-9781119941309)
2. Handbook of Practical Gear Design and Manufacture, 1st Edition. Darle W. Dudley
3. ANSI/AGMA 1010-F14, Appearance of Gear Teeth - Terminology of Wear and Failure
4. Cheng, Harry H., "Derivation of the Explicit Solution of the Inverse Involute Function and its Application in gear Tooth Geometry", Journal of Applied Mechanisms and Robotics, 1996
5. Wolfram MathWorld - Helix (http://mathworld.wolfram.com/Helix.html)
6. Wikipedia - Helix (https://en.wikipedia.org/wiki/Helix)