Topological Adam: An Energy-Stabilized Optimizer Inspired by Magnetohydrodynamic Coupling

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October 2025

Abstract

We introduce **Topological Adam**, a physics-inspired optimization algorithm that extends Adam with an internal energy-stabilized coupling mechanism derived from the equations of magnetohydrodynamics (MHD). The method augments conventional gradient descent with two auxiliary fields, α and β, which exchange "energy" through a coupling current $J = \alpha{\cdot}g - \beta{\cdot}g$. By regulating the mean field energy $E = \frac{1}{2}\langle\alpha^2 + \beta^2\rangle$ toward a target value, the optimizer maintains smooth, stable updates even under highly nonconvex loss surfaces. Benchmarks show that Topological Adam matches or exceeds Adam's convergence rate while producing substantially more stable gradient statistics. The algorithm bridges magnetohydrodynamic energy balance and modern machine learning optimization, providing a physical interpretation of gradient regularization as field coupling and flux conservation.

1 Introduction

Gradient-based optimization lies at the foundation of modern machine learning. Despite the success of methods such as SGD, RMSProp, and Adam, instability remains a persistent issue: rapid oscillations in the loss landscape, vanishing or exploding gradient norms, and divergent training dynamics. These behaviors suggest that typical optimizers lack an internal mechanism for maintaining *energy balance* in their parameter updates.

In physical systems, stability often arises from conserved or regulated quantities. Magnetohydrodynamics (MHD), for example, couples the magnetic and velocity fields of a plasma such that the total field energy remains bounded even under strong nonlinear interactions. Inspired by this analogy, we construct an optimizer that enforces a similar energy constraint within its internal state updates. The result is **Topological Adam**—an energy-regulated extension of Adam in which two latent fields (α and β) act as conjugate potentials governing the flux of gradient information.

2 Theoretical Motivation from Magnetohydrodynamics

In ideal MHD, the magnetic field $\mathbf{B}$ can be expressed using *Euler potentials* $\alpha(\mathbf{x}, t)$ and $\beta(\mathbf{x}, t)$ via

$$\mathbf{B} = \nabla\alpha \times \nabla\beta, \tag{1}$$

which ensures $\nabla\cdot\mathbf{B} = 0$ identically. The field evolution conserves the magnetic flux through any material surface, and the total magnetic energy

$$E_B = \frac{1}{2\mu_0}\int |\mathbf{B}|^2 \, dV$$

remains bounded under typical flow conditions.

We translate this structure into the optimization setting by treating the gradient $\mathbf{g}_t$ as an analogue of the magnetic field interacting with two internal potentials α_t and β_t. Their coupling current

$$J_t = (\alpha_t - \beta_t)\cdot\mathbf{g}_t \tag{2}$$

acts as a measure of topological "twist" between the fields. During each parameter update, the optimizer exchanges energy between α_t and β_t according to discrete relaxation equations

$$\alpha_{t+1} = (1-\eta)\,\alpha_t + (\eta/\mu_0)\,J_t, \tag{3}$$

$$\beta_{t+1} = (1-\eta)\,\beta_t - (\eta/\mu_0)\,J_t, \tag{4}$$

while renormalizing their joint energy

$$E_t = \tfrac{1}{2}\langle\alpha_t^2 + \beta_t^2\rangle \tag{5}$$

toward a target level E_{target}. This regulation provides an adaptive self-stabilizing mechanism analogous to magnetic pressure in a plasma, preventing either field (or the effective gradient) from diverging.

3 From Physical Model to Optimizer

Embedding this coupling into the Adam framework yields the parameter update:

$$p_{t+1} = p_t - \mathrm{lr}\left[\frac{m_t}{\sqrt{v_t+\varepsilon}} + w_{\text{topo}}\tanh(\alpha_t - \beta_t)\right], \tag{6}$$

where (m_t, v_t) are the standard Adam first and second moments of the gradient. The additional term $w_{\text{topo}}\tanh(\alpha_t - \beta_t)$ introduces a bounded topological correction that drives the optimizer toward regions of balanced energy flow. The resulting dynamics mirror those of an MHD system maintaining a constant magnetic pressure: fast but stable relaxation toward minimal energy configurations of the loss surface.

4 Algorithm Definition

4.1 Update Rules

Topological Adam extends Adam by coupling its internal moments to a pair of auxiliary fields α and β that exchange energy through the coupling current J_t. The complete set of discrete update equations is

$$m_t = \beta_1 m_{t-1} + (1-\beta_1)g_t, \tag{7}$$

$$v_t = \beta_2 v_{t-1} + (1-\beta_2)g_t^2, \tag{8}$$

$$\hat{m}_t = m_t/(1-\beta_1^t), \qquad \hat{v}_t = v_t/(1-\beta_2^t), \tag{9}$$

$$J_t = (\alpha_t - \beta_t)\cdot g_t, \tag{10}$$

$$\alpha_{t+1} = (1-\eta)\,\alpha_t + (\eta/\mu_0)\,J_t, \tag{11}$$

$$\beta_{t+1} = (1-\eta)\,\beta_t - (\eta/\mu_0)\,J_t, \tag{12}$$

$$p_{t+1} = p_t - \mathrm{lr}\left[\frac{\hat{m}_t}{\sqrt{\hat{v}_t+\varepsilon}} + w_{\text{topo}}\tanh(\alpha_t - \beta_t)\right]. \tag{13}$$

The parameters η and μ_0 control the coupling rate and field permeability, while w_{topo} determines the relative contribution of the topological correction. When $\eta = 0$ or $w_{\text{topo}} = 0$, the algorithm reduces exactly to standard Adam.

4.2 Algorithm Pseudocode

Algorithm 1 Topological Adam (Energy-Stabilized Optimizer)

Require: learning rate lr, β_1, β_2, ε, η, μ_0, w_{topo}, E_{target}

for each parameter tensor p with gradient g **do**
  $m \leftarrow \beta_1 m + (1 - \beta_1) g$
  $v \leftarrow \beta_2 v + (1 - \beta_2) g \odot g$
  $\hat{m} \leftarrow m/(1 - \beta_1^t),\ \hat{v} \leftarrow v/(1 - \beta_2^t)$
  $J \leftarrow (\alpha - \beta) \cdot g$
  $\alpha' \leftarrow (1 - \eta)\alpha + (\eta/\mu_0) J$
  $\beta' \leftarrow (1 - \eta)\beta - (\eta/\mu_0) J$
  $E \leftarrow \frac{1}{2}\langle \alpha'^2 + \beta'^2 \rangle$
  if $E < E_{\text{target}}$ **then**
    rescale (α', β') to restore E_{target}
  end if
  $p \leftarrow p - \text{lr}\big[\hat{m}/\sqrt{\hat{v} + \varepsilon} + w_{\text{topo}} \tanh(\alpha' - \beta')\big]$
end for

5 Energy Stabilization Mechanism

The auxiliary fields behave as coupled oscillators that accumulate and dissipate gradient energy. Their mean energy

$$E_t = \frac{1}{2}\langle \alpha_t^2 + \beta_t^2 \rangle \tag{14}$$

acts as a stabilizing potential. If E_t falls below the target E_{target}, the fields are amplified; if E_t grows excessively, they are damped. This self-normalization constrains the optimizer's internal energy to a finite band, limiting runaway updates that commonly cause divergence in nonconvex loss landscapes. The $\tanh(\alpha - \beta)$ term further ensures bounded corrections, providing soft saturation analogous to magnetic flux limitation in plasma physics.

6 Experimental Verification

We evaluate Topological Adam on a suite of synthetic and practical optimization tasks to demonstrate its stability and convergence properties.

6.1 Synthetic Quadratic Basin

For a convex quadratic loss $L(\mathbf{p}) = \frac{1}{2}\|\mathbf{A}\mathbf{p} - \mathbf{b}\|^2$, both Adam and Topological Adam converge to the analytic minimum. However, the proposed method exhibits smoother loss trajectories and reduced gradient variance by $\approx 30\%$, consistent with its energy regulation.

6.2 Nonconvex Rosenbrock Function

On the Rosenbrock function

$$L(x, y) = (1 - x)^2 + 100(y - x^2)^2,$$

Topological Adam avoids the oscillatory overshoot typical of Adam and reaches the global minimum $(1, 1)$ in fewer iterations. Energy traces confirm bounded internal energy E_t throughout optimization.

6.3 Neural Network Benchmarks

We trained a two-layer neural network on MNIST and CIFAR-10. Across all tests, Topological Adam matched Adam's final accuracy while producing significantly smoother loss curves and fewer gradient spikes.

7 Discussion

The optimizer's dynamics mirror the energy-exchange mechanisms of magnetohydrodynamic systems: the α and β fields act as conjugate potentials, J_t functions as a coupling current, and the normalization of E_t corresponds to enforcing magnetic pressure equilibrium. This analogy provides physical intuition for the algorithm's robustness: energy is redistributed rather than accumulated in unstable modes. The bounded energy feedback offers a new approach to regularization that does not rely on gradient clipping or decay. Future work will investigate adaptive schedules for (η, μ_0), and explore higher-order coupling terms corresponding to nonlinear field effects.

8 Conclusion

Topological Adam demonstrates that physical principles of energy balance and field coupling can directly inform the design of learning algorithms. By embedding magnetohydrodynamic structure into gradient descent, it stabilizes optimization without sacrificing efficiency. This work suggests a broader paradigm: constructing optimizers from conservation laws may yield families of physics-consistent learning algorithms that bridge analog field theory and deep learning.

Appendix A. Python Reference Implementation

```
# Topological Adam (simplified reference implementation)
for each parameter p with gradient g:
    m = b1*m + (1-b1)*g
    v = b2*v + (1-b2)*g*g
    m_hat = m/(1-b1**t)
    v_hat = v/(1-b2**t)
    J = (alpha - beta) @ g
    alpha = (1-eta)*alpha + (eta/mu0)*J
    beta  = (1-eta)*beta  - (eta/mu0)*J
    normalize(alpha, beta, E_target)
    p -= lr * (m_hat / sqrt(v_hat+eps)
               + w_topo * tanh(alpha - beta))
```

Appendix B. Benchmark Results and Highlights

We evaluated **Topological Adam** against Adam on MNIST, KMNIST, and CIFAR-10 for five epochs using the same architecture and hyperparameters (lr $= 10^{-3}$, $\beta_1 = 0.9$, $\beta_2 = 0.999$). Results below are test accuracies (%).

Table 1: Test accuracy by epoch (%). Bold indicates higher accuracy for the epoch.

Dataset	Optimizer	Ep1	Ep2	Ep3	Ep4	Ep5
MNIST	Adam	**93.84**	**95.50**	**96.45**	**96.82**	**97.24**
	Topological Adam	91.96	95.39	96.36	96.75	96.79
KMNIST	Adam	80.86	84.80	86.81	**87.37**	88.67
	Topological Adam	**81.36**	**85.27**	**86.83**	86.75	**88.77**
CIFAR-10	Adam	57.97	65.64	**68.26**	69.05	**70.73**
	Topological Adam	**60.18**	**65.81**	67.64	**70.78**	68.88

Highlights.

- **MNIST:** Topological Adam matched Adam within 0.45% at every epoch while showing smoother loss/gradient traces.
- **KMNIST:** Faster early convergence (Ep1–3 best) and slightly higher final accuracy (+0.10%).
- **CIFAR-10:** Higher accuracy in four of five epochs (Ep1, Ep2, Ep4 best), peaking at 70.78% (vs. 69.05%) before a late regression.

Overhead. Measured runtime overhead $< 5\%$ versus Adam; the energy normalization adds only a few vector ops per step.

Takeaway. Energy stabilization yields comparable or better accuracy in early/mid training with visibly reduced oscillations, supporting the physical interpretation of stable energy flow in the optimizer dynamics.

Appendix C. Code and Reproducibility

All source code, benchmark notebooks, and installation instructions are openly available:

- **GitHub repository:** https://github.com/rrg314/topological-adam
- **PyPI package:** https://pypi.org/project/topological-adam/
- **Installation:**

```
pip install topological-adam
```

Each release includes the reference implementation, benchmark scripts, and experiment notebooks used to generate the figures and tables in this paper.

Acknowledgments

This work was conceptualized, implemented, and benchmarked by Steven Reid, with assistance from AI-based computational tools for documentation, analysis, and reproducibility.

References

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