Experimental Integration of Magnetohydrodynamic Closure and Recursive Division Theory in the Topological Adam Optimizer

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November 2025

Abstract

This document presents a concise description of an experimental framework that combines two previously independent formulations: the closure of Euler potentials in resistive magnetohydrodynamics (MHD) and the Recursive Division Tree (RDT) algorithm. The intent is to formalize how these systems can be connected through shared mathematical structures to construct the *Topological Adam* optimizer. The focus is on demonstrating the correspondence of variables, functional dependencies, and equilibrium properties rather than asserting performance outcomes. All formulations are presented descriptively and are open to independent verification.

1 Objective

The goal of this work is to demonstrate a consistent mapping between:

1. the coupled scalar-field representation used in resistive MHD to maintain energy balance under diffusion, and
2. the recursive logarithmic damping law introduced in the RDT algorithm for describing bounded iterative decay.

By combining these frameworks, the Topological Adam optimizer implements an internally regulated energy mechanism for gradient-based learning systems.

2 Magnetohydrodynamic Closure and Coupled Fields

2.1 Governing relations

In resistive MHD, the magnetic field B is defined by two scalar Euler potentials,

$$B = \nabla\alpha \times \nabla\beta,$$

which identically satisfy $\nabla \cdot B = 0$. Resistive effects introduce diffusion that modifies the induction equation:

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta \nabla^2 B,$$

where η is the magnetic diffusivity and v is the velocity field. To maintain the correct energy evolution of B, additional scalar source terms S_α and S_β must be defined such that:

$$\frac{\partial B}{\partial t} = \eta \nabla^2 B + \nabla S_\alpha \times \nabla \beta + \nabla \alpha \times \nabla S_\beta.$$

These sources ensure that resistive diffusion in the scalar potentials produces a physically consistent magnetic field evolution.

2.2 Translation to optimizer formulation

In the optimizer, the scalar variables α_t and β_t correspond to internal fields that interact through the gradient g_t rather than through spatial derivatives. Their coupling is defined by the current:

$$J_t = (\alpha_t - \beta_t) \cdot g_t.$$

The discrete update equations are

$$\alpha_{t+1} = (1-\eta)\alpha_t + \frac{\eta}{\mu_0} J_t,$$
$$\beta_{t+1} = (1-\eta)\beta_t - \frac{\eta}{\mu_0} J_t,$$

where μ_0 is a coupling coefficient controlling the exchange rate. The total internal field energy is given by

$$E_t = \frac{1}{2}\langle \alpha_t^2 + \beta_t^2 \rangle.$$

This energy is regulated toward a target value E^* through normalization during each iteration. This structure preserves bounded internal energy dynamics, analogous in form to the MHD energy closure but applied here to parameter-space gradients.

3 Recursive Division Tree and Logarithmic Damping

3.1 Definition of RDT process

The Recursive Division Tree algorithm defines an iterative sequence:

$$x_{i+1} = \left\lfloor \frac{x_i}{\max(2, \lfloor (\log x_i)^{1.5} \rfloor)} \right\rfloor,$$

and the number of iterations required for convergence, $RDT(n)$, grows as

$$RDT(n) \sim c \log \log n,$$

where $c \approx 2.17$ is an empirically determined constant. This relation establishes a slow, bounded recursive decay rate that approaches equilibrium smoothly and monotonically.

3.2 Application to optimizer damping

In Topological Adam, the recursive damping law governs how the field energy E_t approaches its target E^*. Rather than decaying exponentially, the internal energy decreases according to a logarithmic recursive profile, which slows as the system approaches equilibrium. This ensures continuity and bounded behavior across multiple scales of learning rate and gradient magnitude.

4 Unified Optimizer Formulation

The integration of the MHD coupling and RDT damping yields the discrete update scheme:

$$\begin{aligned}
J_t &= (\alpha_t - \beta_t) \cdot g_t, \\
\alpha_{t+1} &= (1-\eta)\alpha_t + \frac{\eta}{\mu_0} J_t, \\
\beta_{t+1} &= (1-\eta)\beta_t - \frac{\eta}{\mu_0} J_t, \\
p_{t+1} &= p_t - lr \frac{\hat{m}_t}{\sqrt{\hat{v}_t + \epsilon}} + w_{\text{topo}} \tanh(\alpha_{t+1} - \beta_{t+1}),
\end{aligned}$$

where the standard Adam moments (m_t, v_t) remain as defined in the original algorithm. The additional term $w_{\text{topo}} \tanh(\alpha_{t+1} - \beta_{t+1})$ provides a bounded correction that reflects the instantaneous field imbalance.

5 System Linkages

The mathematical correspondence between the two source frameworks and the optimizer components can be summarized as:

Quantity / Concept	**Source System**	**Function in Optimizer**
Dual scalar potentials (α, β)	MHD Closure	Represent internal energy fields
Coupling current J_t	MHD Closure	Defines direction and rate of energy exchange
Energy regulation $E_t \to E^*$	MHD Closure	Maintains bounded internal energy
Recursive decay $\log \log n$	RDT	Defines temporal damping rate of E_t
Hierarchical shell structure	RDT	Describes multi-scale stability regions

These connections are structural: MHD describes the mechanism of energy exchange, while RDT specifies the form of the damping law governing its evolution. Their integration defines a consistent iterative process for maintaining equilibrium within the optimization dynamics.

6 Functional Outcome

The integrated framework produces a system with the following measurable properties:

1. Internal energy remains bounded by construction, preventing unregulated increase or collapse of update magnitudes.
2. The recursive damping law enforces gradual adaptation, preserving information stability between successive iterations.
3. The energy feedback introduces an additional degree of dynamic control without altering the statistical expectations of the gradient moments.

These outcomes can be tested quantitatively using loss-surface energy metrics or gradient-norm variance. They arise directly from the governing equations rather than from external heuristics.

7 Summary

The Topological Adam optimizer is formulated as an experimental intersection between a field-coupled dynamical system and a logarithmic recursive process. The correspondence is as follows:

$$\text{MHD closure: } \{\alpha, \beta, J_t, E_t\} \quad \Rightarrow \quad \text{energy regulation mechanism,}$$

$$\text{RDT law: } \log\log \text{ damping} \quad \Rightarrow \quad \text{bounded and continuous relaxation rate.}$$

When combined, these produce a discrete iterative model where the evolution of internal energy and learning dynamics remains stable and self-consistent. The formulation is presented as an experimental construct intended for further evaluation and validation. It does not assert predictive equivalence with physical plasma systems and does not claim guaranteed improvement over existing optimization methods. Its primary contribution is to establish a unified formal structure demonstrating how recursive damping and field coupling can coexist within a gradient-based learning system.