Week 1 - Written Exercises

1. (Not from the book) Consider the sample space $\Omega = \{1, 2, 3, 4, 5\}$ and its subsets $A = \{3, 5\}$, $B = \{1, 2, 4\}$ and $C = \{1, 2, 5\}$.

   (a) List the outcomes/elements that are in the intersection $B \cap C$.

   (b) List the outcomes/elements that are in the union $A \cup C$.

   (c) List the outcomes/elements that are in the complement B^c of the set B. What do you notice about this complement?

   (d) List the outcomes/elements that are in the union $A \cup (B \cap C^c)$.

   (e) Are the sets A and B disjoint? What about the sets A and C? Justify your answer.

a) $B \cap C = \{1, 2\}$

b) $A \cup C = \{1, 2, 3, 5\}$

c) $B^C = \{3, 5\}$ -> I notice that it's the same as set A

d) $A \cup (B \cap C^C) = ?$

Let's do this in parts till we get to the final result:

$C^C = \{3, 4\}$ and $B \cap C^C = \{4\}$

Therefore, $A \cup (B \cap C^C) = \{3, 4, 5\}$

e) - Yes, the sets A and B are disjoint because they are the complement set of each other, which means that the events contained within each of the two sets cannot happen simultaneously. Their intersection is a null set.

- No, the sets A and C are not disjoint because they contain a common element so their intersection is not a null set. $A \cap C = \{5\}$

2. (page 157, problem 4) The following table lists the 10 countries with the highest production of meat. Suppose a World Health Organization committee is formed to discuss the long-term ramifications of producing such quantities of meat. Suppose further that it consists of one representative from each of these countries. If the chair of this committee is then randomly chosen, find the probability that this person will be from a country whose production of meat (in thousands of metric tons)

   (a) exceeds 10 000

   (b) is under 3500

   (c) is between 4000 and 6000

   (d) is less than 2000.

Country	Meat production (thousands of metric tons)
China	20 136
United States	17 564
Russia	12 698
Germany	6395
France	3853
Brazil	3003
Argentina	2951
Britain	2440
Italia	2413
Australia	2373

Total number of countries = 10

M = "Meat Production (thousands of metric tons)

a) number of countries with M > 10 000 = 3 (China, United States, Russia)

P = $\frac{number\ of\ countries\ with\ M>10\ 000}{total\ number\ of\ countries} = \frac{3}{10}$ = 0.3 = 30%

b) number of countries with M < 3500 = 5 (Brazil, Argentina, Britain, Italy, Australia)

P = $\frac{number\ of\ countries\ with\ M<3\ 500}{total\ number\ of\ countries} = \frac{5}{10}$ = 0.5 = 50%

c) number of countries with 4000 ≤ M ≤ 6000 = 0 (France does not have more than 4000, and Germany does not have less than 6000. There are not countries listed between those two)

P = $\frac{number\ of\ countries\ with\ 4000 \leq M \leq 6000}{total\ number\ of\ countries} = \frac{0}{10}$ = 0 = 0%

d) number of countries with M <2000 = 0 (the smallest M is for Australia with 2373, which is not less than 2000)

P = $\frac{number\ of\ countries\ with\ M<2000}{total\ number\ of\ countries} = \frac{0}{10}$ = 0 = 0%

3. (page 153, problem 16) Anita has a 40 percent chance of receiving an A grade in statistics, a 60 percent chance of receiving an A in physics, and an 86 percent chance of receiving an A in statistics or physics. Find the probability that she

(a) does not receive an A in either statistics or physics

(b) receives A's in both statistics and physics.

Note The word "or" above is inclusive or (common meaning in mathematics) and not the exclusive or (i.e. "either ... or"), which is common interpretation. However, the "... not ... either .. or" is interpreted as usual.

A = event where Anita gets an A in Statistics ; *~A = A doesn't happen*

B = event where Anita gets an A in Physics ; *~B = B doesn't happen*

P(A) = 40% ; P(B) = 60% ; P(A ∪ B) = 80% ; *P(~A) = 1-P(A) ; P(~B) = 1-P(B)*

a) P(~A ∩ ~B) = P(~A) * P(~B) = [1-P(A)] * [1-P(B)] = (1-0.4) * (1-0.6) = 0.24 = 24%

b) P(A ∩ B) = P(A) * P(B) = 0.4 * 0.6 = 0.24 = 24%

4. (Page 150, problem 4) A certain person encounters three traffic lights when driving to work. Suppose that the following represent the probabilities of the total number of red lights that she has to stop for:

$$\mathbb{P}(\text{“0 red lights”}) = 0.14$$
$$\mathbb{P}(\text{“1 red light”}) = 0.36$$
$$\mathbb{P}(\text{“2 red lights”}) = 0.34$$
$$\mathbb{P}(\text{“3 red lights”}) = 0.16$$

(a) What is the probability that she stops for at least one red light when driving to work?

(b) What is the probability that she stops for more than two red lights?

a) P("at least one red light") = P("1 red light") + P("2 red lights") + P("3 red lights") = 0.36 + 0.34 + 0.16 = 0.86 = 86%

b) P("more than 2 red lights") = P("3 red lights") = 0.16