Problems are from sections 4.5–4.7 from Introductory Statistics by Ross.

1. (Page 189, problem 12) A delivery company has 10 trucks, of which 3 have faulty brakes. If an inspector randomly chooses 2 of the trucks for a brake check, what is the probability that **none** of the trucks with faulty brakes are chosen?

P("faulty brakes") = 3/10 = 0.3 = 30%

P("not faulty brakes") = 7/10 = 0.7 = 70%

P("2 randomly chosen trucks do not have faulty brakes") = P("not faulty") x P("not faulty") = 0.7 x 0.7 = 0.49 = 49%

2. (Page 170, problem 8) The following table lists the number of students enrolled in a California State College, categorized by sex and age. (see table 4.4 on page 162)

	Male	Female
14 to 17 years old	91	119
18 to 19 years old	1 309	1 455
20 to 21 years old	1 089	1 135
22 to 24 years old	1 080	968
25 to 29 years old	1 016	931
30 to 34 years old	613	716
35 years old and over	683	1 339
Total	5 881	6 663

Determine the conditional probability that a randomly chosen student for these 12 544 students is

(a) Less than 25 years old, given that the student is a man
(b) A man, given that this student is less than 25 years old
(c) Less than 25 years old, given that the student is a woman
(d) A woman, given that this student is less than 25 years old

Conditional probability: P(A | B) = P(A ∩ B) / P(B)

P("student is a man") = 5881 / 12544 = 0.4688 = 46.88%

P("student is a woman") = 6663 / 12544 = 0.5312 = 53.12 %

P("student is less than 25 years old – either gender") = (91 + 119 + 1309 + 1455 + 1089 + 1135 + 1080 + 968) / 12544 = 0.5776 = 57.76%

P("student is a man AND he is under 25 years old") = (91 + 1309 + 1089 + 1080) / 12544 = 0.2845 = 28.45%

P("student is a woman AND she is under 25 years old") = (119 + 1455 + 1135 + 968) / 12544 = 0.2931 = 29.31%

a) A = "student is less than 25 years old"

B = "student is a man"

A ∩ B = ("student is a man AND he is under 25 years old"

P(A | B) = P(A ∩ B) / P(B) = 0.2845 / 0.4688 = 0.6069 = 60.69%

b) A = “student is a man”

B = “student is less than 25 years old”

A ∩ B = (“student is a man AND he is under 25 years old”

P(A | B) = P(A ∩ B) / P(B) = 0.2845 / 0.5776 = 0.4926 = 49.26 %

c) A = “student is less than 25 years old”

B = “student is a woman”

A ∩ B = (“student is a woman AND she is under 25 years old”

P(A | B) = P(A ∩ B) / P(B) = 0.2931 / 0.5312 = 0.5518 = 55.18%

d) A = “student is a woman”

B = “student is less than 25 years old”

A ∩ B = (“student is a woman AND she is under 25 years old”

P(A | B) = P(A ∩ B) / P(B) = 0.2931 / 0.5776 = 0.5074 = 50.74%

3. Let us consider events A, B, and C, assosiated with the following probabilities

$$\begin{aligned} \mathbb{P}(A) &= 0.5 \\ \mathbb{P}(B) &= 0.8 \\ \mathbb{P}(C) &= 0.6 \\ \mathbb{P}(A \cap B) &= 0.4 \\ \mathbb{P}(A \cap C) &= 0.2 \end{aligned}$$

(a) Are the events A and B independent? Justify your answer using the definition of independence (from the page 166 in the book).

(b) Are the events A and C independent? Justify your answer using the definition of independence.

(c) Calculate the conditional probability $\mathbb{P}(A|C)$.

(d) Calculate the conditional probability $\mathbb{P}(C|A)$.

From page 166 in the book, we get:

Events A and B are *independent* if

$P(A \cap B) = P(A)P(B)$

If A and B are independent, then the probability that a given one of them occurs is unchanged by information as to whether the other one has occurred.

a) P(A ∩ B) = 0.4 and P(A) x P(B) = 0.5 x 0.8 = 0.4

Since P(A ∩ B) **=** P(A) x P(B), the events A and B <u>are</u> independent

b) $P(A \cap C) = 0.2$ and $P(A) \times P(C) = 0.5 \times 0.6 = 0.3$

Since $P(A \cap C) \neq P(A) \times P(C)$, the events A and C <u>are not</u> independent

c) $P(A \mid C) = P(A \cap C) / P(C) = 0.2 / 0.6 = 0.3333 = 33.33\%$

d) $P(C \mid A) = P(C \cap A) / P(A) = 0.2 / 0.5 = 0.4 = 40\%$

4. (Page 180, problem 3) The inspector in charge of a criminal investigation is 60 percent certain of the guilt of a certain suspect. A new piece of evidence proving that the criminal was left-handed has just been discovered. Whereas the inspector knows that 18 percent of the population is left-handed, she is waiting to find out whether the suspect is left-handed.

   (a) What is the probability that the suspect is left-handed?

   (b) If the suspect turns out to be left-handed, what is the probability that the suspect is guilty?

P("suspect is guilty") = 60% = 0.6

P("a person is left-handed") = 18% = 0.18

a) P("suspect is left-handed") = P(" a person is left-handed") = 18%

Even if we would take this reasoning further, we would get still get the same answer:

P("suspect is left-handed") = P("guilty and left-handed) + P("not guilty and left-handed") = 0.6x0.18 + 0.4x0.18 = 0.108 + 0.072 = 0.18 = 18%

b) P("suspect is guilty" given that "suspect is left-handed") = ?

We can calculate:

P("guilty" ∩ "left-handed") = P("guilty") x P("left-handed") = 0.6 x 0.18 = 0.108

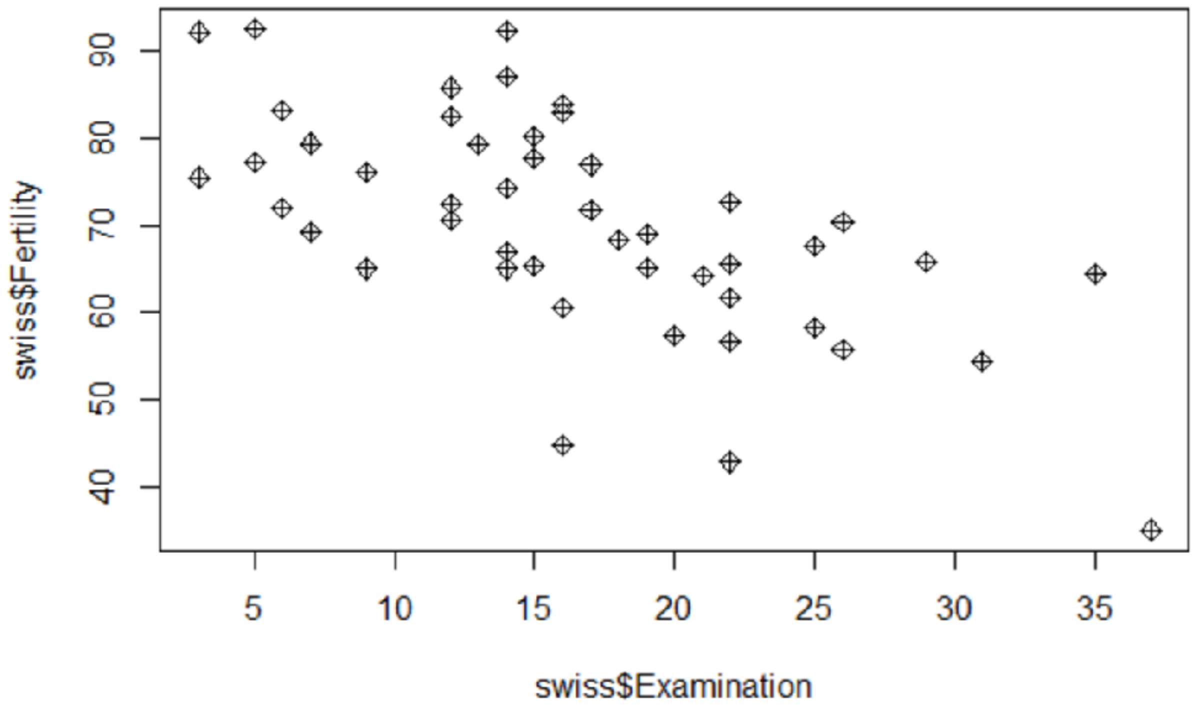
So, finally, we can replace and calculate:

P("guilty" | "left-handed") = P("guilty" ∩ "left-handed") / P("left-handed") = 0.108 / 0.18 = 0.60 = 60%

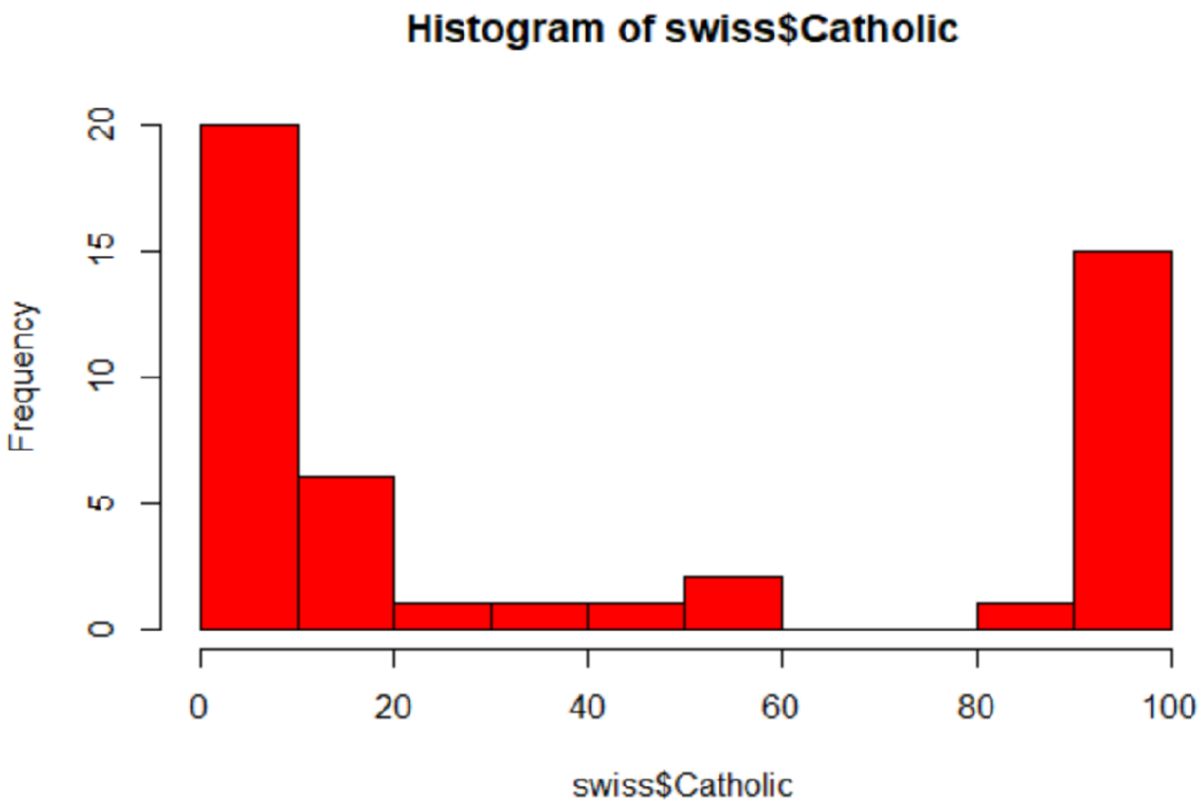
5. (Page 200, problem 30) Two percent of women of age 45 who participate in routine screening have breast cancer. Ninety percent of those with breast cancer have positive mammographies. Ten percent of the women who do not have breast cancer will also have positive mammographies. Given a woman has a positive mammography, what is the probability she has breast cancer?

We are told in the problem that 90% of positive test results are true positives, so if the woman tested positive for cancer there is a 90% probability that it is correct and she has cancer.

R exercise 3a – scatter plot of Examination and Fertility, using diamonds instead of circles:



R exercise 3b – Red histogram of variable Catholic:



R programming exercise 3c – Box plot of Fertility: