1. (Page 267, problem 6) Suppose that the number of minutes of playing time of a certain college basketball player in a randomly chosen game has the following density curve (fig 1).

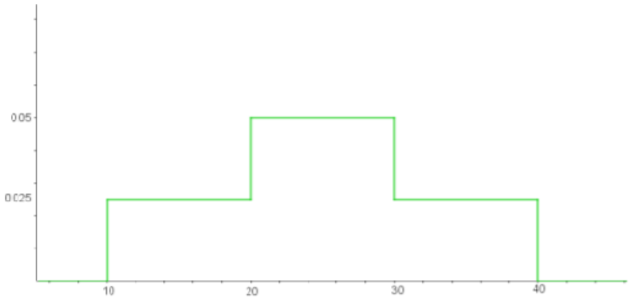


Figure 1: The curve of the density function

Find the probability that the player plays

(a) Over 20 minutes

(b) Less than 25 minutes

(c) Between 15 and 35 minutes

(d) More than 35 minutes

To find out the probabilities all I need to do is to calculate the area contained under the probability density curve:

a) P("plays over 20 minutes") = area of density curve beyond 20 minutes = (30-20) * 0.05 + (40-30) * 0.025 = 0.5 + 0.25 = 0.75 = 75%

b) P("plays under 25 minutes") = area of density curve before 25 minutes = (20-10) * 0.025 + (25-20) * 0.05 = 0.25 + 0.25 = 0.5 = 50%

c) P("plays between 15 and 35 minutes") = 5 * 0.025 + (30-20) * 0.05 + 5 * 0.025 = 0.75 = 75%

d) P("plays more than 35 minutes") = 5 * 0.025 = 0.125 = 12.5%

2. (Page 289, problem 12) Scores on the quantitative part of the Graduate Record Examination were normally distributed with a mean score of 510 and a standard deviation of 92. How high a score was necessary to be in the top

(a) 10

(b) 5

(c) 1

percent of all scores?

SD = 92 => Variance = $SD^2 = 92^2 = 8464$; mean = 510

Our scores X have a Normal distribution **N(510, 8464)**

In this problem we are working backwards: we have the probability/percentage result from the Z table, from which we will get the Z value, from which we will calculate the X value for the exam scores.

a) Top 10% means a probability that 90% (or p = 0.9) of people had lower scores. Looking up 0.9 in the Z table, we get Z = 1.28 as being the closest one.

Z = (X – Mean)/ SD ⇔ X = (Z * SD) + Mean = (1.28 * 92) + 510 = 117.76 + 510 = 627.76

One needed a minimum score of <u>627.76</u> to be in the top 10% of all scores.

b) Top 5% means a probability that 95% (or p = 0.95) of people had lower scores. Looking up 0.95 in the Z table, we get Z being between 1.64 and 1.65 so we'll do Z = 1.645

Z = (X – Mean)/ SD ⇔ X = (Z * SD) + Mean = (1.645 * 92) + 510 = 151.34 + 510 = 661.34

One needed a minimum score of <u>661.34</u> to be in the top 5% of all scores.

c) Top 1% means a probability that 99% (or p = 0.9) of people had lower scores. Looking up 0.99 in the Z table, we get Z = 2.33 as being the closest one.

Z = (X – Mean)/ SD ⇔ X = (Z * SD) + Mean = (2.33 * 92) + 510 = 214.36 + 510 = 724.36

One needed a minimum score of <u>724.36</u> to be in the top 1% of all scores.

3. (Page 283, problem 15 beginning) The height of adult women in the United States is normally distributed with mean 64.5 inches and standard deviation 2.4 inches. Find the probability that a randomly chosen woman is

(a) Less than 63 inches tall
(b) Less than 70 inches tall
(c) Between 63 and 70 inches tall
(d) Alice is 72 inches tall. What percentage of women are shorter than Alice?

Mean = 64.5 inches ; SD = 2.4 inches => Var = $SD^2 = 2.4^2 = 5.76$ $inches^2$

Normal distribution X ~ N(64.5 , 5.76)

Z score calculated by: Z = (X – Mean)/ SD

a) P(X < 63) = ?

Z = (63 – 64.5) / 2.4 = -0.625

Looking up in a Z table **-** 0.63, we get P(Z < 0.625) = 0.26435

(the Z table that I used contained also negative Z values; if it had only contained positive values, I would have used P = 1 - |Z|)

P(X < 63) = P(Z < -0.63) = 0.26435 = 26.435%

b) P(X < 70) = ?

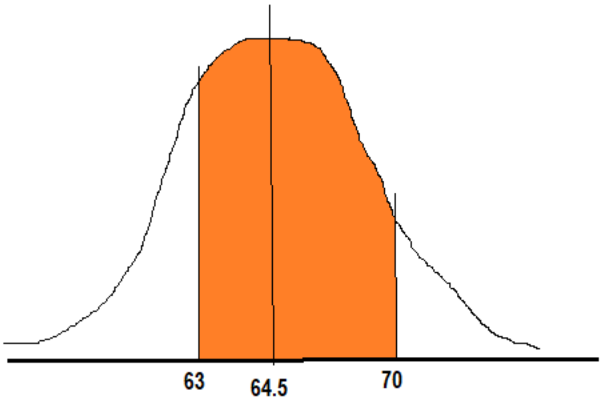
Z = (70 – 64.5) / 2.4 = 2.29

Looking up in a Z table **+ 2.29**, we get P(Z < 2.29) = 0.98899

P(X < 70) = P(Z < 2.29) = 0.98899 = 98.899%

c) $P(63 < X < 70) = ?$

This problem is easier if we visualize it:



We have already calculated in 3a) the area to the left of 63 and, in 3b), the area to the left of 70. The area between 63 and 70 is equal to the area to the left of 70 minus the area to the left of 63:

P(63 < X < 70) = P(X < 70) – P(X < 63) = **0.98899 - 0.26435 = 0.72464 = 72.464 %**

d) $P(X < 72) = ?$

$Z = (72 – 64.5) / 2.4 = 3.125$

Looking up in a Z table **+ 3.123**, we get $P(Z < 3.123) = 0.99913$

P(X < 72) = P(Z < 3.123) = 0.99913 = 99.913%

99.913% of women are shorter than Alice.

4. (Page 283, problem 15 last part modified) The height of adult women in the United States is normally distributed with mean 64.5 inches and standard deviation 2.4 inches.

   (a) Find the probability that the average of the heights of two randomly chosen women is greater than 67.5 inches.

   (b) What is the probability that the heights of two randomly chosen women deviates less than 6 inches from each other?

a) $P((X_1 + X_2)/2 > 67.5) = ?$

Both X_1 and and X_2 have a normal distribution $N(64.5 , 2.4^2)$

From the properties of the normal distribution, we then get:

$Y = X_1 + X_2$ follows a normal distribution with:

$\text{mean}(Y) = \text{mean}(X_1) + \text{mean}(X_2) = 64.5 + 64.5 = 129$

$\text{Var}(Y) = \text{Var}(X_1) + \text{Var}(X_2) = 2.4^2 + 2.4^2 = 11.52$ => $SD(Y) = \text{sqrt}(\text{Var}(Y)) = 3.39$

The average for the two heights X_1 and X_2 is 67.5 inches, so the sum of the two heights is $Y = X_1 + X_2 = 135$ inches.

$P((X_1 + X_2)/2 > 67.5) = P((X_1 + X_2) > 135) = P(Y > 135)$

$= P(Z > (135 – \text{mean}) / SD) = P(Z > (135 – 129)/3.39) = P(Z > 1.77)$

Looking up in a Z table **+ 1.77**, we get 0.96164. This is the cumulative probability to the left. Since we want the probability to the right, we get:

$P(Z > 1.77) = 1 - 0.96164 = 0.03836 =$ **3.836 %**

b) **$P(| X_1 - X_2| < 6) = P(-6 < X_1 - X_2 < +6) =?$**

Both X_1 and and X_2 have a normal distribution $N(64.5 , 2.4^2)$

From the properties of the normal distribution, we then get:

$Y = X_1 - X_2$ follows a normal distribution with:

$\text{mean}(Y) = \text{mean}(X_1) - \text{mean}(X_2) = 64.5 - 64. 5 = 0$

$Var(Y) = Var(X_1) + Var(X_2) = 2.4^2 + 2.4^2 = 11.52 \Rightarrow SD(Y) = \text{sqrt}(Var(Y)) = 3.39$

Calculating the two parts:

- $P(X_1 - X_2 = +6) = P(Y = 6) = P(Z = (6 – \text{mean}) / SD) = P(Z = (6 – 0)/3.39) = P(Z = 1.77)$

Looking up in a Z table **+ 1.77**, we get **0.96164**

- $P(X_1 - X_2 = -6) = P(Y = -6) = P(Z = (-6 – \text{mean}) / SD) = P(Z = (-6 – 0)/3.39) = P(Z = -1.77)$

Looking up in a Z table **- 1.77**, we get **0.03836** (yes, I consult a table that has also negative Z values: https://www.math.arizona.edu/~rsims/ma464/standardnormaltable.pdf)

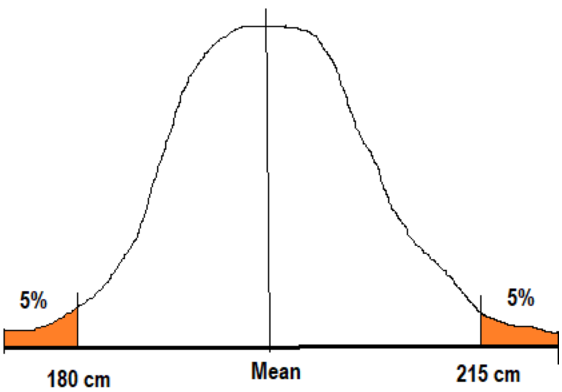
Final calculation:

We want the area of the density curve to the right of -6 and to the left of +6. This means:

$P(| X_1 - X_2| < 6) = P(-6 < X_1 - X_2 < +6) = P(X_1 - X_2 < +6) – P(X_1 - X_2 > -6) =$ $0.96164 - 0.03836 = 0.92328$ = **92.328%**

5. (Not from the book) The lengths of professional basketball players in a certain country roughly follow a normal distribution. If 5 % are more than 215 cm tall and 5 % are less than 180 cm tall, then what proprotion of them are between 200 cm and 210 cm?

To start with, it helps to visualize the information that we are given:



Since the Normal distribution is symmetrical, the Mean of this distribution will be exactly between 180 and 215 cm since the area to the left of 180 cm and the area to the right of 215 are equal (5%).

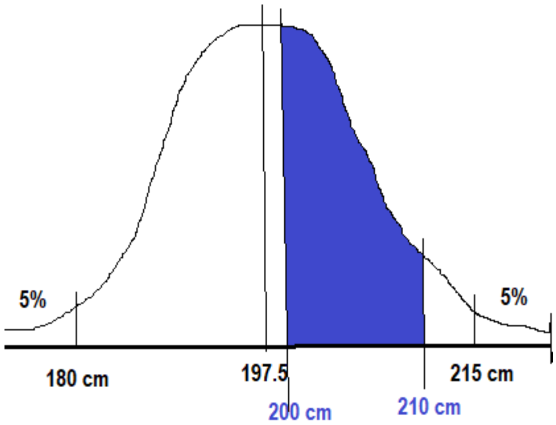
Mean = (215 + 180) / 2 = 197.5 cm

Next we want to calculate the Standard Deviation.

For the case of 215, we know that the P(Z) will be 0.95 (since the area to the left of 215 is 95% given that the area to the right is 5% and the total area is 100%). Looking up in a Z table, we get Z being between 1.64 and 1.65 so we'll do Z = 1.645

Z = (X – Mean)/ SD ⇔ **SD =** (X – Mean)/ Z = (215 – 197.5) / 1.645 = **10.638**

We want to find out the percentage of players with heights between 200 and 210 cm. This is:



This is equivalent to the area to the left of 210 minus the area to the left of 200:

P(200 < X < 210) = P(X < 210) – P(X < 200)

Let's calculate each of them:

- **P(X < 210) = ?**

  Z = (X – Mean)/ SD => Z = (210 – 197.5) / 10.638 = 1.175

  Looking up in a Z table **+ 1.175**, we get P(Z < 1.175) = 0.88

- **P(X < 200) = ?**

  Z = (X – Mean)/ SD => Z = (200 – 197.5) / 10.638 = 0.235

  Looking up in a Z table **+ 0.235**, we get P(Z < 1.175) = 0.59289

So now we can get: P(200 < X < 210) = P(X < 210) – P(X < 200) = 0.88 - 0.59289 = 0.28711

There are 28.711% of players with heights between 200 and 210 cm.