1. (Page 218, problem 4) In a certain liability suit, a lawyer has to decide whether to charge a straight fee of $1200 or to take the case on a contingency basis, in which case she will receive a fee of $5000 only if her client wins the case. Determine whether the straight fee or the contingency arrangement will result in a higher expected fee when the probability that the client will win the case is

   (a) $\frac{1}{2}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{4}$
   (d) $\frac{1}{5}$

X = earnings and P(X) = probability of earning the amount X

The expected earning if charging a straight fee is: E[X] = 1200 x 1 = 1200

possible values of X if contingency case: {0, 5000}

a) probability of each x: { ½, ½ }. Expected contingency earnings:

$$E[X] = \sum_{i=1}^{2} x_i . p(x_i) = 0 * \frac{1}{2} + 5000 * \frac{1}{2} = 2500$$

The expected contingency earnings are greater than the straight fee.

b) probability of each x: { 2/3, 1/3 }. Expected contingency earnings:

$$E[X] = \sum_{i=1}^{2} x_i . p(x_i) = 0 * \frac{2}{3} + 5000 * \frac{1}{3} = 1666.67$$

The expected contingency earnings are greater than the straight fee.

c) probability of each x: { 3/4, 1/4 }. Expected contingency earnings:

$$E[X] = \sum_{i=1}^{2} x_i . p(x_i) = 0 * \frac{3}{4} + 5000 * \frac{1}{4} = 1250$$

The expected contingency earnings are greater than the straight fee.

d) probability of each x: { 4/5, 1/5 }. Expected contingency earnings:

$$E[X] = \sum_{i=1}^{2} x_i . p(x_i) = 0 * \frac{4}{5} + 5000 * \frac{1}{5} = 1000$$

The expected contingency earnings are <u>smaller</u> than the straight fee.

2. (Page 233, problem 16) The amount of money that Robert earns has expected value \$30,000 and standard deviation \$3000. The amount of money that his wife Sandra earns has expected value \$32,000 and standard deviation \$5000. Determine the

(a) Expected value
(b) Standard deviation

of the total earnings of this family. In answering part (b), assume that Robert's earnings and Sandra's earnings are independent. (Hint: In answering part (b), first find the variance of the family's total earnings.)

E[A] = 30,000 ; SD(A) = 3000 ; A = what Robert earns

E[B] = 32,000 ; SD(B) = 5000 ; B = what Robert's wife earns

(a) E(A + B) = E(A) + E(B) = 30,000 + 32,000 = 62,000

(b) SD = sqrt(Var) => Var = SD^2

Var(A + B) = Var(A) + Var(B) = $[SD(A)]^2 + [SD(B)]^2$ = 9 000 000 + 25 000 000 = 34 000 000

SD(A + B) = sqrt(Var(A + B)) = sqrt(34 000 000) = 5830.95

3. (Page 247, problem 10) A multiple-choice examination has 3 possible answers for each of 5 questions. What is the probability that a student will get 4 or more correct answers just by guessing?

P("correct answer in a question") = 1/3 = 33.33%

P("4 correct answers") = 1/3 x 1/3 x 1/3 x 1/3 = $(1/3)^4$ = 0.012

P("5 correct answers") = 1/3 x 1/3 x 1/3 x 1/3 x 1/3= $(1/3)^5$ = 0.004

P("4 or more correct answers) = P("4 correct") + P("5 correct") = 0.012 + 0.004 = 0.016 = 1.6%

4. Let Z be a binomial random variable with expected value 7 and variance 2.1. Find

(a) $\mathbb{P}(Z = 3)$
(b) $\mathbb{P}(Z > 9)$
(c) $\mathbb{P}(4 \leq Z \leq 15)$

We are given:

n * p = 7 (the mean or expected value)

n * p * (1-p) = 2.1 (the variance)

Thus, **(1 – p) = 2.1 / 7 = 0.3**, **p = 1 – 0.3 = 0.7** and **n = 7 / 0.7 = 10**

a) Assuming Z represents the number of successes in a sequence of n Bernoulli trials and p the probability of a success in a single trial, then:

P(Z = 3) = $\binom{n}{3}(P)^2(1-P)^2 = \frac{10!}{(10-3)!\ 3!}.(0.7)^2.(0.3)^2 = 5.292$ (????)

Since $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

b) P(Z > 9) = 1 – [P(Z = 9) + P(Z = 8) + … + P(Z= 1) + P(Z = 0)] =

c) P(4 ≤ Z ≤ 15) = P(Z = 4) + P(Z = 5) + … P(Z = 14) + P(Z = 15) =

Problem 5 is an R theory exercise.

5. (a) Consider a function in R defined as follows

```
myfunction <- function(n) {
  result <- 1
  for(i in 1:n) {
    result <- result*i
  }
  result
}
```

Assume the argument `n` is a positive integer, i.e. the function is called like `myfunction(5)` or `myfunction(8)`. What does this function do? Which function in R does the same operation?

(b) Consider a function in R defined as follows

```
new_price_calc <- function(h, p) {
  coef <- 1 - p / 100
  h * coef
}
```

The function can be called, for example, with the commands `new_price_calc(50, 25)` or `new_price_calc(50, 30)`. What could this function be used for?

(a) The function multiplies all the integrals from 1 up to the number n for which the function is called **myfunction(n)** . The function in R that does the same operation is the factorial, which for a number n is called like **factorial(n)** .

(b) Function priceCalc calculates a new price based upon the initial price **h**, after a percentual price change of **p**.

Example: The initial price of a pair of shoes is 50 euros. There is a sale going on so the shoes will now be 30% off. We call the function with priceCalc(50, 30) and the function gives us 35 euros as the new price for the shoes.