Fintech545 project3

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1 Problem1

1.1 Classic Brownian Motion

$$p_t = p_{t-1} + r_t \tag{1}$$

$$r_t \sim N(0, \sigma^2 = 0.01) \tag{2}$$

From (1) and (2), we can obtain that,

$$p_t \sim N(p_{t-1}, \sigma^2 = 0.01) \tag{3}$$

Then, the mean of p_t is p_{t-1}, and the standard deviation is $\sigma = 0.1$.

1.2 Arithmetic Return System

$$p_t = p_{t-1}(1 + r_t) \tag{4}$$

From (1) and (4), we can get that,

$$p_t \sim N(p_{t-1}, 0.01 * p_{t-1}^2) \tag{5}$$

Then, the mean of p_t is p_{t-1}, and the standard deviation is $0.1 * p_{t-1}$.

1.3 Log Return or Geometric Brownian Motion

$$p_t = p_{t-1} e^{r_t} \tag{6}$$

From (1) and (6), we can get that,

$$\ln p_t \sim N(\ln p_{t-1}, \sigma^2 = 0.01) \tag{7}$$

Then, the mean of p_t is $p_{t-1} e^{0.005}$, and the standard deviation is $p_{t-1}^2 e^{0.01}(e^{0.01} - 1)$.

2 Problem2

Method	normal_N	normal_N with ($\lambda = 0.\ 94$)	MLE fitted T	fitted AR(1)	Historic Simulation
VaR	0.038	0.031	0.032	0.038	0.035

Table 1: VaR with different methods.

Comparison:

The normal distribution and AR(1) model provide the highest VaR estimates, suggesting a more conservative approach, assuming higher potential losses.

The normal distribution with $\lambda = 0.94$ and MLE fitted T-distribution show lower VaR estimates, potentially reflecting lower risk under recent market conditions.

The historic simulation gives a mid-range estimate, which is purely based on past data and does not assume any specific distribution.

3 Problem3

VaR	A	B	C T	Total
VaR	51.8	56.8	46.1	145.0

Table 2: different portfolio's VaR based on Arithmetic Return System.

We can tell that the sum of VaR of portfolio A, B and C is larger than the total VaR. It signs that VaR is a non-coherent risk measure.

VaR	A	B	C T	Total
VaR	52.0	57.2	42.9	145.4

Table 3: different portfolio's VaR based on Log Return.

I chose Log Returns because they are additive over time, handle large price movements more symmetrically. Log returns also provide a more consistent risk estimate, especially for long-term analysis or in volatile markets.

Switching from Arithmetic Returns to Log Returns typically results in higher VaR estimates, reflecting a more conservative approach. This happens because log returns capture downside risk more effectively, especially during periods of high volatility, providing a more realistic assessment of potential losses.