$$
\begin{aligned}
f\left(x\right) =& x^{x} \\
\ln f\left(x\right) =& x\ln x \\
\frac{\mathrm{d}}{\mathrm{d}x}\ln f\left(x\right) =& \frac{\mathrm{d}}{\mathrm{d}x}\left[x\ln x\right] \\
\frac{f'\left(x\right)}{f\left(x\right)} =& \left[x\ln x\right]' \\
f'\left(x\right) =& f\left(x\right)\left[x\ln x\right]' = x^{x}\left[\left(x\right)'\ln x + x\left(\ln x\right)'\right] \\
=& x^{x}\left[1\ln x + x\frac{1}{x}\right] = x^{x}\left[\ln x + 1\right] = x^{x}\left[1 + \ln x\right] \\
=& x^{x} + x^{x}\ln x
\end{aligned}
$$