$$f(x) = x^{x}$$

$$\ln f(x) = x \ln x$$

$$\frac{d}{dx} \ln f(x) = \frac{d}{dx} [x \ln x]$$

$$\frac{f'(x)}{f(x)} = [x \ln x]'$$

$$f'(x) = f(x) [x \ln x]' = x^{x} [(x)' \ln x + x (\ln x)']$$

$$= x^{x} \left[1 \ln x + x \frac{1}{x} \right] = x^{x} [\ln x + 1] = x^{x} [1 + \ln x]$$

$$= x^{x} + x^{x} \ln x$$