

$$\begin{array}{ccc} \{\boldsymbol{v}_j\}_{j=1}^n = \mathfrak{V} \subseteq \mathcal{V} & \xrightarrow{T(\cdot)} & \mathcal{W} \supseteq \mathfrak{W} = \{\boldsymbol{w}_j\}_{j=1}^m \\ \Downarrow & & \Downarrow \end{array}$$

$$V[\boldsymbol{v}]_V = v^j \boldsymbol{v}_j = \boldsymbol{v} \begin{array}{c} \xleftarrow{T^{-1}(\cdot)} \\ \xrightarrow{T(\cdot)} \end{array} \boldsymbol{w} = w^j \boldsymbol{w}_j = W[\boldsymbol{w}]_W = T(\boldsymbol{v}) = T(V)[\boldsymbol{v}]_V$$

$$\begin{array}{ccccc} VT^{-1}W^{-1}\boldsymbol{w} = & \begin{array}{c} V^{-1} \updownarrow V \\ \downarrow \end{array} & \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \end{array} & \begin{array}{c} \updownarrow W \\ \downarrow \end{array} & = WTV^{-1}\boldsymbol{v} \\ & & \begin{array}{c} \xleftarrow{T^{-1}} \\ \xrightarrow{T} \end{array} & & \\ V^{-1}\boldsymbol{v} = [\boldsymbol{v}]_V & & [\boldsymbol{w}]_W & = W^{-1}\boldsymbol{w} = [T(\boldsymbol{v})]_W = T[\boldsymbol{v}]_V & \\ \Downarrow & & \Downarrow & & \end{array}$$

$$\begin{array}{ccc} \boldsymbol{t}_j = [T(\boldsymbol{v}_j)]_W & \mathbb{F}^n \xrightarrow{T} \mathbb{F}^m & T = [T(V)]_W \\ \Downarrow & & \Downarrow \end{array}$$

$$\{\mathbb{R}^n, \mathbb{C}^m, \dots\} \quad \{\mathbb{R}^m, \mathbb{C}^m, \dots\}$$