math

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index

math on bookdown started on 2024/01/28

Part I

A Minimal Book Example

Chapter 1

About

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports; for example, a math equation $a^2 + b^2 = c^2$.

1.1 Usage

Each **bookdown** chapter is an .Rmd file, and each .Rmd file can contain one (and only one) chapter. A chapter *must* start with a first-level heading: `# A good chapter`, and can contain one (and only one) first-level heading.

Use second-level and higher headings within chapters like: `## A short section` or `### An even shorter section`.

The `index.Rmd` file is required, and is also your first book chapter. It will be the homepage when you render the book.

1.2 Render book

You can render the HTML version of this example book without changing anything:

1. Find the **Build** pane in the RStudio IDE, and
2. Click on **Build Book**, then select your output format, or select "All formats" if you'd like to use multiple formats from the same book source files.

Or build the book from the R console:

```
bookdown::render_book()
```

To render this example to PDF as a `bookdown::pdf_book`, you'll need to install XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.org/tinytex/.

1.3 Preview book

As you work, you may start a local server to live preview this HTML book. This preview will update as you edit the book when you save individual .Rmd files. You can start the server in a work session by using the RStudio add-in "Preview book", or from the R console:

```
bookdown::serve_book()
```

Chapter 2

Hello bookdown

All chapters start with a first-level heading followed by your chapter title, like the line above. There should be only one first-level heading (`#`) per .Rmd file.

2.1 A section

All chapter sections start with a second-level (`##`) or higher heading followed by your section title, like the sections above and below here. You can have as many as you want within a chapter.

An unnumbered section

Chapters and sections are numbered by default. To un-number a heading, add a `{.unnumbered}` or the shorter `{-}` at the end of the heading, like in this section.

Chapter 3

Cross-references

Cross-references make it easier for your readers to find and link to elements in your book.

3.1 Chapters and sub-chapters

There are two steps to cross-reference any heading:

1. Label the heading: `# Hello world {#nice-label}`.
    - Leave the label off if you like the automated heading generated based on your heading title: for example, `# Hello world` = `# Hello world {#hello-world}`.
    - To label an un-numbered heading, use: `# Hello world {-#nice-label}` or `{# Hello world .unnumbered}`.
2. Next, reference the labeled heading anywhere in the text using `\@ref(nice-label)`; for example, please see Chapter 3.
    - If you prefer text as the link instead of a numbered reference use: any text you want can go here.

3.2 Captioned figures and tables

Figures and tables *with captions* can also be cross-referenced from elsewhere in your book using `\@ref(fig:chunk-label)` and `\@ref(tab:chunk-label)`, respectively.

See Figure 3.1.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Don't miss Table 3.1.

```
knitr::kable(
  head(pressure, 10), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

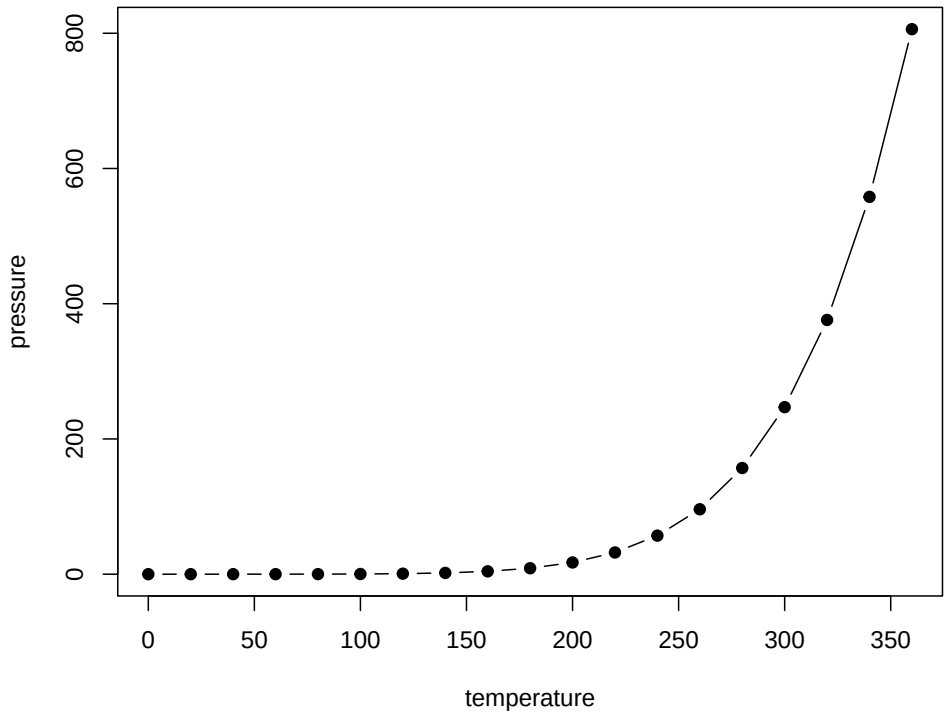


Figure 3.1: Here is a nice figure!

Table 3.1: Here is a nice table!

temperature	pressure
0	0.0002
20	0.0012
40	0.0060
60	0.0300
80	0.0900
100	0.2700
120	0.7500
140	1.8500
160	4.2000
180	8.8000

Chapter 4

Parts

You can add parts to organize one or more book chapters together. Parts can be inserted at the top of an .Rmd file, before the first-level chapter heading in that same file.

Add a numbered part: `# (PART) Act one {-}` (followed by `# A chapter`)

Add an unnumbered part: `# (PART\*) Act one {-}` (followed by `# A chapter`)

Add an appendix as a special kind of un-numbered part: `# (APPENDIX) Other stuff {-}` (followed by `# A chapter`). Chapters in an appendix are prepended with letters instead of numbers.

Chapter 5

Footnotes and citations

5.1 Footnotes

Footnotes are put inside the square brackets after a caret `^[]`. Like this one [1].

5.2 Citations

Reference items in your bibliography file(s) using `@key`.

For example, we are using the **bookdown** package[1] (check out the last code chunk in index.Rmd to see how this citation key was added) in this sample book, which was built on top of R Markdown and **knitr**[2] (this citation was added manually in an external file book.bib). Note that the `.bib` files need to be listed in the index.Rmd with the YAML `bibliography` key.

The RStudio Visual Markdown Editor can also make it easier to insert citations: https://rstudio.github.io/visual-markdown-editing/#/citations

[1] This is a footnote.

Chapter 6

Blocks

6.1 Equations

Here is an equation.

$$f\left(k\right) = \binom{n}{k} p^k \left(1-p\right)^{n-k} \tag{6.1}$$

You may refer to using `\@ref(eq:binom)`, like see Equation (6.1).

6.2 Theorems and proofs

Labeled theorems can be referenced in text using `\@ref(thm:tri)`, for example, check out this smart theorem 6.1.

Theorem 6.1. *For a right triangle, if c denotes the* length *of the hypotenuse and a and b denote the lengths of the* ***other*** *two sides, we have*

$$a^2 + b^2 = c^2$$

Read more here https://bookdown.org/yihui/bookdown/markdown-extensions-by-bookdown.html.

6.3 Callout blocks

The R Markdown Cookbook provides more help on how to use custom blocks to design your own callouts: https://bookdown.org/yihui/rmarkdown-cookbook/custom-blocks.html

Chapter 7

Sharing your book

7.1 Publishing

HTML books can be published online, see: https://bookdown.org/yihui/bookdown/publishing.html

7.2 404 pages

By default, users will be directed to a 404 page if they try to access a webpage that cannot be found. If you'd like to customize your 404 page instead of using the default, you may add either a `_404.Rmd` or `_404.md` file to your project root and use code and/or Markdown syntax.

7.3 Metadata for sharing

Bookdown HTML books will provide HTML metadata for social sharing on platforms like Twitter, Facebook, and LinkedIn, using information you provide in the `index.Rmd` YAML. To setup, set the `url` for your book and the path to your `cover-image` file. Your book's `title` and `description` are also used.

This `gitbook` uses the same social sharing data across all chapters in your book- all links shared will look the same.

Specify your book's source repository on GitHub using the `edit` key under the configuration options in the `_output.yml` file, which allows users to suggest an edit by linking to a chapter's source file.

Read more about the features of this output format here:

https://pkgs.rstudio.com/bookdown/reference/gitbook.html

Or use:

```
?bookdown::gitbook
```

Part II

by discipline

Chapter 8

test

8.1 superscript and subscript

$\text{script}^{\text{superscript}}_{\text{subscript}}$

8.2 equation term coloring

https://bookdown.org/yihui/rmarkdown-cookbook/font-color.html

LaTex color

https://latexcolor.com/

https://www.overleaf.com/learn/latex/Using_colors_in_LaTeX

https://latex-tutorial.com/color-latex/#:~:text=To%20summarize%2C%20pyellow!50efined%20colors%20in,w

LaTex color methods

color frame

https://tex.stackexchange.com/questions/582748/highlight-equation-with-boxes-and-arrows

color box

https://tex.stackexchange.com/questions/567739/how-to-move-and-size-colorbox

color box with round corners

https://tex.stackexchange.com/questions/568880/color-box-with-rounded-corners

highlighting

https://tex.stackexchange.com/questions/318991/highlighting-math

https://forum.remnote.io/t/highlighting-latex-formulas/149

LyX

https://tex.stackexchange.com/questions/250069/create-a-color-box https://latexlyx.blogspot.com/2013/12/lyx.html

https://tex.stackexchange.com/questions/635486/prevent-lyx-from-escaping-math-in-color-box-title

Bookdown - conditional display of text and code blocks (LaTeX/PDF vs. HTML) https://stackoverflow.com/questions/76240244/bookdown-conditional-display-of-text-and-code-blocks-latex-pdf-vs-html

$$F = ma$$

https://community.rstudio.com/t/highlighting-text-inline-in-rmarkdown-or-bookdown-pdf/35118/4

$$F = ma$$

$$F = F$$

$$F = ma \tag{8.1}$$

$$F = ma$$

$$Y = \beta_0 + \beta_1 X_1 + \ldots + \beta_n X_n$$

8.3 link and reference

$$E = mc^2 \tag{8.2}$$

`\@ref(nice-label)` 10

`[link to partition][partition]` link to partition

`[partition] \@ref(partition)`

partition [#partition] (10) @ref(#partition)

`[equivalence class] \@ref(equivalence class)`

equivalence class [#equivalence class] (@ref(equivalence class)) @ref(#equivalence class)

[equivalence-class] [#equivalence-class] (10) @ref(#equivalence-class)

[equivalence-class.html] [equivalence-class.html#equivalence-class] (@ref(equivalence-class.html)) @ref(equivalence-class.html#equivalence-class)

equivalence relation [#equivalence relation] (@ref(equivalence relation)) @ref(#equivalence relation)

[equivalence-relation] [#equivalence-relation] (10) @ref(#equivalence-relation)

[equivalence-relation.html] [equivalence-relation.html#equivalence-relation] (@ref(equivalence-relation.html)) @ref(equivalence-relation.html#equivalence-relation)

8.4 number and reference equations

https://bookdown.org/yihui/rmarkdown/bookdown-markdown.html#equations

`\#eq:emc \@ref(eq:emc)`

$$
\begin{aligned}
& C \text{ is an equivalence class of } a \text{ on } A \\
\Leftrightarrow [a]_\sim = C = & \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation over } A \times A = A^2 \end{cases} \right\} \subseteq A \neq \emptyset \\
\Leftrightarrow [a] = [a]_\sim = & \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation on } A \end{cases} \right\} \subseteq A \neq \emptyset \\
\Rightarrow [a]_\sim = & \{x | x \sim a\} \subseteq A \neq \emptyset
\end{aligned}
$$

https://bookdown.org/yihui/rmarkdown/bookdown-markdown.html#cross-referencing

This cross reference is the Fig. 8.1

https://stackoverflow.com/questions/51595939/bookdown-cross-reference-figure-in-another-file

I ran into the same issue and came up with this solution if you aim at compiling 2 different pdfs. It relies on LaTeX's xr package for cross references: https://stackoverflow.com/a/52532269/576684

8.5 footnote

noun[1]

8.6 citation

https://stackoverflow.com/questions/48965247/use-csl-file-for-pdf-output-in-bookdown/49145699#49145699

citation 1[3] citation 2[3]

citation 3[4] citation 4[4]

8.7 bookdown environment for definition, theorem, proof

https://bookdown.org/yihui/rmarkdown/bookdown-markdown.html

Theorem 8.1 (Theorem Name). *Here is my theorem.*

Proof Name. Here is my proof. □

[1] This is a footnote.

Theorem 8.2 (Pythagorean theorem)**.** *For a right triangle, if c denotes the length of the hypotenuse and a and b denote the lengths of the other two sides, we have*

$$a^2 + b^2 \overset{8.2}{=} c^2$$

Definition 8.1 (Definition Name)**.** Here is my definition.

number and reference equations

(**??**)

(8.2)

8.2

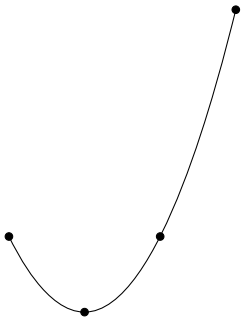


Figure 8.1: parabola arc with points

Chapter 9

test cross-link 2

9.1 verbatim

https://bookdown.org/yihui/rmarkdown-cookbook/verbatim-code-chunks.html

````
```r
1 + 1
```

```
## [1] 2
```
````

````
We can output arbitrary content **verbatim**.


```r
1 + 1
```

```
## [1] 2
```

The content can contain inline code like
78.5398163, too.
````

Chapter 10

math

- formula typesetting
  - TeX
    - LaTeX
      - pdfLaTeX
      - XeLaTeX
      - editor/tool:
        - LyX
        - OverLeaf
        - MathPix Snip
        - Micro$oft Office Word
  - MathML
  - MathJax: JavaScript
- symbolic computing
  - Maple: by MapleSoft
  - Mathematica: by Wolfram
- numeric computing
  - MatLab: by MathWorks

equivalence relation 10

equivalence class 10

partition 10

equivalence relation

https://stackoverflow.com/questions/76240244/bookdown-conditional-display-of-text-and-code-blocks-latex-pdf-vs-html

等價關係 equivalence relation

R is an equivalence relation over $A \times B$

$$\Leftrightarrow \begin{cases} R =\sim= \{\langle x,y\rangle | x \sim y\} \subseteq A \times B & \text{(e) equivalence 等價} \\ \vdots & \vdots \end{cases}$$

$$\Leftrightarrow \begin{cases} R = \{\langle x,y\rangle | xRy\} \subseteq A \times B & (R)\,\text{relation} \\ \forall \langle x,y\rangle \in R\,(xRx) & (r)\,\text{reflexive} \\ \forall \langle x,y\rangle \in R\,(xRy \Rightarrow yRx) & (s)\,\text{symmetric} \\ \forall \langle x,y\rangle, \langle y,z\rangle \in R \left(\begin{cases} xRy \\ yRz \end{cases} \Rightarrow xRz \right) & (t)\,\text{transitive} \end{cases} \Leftrightarrow \begin{cases} R = \{\langle x,y\rangle | xRy\} \subseteq A \times B & \text{關係} \\ \forall \langle x,y\rangle \in R\,(\langle x,x\rangle \in R) & \text{自反} \\ \forall \langle x,y\rangle \in R\,(\langle y,x\rangle \in R) & \text{對稱} \\ \forall \langle x,y\rangle, \langle y,z\rangle \in R\,(\langle x,z\rangle \in R) & \text{遞移} \end{cases}$$

equivalence class

$$
\begin{aligned}
&C \text{ is an equivalence class of } a \text{ on } A \\
\Leftrightarrow [a]_\sim = C &= \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation over } A \times A = A^2 \end{cases} \right\} \subseteq A \neq \emptyset \\
\Leftrightarrow [a] = [a]_\sim &= \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation on } A \end{cases} \right\} \subseteq A \neq \emptyset \\
\Rightarrow [a]_\sim &= \{x | x \sim a\} \subseteq A \neq \emptyset
\end{aligned}
$$

where the definition of equivalence relation can be found in 10.

number and reference equations

(**??**)

(8.2)

8.2

partition

$$\{A_i\}_{i \in I} = \{A_i | i \in I\} \text{ is a partition of a set } A$$
$$\Leftrightarrow \begin{cases} \forall i \in I \, (A_i \neq \emptyset) \\ A = \bigcup\limits_{i \in I} A_i \\ \forall i, j \in I \, (i \neq j \Rightarrow A_i \cap A_j = \emptyset) \end{cases}$$

https://proofwiki.org/wiki/Definition:Set_Partition

Chapter 11

statistics

11.1 covariance matrix

5

11.1.1 calculation

$$\begin{aligned}
\mathrm{C}[\boldsymbol{X}] = \mathrm{Cov}[\boldsymbol{X}] = \mathrm{V}[\boldsymbol{X}] =& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]\left[\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right]\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}} - \boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] + \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}\left[\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}[\boldsymbol{X}]\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{X} = [X]_{1\times 1} = X \Rightarrow C(X) = \mathrm{C}[\boldsymbol{X}] =& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}[XX] - \mathrm{E}(X)\mathrm{E}(X) \\
=& \mathrm{E}\left(X^2\right) - [\mathrm{E}(X)]^2 = \mathrm{V}(X)
\end{aligned}$$

11.1.2 $\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[\boldsymbol{X}]$

$$\begin{aligned}
\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] =& \mathrm{E}\left[[(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})][(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(\boldsymbol{X}+\boldsymbol{b})=\mathrm{E}(\boldsymbol{X})+\boldsymbol{b}}{=}& \mathrm{E}\left[[\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}][\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] = \mathrm{V}[\boldsymbol{X}]
\end{aligned}$$

11.1.3 $\mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\begin{aligned}
\mathrm{V}[A\boldsymbol{X}] =& \mathrm{E}\left[[(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})][(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(A\boldsymbol{X})=A\mathrm{E}(\boldsymbol{X})}{=}& \mathrm{E}\left[[A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})][A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}A^{\mathrm{T}}\right] \\
=& A\mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right]A^{\mathrm{T}} = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}
\end{aligned}$$

11.1.4 $\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$$

Chapter 12

physics

Chapter 13

plot

- LaTeX
  - TikZ
    - TikZ-3Dplot
    - PGFplots
  - xypic = xy-pic
- OverLeaf
- MathCha
- GeoGebra
- Python
  - MatPlotLib
  - Seaborn
  - Plotly

neural network plot/draw https://github.com/ashishpatel26/Tools-to-Design-or-Visualize-Architecture-of-Neural-Network

TikZ

How to speed up bookdown generation?

https://stackoverflow.com/questions/56541371/how-to-speed-up-bookdown-generation

TikZ and PGFplots

What's the relation between packages PGFplots and TikZ?

https://tex.stackexchange.com/questions/285925/whats-the-relation-between-packages-pgfplots-and-tikz

https://www.youtube.com/watch?v=bQugbYq0BVA

https://www.youtube.com/watch?v=ft4Kg9emK1k&list=PLg5nrpKdkk2DWcg3scb75AknF7DJXs8lk&index=18

```
\begin{tikzpicture}
  \def\a{1.5} % amplitude
  \def\b{2}   % frequency
  \draw[->] (-0.2,0)--(4.2,0) node[right, font=\small] {$x$};
  \draw[->] (0,-4)--(0,0.5) node[above] {$y$};
  \draw[domain=0:4,smooth,variable=\t,blue,thick]
    plot ({\a * (\b*\t - sin(deg(\b*\t)))},{-\a * (1 - cos(deg(\b*\t)))});
  % \node[above] at (2, 0.5) {Brachistochrone Curve};
  \node[above, font=\footnotesize] at (2, 1) {Brachistochrone Curve};
  \node[above, font=\footnotesize] at (2, 0) {$\begin{aligned}
& x=r(t-\sin t) \\
& y=r(1-\cos t)
\end{aligned}$};
\end{tikzpicture}
```

13.1 2D

https://zhuanlan.zhihu.com/p/127155579?utm_psn=1741479950987960320

1

```
\begin{tikzpicture}
  \draw (-1,1)--(0,0)--(1,2);
\end{tikzpicture}
```

2

3

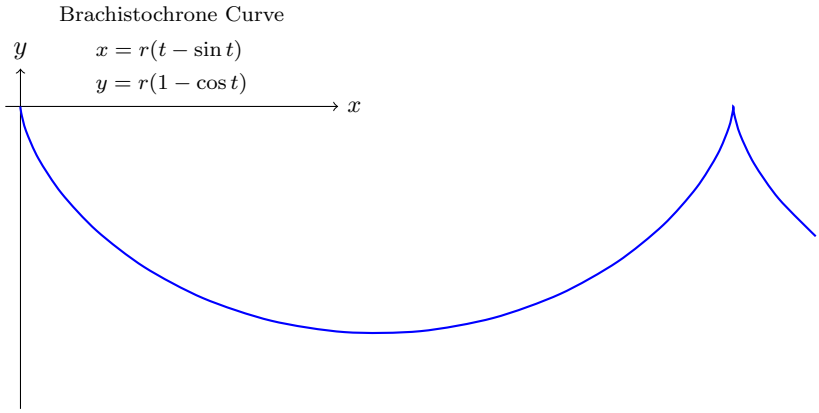


Figure 13.1: Brachistochrone Curve

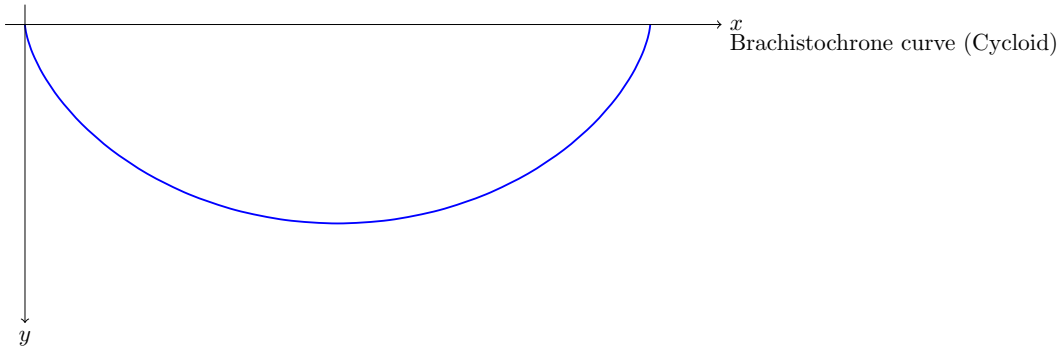
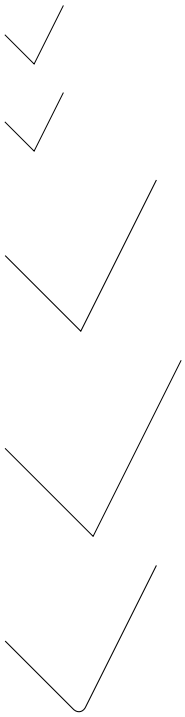


Figure 13.2: Brachistochrone Curve



```
\begin{tikzpicture}
  \draw[rounded corners] (-1,1)--(0,0)--(1,2)--(-1,1);
\end{tikzpicture}
```

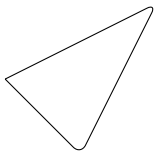


Figure 13.3: rounded corner pseudo-closed triangle

```
\begin{tikzpicture}
  \draw[rounded corners] (-1,1)--(0,0)--(1,2)--cycle;
\end{tikzpicture}
```

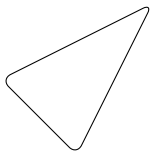


Figure 13.4: rounded corner triangle

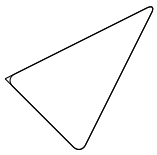


Figure 13.5: triangle vs. pseudo-closed triangle

```
\begin{tikzpicture}
  \draw (0,0) rectangle (4,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) circle (1);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) circle (1);
  \draw (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (1,1) ellipse (2 and 1);
```

Figure 13.6: rectangle

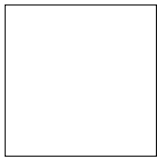


Figure 13.7: square

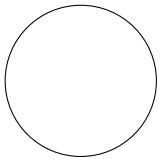


Figure 13.8: circle

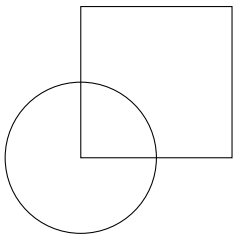


Figure 13.9: circle and square

```
\end{tikzpicture}
```

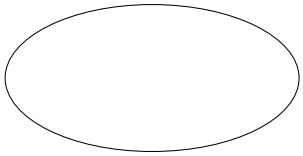


Figure 13.10: ellipse

```
\begin{tikzpicture}
  \draw (1 ,1) arc (0:270:1);
  \draw (6 ,1) arc (0:270:2 and 1);
\end{tikzpicture}
```

Figure 13.11: circle and ellipse arcs

```
\begin{tikzpicture}
  \draw (-1,1) parabola bend (0,0) (2,4);
\end{tikzpicture}
```

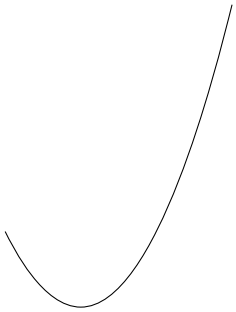


Figure 13.12: parabola arc

```
\begin{tikzpicture}
  \draw (-1,1) parabola bend (0,0) (2,4);
  \filldraw
    (-1,1) circle (.05)
    ( 0,0) circle (.05)
    ( 1,1) circle (.05)
    ( 2,4) circle (.05);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw[step=20pt] (0,0) grid (3,2);
  \draw[help lines ,step=20pt] (4,0) grid (7,2);
\end{tikzpicture}
```

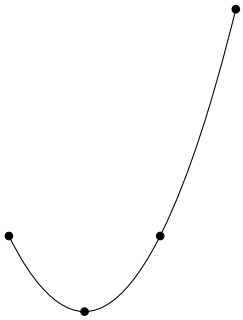


Figure 13.13: parabola arc with points

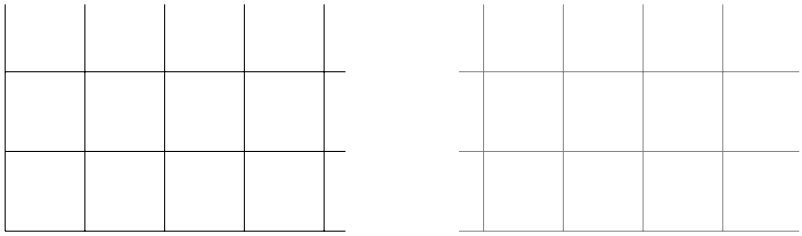


Figure 13.14: grid and help lines

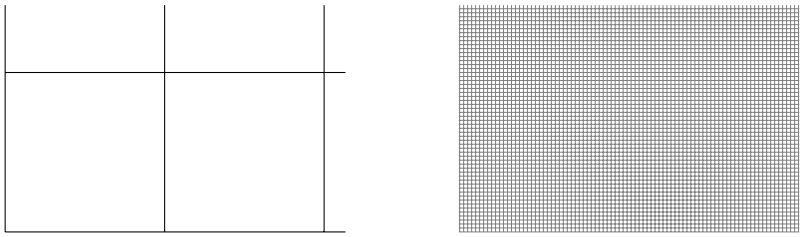


Figure 13.15: grid and help lines

```
\begin{tikzpicture}[scale=0.25]
  \draw[->] (0,0)--(9,0);
  \draw[<-] (0,1)--(9,1);
  \draw[<->] (0,2)--(9,2);
  \draw[>->>] (0,3)--(9,3);
  \draw[|<->|] (0,4)--(9,4);
\end{tikzpicture}
```

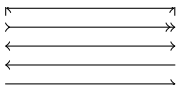


Figure 13.16: arrows

```
\begin{tikzpicture}
  \draw[line width =2pt] (0,6)--(9,6);
  \draw[dotted]          (0,5)--(9,5);
  \draw[densely dotted]  (0,4)--(9,4);
  \draw[loosely dotted]  (0,3)--(9,3);
  \draw[dashed]          (0,2)--(9,2);
  \draw[densely dashed]  (0,1)--(9,1);
  \draw[loosely dashed]  (0,0)--(9,0);
\end{tikzpicture}
```

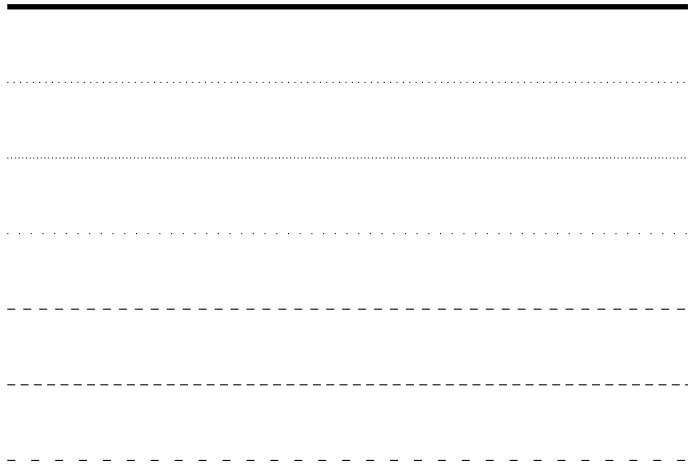


Figure 13.17: arrows

```
\begin{tikzpicture}[dline/.style={color= blue, line width=2pt}]
  \draw[dline] (0,0)--(9,0);
\end{tikzpicture}
```


Figure 13.18: head styling

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
```

```
  \draw[shift={( 3, 0)}] (0,0) rectangle (2,2);
  \draw[shift={( 0, 3)}] (0,0) rectangle (2,2);
  \draw[shift={( 0,-3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3, 0)}] (0,0) rectangle (2,2);
  \draw[shift={( 3, 3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3, 3)}] (0,0) rectangle (2,2);
  \draw[shift={( 3,-3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3,-3)}] (0,0) rectangle (2,2);
\end{tikzpicture}
```

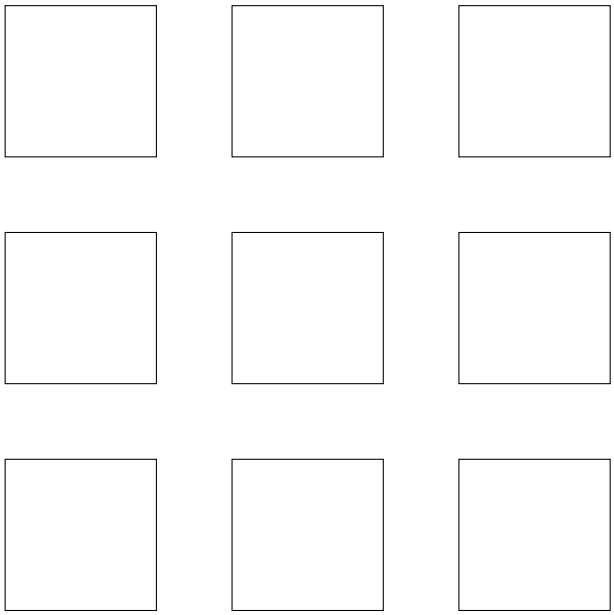


Figure 13.19: transform: shift

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt] (0,0) rectangle (2,2);
  \draw[xshift=-100pt] (0,0) rectangle (2,2);
  \draw[yshift= 100pt] (0,0) rectangle (2,2);
  \draw[yshift=-100pt] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt, xscale=1.5] (0,0) rectangle (2,2);
  \draw[yshift= 100pt, xscale=0.5] (0,0) rectangle (2,2);
  \draw[xshift=-100pt, yscale=1.5] (0,0) rectangle (2,2);
  \draw[yshift=-100pt, yscale=0.5] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt, xscale=1.5] (0,0) rectangle (2,2);
```

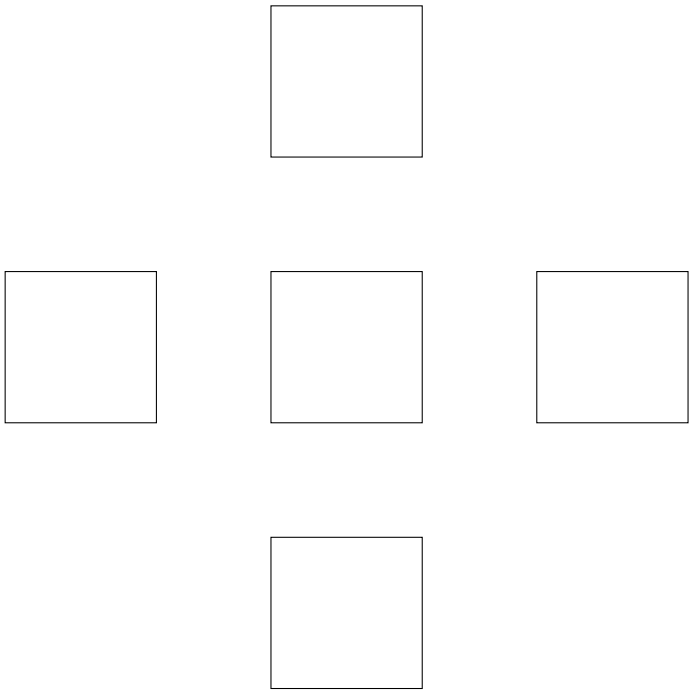


Figure 13.20: transform: shift x, y

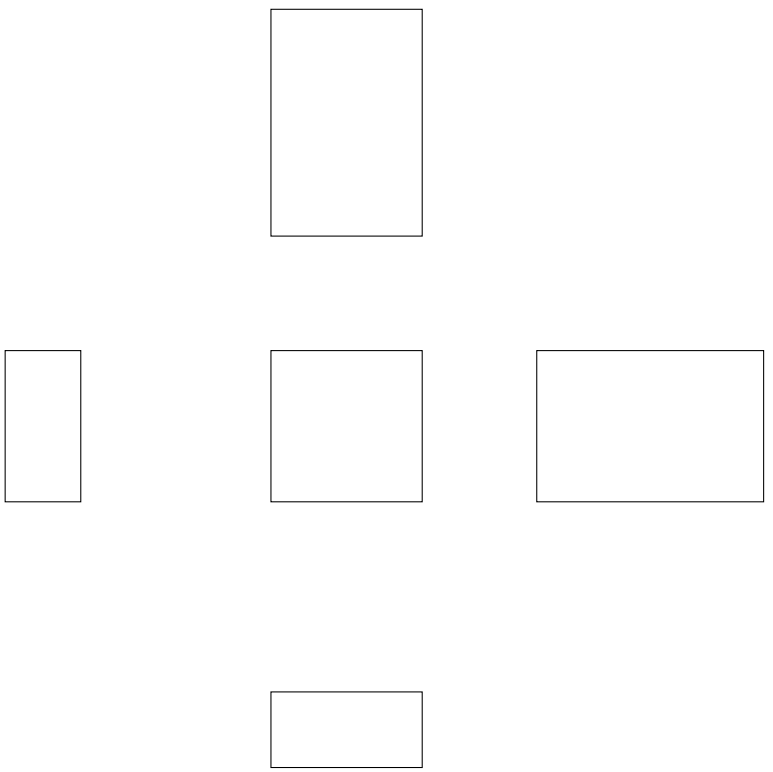


Figure 13.21: transform: scale x, y

```
  \draw[yshift= 100pt, yscale=1.5] (0,0) rectangle (2,2);
  \draw[xshift=-100pt, xscale=0.5] (0,0) rectangle (2,2);
  \draw[yshift=-100pt, yscale=0.5] (0,0) rectangle (2,2);
\end{tikzpicture}
```

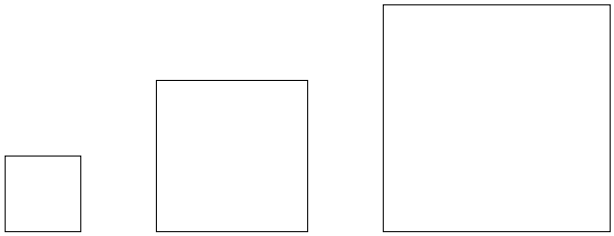


Figure 13.22: transform: scale

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift=125pt,rotate=45] (0,0) rectangle (2,2);
  \draw[xshift=175pt,rotate around={45:(2 ,2)}] (0,0) rectangle (2,2);
\end{tikzpicture}
```

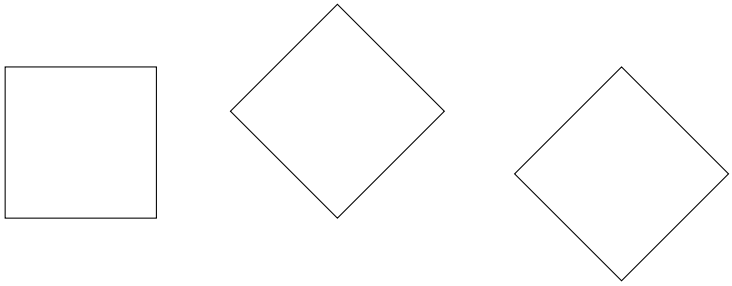


Figure 13.23: transform: rotate

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift=70pt,xslant=1] (0,0) rectangle (2,2);
  \draw[yshift=70pt,yslant=1] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\tikzset{
  box/.style={
    draw=blue,
    rectangle,
    rounded corners=5pt,
    minimum width=50pt,
    minimum height=20pt,
    inner sep=5pt
  }
}
\begin{tikzpicture}
  \node[box] (1) at (0,0) {1};
```

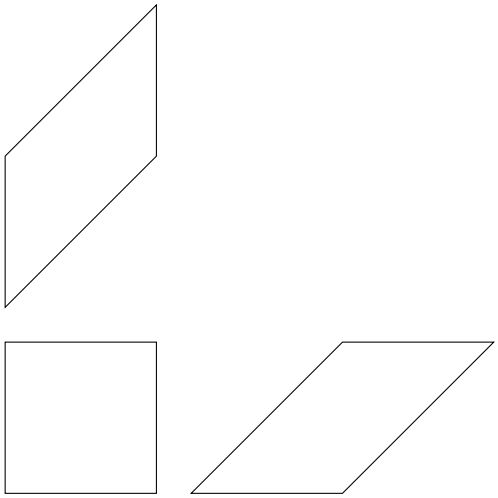


Figure 13.24: transform: slant

```
  \node[box] (2) at (4,0) {2};
  \node[box] (3) at (8,0) {3};
  \draw[->] (1)--(2);
  \draw[->] (2)--(3);
  \node at (2,1) {a};
  \node at (6,1) {b};
\end{tikzpicture}
```

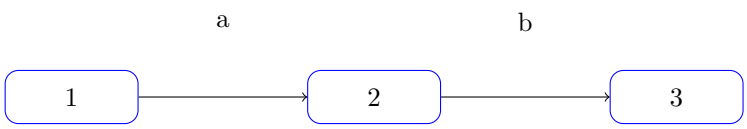


Figure 13.25: flowchart

```
\tikzset{
  box/.style={
    draw=blue,
    fill=blue!20,
    rectangle,
    rounded corners=5pt,
    minimum height=20pt,
    inner sep=5pt
  }
}
\begin{tikzpicture}
  \node[box] {1}
      child {node[box] {2}}
      child {node[box] {3}
          child {node[box] {4}}
          child {node[box] {5}}
          child {node[box] {6}}
```

```
      };
\end{tikzpicture}
```

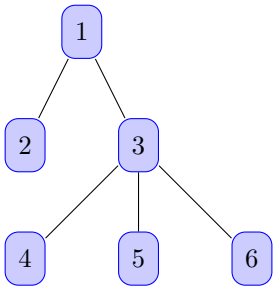


Figure 13.26: tree

```
\begin{tikzpicture}
  \draw[->] (-0.2,0)--(6,0) node[right] {$x$};
  \draw[->] (0,-0.2)--(0,6) node[above] {$f(x)$};
  \draw[domain=0:4] plot (\x ,{0.1* exp(\x)}) node[right] {$f(x)=\frac{1}{10}e^x$};
\end{tikzpicture}
```

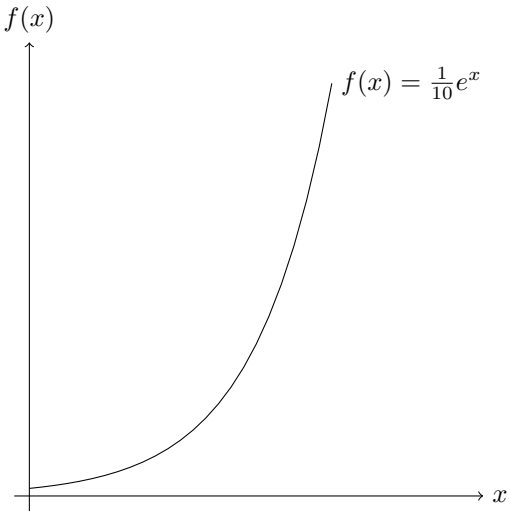


Figure 13.27: tree

https://stackoverflow.com/questions/64897575/tikz-libraries-in-bookdown

It turns out that you can simply put the `\usetikzlibrary{...}` command directly before the `\begin{tikzpicture}` and everything works fine :)

https://stackoverflow.com/questions/56211210/r-markdown-document-with-html-docx-output-using-latex-package-bbm

https://tex.stackexchange.com/questions/171711/how-to-include-latex-package-in-r-markdown

13.2 3D

https://zhuanlan.zhihu.com/p/431732330?utm_psn=1741857547550638080

https://github.com/RRWWW/Stereometry

```
\begin{tikzpicture}
  \coordinate (A) at ( 1, 1, 1);
  \coordinate (B) at ( 1, 1,-1);
  \coordinate (C) at ( 1,-1,-1);
  \coordinate (D) at ( 1,-1, 1);
  \coordinate (E) at (-1,-1, 1);
  \coordinate (F) at (-1,-1,-1);
  \coordinate (G) at (-1, 1,-1);
  \coordinate (H) at (-1, 1, 1);
  \draw (A) node[right=1pt] {$A$}--
        (B) node[right=1pt] {$B$}--
        (C) node[right=1pt] {$C$}--
        (D) node[right=1pt] {$D$}--
        (E) node[left= 1pt] {$E$}--
        (F) node[right=1pt] {$F$}--
        (G) node[right=1pt] {$G$}--
        (H) node[left= 1pt] {$H$}--
        (A) node[right=1pt] {$A$};
\end{tikzpicture}
```

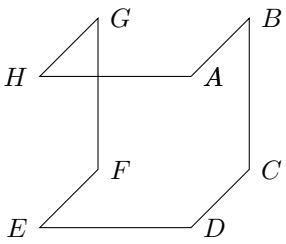


Figure 13.28: cube

https://tex.stackexchange.com/questions/388621/optimizing-perspective-tikz-graphic

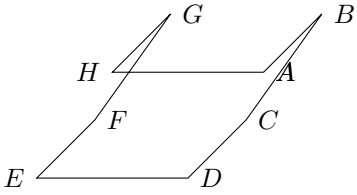


Figure 13.29: cube rotate

https://github.com/XiangyunHuang/bookdown-broken/blob/master/index.Rmd

```
\smartdiagramset{planet color=gray!40!white, uniform color list=gray!40!white for 10
↪  items}
\smartdiagram[bubble diagram]{Basic skills,
```

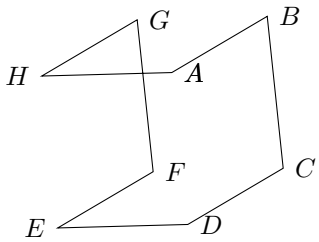


Figure 13.30: cube rotate

```
  Edit~/\\ (RStudio), Organize~/\\ (bookdown), Cooperate~/\\ (Git), Typeset~/\\
↪  (LaTeX/Pandoc), Compile~/\\ (GitHub Action)}
```

Figure 13.31: modern statistics plot skills

```
\usetikzlibrary{patterns}
\usetikzlibrary{3d,calc}
\tdplotsetmaincoords{45}{45}
\begin{tikzpicture}[tdplot_main_coords]
  \coordinate (A) at ( 1, 1, 1);
  \coordinate (B) at ( 1, 1,-1);
  \coordinate (C) at ( 1,-1,-1);
  \coordinate (D) at ( 1,-1, 1);
  \coordinate (E) at (-1,-1, 1);
  \coordinate (F) at (-1,-1,-1);
  \coordinate (G) at (-1, 1,-1);
  \coordinate (H) at (-1, 1, 1);
  \draw (A) node[right=1pt] {$A$}--
        (B) node[right=1pt] {$B$}--
```

```
        (C) node[right=1pt] {$C$}--
        (D) node[right=1pt] {$D$}--
        (E) node[left= 1pt] {$E$}--
        (F) node[right=1pt] {$F$}--
        (G) node[right=1pt] {$G$}--
        (H) node[left= 1pt] {$H$}--
        (A) node[right=1pt] {$A$};
\end{tikzpicture}
```

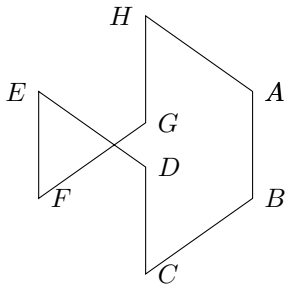


Figure 13.32: cube rotate

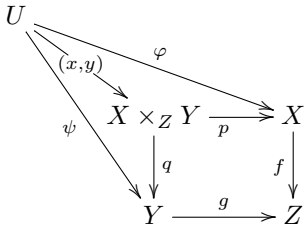
xy-pic

https://bookdown.org/yihui/rmarkdown-cookbook/install-latex-pkgs.html

`tinytex::install_tinytex()`

the following xymatrix from LaTeX package xy for xy-pic is not shown or rendered in HTML:

`$\LaTeX$` can only be used in HTML, not PDF



Chapter 14

programming language

- Python
- JavaScript
- SQL
- R
  - knitr: engine
    - TikZ
  - reticulate: Python
- C#
  - web
    - MVC
    - .NET
  - desktop
    - UWP = Universal Windows Platform
    - WPF = Windows Presentation Foundation
    - WinForms = Windows Forms
  - 3D/game
    - Unity

Part III

by date

Chapter 15

by date

partition

$$\{A_i\}_{i\in I} = \{A_i | i \in I\} \text{ is a partition of a set } A$$
$$\Leftrightarrow \begin{cases} \forall i \in I \, (A_i \neq \emptyset) \\ A = \bigcup\limits_{i \in I} A_i \\ \forall i, j \in I \, (i \neq j \Rightarrow A_i \cap A_j = \emptyset) \end{cases}$$

https://proofwiki.org/wiki/Definition:Set_Partition

202401281000

equivalence class

$$
\begin{aligned}
&C \text{ is an equivalence class of } a \text{ on } A \\
\Leftrightarrow [a]_\sim = C &= \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation over } A \times A = A^2 \end{cases} \right\} \subseteq A \neq \emptyset \\
\Leftrightarrow [a] = [a]_\sim &= \left\{ x \middle| \begin{cases} a \in A \\ x \in A \\ x \sim a \\ \sim \text{ is an equivalence relation on } A \end{cases} \right\} \subseteq A \neq \emptyset \\
\Rightarrow [a]_\sim &= \{x | x \sim a\} \subseteq A \neq \emptyset
\end{aligned}
$$

where the definition of equivalence relation can be found in 10.

number and reference equations

(**??**)

(8.2)

8.2

equivalence relation

https://stackoverflow.com/questions/76240244/bookdown-conditional-display-of-text-and-code-blocks-latex-pdf-vs-html

等價關係 equivalence relation

R is an equivalence relation over $A \times B$

$$\Leftrightarrow \begin{cases} R =\sim= \{\langle x,y\rangle | x \sim y\} \subseteq A \times B & \text{(e) equivalence 等價} \\ \vdots & \vdots \end{cases}$$

$$\Leftrightarrow \begin{cases} R = \{\langle x,y\rangle | xRy\} \subseteq A \times B & (R)\text{ relation} \\ \forall \langle x,y\rangle \in R\,(xRx) & (r)\text{ reflexive} \\ \forall \langle x,y\rangle \in R\,(xRy \Rightarrow yRx) & (s)\text{ symmetric} \\ \forall \langle x,y\rangle, \langle y,z\rangle \in R \left(\begin{cases} xRy \\ yRz \end{cases} \Rightarrow xRz \right) & (t)\text{ transitive} \end{cases} \Leftrightarrow \begin{cases} R = \{\langle x,y\rangle | xRy\} \subseteq A \times B & \text{關係} \\ \forall \langle x,y\rangle \in R\,(\langle x,x\rangle \in R) & \text{自反} \\ \forall \langle x,y\rangle \in R\,(\langle y,x\rangle \in R) & \text{對稱} \\ \forall \langle x,y\rangle, \langle y,z\rangle \in R\,(\langle x,z\rangle \in R) & \text{遞移} \end{cases}$$

Chapter 16

Python

https://bookdown.org/yihui/rmarkdown/language-engines.html

```
names(knitr::knit_engines$get())
```

```
##  [1] "awk"         "bash"        "coffee"      "gawk"        "groovy"
##  [6] "haskell"     "lein"        "mysql"       "node"        "octave"
## [11] "perl"        "php"         "psql"        "Rscript"     "ruby"
## [16] "sas"         "scala"       "sed"         "sh"          "stata"
## [21] "zsh"         "asis"        "asy"         "block"       "block2"
## [26] "bslib"       "c"           "cat"         "cc"          "comment"
## [31] "css"         "ditaa"       "dot"         "embed"       "eviews"
## [36] "exec"        "fortran"     "fortran95"   "go"          "highlight"
## [41] "js"          "julia"       "python"      "R"           "Rcpp"
## [46] "sass"        "scss"        "sql"         "stan"        "targets"
## [51] "tikz"        "verbatim"    "theorem"     "lemma"       "corollary"
## [56] "proposition" "conjecture"  "definition"  "example"     "exercise"
## [61] "hypothesis"  "proof"       "remark"      "solution"    "glue"
## [66] "glue_sql"    "gluesql"
```

https://rstudio.github.io/reticulate/articles/python_packages.html

```
x = 'hello, python world!'
print(x.split(' '))
```

```
## ['hello,', 'python', 'world!']
```

```
library(reticulate)
virtualenv_python()
```

```
library(reticulate)
conda_list()
```

```
library(reticulate)
virtualenv_list()
```

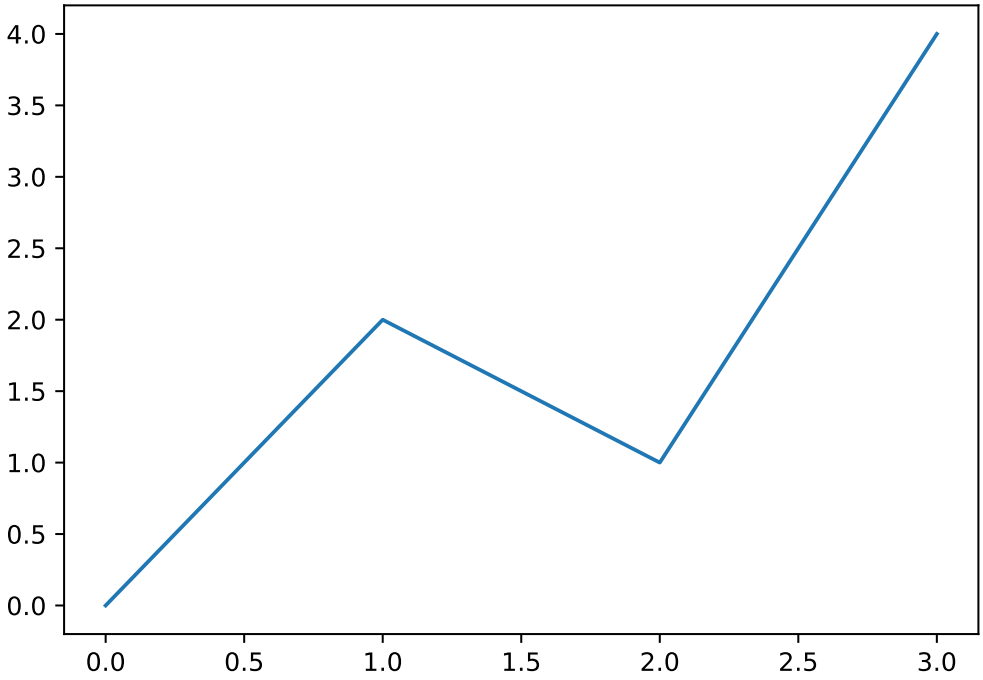
https://rstudio.github.io/reticulate/reference/install_python.html

```r
library(reticulate)
version <- "3.9.12"
# install_python(version)

# create a new environment
# virtualenv_create("r-reticulate", version = version)

# use_virtualenv("r-reticulate")

# install MatPlotLib
# virtualenv_install("r-reticulate", "matplotlib")

# import MatPlotLib (it will be automatically discovered in "r-reticulate")
matplotlib <- import("matplotlib")
```

copy `C:\Users\RW\AppData\Local\r-reticulate\r-reticulate\pyenv\pyenv-win\versions\3.9.12\tc`
and `C:\Users\RW\AppData\Local\r-reticulate\r-reticulate\pyenv\pyenv-win\versions\3.9.12\tcl`
two folders to the folder `C:\Users\RW\AppData\Local\r-reticulate\r-reticulate\pyenv\pyenv-win\ver`

```r
# library(reticulate)
# use_virtualenv("r-reticulate")
# # matplotlib <- import("matplotlib")
# matplotlib$use("Agg", force = TRUE)
```

```
import matplotlib.pyplot as plt
plt.plot([0, 2, 1, 4])
plt.show()
```


4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

TikZ

How to speed up bookdown generation?

https://stackoverflow.com/questions/56541371/how-to-speed-up-bookdown-generation

TikZ and PGFplots

What's the relation between packages PGFplots and TikZ?

https://tex.stackexchange.com/questions/285925/whats-the-relation-between-packages-pgfplots-and-tikz

https://www.youtube.com/watch?v=bQugbYq0BVA

https://www.youtube.com/watch?v=ft4Kg9emK1k&list=PLg5nrpKdkk2DWcg3scb75AknF7DJXs8lk&index=18

```
\begin{tikzpicture}
  \def\a{1.5} % amplitude
  \def\b{2}   % frequency
  \draw[->] (-0.2,0)--(4.2,0) node[right, font=\small] {$x$};
  \draw[->] (0,-4)--(0,0.5) node[above] {$y$};
  \draw[domain=0:4,smooth,variable=\t,blue,thick]
    plot ({\a * (\b*\t - sin(deg(\b*\t)))},{-\a * (1 - cos(deg(\b*\t)))});
  % \node[above] at (2, 0.5) {Brachistochrone Curve};
  \node[above, font=\footnotesize] at (2, 1) {Brachistochrone Curve};
  \node[above, font=\footnotesize] at (2, 0) {$\begin{aligned}
& x=r(t-\sin t) \\
& y=r(1-\cos t)
\end{aligned}$};
\end{tikzpicture}
```

16.1 2D

https://zhuanlan.zhihu.com/p/127155579?utm_psn=1741479950987960320

1

```
\begin{tikzpicture}
  \draw (-1,1)--(0,0)--(1,2);
\end{tikzpicture}
```

2

3

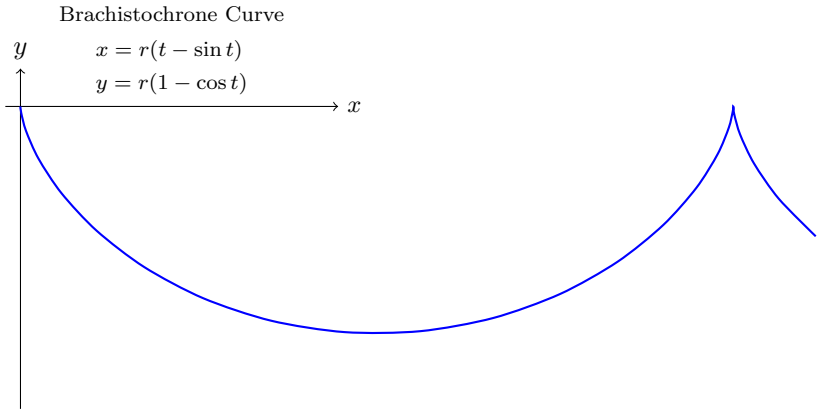


Figure 16.1: Brachistochrone Curve

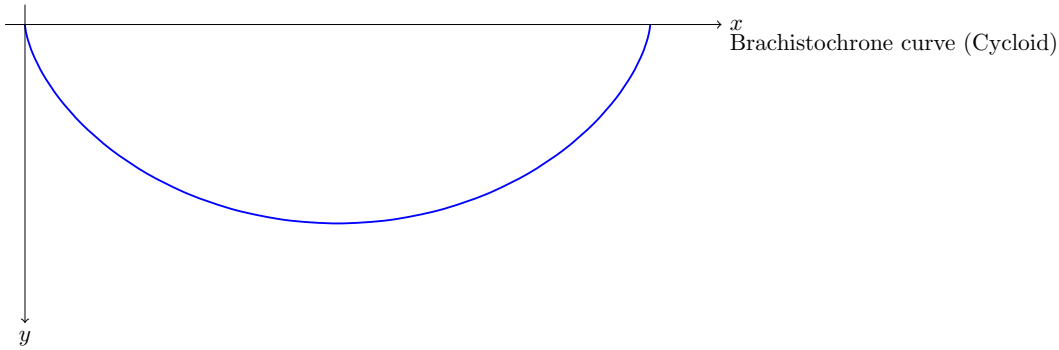
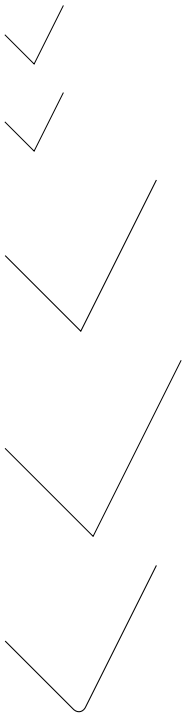


Figure 16.2: Brachistochrone Curve



```
\begin{tikzpicture}
  \draw[rounded corners] (-1,1)--(0,0)--(1,2)--(-1,1);
\end{tikzpicture}
```

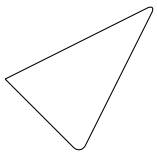


Figure 16.3: rounded corner pseudo-closed triangle

```
\begin{tikzpicture}
  \draw[rounded corners] (-1,1)--(0,0)--(1,2)--cycle;
\end{tikzpicture}
```

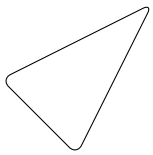


Figure 16.4: rounded corner triangle

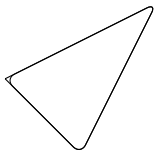


Figure 16.5: triangle vs. pseudo-closed triangle

```
\begin{tikzpicture}
  \draw (0,0) rectangle (4,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) circle (1);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) circle (1);
  \draw (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (1,1) ellipse (2 and 1);
```

Figure 16.6: rectangle

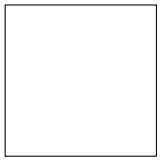


Figure 16.7: square

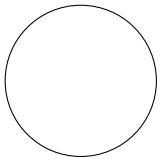


Figure 16.8: circle

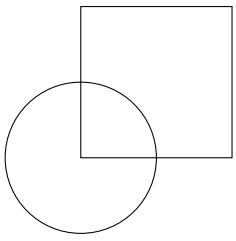


Figure 16.9: circle and square

```
\end{tikzpicture}
```

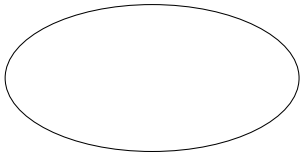


Figure 16.10: ellipse

```
\begin{tikzpicture}
  \draw (1 ,1) arc (0:270:1);
  \draw (6 ,1) arc (0:270:2 and 1);
\end{tikzpicture}
```

Figure 16.11: circle and ellipse arcs

```
\begin{tikzpicture}
  \draw (-1,1) parabola bend (0,0) (2,4);
\end{tikzpicture}
```

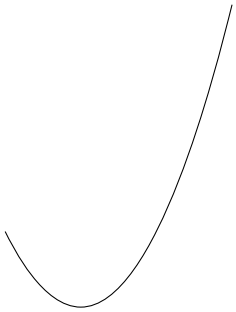


Figure 16.12: parabola arc

```
\begin{tikzpicture}
  \draw (-1,1) parabola bend (0,0) (2,4);
  \filldraw
    (-1,1) circle (.05)
    ( 0,0) circle (.05)
    ( 1,1) circle (.05)
    ( 2,4) circle (.05);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw[step=20pt] (0,0) grid (3,2);
  \draw[help lines ,step=20pt] (4,0) grid (7,2);
\end{tikzpicture}
```

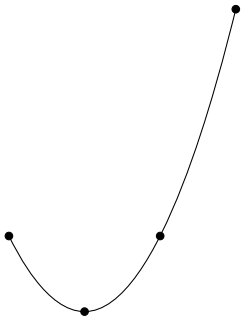


Figure 16.13: parabola arc with points

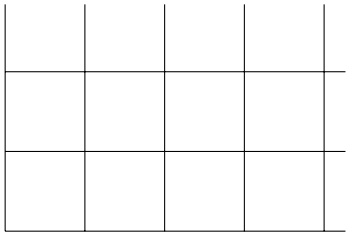


Figure 16.14: grid and help lines

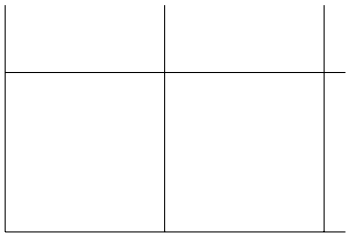


Figure 16.15: grid and help lines

```
\begin{tikzpicture}[scale=0.25]
  \draw[->] (0,0)--(9,0);
  \draw[<-] (0,1)--(9,1);
  \draw[<->] (0,2)--(9,2);
  \draw[>->>] (0,3)--(9,3);
  \draw[|<->|] (0,4)--(9,4);
\end{tikzpicture}
```

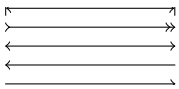


Figure 16.16: arrows

```
\begin{tikzpicture}
  \draw[line width =2pt] (0,6)--(9,6);
  \draw[dotted]          (0,5)--(9,5);
  \draw[densely dotted]  (0,4)--(9,4);
  \draw[loosely dotted]  (0,3)--(9,3);
  \draw[dashed]          (0,2)--(9,2);
  \draw[densely dashed]  (0,1)--(9,1);
  \draw[loosely dashed]  (0,0)--(9,0);
\end{tikzpicture}
```

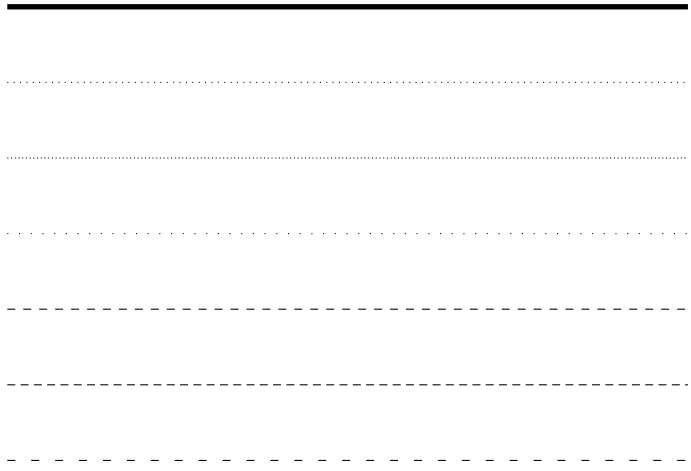


Figure 16.17: arrows

```
\begin{tikzpicture}[dline/.style={color= blue, line width=2pt}]
  \draw[dline] (0,0)--(9,0);
\end{tikzpicture}
```


Figure 16.18: head styling

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
```

```
  \draw[shift={( 3, 0)}] (0,0) rectangle (2,2);
  \draw[shift={( 0, 3)}] (0,0) rectangle (2,2);
  \draw[shift={( 0,-3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3, 0)}] (0,0) rectangle (2,2);
  \draw[shift={( 3, 3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3, 3)}] (0,0) rectangle (2,2);
  \draw[shift={( 3,-3)}] (0,0) rectangle (2,2);
  \draw[shift={(-3,-3)}] (0,0) rectangle (2,2);
\end{tikzpicture}
```

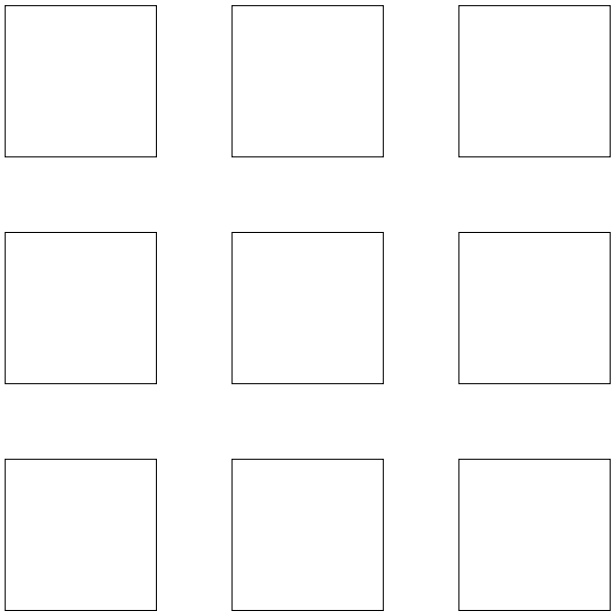


Figure 16.19: transform: shift

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt] (0,0) rectangle (2,2);
  \draw[xshift=-100pt] (0,0) rectangle (2,2);
  \draw[yshift= 100pt] (0,0) rectangle (2,2);
  \draw[yshift=-100pt] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt, xscale=1.5] (0,0) rectangle (2,2);
  \draw[yshift= 100pt, xscale=0.5] (0,0) rectangle (2,2);
  \draw[xshift=-100pt, yscale=1.5] (0,0) rectangle (2,2);
  \draw[yshift=-100pt, yscale=0.5] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift= 100pt, xscale=1.5] (0,0) rectangle (2,2);
```

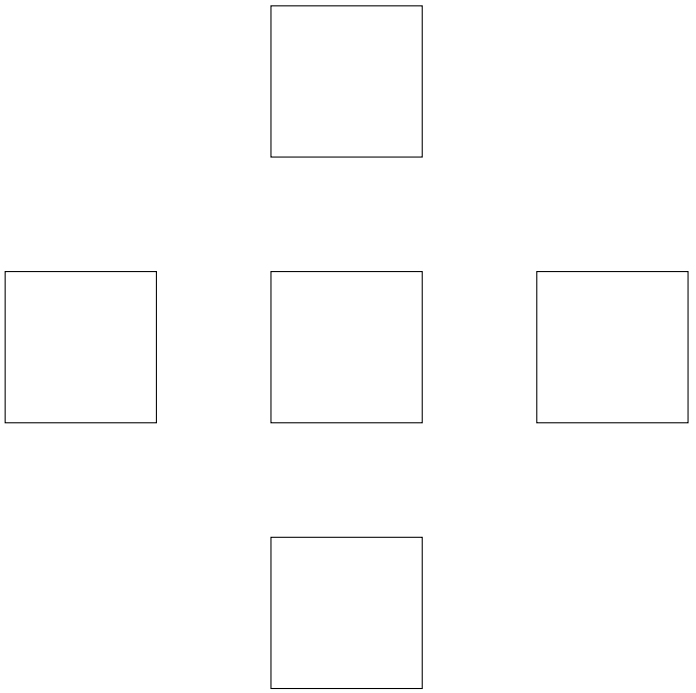


Figure 16.20: transform: shift x, y

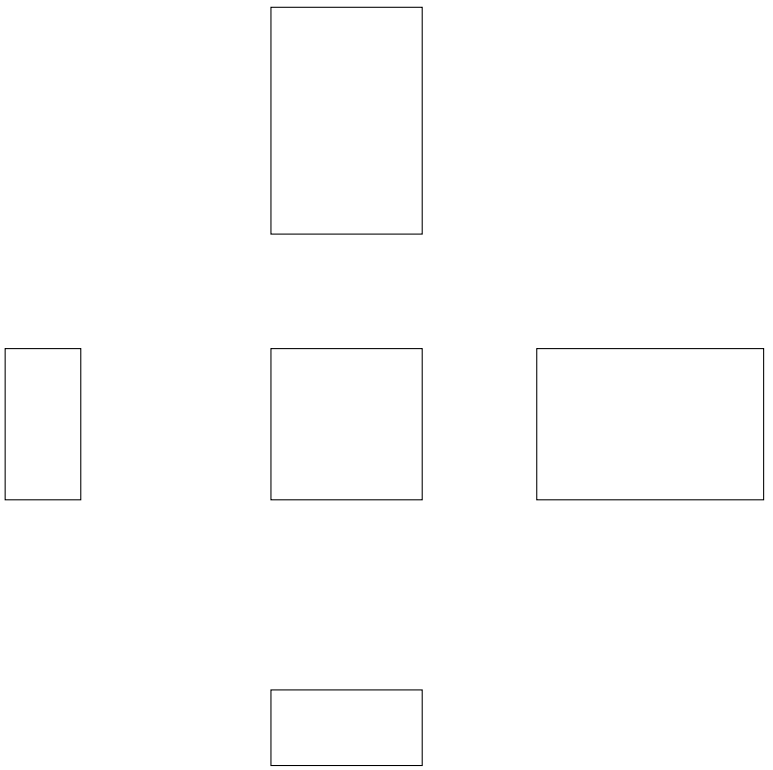


Figure 16.21: transform: scale x, y

```
  \draw[yshift= 100pt, yscale=1.5] (0,0) rectangle (2,2);
  \draw[xshift=-100pt, xscale=0.5] (0,0) rectangle (2,2);
  \draw[yshift=-100pt, yscale=0.5] (0,0) rectangle (2,2);
\end{tikzpicture}
```

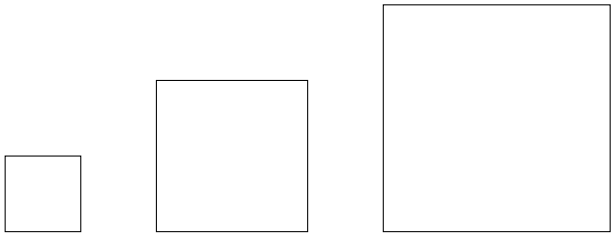


Figure 16.22: transform: scale

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift=125pt,rotate=45] (0,0) rectangle (2,2);
  \draw[xshift=175pt,rotate around={45:(2 ,2)}] (0,0) rectangle (2,2);
\end{tikzpicture}
```

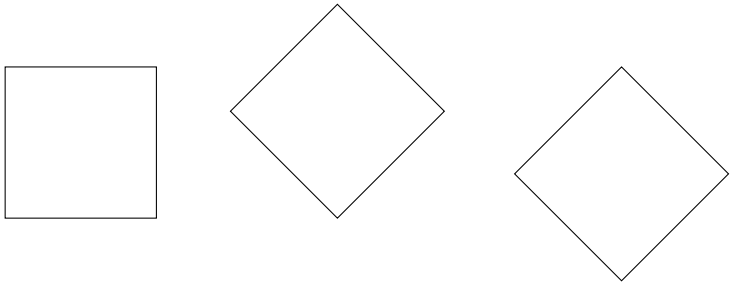


Figure 16.23: transform: rotate

```
\begin{tikzpicture}
  \draw (0,0) rectangle (2,2);
  \draw[xshift=70pt,xslant=1] (0,0) rectangle (2,2);
  \draw[yshift=70pt,yslant=1] (0,0) rectangle (2,2);
\end{tikzpicture}
```

```
\tikzset{
  box/.style={
    draw=blue,
    rectangle,
    rounded corners=5pt,
    minimum width=50pt,
    minimum height=20pt,
    inner sep=5pt
  }
}
\begin{tikzpicture}
  \node[box] (1) at (0,0) {1};
```

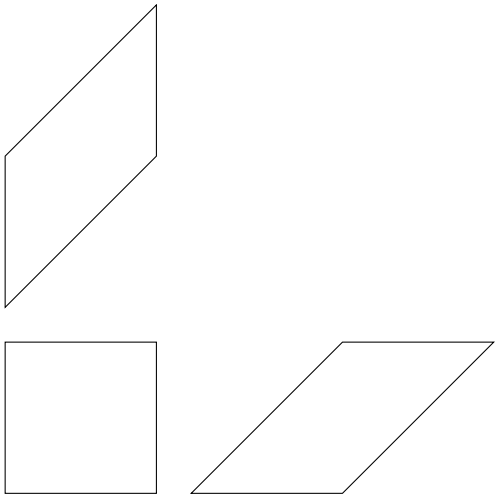


Figure 16.24: transform: slant

```
  \node[box] (2) at (4,0) {2};
  \node[box] (3) at (8,0) {3};
  \draw[->] (1)--(2);
  \draw[->] (2)--(3);
  \node at (2,1) {a};
  \node at (6,1) {b};
\end{tikzpicture}
```

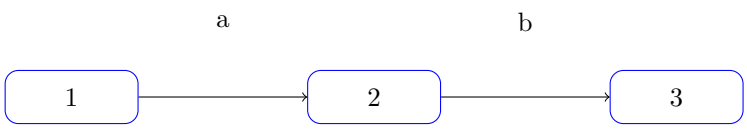


Figure 16.25: flowchart

```
\tikzset{
  box/.style={
    draw=blue,
    fill=blue!20,
    rectangle,
    rounded corners=5pt,
    minimum height=20pt,
    inner sep=5pt
  }
}
\begin{tikzpicture}
  \node[box] {1}
      child {node[box] {2}}
      child {node[box] {3}
          child {node[box] {4}}
          child {node[box] {5}}
          child {node[box] {6}}
```

```
      };
\end{tikzpicture}
```

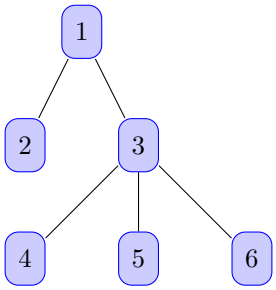


Figure 16.26: tree

```
\begin{tikzpicture}
  \draw[->] (-0.2,0)--(6,0) node[right] {$x$};
  \draw[->] (0,-0.2)--(0,6) node[above] {$f(x)$};
  \draw[domain=0:4] plot (\x ,{0.1* exp(\x)}) node[right] {$f(x)=\frac{1}{10}e^x$};
\end{tikzpicture}
```

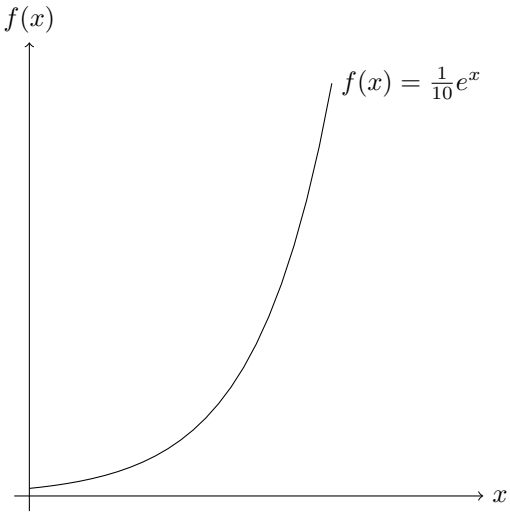


Figure 16.27: tree

https://stackoverflow.com/questions/64897575/tikz-libraries-in-bookdown

It turns out that you can simply put the `\usetikzlibrary{...}` command directly before the `\begin{tikzpicture}` and everything works fine :)

https://stackoverflow.com/questions/56211210/r-markdown-document-with-html-docx-output-using-latex-package-bbm

https://tex.stackexchange.com/questions/171711/how-to-include-latex-package-in-r-markdown

16.2 3D

https://zhuanlan.zhihu.com/p/431732330?utm_psn=1741857547550638080

https://github.com/RRWWW/Stereometry

```
\begin{tikzpicture}
  \coordinate (A) at ( 1, 1, 1);
  \coordinate (B) at ( 1, 1,-1);
  \coordinate (C) at ( 1,-1,-1);
  \coordinate (D) at ( 1,-1, 1);
  \coordinate (E) at (-1,-1, 1);
  \coordinate (F) at (-1,-1,-1);
  \coordinate (G) at (-1, 1,-1);
  \coordinate (H) at (-1, 1, 1);
  \draw (A) node[right=1pt] {$A$}--
        (B) node[right=1pt] {$B$}--
        (C) node[right=1pt] {$C$}--
        (D) node[right=1pt] {$D$}--
        (E) node[left= 1pt] {$E$}--
        (F) node[right=1pt] {$F$}--
        (G) node[right=1pt] {$G$}--
        (H) node[left= 1pt] {$H$}--
        (A) node[right=1pt] {$A$};
\end{tikzpicture}
```

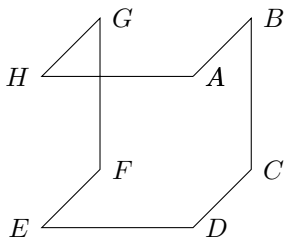


Figure 16.28: cube

https://tex.stackexchange.com/questions/388621/optimizing-perspective-tikz-graphic

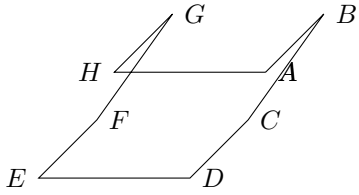


Figure 16.29: cube rotate

https://github.com/XiangyunHuang/bookdown-broken/blob/master/index.Rmd

```
\smartdiagramset{planet color=gray!40!white, uniform color list=gray!40!white for 10
↪  items}
\smartdiagram[bubble diagram]{Basic skills,
```

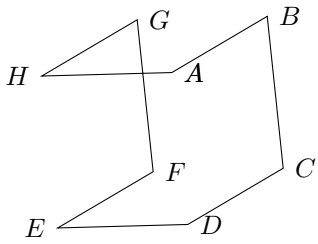


Figure 16.30: cube rotate

```
  Edit~/\\ (RStudio), Organize~/\\ (bookdown), Cooperate~/\\ (Git), Typeset~/\\
↪  (LaTeX/Pandoc), Compile~/\\ (GitHub Action)}
```

Figure 16.31: modern statistics plot skills

```
\usetikzlibrary{patterns}
\usetikzlibrary{3d,calc}
\tdplotsetmaincoords{45}{45}
\begin{tikzpicture}[tdplot_main_coords]
  \coordinate (A) at ( 1, 1, 1);
  \coordinate (B) at ( 1, 1,-1);
  \coordinate (C) at ( 1,-1,-1);
  \coordinate (D) at ( 1,-1, 1);
  \coordinate (E) at (-1,-1, 1);
  \coordinate (F) at (-1,-1,-1);
  \coordinate (G) at (-1, 1,-1);
  \coordinate (H) at (-1, 1, 1);
  \draw (A) node[right=1pt] {$A$}--
        (B) node[right=1pt] {$B$}--
```

```
        (C) node[right=1pt] {$C$}--
        (D) node[right=1pt] {$D$}--
        (E) node[left= 1pt] {$E$}--
        (F) node[right=1pt] {$F$}--
        (G) node[right=1pt] {$G$}--
        (H) node[left= 1pt] {$H$}--
        (A) node[right=1pt] {$A$};
\end{tikzpicture}
```

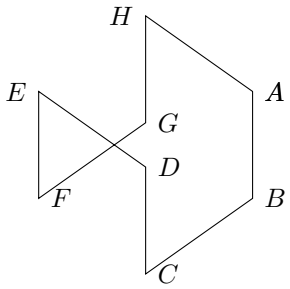


Figure 16.32: cube rotate

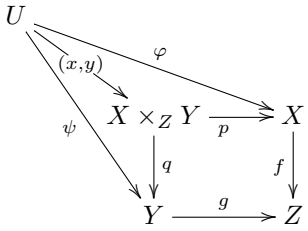
xy-pic

https://bookdown.org/yihui/rmarkdown-cookbook/install-latex-pkgs.html

`tinytex::install_tinytex()`

the following xymatrix from LaTeX package xy for xy-pic is not shown or rendered in HTML:

`$\LaTeX$` can only be used in HTML, not PDF



Chapter 17

RMarkdown

https://bookdown.org/yihui/rmarkdown-cookbook/verbatim-code-chunks.html

17.1 Markdown

https://www.rstudio.com/wp-content/uploads/2015/02/rmarkdown-cheatsheet.pdf

```
script^superscript^
```

script^{superscript}

```
~subscript~
```

script$_{subscript}$

```
***
```

horizontal rule (or slide break)

```
dim(iris)
```

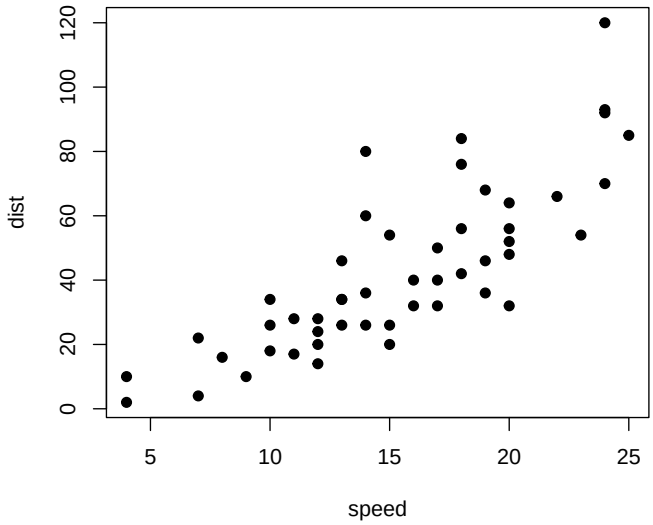
```
## [1] 150   5
```

17.2 Bookdown

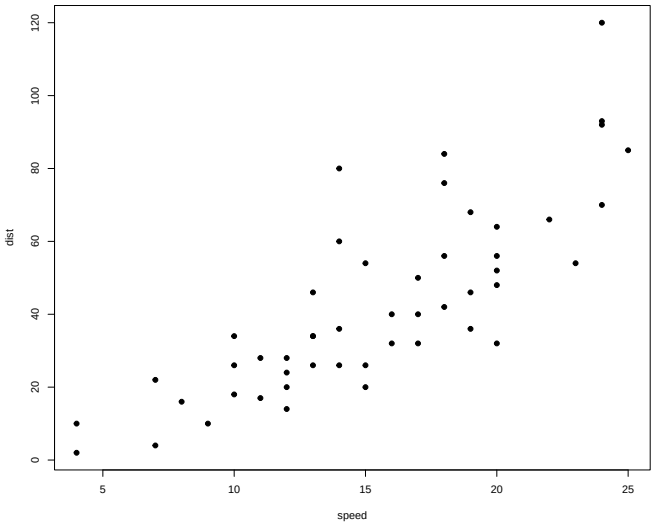
https://bookdown.org/yihui/rmarkdown-cookbook/multi-column.html

17.2.1 Two columns

Below is a Div containing three child Divs side by side. The Div in the middle is empty, just to add more space between the left and right Divs.



The figure on the left-hand side shows the `cars` data.
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur.



The figure on the left-hand side shows the `cars` data.
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur.

Chapter 18

statistics

18.1 covariance matrix

[5]

18.1.1 calculation

$$\begin{aligned}
\mathrm{C}[\boldsymbol{X}] = \mathrm{Cov}[\boldsymbol{X}] = \mathrm{V}[\boldsymbol{X}] =& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]\left[\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right]\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}} - \boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] + \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}\left[\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}[\boldsymbol{X}]\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{X} = [X]_{1\times 1} = X \Rightarrow C(X) = \mathrm{C}[\boldsymbol{X}] =& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}[XX] - \mathrm{E}(X)\mathrm{E}(X) \\
=& \mathrm{E}\left(X^2\right) - [\mathrm{E}(X)]^2 = \mathrm{V}(X)
\end{aligned}$$

18.1.2 $\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[\boldsymbol{X}]$

$$\begin{aligned}
\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] =& \mathrm{E}\left[[(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})][(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(\boldsymbol{X}+\boldsymbol{b})=\mathrm{E}(\boldsymbol{X})+\boldsymbol{b}}{=}& \mathrm{E}\left[[\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}][\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] = \mathrm{V}[\boldsymbol{X}]
\end{aligned}$$

18.1.3 $\mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\begin{aligned}
\mathrm{V}[A\boldsymbol{X}] =& \mathrm{E}\left[[(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})][(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(A\boldsymbol{X})=A\mathrm{E}(\boldsymbol{X})}{=}& \mathrm{E}\left[[A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})][A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}A^{\mathrm{T}}\right] \\
=& A\mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right]A^{\mathrm{T}} = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}
\end{aligned}$$

18.1.4 $\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$$

18.2 covariance matrix

5

18.2.1 calculation

$$\begin{aligned}
\mathrm{C}[\boldsymbol{X}] = \mathrm{Cov}[\boldsymbol{X}] = \mathrm{V}[\boldsymbol{X}] =& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]\left[\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right]\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}} - \boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}\left[\boldsymbol{X}\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] + \mathrm{E}\left[\mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}\left[\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}[\boldsymbol{X}]\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} + \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{X} = [X]_{1\times 1} = X \Rightarrow C(X) = \mathrm{C}[\boldsymbol{X}] =& \mathrm{E}\left[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}\right] - \mathrm{E}(\boldsymbol{X})\mathrm{E}(\boldsymbol{X})^{\mathrm{T}} \\
=& \mathrm{E}[XX] - \mathrm{E}(X)\mathrm{E}(X) \\
=& \mathrm{E}\left(X^2\right) - [\mathrm{E}(X)]^2 = \mathrm{V}(X)
\end{aligned}$$

18.2.2 $\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[\boldsymbol{X}]$

$$\begin{aligned}
\mathrm{V}[\boldsymbol{X} + \boldsymbol{b}] =& \mathrm{E}\left[[(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})][(\boldsymbol{X} + \boldsymbol{b}) - \mathrm{E}(\boldsymbol{X} + \boldsymbol{b})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(\boldsymbol{X}+\boldsymbol{b})=\mathrm{E}(\boldsymbol{X})+\boldsymbol{b}}{=}& \mathrm{E}\left[[\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}][\boldsymbol{X} + \boldsymbol{b} - \mathrm{E}(\boldsymbol{X}) - \boldsymbol{b}]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] = \mathrm{V}[\boldsymbol{X}]
\end{aligned}$$

18.2.3 $\mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\begin{aligned}
\mathrm{V}[A\boldsymbol{X}] =& \mathrm{E}\left[[(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})][(A\boldsymbol{X}) - \mathrm{E}(A\boldsymbol{X})]^{\mathrm{T}}\right] \\
\overset{\mathrm{E}(A\boldsymbol{X})=A\mathrm{E}(\boldsymbol{X})}{=}& \mathrm{E}\left[[A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})][A\boldsymbol{X} - A\mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]]^{\mathrm{T}}\right] \\
=& \mathrm{E}\left[A[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}A^{\mathrm{T}}\right] \\
=& A\mathrm{E}\left[[\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})][\boldsymbol{X} - \mathrm{E}(\boldsymbol{X})]^{\mathrm{T}}\right]A^{\mathrm{T}} = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}
\end{aligned}$$

18.2.4 $\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$

$$\mathrm{V}[A\boldsymbol{X} + \boldsymbol{b}] = \mathrm{V}[A\boldsymbol{X}] = A\mathrm{V}[\boldsymbol{X}]A^{\mathrm{T}}$$

Chapter 19

Gosper algorithm

https://stackoverflow.com/questions/42543206/r-markdown-compile-error

always_allow_html: true

```
install.packages("webshot")
webshot::install_phantomjs()
```

however webshot not work

Error: cannot find bilibili.com

https://cran.r-project.org/web/packages/vembedr/vignettes/embed.html

```
## Warning: package 'vembedr' was built under R version 4.2.3
```

```
## embed_youtube("qeMqtt7NFDM")
```

references

1. Xie, Y. *Bookdown: Authoring Books and Technical Documents with r Markdown*. (2023).
2. Xie, Y. *Dynamic Documents with R and Knitr*. (Chapman; Hall/CRC, Boca Raton, Florida, 2015).
3. Bookdown books on the web: Downloading and converting to pdf - R Markdown. *Posit Community* (2019).
4. ccjou. 二次型與正定矩陣. (2009).
5. ccjou. 多變量常態分布. (2014).