$$\begin{aligned}
f(x) &= x^x \\
\ln f(x) &= x \ln x \\
\frac{\mathrm{d}}{\mathrm{d}x} \ln f(x) &= \frac{\mathrm{d}}{\mathrm{d}x} [x \ln x] \\
\frac{f'(x)}{f(x)} &= [x \ln x]' \\
f'(x) &= f(x) [x \ln x]' = x^x \left[(x)' \ln x + x (\ln x)' \right] \\
&= x^x \left[1 \ln x + x \frac{1}{x} \right] = x^x [\ln x + 1] = x^x [1 + \ln x] \\
&= x^x + x^x \ln x
\end{aligned}$$