

$$\begin{array}{ccc} \{v_j\}_{j=1}^n = \mathfrak{V} \subseteq \mathcal{V} & \xrightarrow{T(\cdot)} & \mathcal{W} \supseteq \mathfrak{W} = \{w_j\}_{j=1}^m \\ \Downarrow & & \Downarrow \\ \end{array}$$

$$\begin{array}{ccccc} VT^{-1}W^{-1}w = & v^j v_j = & \mathbf{v} & \begin{array}{c} \xleftarrow{T^{-1}(\cdot)} \\ \xrightarrow{T(\cdot)} \end{array} & \mathbf{w} = w^j w_j = W[\mathbf{w}]_W = T(\mathbf{v}) = T(V)[\mathbf{v}]_V \\ & \downarrow V^{-1} \uparrow V & & \downarrow W^{-1} \uparrow W & \\ & & & & = WTV^{-1}v \\ & V^{-1}v = [\mathbf{v}]_V & \begin{array}{c} \xleftarrow{T^{-1}} \\ \xrightarrow{T} \end{array} & [\mathbf{w}]_W = W^{-1}w = [T(v)]_W = T[\mathbf{v}]_V & \\ & \Downarrow & & \Downarrow & \\ & \mathbb{F}^n & \xrightarrow{T} & \mathbb{F}^m & \\ & \Downarrow & & \Downarrow & \\ & & & & T = [T(V)]_W \end{array}$$

$$t_j = [T(v_j)]_W$$

$$\{\mathbb{R}^n, \mathbb{C}^m, \dots\} \quad \{\mathbb{R}^m, \mathbb{C}^m, \dots\}$$