

group

abelian group

linear space

inner product space

1. vector addition closure
2. identity  $\mathbf{0}$
3. inverse
4. associativity

4 + 1. commutativity

1. scalar multiplication closure
2. identity 1
3. associativity
4. scalar distributivity
5. vector distributivity

inner product  $\langle \cdot | \cdot \rangle$

1.  $\langle f | g + h \rangle = \langle f | g \rangle + \langle f | h \rangle$
2.  $\langle f | \lambda g \rangle = \lambda \langle f | g \rangle$
3.  $\langle f | g \rangle = \overline{\langle g | f \rangle} = \langle g | f \rangle^*$
4.  $\langle f | f \rangle \geq 0$   
 $f=0 \Leftrightarrow \langle f | f \rangle = 0$

completeness:  
complete if  
all Cauchy function sequence convergent

Hilbert space