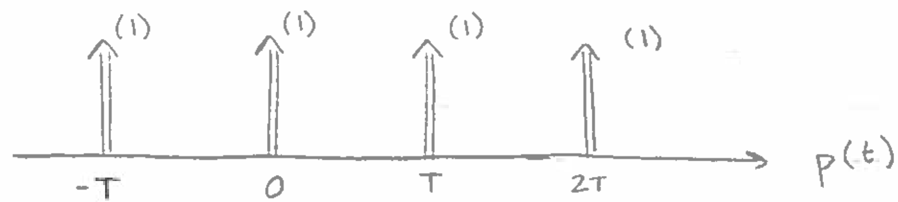
1.

$$p(t) = \sum_{-\infty}^{\infty} \delta(t - kT)$$

a)



b) $\tilde{p}(t) \quad \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-\frac{2\pi jkt}{T}}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t-kT)\, e^{-\frac{2\pi jkt}{T}}\, dt$$

Picking property → $= \left.\frac{e^{-\frac{2\pi jkt}{T}}}{T}\right|_{t=kT} = \frac{e^{-2\pi jk^2}}{T}$

That's awesome

$$\therefore\ \tilde{p}(t) = \sum_{k=-\infty}^{\infty} \frac{e^{-2\pi jk^2}}{T}\, e^{\frac{j2\pi kt}{T}} = \sum_{k=-\infty}^{\infty} \frac{e^{\frac{2\pi jk}{T}(-kT+t)}}{T}$$

c) $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$

$\omega_0 = \frac{2\pi}{\ }$ $\Rightarrow$ $C_k = \frac{\omega_0}{\ } \int^{T/2} x(t)\, e^{-j\omega_0 kt}\, dt$

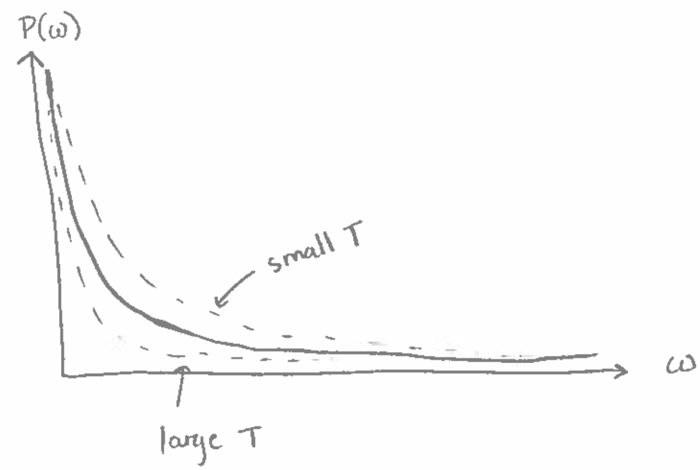
$T = \frac{2\pi}{\omega}$

d) $P(\omega) = \frac{2\pi C_k e^{-k}}{\omega} = \frac{2\pi}{\omega} \frac{e^{-2\pi j k^2}}{T} \cdot e^{-k}$

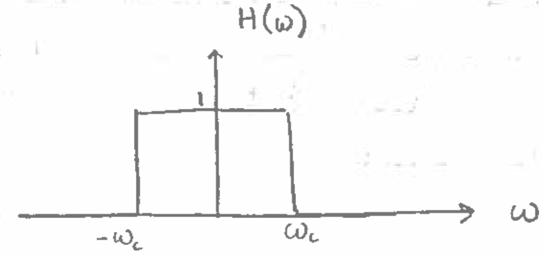
$= \frac{2\pi e^{-2\pi j k^2 - k}}{\omega T} = \frac{2\pi e^{-2\pi j k^2 - k}}{\omega\, 2\pi/\omega_0}$

$= \frac{\omega_0 e^{-2\pi j k^2 - k}}{\omega}$

e) $T = \frac{2\pi}{\omega_0} \Rightarrow$ Increasing T decreases the fundamental frequency of $p(t)$ and decreases the amplitude of $P(\omega)$



2.



a) $h(t)$ = impulse response

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \left[\underbrace{\int_{-\infty}^{-\omega_c} \cancel{H(\omega)} e^{j\omega t}\, d\omega}_{0} + \int_{-\omega_c}^{\omega_c} H(\omega)\, e^{j\omega t}\, dt + \underbrace{\int_{\omega_c}^{\infty} \cancel{H(\omega)} e^{j\omega t}\, dt}_{0} \right]$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} 1 \cdot e^{j\omega t}\, d\omega = \frac{1}{2\pi} \cdot \frac{1}{jt}\, e^{j\omega t} \Big|_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j t} \left(e^{j\omega_c t} - e^{-j\omega_c t} \right) = \frac{1}{\pi t} \cdot \left[\frac{1}{2j} e^{j\omega_c t} - \frac{1}{2j} e^{-j\omega_c t} \right]$$

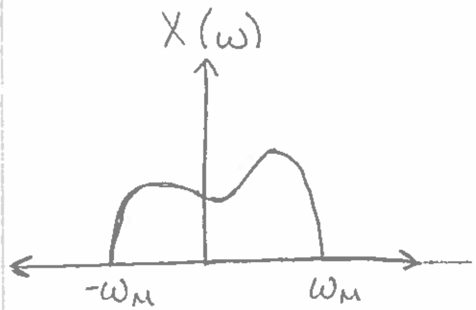
$$= \frac{1}{\pi t} \sin(\omega_c t)$$

b) $X(\omega)$ $\qquad$ $Y(\omega)$

c) In the frequency domain, convolution is multiplication of functions. In this case, $Y(\omega) = X(\omega) * H(\omega)$, and the result cuts off $X(\omega)$ at $\pm\omega_c$, which corresponds to a low-pass filter effect.

3.

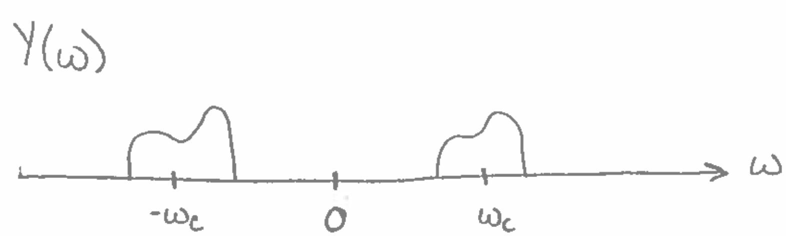
$x(t)$ band limited $[-\omega_M, \omega_M]$

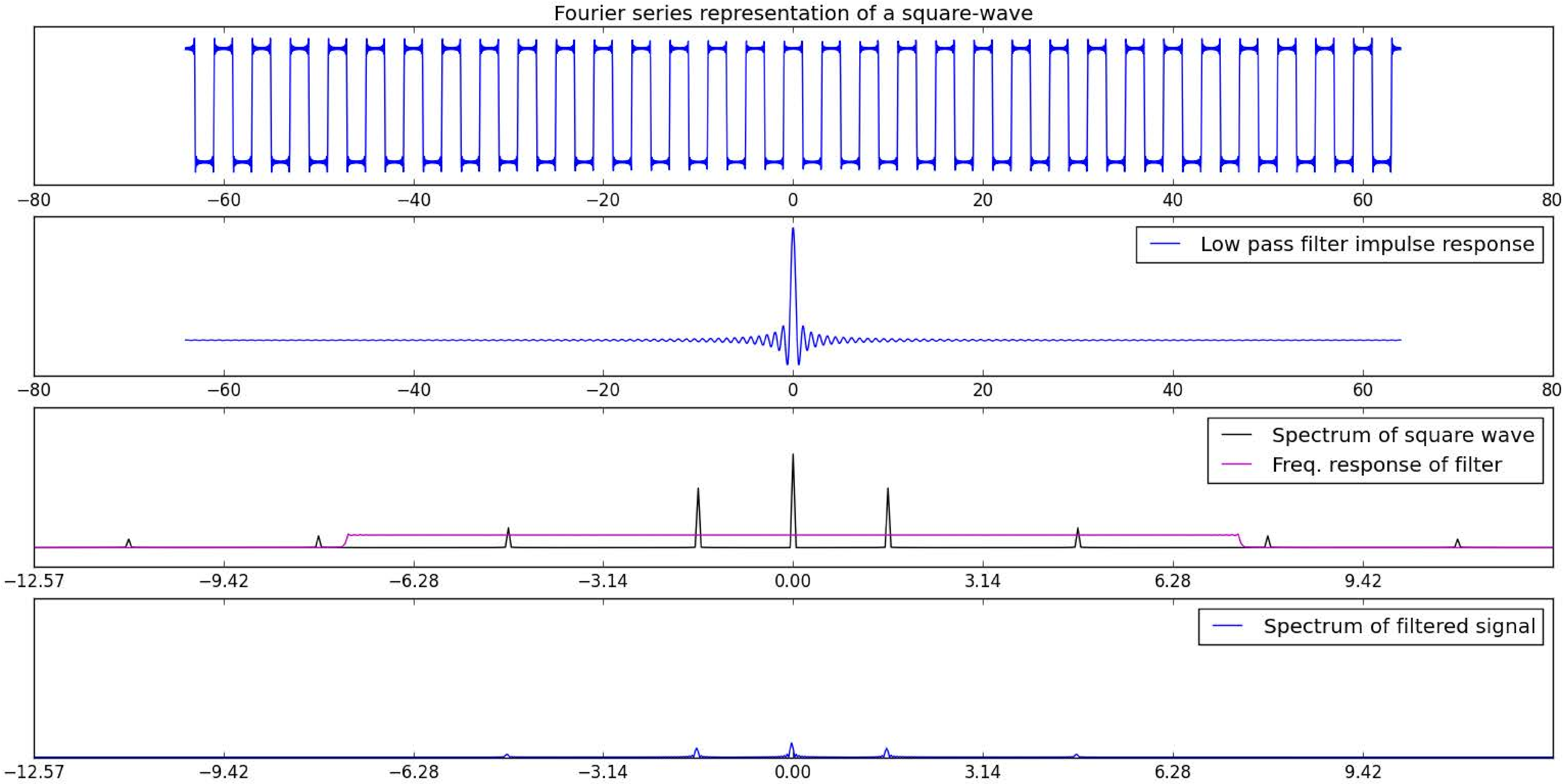


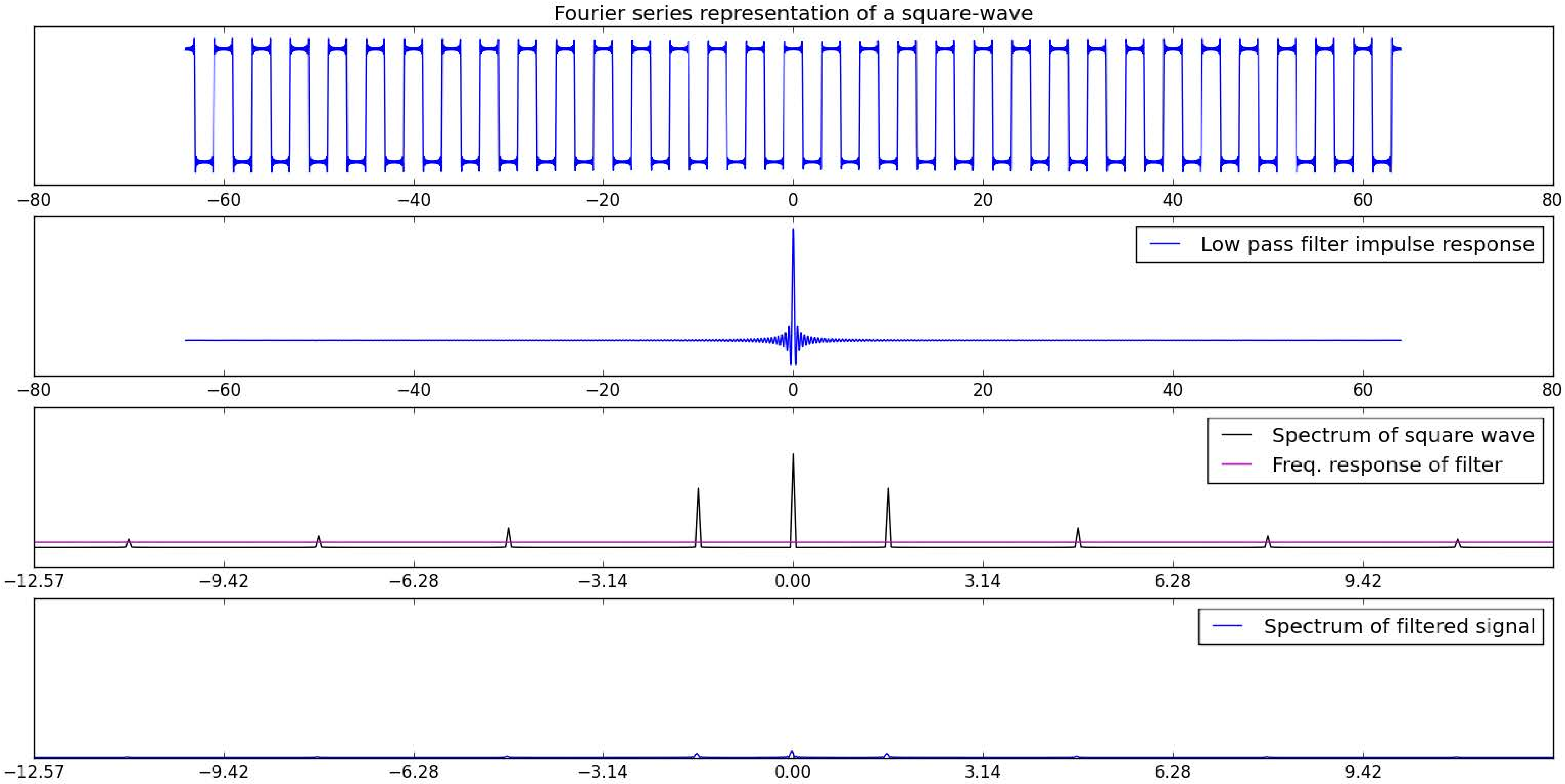
$y(t) = x(t)\cos(\omega_c t)$

$Y(\omega) = X(\omega) * F(\cos(\omega_c t)) = X(\omega) * (\delta(\omega+\omega_c) + \delta(\omega-\omega_c))$

$= X(\omega)$ centered at two impulses




Fourier series representation of a square-wave
−80
−60
−40
−20
0
20
40
60
80
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42
Spectrum of filtered signal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