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3/5/15
SigSys PS06

1. Gunshot + Violin

We know that if we have an LTI system, we can draw the following diagram:

$$\delta(t) \rightarrow \boxed{\text{system}} \rightarrow h(t)$$

$$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$$

In the diagram, $\delta(t)$ is an impulse, $h(t)$ is the impulse response, $x(t)$ is some signal, and $y(t)$ is the output from the system when $x(t)$ is inputted. We also know that $y(t)$ is $x(t)$ convolved with $h(t)$.

In our gunshot example, the system is the ~~gun~~ firing range, and the impulse δ is the gunshot. The audio we hear $h(t)$ is the impulse response. In this case, the firing range amplifies and reverberates the gunshot.

When we input the violin wave $x(t)$ into the system, the output is the convolution of $h(t)$ and $x(t)$ and we hear how the violin would sound in the firing range - amplified with echo.

2.

$x(t) \rightarrow \boxed{\text{channel}} \rightarrow y(t)$

$$y(t) = \frac{1}{2} x(t-1) + \frac{1}{4} x(t-10)$$

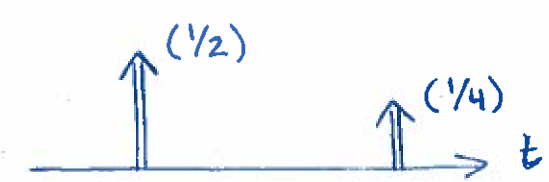
$y(t)$ can be called an echo because it repeats the input with a larger delay and lower ~~ap~~ amplitude, causing a reverberation-like effect.

The impulse response will be two impulses added together, because an addition in $y(t)$ corresponds to an addition in $h(t)$.

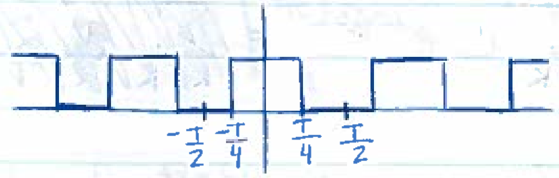
The first impulse will have center 1 and area $1/2$, corresponding to the shift and amplitude of the first addend.

The second impulse will have center 10 and area $1/4$. We therefore have:

$$h(t) = \frac{1}{2} \delta(t-1) + \frac{1}{4} \delta(t-10)$$



3.



a) 1) Coefficients of Fourier expansion

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{\frac{-j2\pi kt}{T}}\, dt$$

We can split $\int_{-T/2}^{T/2}$ into 3 integrals. We therefore have

$$\int_{-T/2}^{T/2} f(t) = \int_{-T/2}^{-T/4} f(t) + \int_{-T/4}^{T/4} f(t) + \int_{T/4}^{T/2} f(t)$$

$$= \emptyset + \int_{-T/4}^{T/4} f(t) + \emptyset$$

$$\therefore \quad C_k = \frac{1}{T}\int_{-T/4}^{T/4} 1 \cdot e^{\frac{-j2\pi kt}{T}}$$

$$= \frac{1}{T}\cdot\frac{-T}{j2\pi k}\; e^{\frac{-j2\pi kt}{T}}\Bigg|_{-T/4}^{T/4}$$

$$= \frac{1}{\pi k}\cdot\left[\frac{1}{2j}e^{\frac{-j2\pi k}{T}} - \frac{1}{2j}e^{\frac{j2\pi k}{T}}\right]$$

$$C_k = \frac{1}{\pi k}\sin\left(\frac{2\pi k}{T}\right) = \frac{T}{T}\cdot\frac{2}{2}\cdot\sin\left(\frac{2\pi k}{T}\right) =$$

$$\frac{2}{T}\,\text{sinc}\left(\frac{2\pi k}{T}\right)$$

2) Fourier expansion

$$\tilde{x}_{K} = \sum_{k=-K}^{K} c_k e^{\frac{j2\pi kt}{T}}$$

$$= \sum_{k=-K}^{K} \frac{2}{T} \operatorname{sinc}\left(\frac{2\pi k}{T}\right) e^{\frac{j2\pi kt}{T}}$$

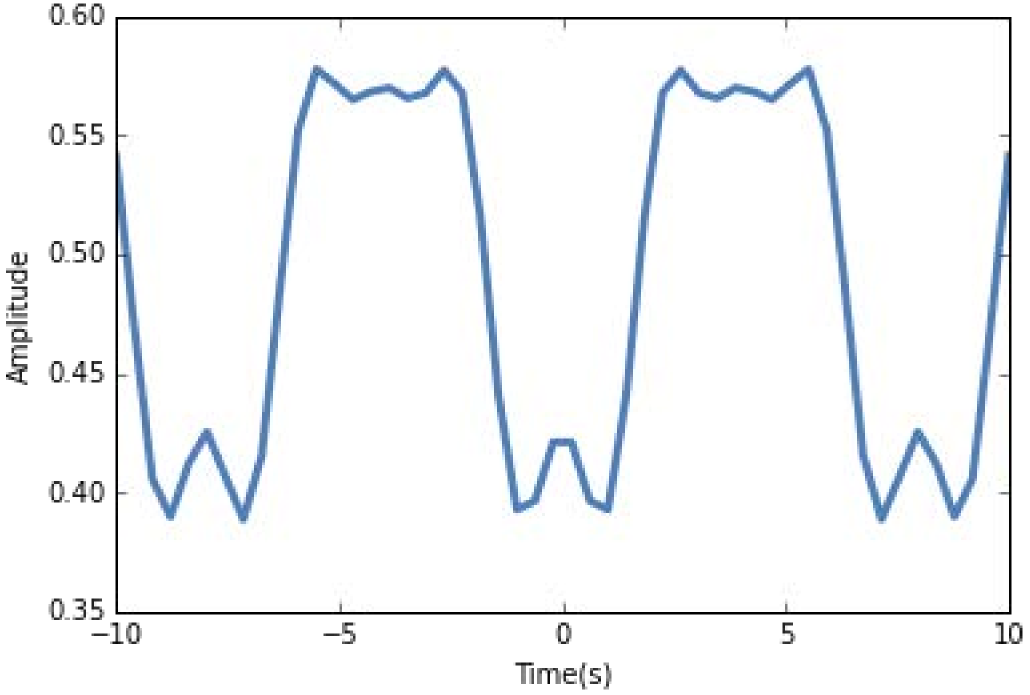
b)

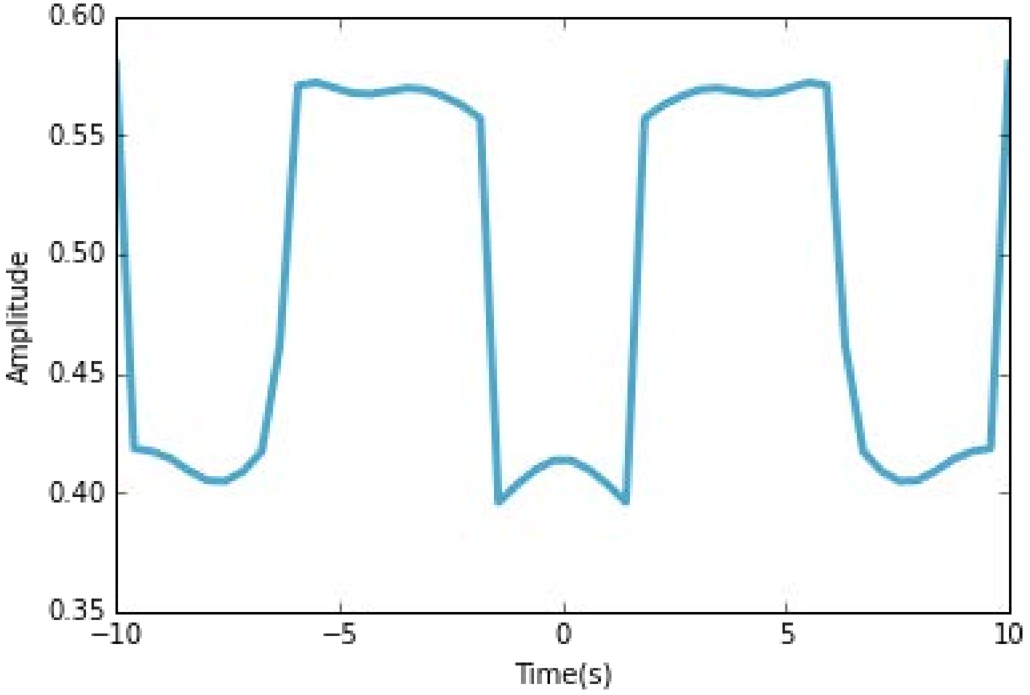
$$T=4 \Rightarrow \tilde{x}_k = \sum_{k=-K}^{K} \frac{1}{2} \operatorname{sinc}\left(\frac{\pi k}{2}\right) e^{\frac{j\pi kt}{2}}$$

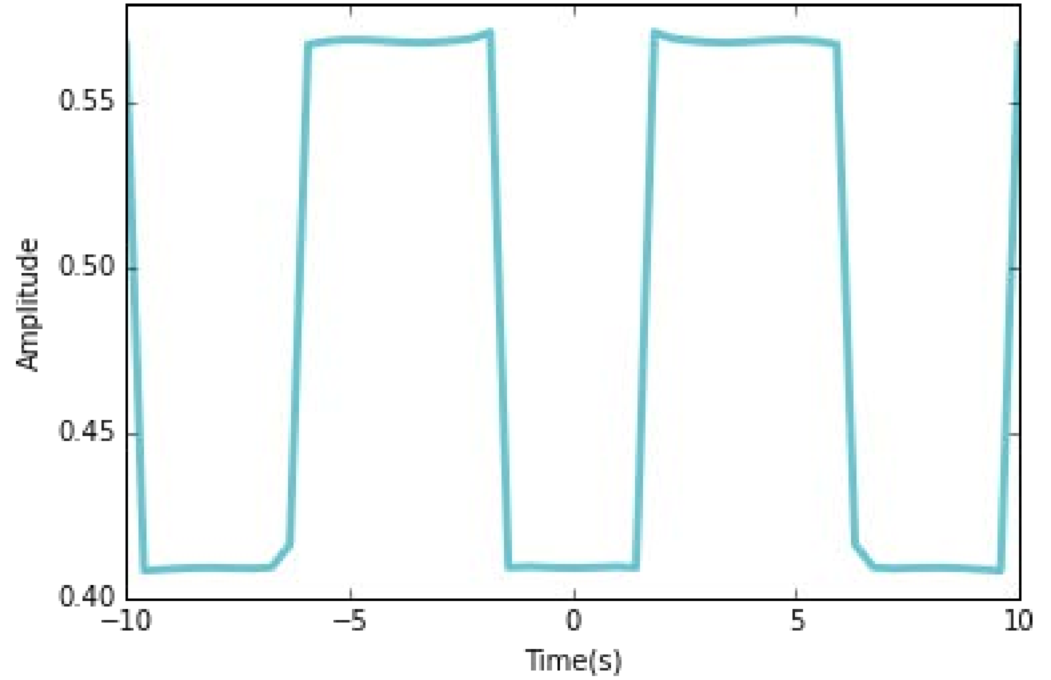
Graphs on next pages

c)

At discontinuities, there is some evidence of sinusoidal oscillations, the remnants of the sin functions that produced the wave. These make sense because as we use more exponentials, the difference between the square wave and our approximation decreases, so the sinusoidal behavior flattens.


0.60
0.55
0.50
0.45
0.40
0.35
Amplitude
−10
−5
0
5
10
Time(s)


0.60
0.55
0.50
0.45
0.40
0.35
Amplitude
−10
−5
0
5
10
Time(s)



0.55
0.50
0.45
0.40
Amplitude
−10
−5
0
5
10
Time(s)

4.

a) Given: $c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{\frac{-j2\pi k t}{T}}\, dt$

For the function $y(t) = x(t-T_1)$, we can say:

$$c'_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t-T_1)\, e^{\frac{-j2\pi k t}{T}}\, dt$$

$$= \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(t-T_1)\, e^{\frac{-j2\pi k t}{T}}\, dt$$

$$= \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(t)\, e^{\frac{-j2\pi k (t-T_1)}{T}}\, dt$$

$$= \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(t)\, e^{\frac{-j2\pi k t}{T}} \cdot e^{\frac{j2\pi k T_1}{T}}\, dt$$

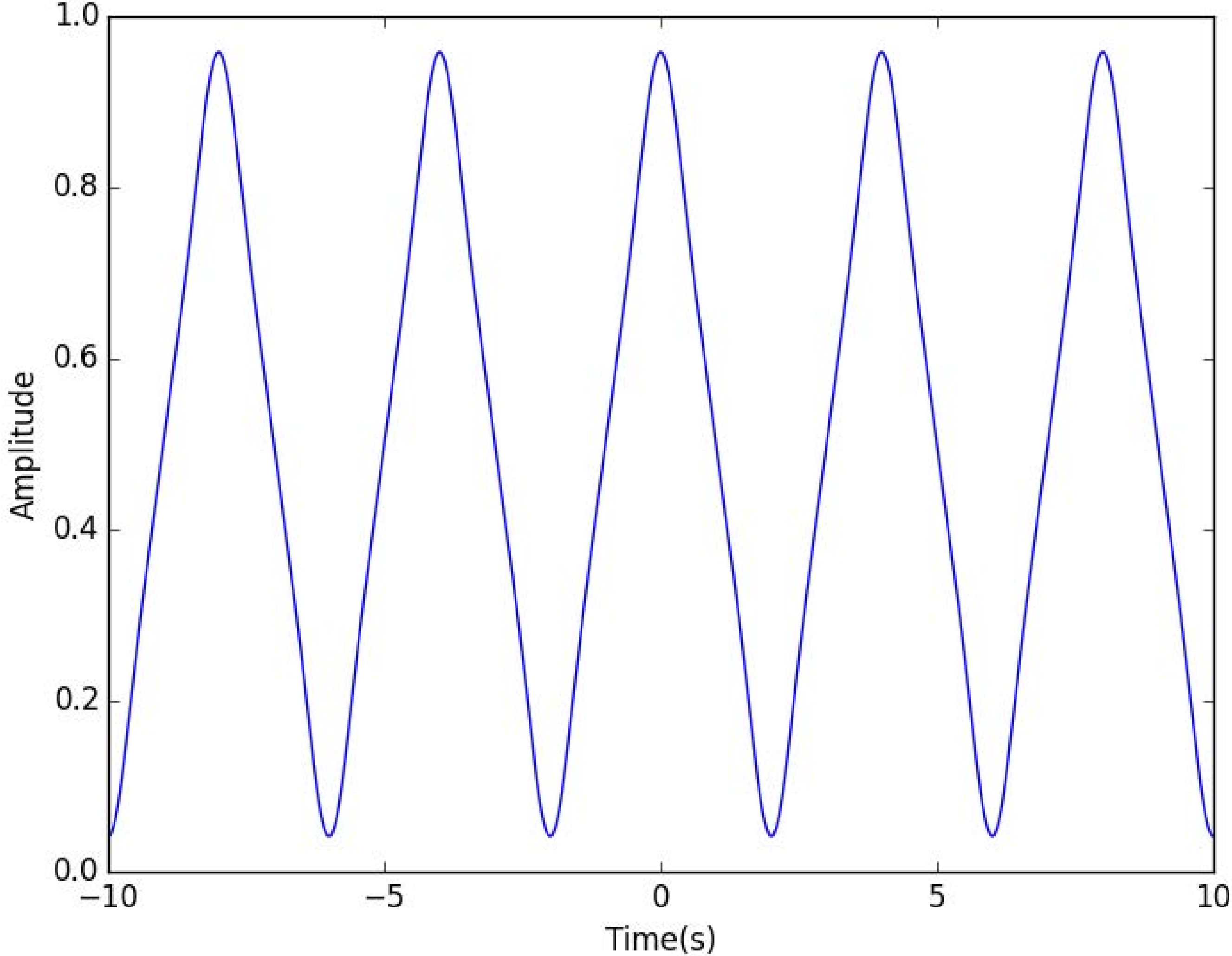
$$c'_k = e^{\frac{j2\pi k T_1}{T}} \cdot c_k$$

b) For the triangle wave, $T_1 = T/2$

$$c_k = \begin{cases} \frac{-2}{\pi^2 k^2}, & k \text{ is odd} \\ 1/2, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$c'_k = \begin{cases} \frac{-2e^{j\pi k}}{\pi^2 k^2}, & k \text{ is odd} \\ \frac{e^{jk\pi}}{2}, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$

Graphs on following pages



1.0
0.8
0.6
0.4
0.2
0.0
Amplitude
−10
−5
0
5
10
Time(s)

```
# coding: utf-8

# In[1]:

from __future__ import print_function
import numpy as np
import matplotlib.pyplot as mplib
import math


# In[2]:

def fs_triangle_shift(ts, M=3, T=4):
    # computes a fourier series representation of a triangle wave
    # with M terms in the Fourier series approximation
    # if M is odd, terms -(M-1)/2 -> (M-1)/2 are used
    # if M is even terms -M/2 -> M/2-1 are used

    # create an array to store the signal
    x = np.zeros(len(ts))

    # if M is even
    if np.mod(M,2) ==0:

        for k in range(-int(M/2), int(M/2)):
            const = np.exp(1j*math.pi*k)
            # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2)) * const
            if np.mod(k,2)==0:
                Coeff = 0
            if k == 0:
                Coeff = 0.5 * const
            x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

    # if M is odd
    if np.mod(M,2) == 1:
        for k in range(-int((M-1)/2), int((M-1)/2)+1):
            const = np.exp(1j*math.pi*k)
           # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2)) * const
            if np.mod(k,2)==0:
                Coeff = 0
            if k == 0:
                Coeff = 0.5 * const
            x = x + Coeff*np.exp(1j*2*np.pi/T*k*ts)

    return x


# In[3]:
```

```
ts = np.linspace(-10,10,2048)

x = fs_triangle_shift(ts, M = 10)

mplib.plot()

line1, = mplib.plot(ts, x, lw = 1)
mplib.xlabel("Time(s)")
mplib.ylabel("Amplitude")
mplib.show()


# In[ ]:
```