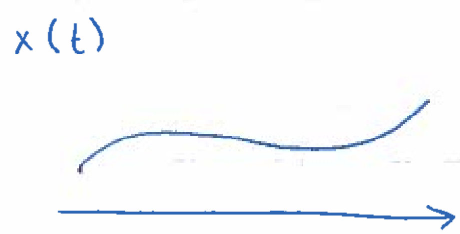
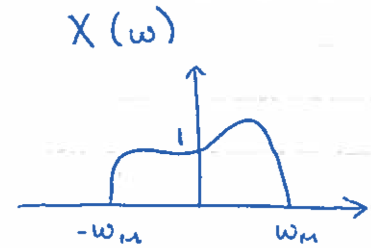
Raagini Rameshwar
3/26/15
PS08

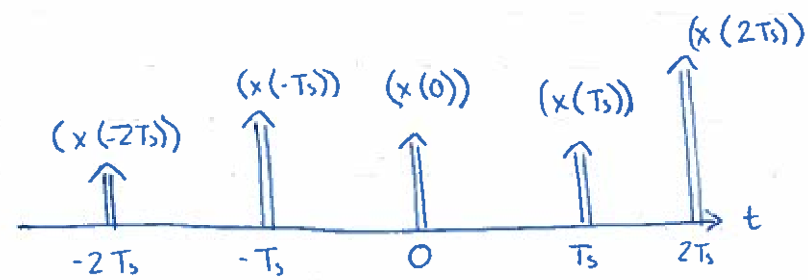




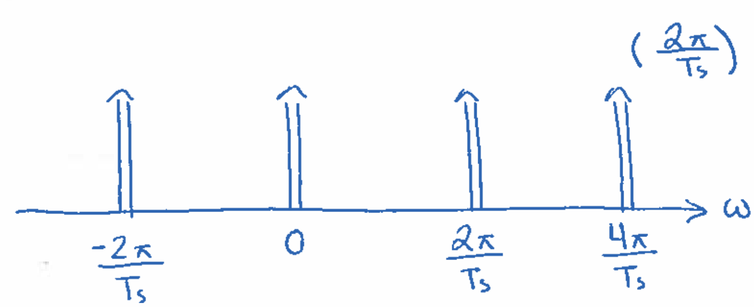
$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ (impulse train)

$x_p(t) = x(t)p(t)$

a) $x_p(t)$



b) $P(\omega)$



c.

$X_p(\omega)$

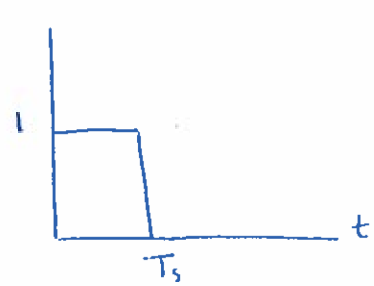
d.

$\frac{2\pi}{T_s} > 2\omega_m$ because any less of a sampling frequency would cause the waves to add and we'd lose information.

e. To recover $x(t)$ from $x_p(t)$, we look at $p(t)$ functions with $T_s \to 0$ so we have an increase in sample size. At an infinite sampling frequency, we will have $x(t)$. Mathematically,

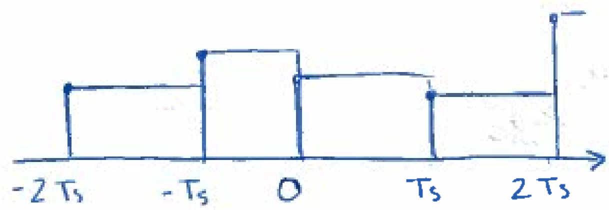
$$x(t) = \lim_{T_s \to 0} x_p(t)$$

f. $z(t)$



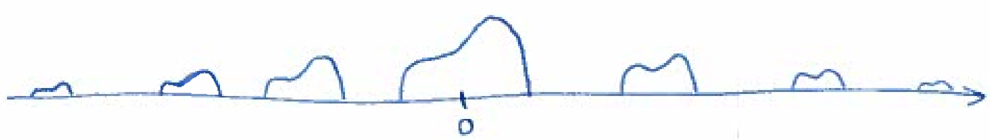
This signal has been very much considered.

g) $x_z(t) = x_p * z(t)$

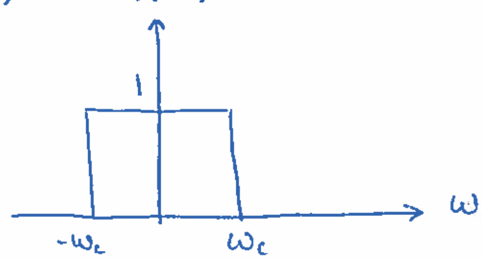


h) $X_z(\omega) = X_p(\omega) * Z(\omega)$

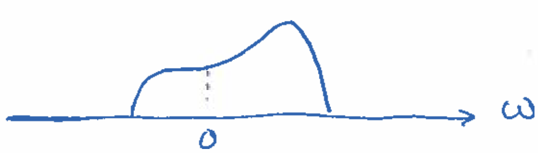
$Z(\omega)$ is a shifted sinc function, so we will see a change in amplitude and dampening for ~~g~~ higher frequencies:



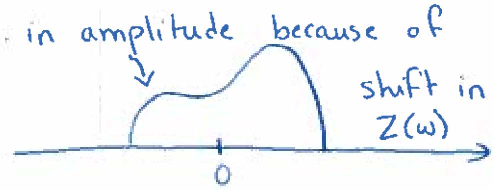
i) $H(\omega)$



$\hat{X}(\omega) = X_p(\omega)H(\omega)$



$\bar{X}(\omega) = X_z(\omega)H(\omega)$



2.

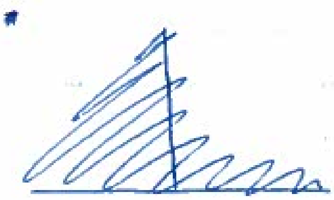
$y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$

$X_1(\omega) = 0$, $X_2(\omega) = 0$ for $|\omega| > \omega_M$

$\omega_1 \gg \omega_M$, $\omega_2 \gg \omega_M$

$\omega_1 + 2\omega_M < \omega_2$

a) $Y(\omega) = X_1(\omega) * (\delta(\omega-\omega_1) + \delta(\omega+\omega_1)) + X_2(\omega) * (\delta(\omega-\omega_2) + \delta(\omega+\omega_2))$



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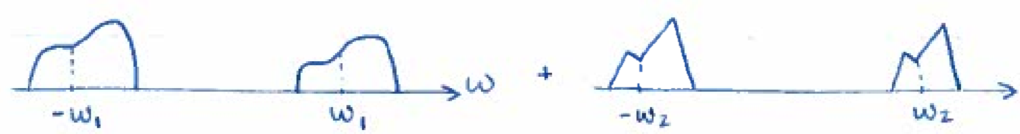

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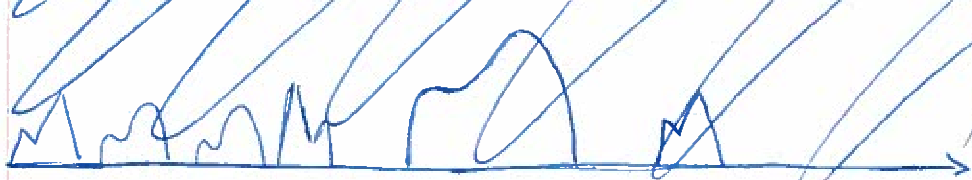
~~b) $\mathcal{F}(y(t)\cos(\omega t))$~~ → convolution of $Y(\omega)$ with impulses

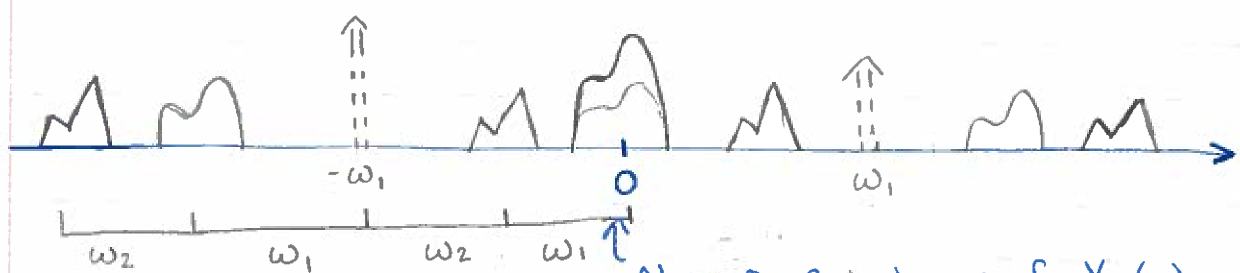
~~We will have 1 $Y(\omega)$ centered around $-\omega_1$ and another centered at $+\omega_1$. This means that $X_1(\omega)$ will add with itself at 0.~~



b.

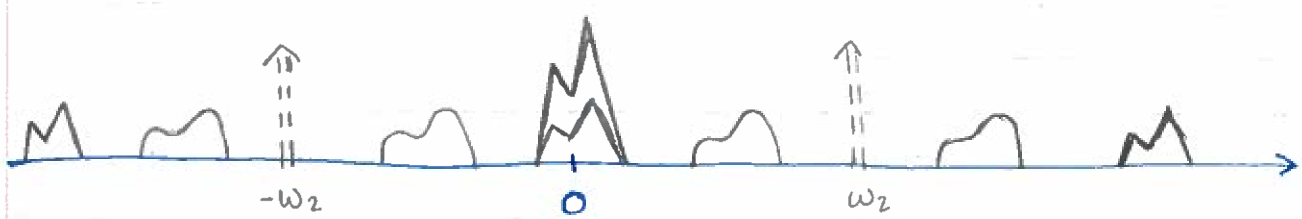
$F(y(t)\cos(\omega_1 t)) = Y(\omega) * (\delta(\omega-\omega_1) + \delta(\omega+\omega_1))$

This equates to a graph of $Y(\omega)$ centered at $-\omega_1$ and another at $+\omega_1$. We thus get:

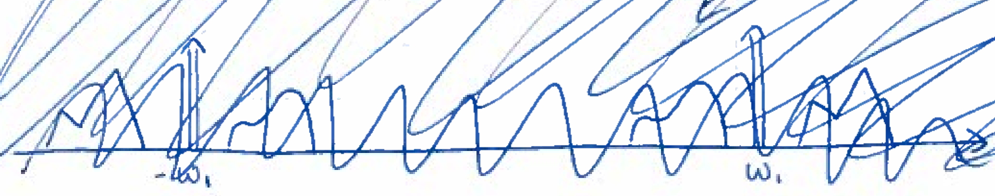


At $\omega=0$, 2 instances of $X_1(\omega)$ coincide, so they add

$F(y(t)\cos(\omega_2 t)) = Y(\omega) * (\delta(\omega-\omega_2) + \delta(\omega+\omega_2))$



b) $\cancel{y(t)\cos(\omega_1 t) = Y(\omega) * (\delta(\omega-\omega_1) + \delta(\omega+\omega_1))}$



c) If we look at $Y(\omega)$ [$y(t)$ in the frequency domain], we can use a band-pass filter from $\omega_1 - \omega_M$ to $\omega_1 + \omega_M$ to grab $X_1(\omega)$, and a filter from $\omega_2 - \omega_M$ to $\omega_2 + \omega_M$ to grab $X_2(\omega)$. Then we can use an IFT to get $x_1(t)$ and $x_2(t)$

3.

a) $V_{in} = V_R + V_L + V_{out}$

$= V_R + L\frac{d}{dt}i(t) + V_{out}$

$= R\,i(t) + L\frac{d}{dt}i(t) + V_{out}$

$= RC\frac{d}{dt}V_{out} + L\frac{d}{dt}\left(C\frac{d}{dt}V_{out}\right) + V_{out}$

$= RC\frac{d}{dt}V_{out} + LC\frac{d^2}{dt^2}V_{out} + V_{out}$

b) Fourier Transform!

$V_{in}(\omega) = RCj\omega V_{out}(\omega) + LCj^2\omega^2 V_{out}(\omega) + V_{out}(\omega)$

$= V_{out}(\omega) \cdot (RCj\omega + LCj^2\omega^2 + 1)$

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{1}{RCj\omega + LCj^2\omega^2 + 1} = \frac{1}{RCj\omega - LC\omega^2 + 1}$$

c) $$|H(\omega)| = \frac{1}{\sqrt{(RCj\omega)^2 + (-LC\omega^2+1)^2}} = \frac{1}{\sqrt{(RC\omega)^2 + (LC\omega^2)^2 - 2(LC\omega^2) + 1}}$$

$$= \frac{1}{\sqrt{R^2C^2\omega^2 + L^2C^2\omega^4 - 2LC\omega^2 + 1}}$$

d) Maximize $H(\omega) \Rightarrow H'(\omega) = 0$

$H'(\omega) = ?$

Let $-R^2C^2\omega^2 + L^2C^2\omega^4 - 2LC\omega^2 + 1 = x$

$H(\omega) = x^{-1/2} \Rightarrow H'(\omega) = \frac{-1}{2} x^{-3/2} \cdot x'$

$= \frac{1}{2} x^{-3/2} \cdot -2R^2C^2\omega + 4L^2C^2\omega^3 - 4LC\omega = 0$

$= \frac{1}{2x^{3/2}} \cdot (2R^2C^2\omega - 4L^2C^2\omega^3 + 4LC\omega) = 0$

$\frac{1}{2x^{3/2}} \neq 0$, so for $H'(\omega) = 0$, we have

$2R^2C^2\cancel{\omega} - 4L^2C^2\omega^{\cancel{3}\,2} + 4LC\cancel{\omega} = 0$

$2R^2C^2 - 4L^2C^2\omega^2 + 4LC = 0$

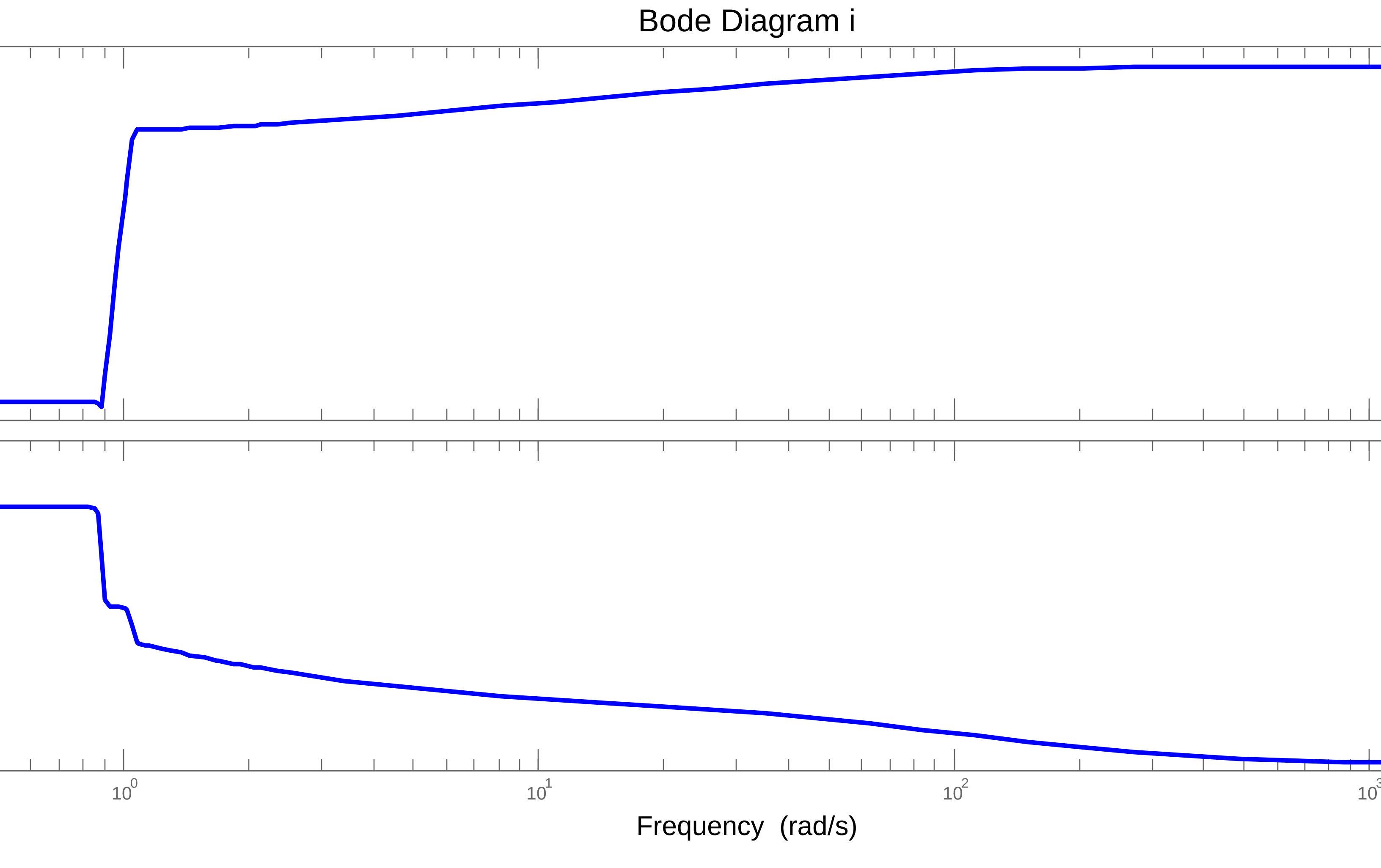
$-4L^2C^2\omega^2 = -4LC - 2R^2C^2$

$\omega^2 = \frac{4L\cancel{C} + 2R^2\cancel{C}}{4L^2C^{\cancel{2}}} = \frac{2L + \cancel{2}RC}{2L^2C}$

$\omega^2 = \pm\sqrt{\frac{2L + RC}{2L^2C}}$

Because in $|H(\omega)|$ all ω's are squared, ~~the~~

$\omega_{max} = \sqrt{\frac{2L + RC}{2L^2C}}$



Bode Diagram i
10^0
10^1
10^2
10^3
Frequency (rad/s)

Bode Diagram ii