CS 161 HW/Assignment 5

U = union = or (V is or)

1. (a) • $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$ • $P \Leftrightarrow \neg Q$, $((P \overset{\text{AND}}{\wedge} \neg Q) \overset{\text{OR}}{\vee} (\neg P \wedge Q))$

P implies NOT Q, Q implies NOT P P iff not Q, ((P and NOT Q) OR (NOT P and Q))

worlds	P	Q	$\neg P$	$\neg Q$	$\Delta_1 = P \Rightarrow \neg Q$	Δ_2: $Q \Rightarrow \neg P$
w_1	T	T	F	F	F	F
w_2	T	F	F	T	T	T
w_3	F	T	T	F	T	T
w_4	F	F	T	T	T	T

$M(P \Rightarrow \neg Q) = M(\neg Q \Rightarrow \neg P)$
OR $M(\Delta_1) = M(\Delta_2)$
$= \{w_2, w_3, w_4\}$

(b) $\underbrace{P \Leftrightarrow \neg Q}_{\Delta_3}$, $((P \wedge \neg Q) \vee (\neg P \wedge Q)) = \Delta 4$

worlds	P	Q	Δ_3: $P \Leftrightarrow \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
w_1	T	T	F	F
w_2	T	F	T	T
w_3	F	T	T	T
w_4	F	F	F	F

Sub-table

$P \wedge \neg Q$	$\neg P \wedge Q$	$\Delta 4$
F	F	F
T	F	T
F	T	T
F	F	F

$\boxed{M(\Delta_3) = M(\Delta_4) = \{w_2, w_3\}}$

2. (a) (Smoke ⇒ Fire) (Smoke ⇒ Fire)
1 2

$M((S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)) = \boxed{\{w_1, w_3, w_4\}}$

Smoke	Fire	¬Smoke	¬Fire	Smoke ⇒ Fire	¬Smoke ⇒ ¬Fire	1 ⇒ 2
T	T	F	F	F	T	T
F	T	T	F	T	F	F
T	F	F	T	T	T	T
F	F	T	T	T	T	T

3/4 worlds are satisfied by this sentence, so its

NEITHER

Satisfiability: compound prop is SAT if ∃ @ least 1 TRUE result, Valid: compound prop which is always true
Unsatisfiability: NOT even 1 "true" in the truth table

2. (b) (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

w	S	F	H	S ∨ H	② S ⇒ F	① (S∨H) ⇒ F	(2 ⇒ 1)
w_1	T	T	T	T	T	T	T
w_2	T	T	F	T	T	T	T
w_3	T	F	T	T	F	F	T
w_4	T	F	F	T	F	F	T
w_5	F	T	T	T	T	T	T
w_6	F	T	F	F	T	T	T
w_7	F	F	T	T	T	F	F
w_8	F	F	F	F	T	T	T

$\boxed{\text{Neither}}$

(c) ((S ∧ H) ⇒ F) ⇔ ((S ⇒ F) ∨ (H ⇒ F))

AND

OR

	S ∧ H	① (S∧H) ⇒ F	S ⇒ F	H ⇒ F	② (S ⇒ F) ∨ (H ⇒ F)	(1 ⇔ 2)
w_1	T	T	T	T	T	T
w_2	F	T	T	T	T	T
w_3	T	F	F	F	F	T
w_4	F	T	F	T	T	T
w_5	F	T	T	T	T	T
w_6	F	T	T	T	T	T
w_7	F	T	T	F	T	T
w_8	F	T	T	T	T	T

Order of Precedence

1) ¬
2) ∧
3) ∨
4) ⇒
5) ⇔

ALL TRUE ⇒ $\boxed{\text{valid}}$

3. (a) (Unicorn is mythical) ⇒ (Immortal)

a b

¬(" " ") ⇒ (mortal mammal)

(Unicorn is (immortal OR Mammal)) ⇒ (horned)

(Horned) ⇒ (Magical)

Mythical, Immortal, Mammal, Horned, Magical
(Mortal)

ANSWER

(1) Mythical ⇒ Immortal

(2) ¬Mythical ⇒ ¬Immortal ∧ Mammal

(3) (Immortal) ∨ Mammal ⇒ horned

(4) horned ⇒ magical

3. (b) CNF = conjunctive normal form = conjunction of clauses where a clause is a disjunction of literals

KB: Knowledge Base

$(\text{Mythical} \Rightarrow \text{Immortal})$

$\neg\text{Mythical} \Rightarrow \neg\text{Immortal} \wedge \text{Mammal}$

$(\text{Immortal} \vee \text{Mammal}) \Rightarrow \text{horned}$

$\text{horned} \Rightarrow \text{magical}$

1. eliminate $\Leftrightarrow$
2. eliminate $\Rightarrow$
3. move $\neg$ inward. $\neg\neg x$, $\neg(x \wedge y)$, $\neg(x \vee y)$
4. distribute $\vee$: $x \vee (y \wedge z)$

Mythical	= My
Mammal	= Mam
Immortal	= I
horned	= h
Magical	= mag

Key

$a \Leftrightarrow b \Rightarrow (a \Rightarrow b) \wedge (b \Rightarrow a)$

* $\boxed{a \Rightarrow b \Rightarrow \neg a \vee b}$ *

$(I \vee Mam) \Rightarrow (h) \Rightarrow \neg(I \vee Mam) \vee (h)$

$H \Rightarrow M$

KB → CNF:

$My \Rightarrow I \Rightarrow \neg My \vee I$

$My \vee (\neg I \wedge Mam)$

* $\boxed{a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)}$ *

$\neg(I \vee Mam) \vee H \Rightarrow (\neg I \wedge \neg Mam) \vee H$

$\neg H \vee Mag$

$\neg My \vee I$

$(My \vee \neg I) \wedge (My \vee Ma)$

$\neg I \vee H$

$\neg Mam \vee H$

$\neg H \vee Mag$

$\Rightarrow$

$(\neg My \vee I)$
$\wedge (My \vee \neg I)$
$\wedge (My \vee Ma)$ ← CNF
$\wedge (\neg I \vee H)$
$\wedge (\neg Mam \vee H)$
$\wedge (\neg H \vee Mag.)$

ANSWER

3. (c) Use that KB to prove Unicorn is mythical? magical? horned? (Use Resolution!)

① $(\neg My \vee I)$
② $\wedge (My \vee \neg I$
③ $\wedge (My \vee Mam)$
④ $\wedge (\neg I \vee H)$
⑤ $\wedge (\neg Mam \vee H)$
⑥ $\wedge (\neg H \vee Mag.)$

Mam, My, I, H, Mag

Δ imply My $\Rightarrow \Delta \wedge \neg My$ unsat
Δ imply Mag $\Rightarrow \Delta \wedge \neg Mag$ unsat
Δ imply H $\Rightarrow \Delta \wedge \neg H$ unsat

"NOT" in logic $= \neg$

Does Δ imply α? equivalent to Is $\Delta \wedge \neg\alpha$ a contradiction?

Δ imply $\alpha \Rightarrow \Delta \wedge \neg\alpha$ unsatisfiable?

Proving the Unicorn is mythical:

we want to prove that $\Delta \wedge \neg My$ is unsatisfiable?

something cancels out the other (H, ¬H)

① $\neg My \vee I$	
② $My \vee \neg I$	
③ $My \vee Mam$	
④ $\neg I \vee H$	
⑤ $\neg Mam \vee H$	
⑥ $\neg H \vee Mag.$	
⑦ $\neg My$	
⑧ Mam.	(3,7)
⑨ H	(5,8)
⑩ Mag.	(6,9)
⑪ I	(2,7)

⇓ SAT

Magical:

① $\neg My \vee I$	
② $My \vee \neg I$	
③ $My \vee Mam$	
④ $\neg I \vee H$	
⑤ $\neg Mam \vee H$	
⑥ $\neg H \vee Mag.$	
⑦ $\neg Mag$	
⑧ $\neg H$	(6,7)
⑨ $\neg I$	(4,8)
⑩ $\neg My$	(1,9)
⑪ $\neg Mam$	(5,8)
⑫ My	(3,11)
⑬ ~~10,~~	(10,12)

⇓ UNSAT

Horned

① $\neg My \vee I$	
② $My \vee \neg I$	
③ $My \vee Mam$	
④ $\neg I \vee H$	
⑤ $\neg Mam \vee H$	
⑥ $\neg H \vee Mag.$	
⑦ $\neg$Horned	
⑧ $\neg Mam$	(5,7)
⑨ My	(3,8)
⑩ I	(1,9)
⑪ H	(4,10)
⑫	(7,11)

⇓ Unsatisfiable

ANSWER ⇓

Not Mythical

Can Prove Magical + Horned

4.) two NNF circuits (Negation Normal Form): decomposable, deterministic, or smooth, and why?

If the sub-circuits of all <u>AND</u>-gates in a circuit share no variables: <u>decomposable</u>

If all sub-circuits of an <u>OR-gate</u> share all variables: <u>Smooth</u>

If inputs to an <u>or-gate</u> are mutually exclusive: <u>deterministic</u>

<u>Figure 1</u>:

decomposable? yes ⇒ $\boxed{\text{DNNF}}$ $\boxed{\text{Decomposable}}$ no shared variables

deterministic? $(\neg A \wedge B) \wedge (\neg B \wedge A)$ $\boxed{\text{NO}}$ all variables NOT shared

~~$(C) \wedge (\neg D \wedge \neg C)$~~

Smoothness? (A, B) (A, B) ✓ $\boxed{\text{NO}}$ (NOT unsatisfiable)
(C) (C, D) ×

<u>Example in Lecture 10</u>

$$\begin{matrix} & \text{or} & \\ \alpha & & B \end{matrix}$$

$$\left.\begin{aligned} \alpha &= \neg A \wedge B \\ B &= \neg B \wedge A \end{aligned}\right\} \alpha \wedge B \text{ are } \underline{\text{unsat}}.$$

If <u>BOTH</u> d-DNNF, then decomposability + determinism.

<u>Figure 2</u>:

decomposable? Yes ⇒ $\boxed{\text{DNNF}}$ No shared variables $\boxed{\text{DECOMPOSABLE + SMOOTH}}$

deterministic? $(\neg A \wedge B) \wedge (\neg A \wedge B)$ $\boxed{\text{NO}}$ all variables NOT shared

smoothness? $\boxed{\text{Yes}}$ mutually exclusive ✓ (unsatisfiable)

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5. a) $\text{Weight}(\neg A \wedge B) \vee (\neg B \wedge A) = 0.9 \times 0.3 + 0.1 \times 0.7 = \boxed{0.34}$

b) Root count is 0.34 ⇒ They match! (we can find the WMC in linear time)

c) $[(w(\neg A))(w(B)) + (w(A))(w(\neg B))] \cdot [w(C))(w(D)) + (w(\neg C))(w(\neg D))]$

$+ [w(\neg A)w(\neg B) + w(A)w(B)] \cdot w(C)w(\neg D) + w(\neg C)w(D)]]$

$= [(0.9)(0.3) + (0.1)(0.7)] \cdot [(0.5)(0.7) + (0.5)(0.3)] + [(0.9)(0.7) + (0.1)(0.3)] \cdot [(0.5)(0.3) +$
$[0.5(0.7)] = 1 \cdot [\ ] \cdot [0.17 + 0.33] = \boxed{0.5}$