Problem 1

What we want to prove: $Pr(\alpha 1, \dots ,\alpha n \mid \beta) =$ [$\prod$(from i = 1 to n) $Pr(\alpha i \mid \alpha i+1, \dots ,\alpha n,\beta)$.
Therefore, we need to use n to do the induction proof:

Base Case: for n = 0 and n = 1, this holds true trivially

for n = 2, Bayes' conditioning rule tells that $Pr(\alpha 1,\alpha 2 \mid \beta) = P\, r(\alpha 1|\alpha 2, \beta)P\, r(\alpha 2|\beta)$

Inductive Step: We make an assumption that $Pr(\alpha 1, \dots ,\alpha n \mid \beta) =$ [$\prod$(from i = 1, to n) $Pr(\alpha i \mid \alpha i+1, \dots ,\alpha n,\beta)$. By the Bayes' conditioning rule, we get the following:

$Pr(\alpha 1, \dots ,\alpha n+1|\beta)$

$=Pr(\alpha n+1|\alpha 1, \dots , \alpha n, \beta) * Pr(\alpha 1|\alpha 2, ..., \alpha n, \beta)...Pr(\alpha n|\beta)$

$=(Pr(\alpha n+1,\alpha 1|\alpha 2,... ,\alpha n,\beta)/Pr(\alpha 1|\alpha 2, ..., \alpha n, \beta)) * (Pr(\alpha 1|\alpha 2, \dots , \alpha n, \beta) \dots Pr(\alpha n|\beta))$

$=((Pr(\alpha 1|\alpha 2, \dots ,\alpha n+1,\beta) * Pr(\alpha n+1|\alpha 2, \dots ,\alpha n,\beta))/(Pr(\alpha 1|\alpha 2, \dots , \alpha n, \beta))) * (Pr(\alpha 1|\alpha 2, \dots , \alpha n, \beta) \dots Pr(\alpha n|\beta))$

$=Pr(\alpha 1|\alpha 2, \dots , \alpha n+1, \beta) * Pr(\alpha n+1|\alpha 2, \dots , \alpha n, \beta) * (Pr(\alpha 2|\alpha 3, \dots , \alpha n, \beta) \dots P\, r(\alpha 1, \dots , \alpha n|\beta))$

$=Pr(\alpha 1|\alpha 2, ..., \alpha n, \beta) * Pr(\alpha 2|\alpha 3, ..., \alpha n, \beta) * Pr(\alpha n+1|\alpha 3, ..., \alpha n, \beta) * (Pr(\alpha 3|\alpha 4, \dots , \alpha n, \beta) \dots Pr(\alpha n|\beta))$

All of the above can be simplified to something like this:

$P\, r(\alpha 1, \dots , \alpha n+1|\beta) = (Pr(\alpha n+1|\beta) / Pr(\alpha n|\beta)) * (P\, r(\alpha 1|\alpha 2, \dots , \alpha n+1, \beta) \dots P\, r(\alpha n-1|\alpha n, \beta)) * Pr(\alpha n|\beta)$

$= Pr(\alpha 1|\alpha 2, \dots , \alpha n+1, \beta) \dots Pr(\alpha n+1|\beta)$

$= \prod$(from i = 1 to n+1) $Pr(\alpha i|\alpha i+1, \dots , \alpha n+1, \beta)$

Problem 2

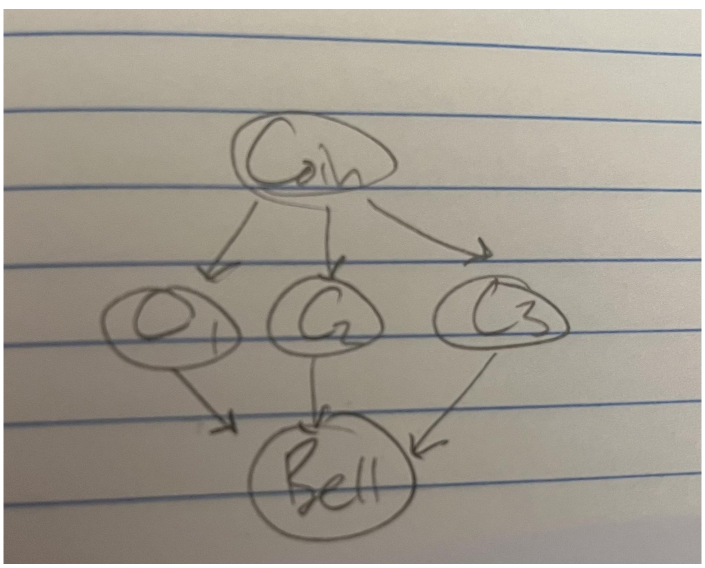
O = oil
N = natural gas
P = positive geographic test

$Pr(o|t) = (Pr(o)/Pr(t)) / Pr(t|o)$
$= (Pr(o)\ Pr(t|o)) / (Pr(t|o)Pr(o) + Pr(t|g)Pr(g) + Pr(t|\bar{o}, \bar{g})P(\bar{o}, \bar{g}))$
$= ((.5)(.9)) / ((.9)(.5) + (.3)(.2) + (.1)(.3)) =$ **.833**

Problem 3

C_n is what we will use for a heads on the coin flip's nth time.

<u>Bayesian Network</u>:



Conditional Probability Tables:

h = heads, t = tails

C	P(C)
a	1/3
b	1/3
c	1/3

C	C_1	$P(C_1 \mid C)$
a	h	.2
a	t	.8
b	h	.4
b	t	.6
c	h	.8
c	t	.2

C	C_2	$P(C_2 \mid C)$
a	h	.2
a	t	.8
b	h	.4
b	t	.6
c	h	.8
c	t	.2

C	C_3	$P(C_3 \mid C)$
a	h	.2
a	t	.8
b	h	.4
b	t	.6
c	h	.8
c	t	.2

C_1	C_2	C_3	Bell	$P(B \mid C_1, C_2, C_3)$
h	h	h	on	1
h	h	h	off	0
h	h	t	on	0
h	h	t	off	1
h	t	h	on	0
h	t	h	off	1
h	t	t	on	0
h	t	t	off	1
t	h	h	on	0
t	h	h	off	1
t	h	t	on	0
t	h	t	off	1
t	t	h	on	0
t	t	h	off	1
t	t	t	on	1
t	t	t	off	0

Problem 4

a. **I(A, φ, BE), I(B, φ, AC), I(C, A, BDE), I(D, AB, CE), I(E, B, ACDFG), I(F, CD, ABEH), I(G, F, ABCDEH), I(H, EF, ABCDG)**
b. d separated(A, F, E) → blocked(ADB)? not → blocked(DBE) not => **False**
   d separated(G, B, E) → blocked(GFH)? not → blocked(FHE)? not => **False**
   d separated(AB, CDE, GH) → blocked(ACF)? yes, blocked(ADF)? yes → blocked(BDF)? yes → blocked(BEH)? yes => **True**
c. Pr(a,b,c,d,e,f,g,h) = **Pr(a) * Pr(b) * Pr(c|a) * Pr(d|a,b) * Pr(e|b) * Pr(f|c,d) * Pr(h|f,e) * Pr(g|f)**
d. For the markovian assumption, I(A, φ, BE):

   Pr(A = 1, B = 1) = Pr(A = 1) * Pr(B = 1) = (0.2)(0.7) = 0.14

   AND: Using I(A, φ, BE): Pr(E =0|A=0) = Pr(E =0)
   = Pr(E = 0|B = 1) * Pr(B=1) + Pr(E=0|B= 0) * Pr(B=0)
   = (.9)(0.7) + (.1)(.3) = **.66**

Problem 5

a. M(α) = **w0, w2, w3**
b. Pr(α) = Pr(A = 0, B = 1) = 1 − Pr(w1) = **0.8**
c. Pr (A,B|α) =

World	P(World)
w0	.375
w1	0
w2	.125
w3	.5

d. Pr(A => ¬B)
   = (Pr((A => ¬B) ∧ (A =⇒ B))) /(Pr(α))
   = (Pr((¬A ∨ ¬B) ∧ (¬A ∨ B))) / (Pr(α))
   = Pr(¬A)/Pr(α)
   = .5/.8 = **.625**