Assistant Professorship of Modeling Spacial Mobility
TUM Department of Civil, Geo and Environmental Engineering
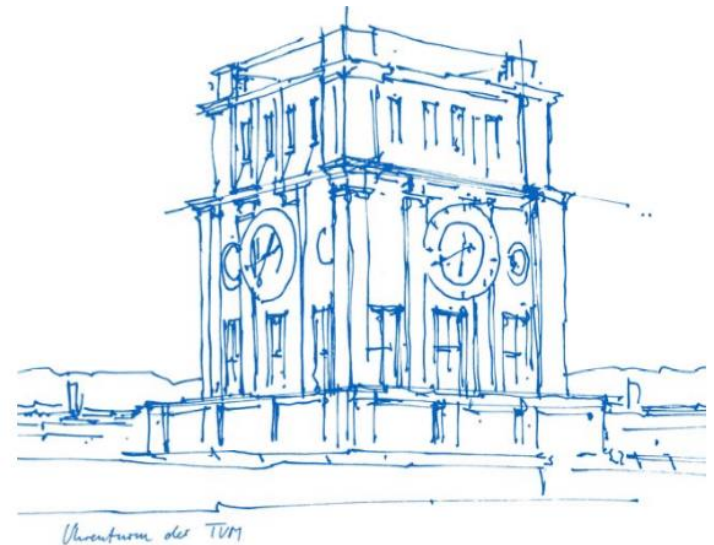Technical University of Munich

Mode Choice in R

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Based on material from Hema Sharanya Rayaprolu

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Discrete Choice Modeling

- Modeling individual choice behaviour econometrically using the principle of **utility maximization**.

- The utility of an alternative is expressed as a function of the alternative's attributes.

$$V_{in} = \beta_{1i}X_{1in} + \beta_{2i}X_{2in} + \cdots + \beta_{ki}X_{kin}$$

where, V_{in} is the (deterministic) utility of individual n choosing alternative i;

$X_{1in}, X_{2in}, \dots, X_{kin}$ are k attributes (independent variables); and

$\beta_{1i}, \beta_{2i}, \dots, \beta_{ki}$ are unknown parameters (to be estimated).

- Assumption: Individuals behave rationally, possess complete information about all alternatives, and are faced with a mutually exclusive and collectively exhaustive choice set.

Discrete Choice Modeling

- **Random utility theory**: To account for model imperfections.

$$U_{in} = V_{in} + \varepsilon_{in}$$

where, U_{in} is the total utility of individual n choosing alternative i;

V_{in} is the deterministic component; and

ε_{in} is the random component.

- ε_{in} is a random variable that caters to the unobserved taste variations among individuals and other observational errors.

Discrete Choice Modeling

- **Probabilistic choice theory**: To accounts for behavioral inconsistencies.

- Probability that an alternative is chosen is now the probability that it has the greatest utility among all the available alternatives.

- Considering a choice set containing two alternatives i and j, the probability that the individual n chooses the alternative i is –

$$P_{in} = Pr \ (\ U_{jn} \leq U_{in}\ )$$

$$P_{in} = Pr \ \{(\varepsilon_{jn} - \varepsilon_{in}) \leq (V_{in} - V_{jn})\}$$

$$P_{in} = Pr \ \{\varepsilon_n \leq \boldsymbol{\beta}'(x_{in} - x_{jn})\}$$

- To solve this, different assumptions on the distribution of the random variable ε_n have been made – e.g. linear, probit, logit,…

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Discrete Choice Modeling

- **Logit models**: Assume a logistic (Gumbel) distribution of the random variable.

$$F(\varepsilon_n) = \frac{1}{1 + e^{-\mu \varepsilon_n}}$$

  where, μ is a positive scale parameter.

- Under this assumption, the probability of individual n choosing the alternative i is –

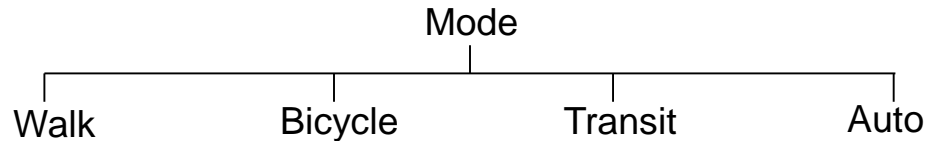$$P_{in} = \frac{1}{1 + e^{-\mu \boldsymbol{\beta}'(x_{in} - x_{jn})}}$$

Assuming $\mu = 1$,
$$P_{in} = \frac{e^{\boldsymbol{\beta}' x_{in}}}{e^{\boldsymbol{\beta}' x_{in}} + e^{\boldsymbol{\beta}' x_{jn}}}$$

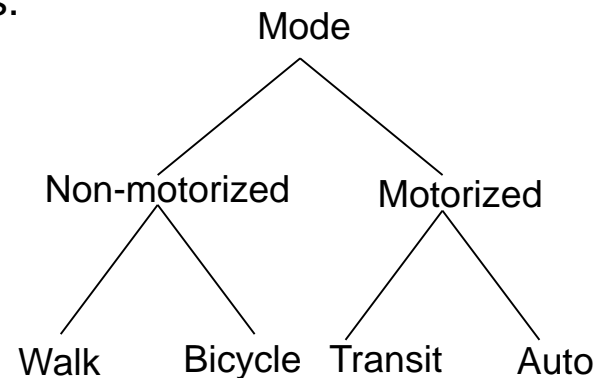- Extending the above to multiple (j) alternatives,

$$P_{in} = \frac{\exp(V_{in})}{\Sigma_j \exp(V_{jn})}$$

Discrete Choice Modeling

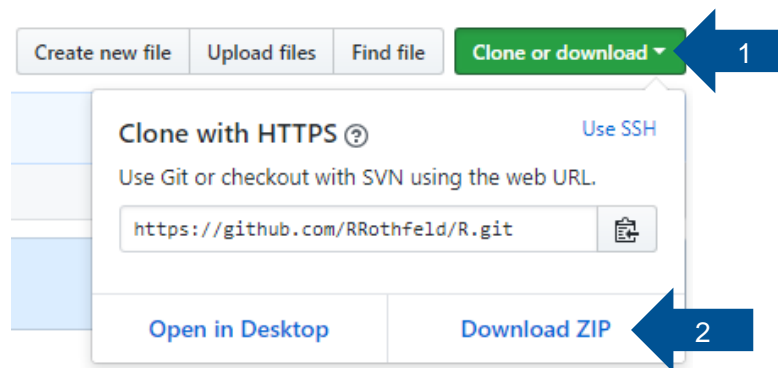- **Multinomial logit (MNL) models**: Assume all alternatives are mutually exclusive (and exhaustive.

```
                          Mode
      ┌──────────────┬─────┴──────┬──────────────┐
    Walk          Bicycle       Transit         Auto
```

- **Nested logit models**: Allow for correlation between alternatives.
- Group correlated alternatives into separate nests.

```
                              Mode
                            ╱      ╲
               Non-motorized        Motorized
                  ╱      ╲            ╱      ╲
               Walk    Bicycle    Transit    Auto
```

Material for Today's Class

https://github.com/RRothfeld/R

Estimation in R

- Using the *mlogit* **package** (though, there are alternatives)

- Model specification:

  - Choice alternatives (modes)

  - Independent variables

  - Data formats: wide (one row for each choice observation) or long (one row for each alternative of each observation)

  - Formula:

  > Choice ~ Part 1 | Part 2 | Part 3

  Part 1: Alternative-specific variables

  Part 2: Individual-specific variables

  Part 3: Alternative-specific variables with coefficients varying across alternatives

References

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Ortúzar, J. d. D., & Willumsen, L. G. (2011). *Modelling Transport* (Fourth Edition). Chichester, West Sussex, United Kingdom: John Wiley & Sons, Ltd.

Viton, P. A. (2015). *Discrete-Choice Logit Models with R*. Retrieved from http://facweb.knowlton.ohio-state.edu/pviton/courses2/crp5700/5700-mlogit.pdf