Matrices

$\underline{A} = [a_{ij}]$ $\qquad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ $\qquad \boxed{a_{\text{row, column}}}$

$\underline{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad a_{ij} = \delta_{ij}$

$\underline{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad a_{ij} = 0$

Some properties

$\underline{A} + \underline{B} = \underline{B} + \underline{A}$ Commute.

$(\underline{A} + \underline{B}) + \underline{C} = \underline{A} + (\underline{B} + \underline{C})$ Assoc.

$\lambda(\underline{A} + \underline{B}) = \lambda \underline{A} + \lambda \underline{B}$ Dist.

etc...

Multiplication $\qquad c_{ij} = \sum_{k=1}^{m} a_{ik} \, b_{kj}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 8 \\ 2 & 9 \\ 3 & 10 \end{pmatrix} = \begin{pmatrix} 1+4+9 & 8+18+30 \\ 3+8+15 & 24+36+50 \end{pmatrix} = \begin{pmatrix} 10 & 56 \\ 26 & 110 \end{pmatrix}$$

Note that $\underline{A} \cdot \underline{B} \neq \underline{B} \cdot \underline{A}$ in general.

Not ~~asso~~ commut.

Square matrix $\qquad$ # rows = # column.

Diagonal $\qquad \begin{pmatrix} a_{ii} & & 0 \\ & \ddots & \\ 0 & & \end{pmatrix}$ upper triangular

lower triangular

$(\underline{A}\,\underline{B})\underline{C} = \underline{A}(\underline{B}\underline{C})$ Assoc.

$\underline{A}(\underline{B} + \underline{C}) = \underline{A}\underline{B} + \underline{A}\underline{C}$ Dist

$\lambda(\underline{A}\underline{B}) = (\lambda\underline{A})\underline{B} = \underline{A}(\lambda\underline{B})$

Inverse

given $\underline{A}$ then inverse A^{-1}

gives $\underline{A}\underline{A}^{-1} = \underline{A}^{-1}\underline{A} = \underline{I}$

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad A^{-1} = \frac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix} \qquad A \cdot A^{-1} = I$

Is useful for solving system of equations

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\underline{A}\,\underline{x} = \underline{B}$$

$$\underline{A}^{-1}\underline{A}\,\underline{x} = \underline{A}^{-1}\underline{B}.$$

$$x = \tfrac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} -8+2 \\ 6-1 \end{pmatrix} = \begin{pmatrix} -3 \\ 5/2 \end{pmatrix}$$

Determinant

M_{ij} = Minor = det of A with i + j rows + column removed.

$$\det A = \sum_{i=1}^{n} a_{ij} M_{ij} (-1)^{i+j} \quad \text{any } j$$

$$= \sum_{j=1}^{n} a_{ij} M_{ij} (-1)^{i+j} \quad \text{any } i$$

Properties of determinants.

Any row or column $= 0$ $\Rightarrow \det A = 0$

Switch rows or colums $\Rightarrow \det \hat{A} = -\det A$

Two rows or colums are $=$ $\Rightarrow \det A = 0$

A row or column is a multiple of another then $\Rightarrow \det A = 0$

A row or column is a linear combination of others $\Rightarrow \det A = 0$

$\det(AB) = \det A \cdot \det B$

$\det(A^T) = \det A.$

$\det A = \prod a_{ii}$ if A is diagonal or triangular.

Equivalent Statements

$Ax=0$ has unique solution $x=0$

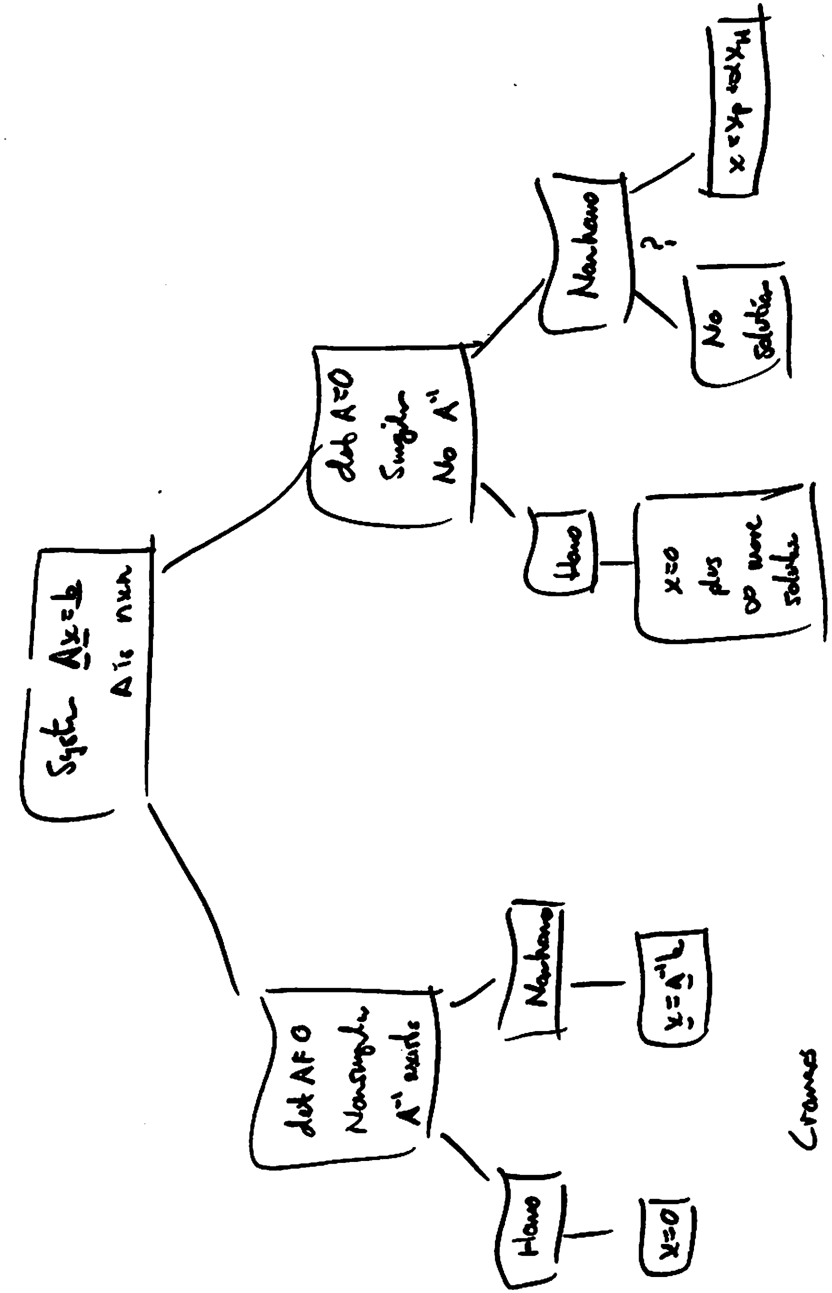
$Ax=b$ has unique solution for $\underline{b}$

A is nonsingular

A^{-1} exists

$\det A \neq 0$

Can use Gauss elimination to find A^{-1}
" Cramers method.



Cramers
Gauss-Jordan
Cofactors.

HW

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \end{array}\right) \rightarrow \frac{1}{9}\begin{pmatrix} -2 & 5 & -1 \\ 4 & -1 & 2 \\ -3 & 3 & 3 \end{pmatrix} = A^{-1}$$

Use Gauss Elimt

END