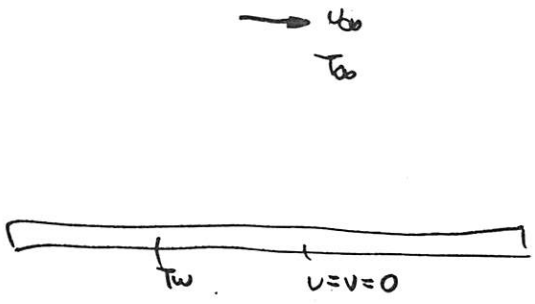
Cooling of Flat Plate



$\bar{V} = u(x,y)\hat{i} + v(x,y)\hat{j}$

$T = T(x,y)$

$u(0,y) = u_\infty$ } leading edge $\eta=\infty$
$T(0,y) = T_\infty$ }

$u(x,0) = 0$
$v(x,0) = 0$ } at wall $\eta=0$
$T(x,0) = T_w$

$T(x,\infty) = T_\infty$ } far field $\eta=\infty$
$u(x,\infty) = u_\infty$ }

Mass cons. $u_x + v_y = 0$

U Mom. cons. $uu_x + vu_y = \nu u_{yy}$

Energy Cons. $uT_x + vT_y = \alpha T_{yy}$

} So called Boundary Layer Equations.

Use the transform $u = u_\infty F'(\eta)$

$v = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}\left[\eta F'(\eta) - F(\eta)\right]$

where $\eta = \frac{y}{\sqrt{\nu x/u_\infty}}$

and for temperature $G(\eta, Pr) = \frac{T - T_\infty}{T_w - T_\infty}$

Pr = $= \nu/\alpha$

$= \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$

$\Rightarrow$ relative thickness of thermal + momentum B.L.s.

Equations become

$$F''' + \frac{1}{2}FF'' = 0$$

$$F(0) = 0$$
$$F'(0) = 0$$
$$F'(\infty) = 1$$

Independent of G!

$$G'' + \frac{Pr\,FG'}{2} = 0$$

$$G(0) = 1$$
$$G(\infty) = 0$$

$f(0) = 0$
$f'(0) = 0$
$f''(0) = 0.332057$

$x = 0$ to 10
$h = \Delta x = 0.1$

Boundary Value Problems.

Must use the technique of 'shooting'

For F problem use $F(0)=0$
$F'(0)=0$
$F''(0)=\text{guess}_1$

~~Adjust $F''(0)=\text{guess}$~~

Solve problem with guess_0 and guess_1. Interpolate to find guess_2 that will force $F'(\infty)\to 1$.
Iterate! By hand or False-Position

Once $F(\eta)$ is solved, then solve G problem.
Pick $G'(0)=\text{guess}_0$ & guess_1. Run the problem to iterate to $G'(0)$ that gives $G(\infty)=0$.

Use $F(0)=0$ $\quad G(0)=1$
$F'(0)=0$ $\quad G'(0)=.574689$
$F''(0)=.332057$

$Pr=5$

$k=0$ to 10 step $=0.1$

$\eta_m = 4.91$

$\eta_t = 2.75$

To use R-K

$F = y_1$
$F' = y_1' = y_2$
$F'' = y_2' = y_3$
$F''' = y_3' = -\frac{1}{2} F F''$

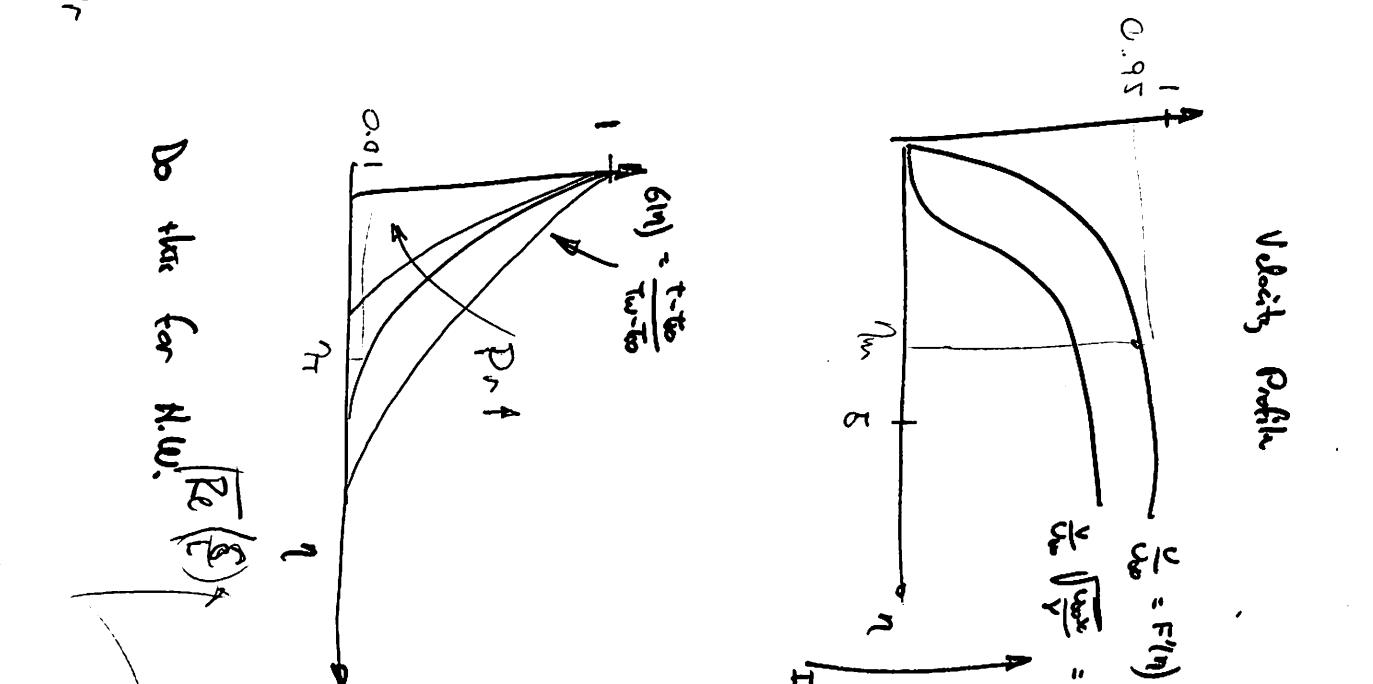
$G = y_4$
$G' = y_4' = y_5$
$G'' = y_5' = -\frac{1}{2} Pr\, F G'$

$y_1' = y_2$	$y_1(0) = 0$
$y_2' = y_3$	$y_2(0) = 0$
$y_3' = -\frac{1}{2} y_1 y_3$	$y_3(0) = \text{guess}$
$y_4' = y_5$	$y_4(0) = 1$
$y_5' = -\frac{1}{2} Pr\, y_1 y_5$	$y_5(0) = \text{guess}$

Run through 4th order R-K.

x	F	F'	F''	G	G'
0	0	0	3.32057-1	1.0	-5.76689-1
0.1	1.66029-3	3.32055-2	3.32048-1	9.42333-1	-5.76609-1
0.2	6.64101-3	6.64078-2	3.31984-1	8.84654-1	-5.76351-1
⋮					
9.8	8.0792	1	1.91607-8	-1.82232-16	-9.97409-32
9.9	8.1792	1	1.27627-8	-1.82232-16	-3.43710-32
10.0	8.2792	1	8.45887-9	-1.82232-16	-1.21701-32

(5)



(6 END)