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MID TERM EXAM

AE - 687 Aircraft Structural Analysis - II

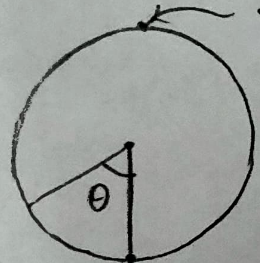
Problem 1: Design Case Study - Helicopter Loops

a) The load factor is not constant during the maneuver, as in the loop maneuver it goes under different phases,

the general load factor formula is,

$$n = \frac{V^2}{Rg} + \cos(\theta)$$

Top point (Pull down maneuver)



Bottom point (Pullup maneuver)

At the bottom point $\theta = 0^\circ$

$$n = \frac{V^2}{Rg} + 1$$

At the top point $\theta = \pi$ rad / 180°

$$n = \frac{V^2}{Rg} - 1$$

The maximum load factor is

$$\boxed{n_{max} = \frac{V^2}{Rg} + 1}$$

b) To calculate the tip deflection,

we will use Euler Bernoulli beam equation

$$EI \frac{d^4 w}{dx^2} = q(x) \qquad (\rho \rightarrow x)$$

→ we know, Shear force differentiation give Load Intensity

$$\frac{dQ}{dx} = q$$

$$Q = \int q \, dx$$

Boundary condition 1:

$Q = 0$ at $x = d/2$

At tip sheer force is zero.

~~Boundary~~ Next,

→ differentiation of moment gives sheer force.

$$\frac{dM}{dx} = Q$$

$$M = \int Q \, dx$$

Boundary condition 2:

$M = 0$ at $x = d/2$

At tip Moment is zero.

→ Differentiation of slope gives moment

$$\frac{dA}{dx} = \frac{d^2 w}{dx^2} = M$$

$$A = \frac{dw}{dx} = \int M \, dx$$

Boundary condition 3:

$$A = \frac{dw}{dx} = \frac{M_{hub}}{K} \quad \text{at} \quad x = 0$$

Here, we know,

$$M_{hub} = K\theta$$

Slope at root will be equal to the θ due to stiffness spring K.

→ Differentiation of deflection gives slope.

$$\frac{dw}{dx} = A$$

$$w = \int A \, dx$$

Boundary condition 4:

$$w = 0 \quad \text{at} \quad x = 0$$

As it is fixed at $x = 0$.

The tip deflection is inversely proportional to stiffness K, as it is increased from 10^4 N-m/rad to 10^6 N-m/rad the tip deflection decreases and when the stiffness reaches ∞ the slope $\frac{dw}{dx}$ at $x=0$ reaches 0. $\left(\frac{M_{hub}}{K}\right)$

c) Value considerations from Research

kappa = 1 N/m3

dia = 9.82 m

Chord = 0.27 m

thickness = 0.1 m

Omega = 41.81 rad/s

Young's modulus = 100×10^9 N/m^2

K = 10^6 N-m/rad

→ Assuming the tip deflection max admissible is 1 m (or it will hit fuselage)

→ I am getting the maximum velocity for the maneuver as 20.5837 m/s which is pretty real.

Now to calculate the variation of Shear stress and bending stress with theta,

$$n(\text{theta}) = \frac{V^2}{gR} + \cos(\text{theta})$$

Shear stress - variation = Shear stress * n (theta)

Bending stress - variation = Bending stress * n(theta)

Here we are multiplying shear stress and bending stress with n(theta) to ge

- a rough idea of how stress increases / changes with theta.

$V = 20.5837$

$R = 30 \longrightarrow$ example value.

$g = 9.8$

d) Minimum radius of R,

$$n = \frac{v^2}{gR} + \cos(\theta)$$

$$n - \cos\theta = \frac{v^2}{gR}$$

$$R = \frac{v^2}{g(n - \cos\theta)}$$

For R to be min $n - \cos\theta \rightarrow$ should be max

taking $n_{max} = 2.5$ (practical)

and $V = 20.5837$

$$R = \frac{(20.5837)^2}{9.8 \times 1.5}$$

$$\boxed{R = 64.85 \text{ m}}$$

Problem 1: Design case study- Helicopter Loops

```
clc ; clear;
syms kappa omega x C1 C2 C3 C4 Mhub K E I theta R d t real
syms V positive real
```

b) Method to calculate Tip-Deflection

Using Euler Bernaulli Beam equation.

```
q = kappa*(V+x*omega)^2;

% Integration of Load-Intensity gives Shear-Force
Q = int(q,x) + C1;
Q_bound = subs(Q,x,d/2) == 0;
C1_val = solve(Q_bound,C1);
Q_val = subs(Q,C1,C1_val);

% Integration of Shear-Frce gives Bending-Moment
M = int(Q_val,x) + C2;
M_bound = subs(M,x,d/2) == 0;
C2_val = solve(M_bound,C2);
M_val = subs(M,C2,C2_val);

% Integration of Bending-Moment gives Slope(theta)
A = int(M_val,x) + C3;
A_bound = subs(A,x,0) == Mhub/K;
C3_val = solve(A_bound,C3);
A_val = subs(A,C3,C3_val);

% Integration of Slope(theta) gives Deflection
w = int(A_val,x) + C4;
w_bound = subs(w,x,0) == 0;
C4_val = solve(w_bound,C4);
w_val = subs(w,C4,C4_val)
```

w_val =

$$\frac{V^2 \kappa x^4}{24} + \frac{\kappa \omega^2 x^6}{360} + \frac{\mathrm{Mhub}\, x}{K} + \frac{d^2 \kappa x^2 \,(24 V^2 + 16 V d \omega + 3 d^2 \omega^2)}{384} + \frac{V \kappa \omega x^5}{60} - \frac{d \kappa x^3 \,(12 V^2 + 6 V d \omega +}{144}$$

```
% Deflection at Tip
w_tip = simplify(subs(w_val,x,d/2))/(E*I)
```

w_tip =

$$\frac{d\,(90 K \kappa V^2 d^3 + 66 K \kappa V d^4 \omega + 13 K \kappa d^5 \omega^2 + 5760\, \mathrm{Mhub})}{11520\, \mathrm{E}\, \mathrm{I}\, K}$$

c) Sketch that explains how bending stress and shear stress varies at the root.

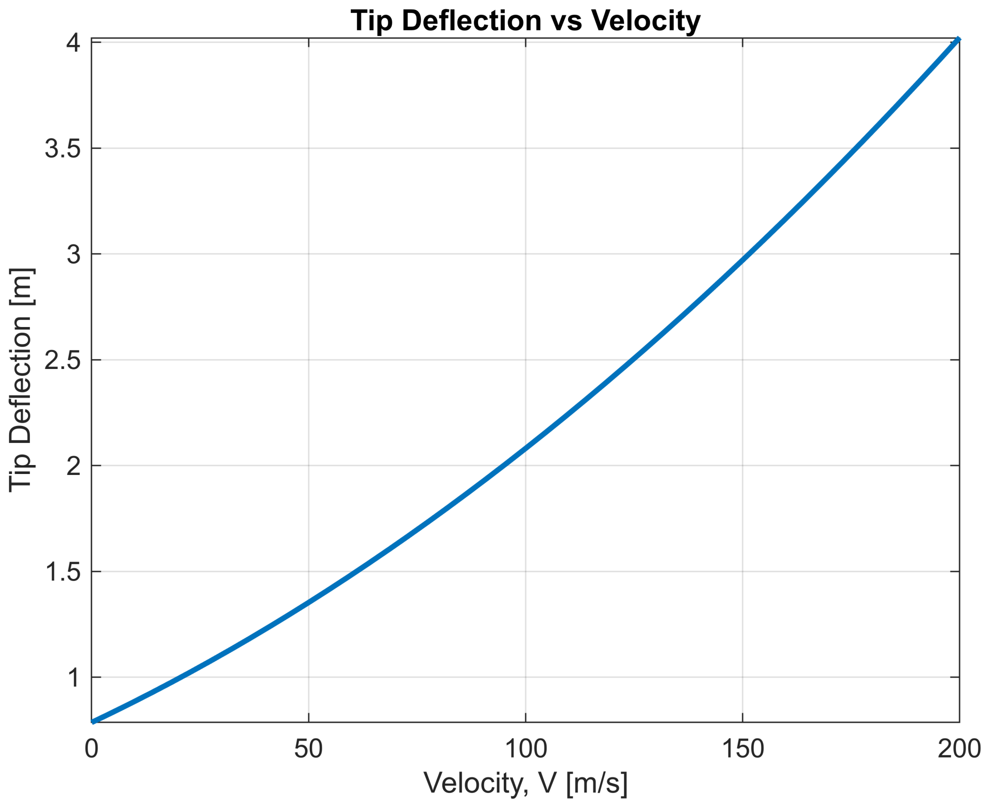
NOTE : Taking corresponding values to the different parameters according to the information available on the internet.

Omega (41.81 rad/s) is mostly kept constant in flight and just the pitch of the rotor blades are changed to vary the CT(coefficient of thrust).

```
% Values corresponding to MBB Bo 105 Helicopter
K_val = 10^6;                  % stiffness of the hub [N-m/rad]
d_val = 9.82;                  % Rotor diameter [m]
c_val = 0.27;                  % Blade chord [m]
t_val = 0.1;                   % Blade thickness [m]
omega_val = 41.81;             % Rotor speed [rad/s]
kappa_val = 1;                 % Lift distribution parameter [N/m^3]
E_val = 100 * 10^9;            % Young's modulus of the blade material [Pa = N/m^2]
Ar = c_val * t_val;            % Cross-sectional area of the blade [m^2]
I_val = c_val * t_val^3 / 12; % Mmoment of inertia of the blade cross-section [m^4]

%Moment generated by Stiffness
Mhub_val = subs(int(q*x,x,0,d/2),[d,K,omega,kappa],
[d_val,K_val,omega_val,kappa_val]);

%Tip deflection variation with V
tip_deflection = simplify(subs(w_tip,[kappa,K,d,omega,E,I,Mhub],
[kappa_val,K_val,d_val,omega_val,E_val,I_val,Mhub_val]));
figure;
fplot(tip_deflection,[0,200], 'LineWidth', 2);
title('Tip Deflection vs Velocity');
xlabel('Velocity, V [m/s]');
ylabel('Tip Deflection [m]');
grid on;
```



I am assuming the tip deflection should not surpass 1m, the corresponding Velocity would be the permissible maneuver speed.

```
eq1 = tip_deflection == 1;
double(solve(eq1,V))
```

```
ans = 20.5837
```

The permissible maneuver speed V = 20.5837 m/s.

```
%Load factor
V_val = 20.5837
```

```
V_val = 20.5837
```

```
R_val = 30
```

```
R_val = 30
```

```
g = 9.8
```

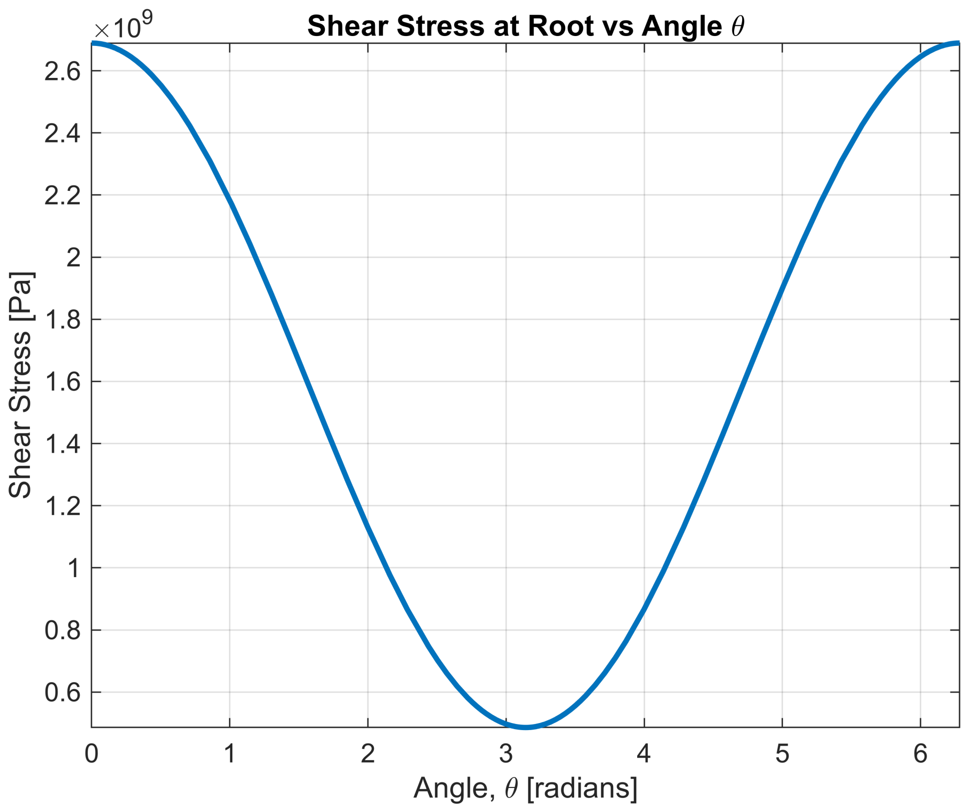
```
g = 9.8000
```

```
n_theta = (V_val^2 / (g * R_val)) +cos(theta);

%shear stress calculation
sforce = int(q,x,0,d/2);
Shear_stress = simplify(sforce*Ar/(I*t)*n_theta);
SS_root = simplify(subs(Shear_stress, [kappa, K, d, omega, E, I, Mhub, x, t, V], ...
```

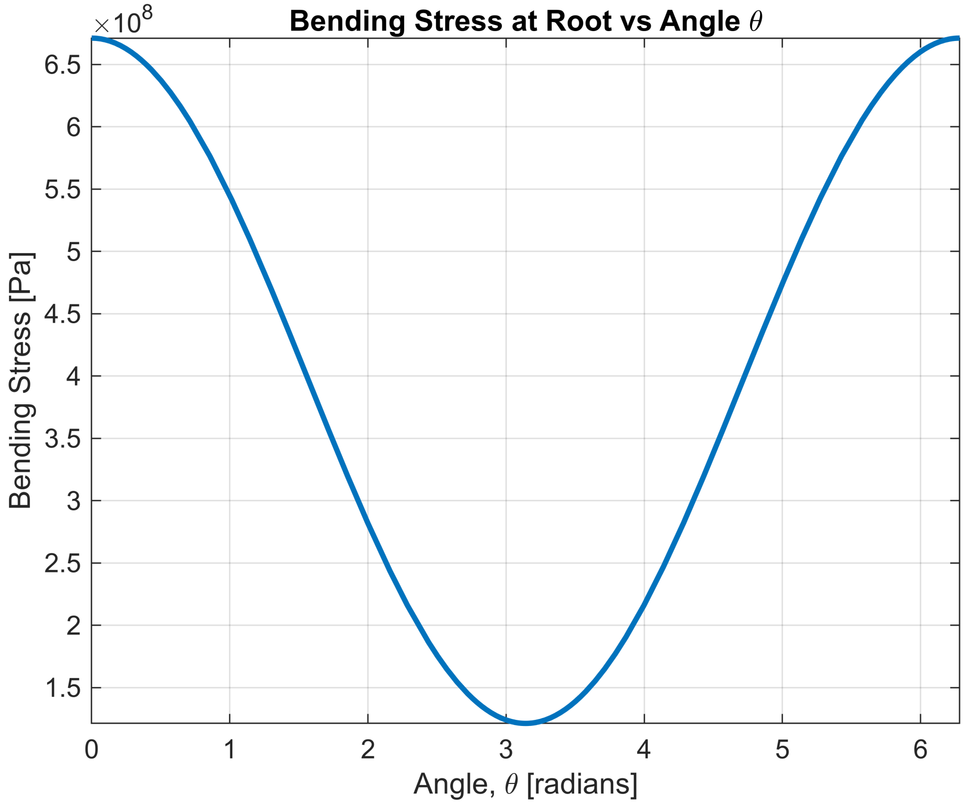
```
    [kappa_val, K_val, d_val, omega_val, E_val, I_val, Mhub_val, 0,
t_val,20.5837]));

figure;
fplot(SS_root, [0, 2*pi], 'LineWidth', 2)
title('Shear Stress at Root vs Angle \theta');
xlabel('Angle, \theta [radians]');
ylabel('Shear Stress [Pa]');
grid on;
```




```
% Bending stress calculation
M_eq = int(int(q, x), 0, d/2);
Bending_stress = simplify(M_eq * (t / 2) / I * n_theta);

BS_root = simplify(subs(Bending_stress, [kappa, K, d, omega, E, I, Mhub, x, t, V],
...
    [kappa_val, K_val, d_val, omega_val, E_val, I_val, Mhub_val, 0, t_val, V_val]));
figure;
fplot(BS_root, [0, 2*pi], 'LineWidth', 2);
title('Bending Stress at Root vs Angle \theta');
xlabel('Angle, \theta [radians]');
ylabel('Bending Stress [Pa]');
grid on;
```


Bending Stress at Root vs Angle θ
×10^8
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
Bending Stress [Pa]
0
1
2
3
4
5
6
Angle, θ [radians]