Objective: To find the effective properties of the composite using:

1. Strength of Materials Approach
2. Hill's Concentration Factors Approach
    - Voigt Approximation
    - Reuss Approximation
3. Concentric Cylinder Assemblage Model
4. Self-consistent Method
5. Mori-Tanaka Method
6. Halpin-Tsai method
7. Hashin Shtrikman Bounds

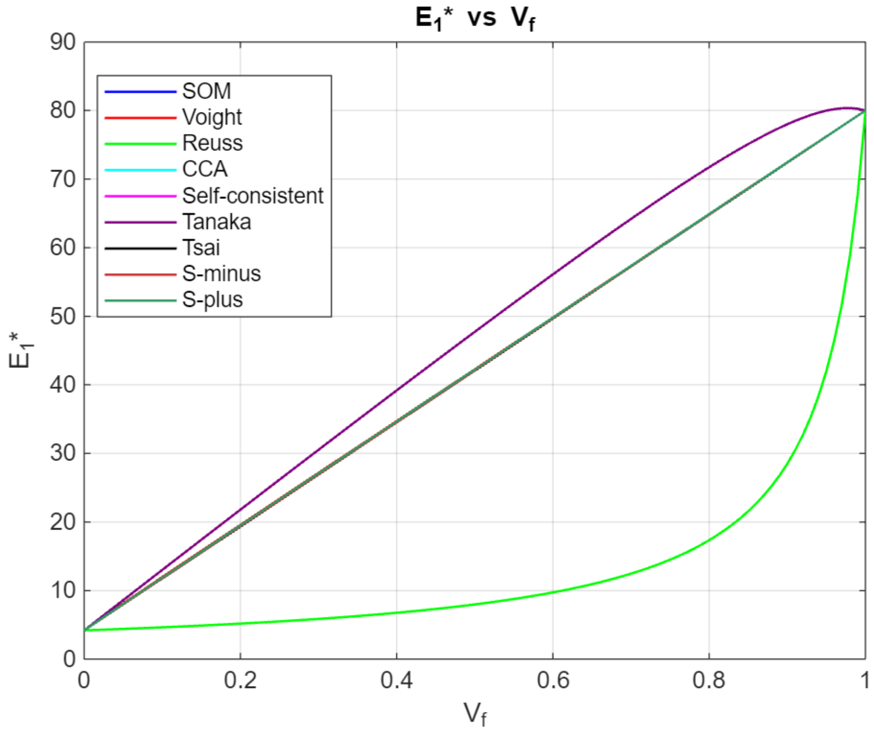
Given Materials:

Fiber type	**E-glass 21xK43 Gevetex**
Longitudinal modulus, E_1 (GPa)	*80*
Transverse modulus, E_2 (GPa)	*80*
In-plane shear modulus, G_{12} (GPa)	*33.33*
Major Poisson's ratio, ν_{12}	*0.2*
Transverse shear modulus, G_{23} (GPa)	*33.33*
Longitudinal thermal coefficient, α_1 (10−6 /°C)	*4.9*
Transverse thermal coefficient, α_2 (10−6 /°C)	*4.9*

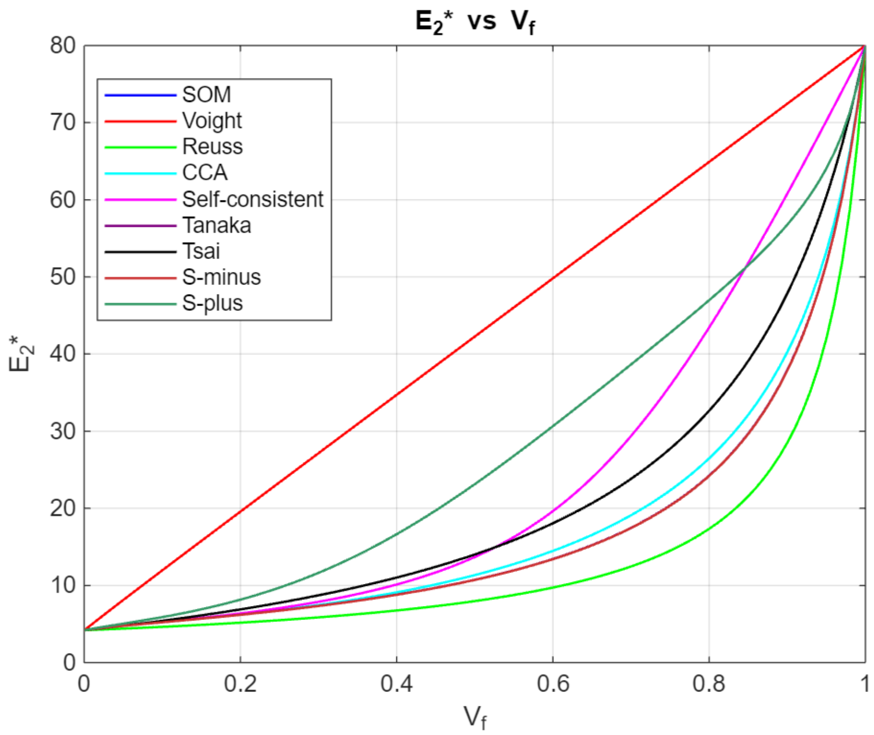
Matrix type	**3501-6 epoxy**
Modulus, E_m (GPa)	*4.2*
Poisson's ratio, ν_m	*0.34*
Thermal coefficient, α_m (10−6 /°C)	*45*

Results: (All Methods Together)

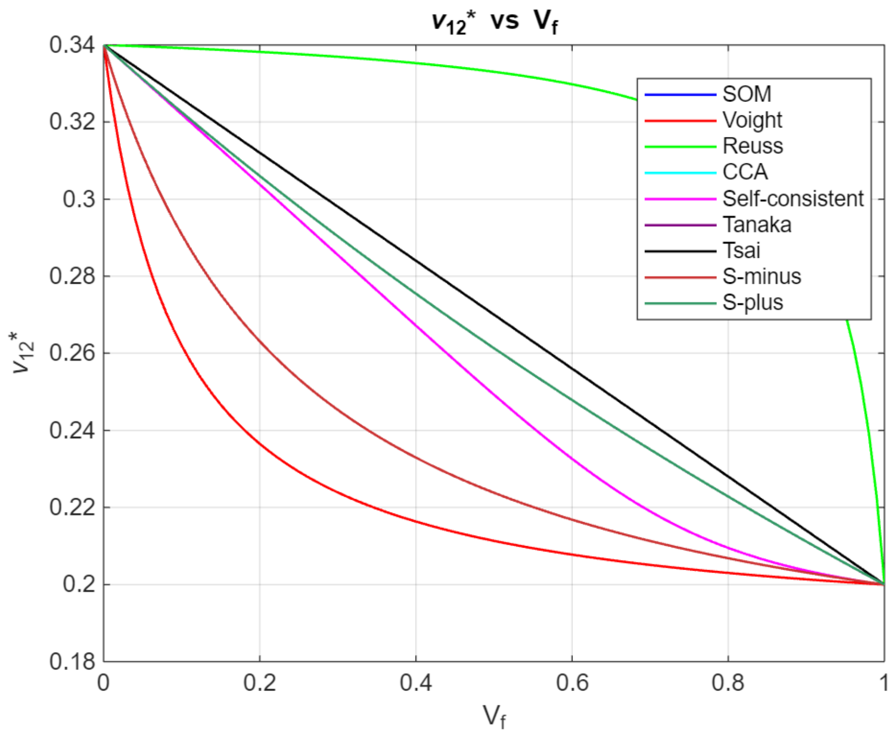
Effective Axial Modulus, E_1^* :



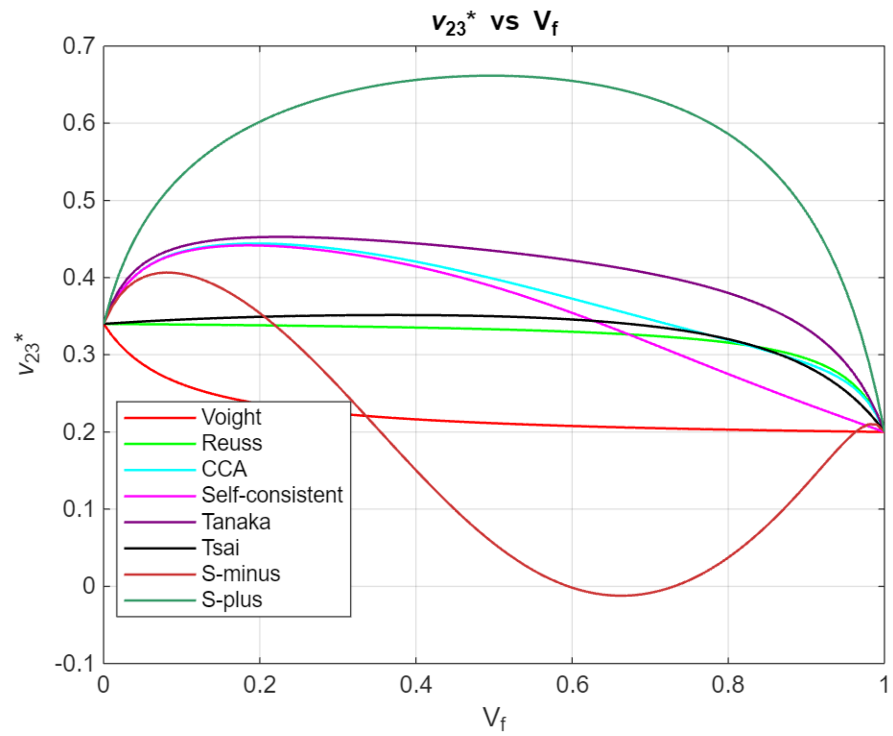
Effective Transverse Modulus, E_2^* :



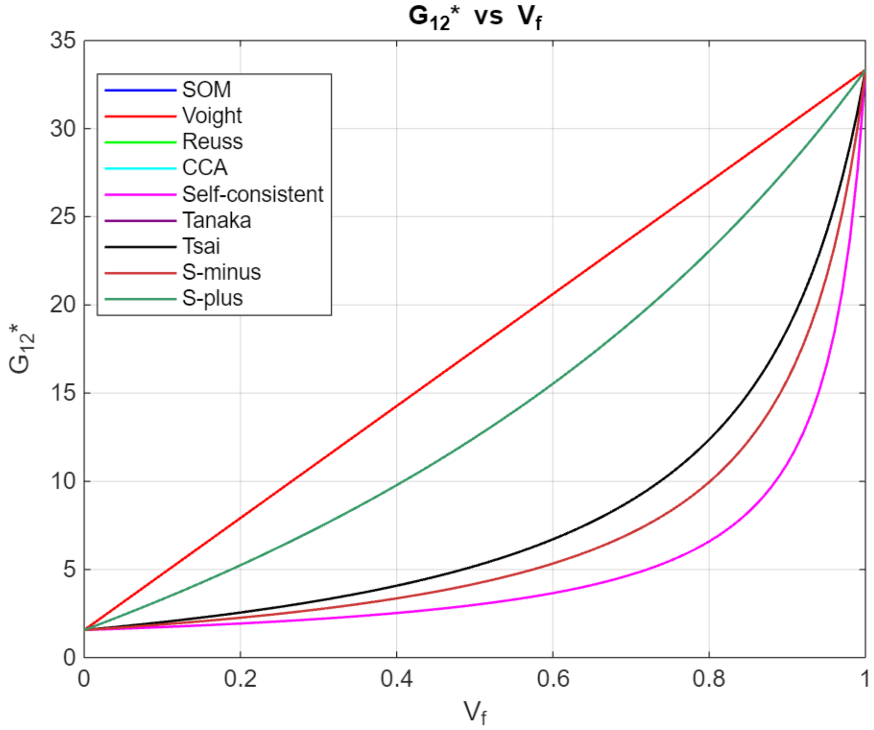
Effective Axial (Major) Poison's Ratio, v_{12}^{*} :



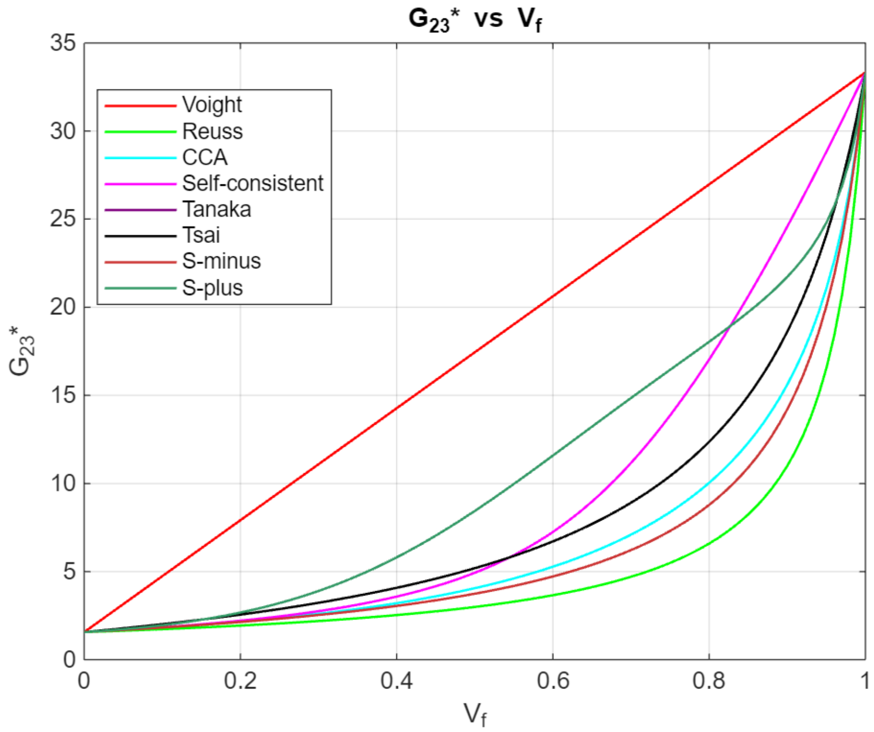
Effective Transverse Poison's Ratio, v_{23}^{*} :



Effective Axial Shear Modulus, G_{12}^{*} :

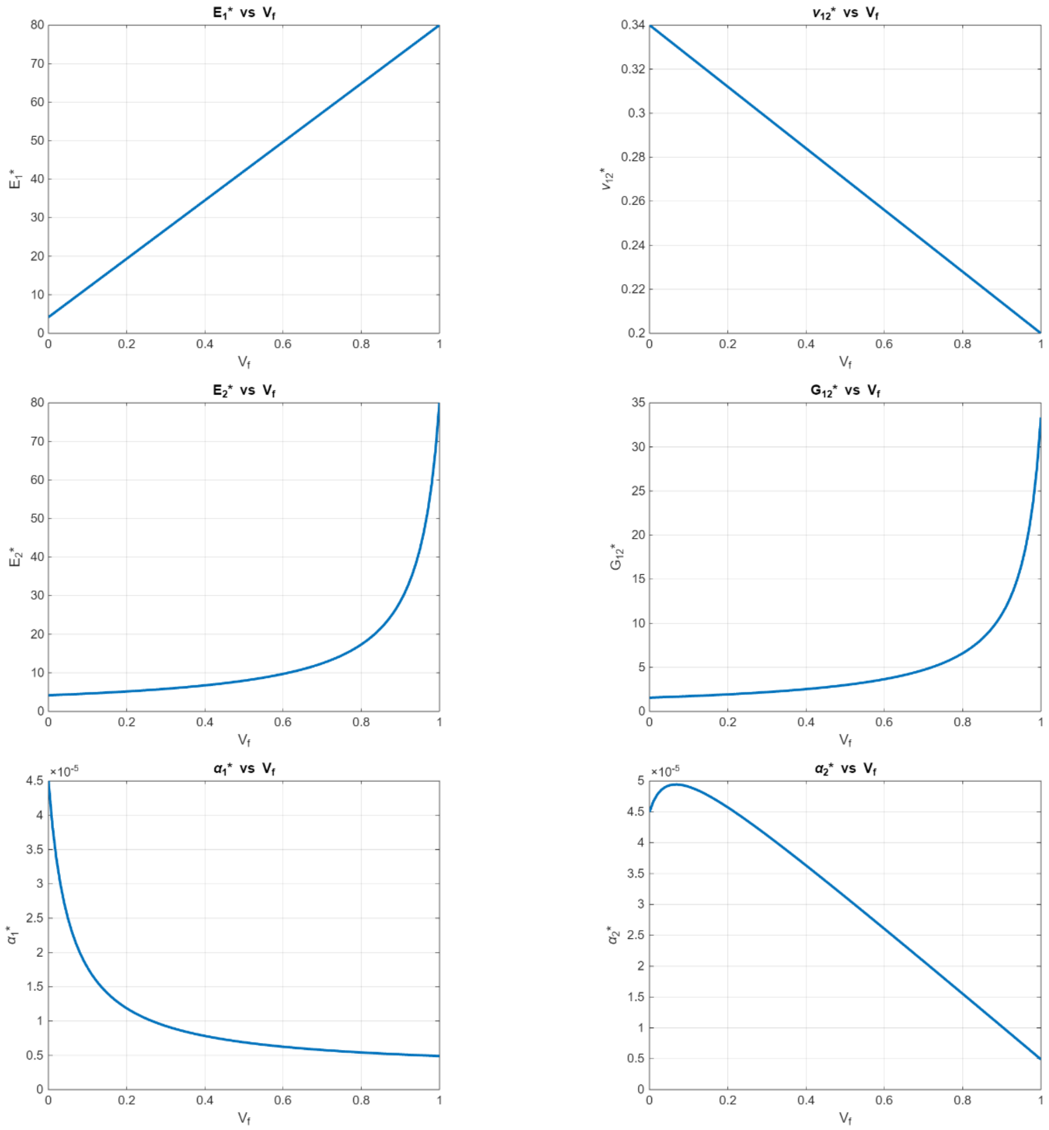


Effective Transverse Shear Modulus, G_{23}^{*} :



Observation:

1. Strength of Materials Approach:



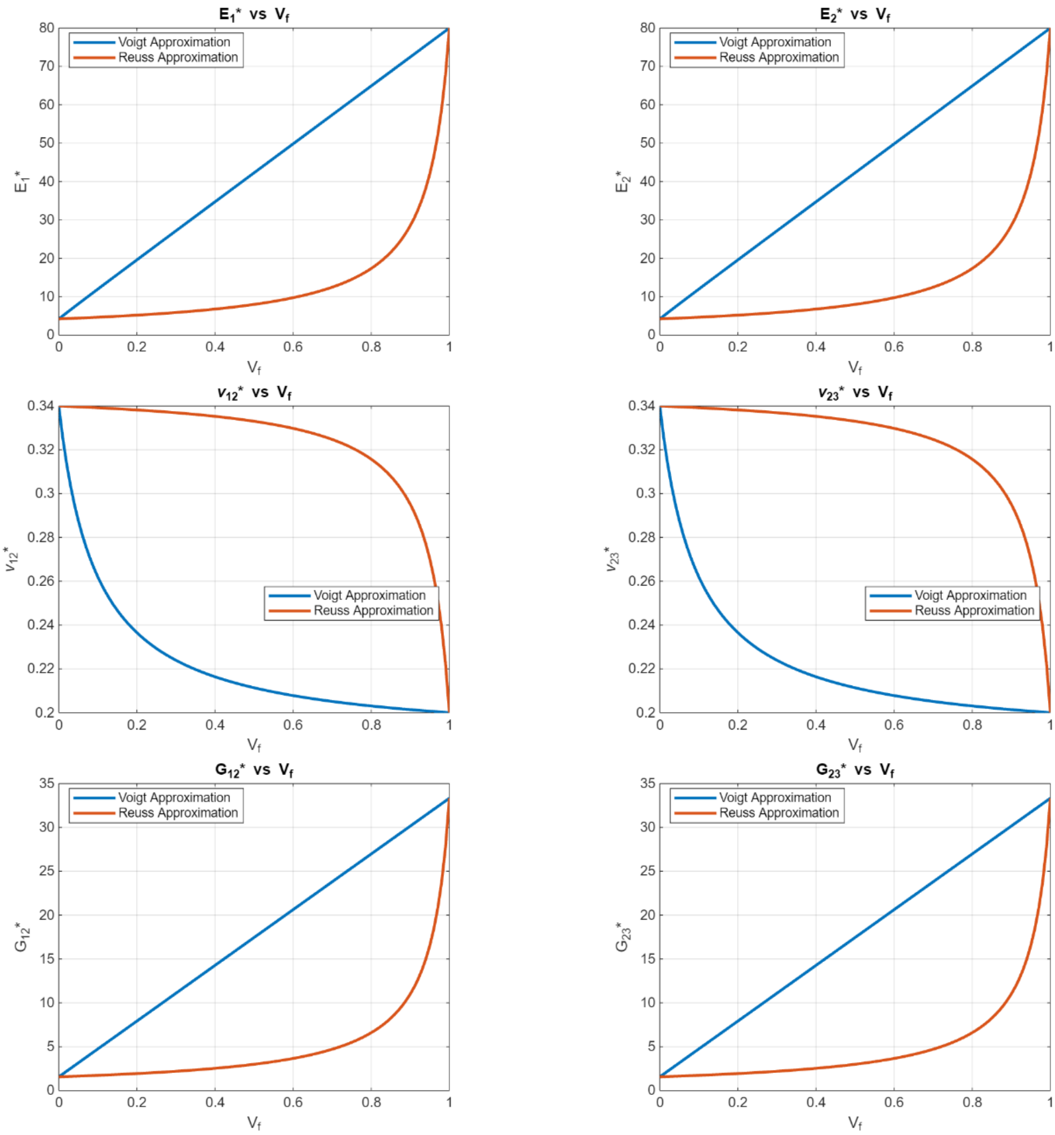
Advantages:

- Simple and straightforward implementation.
- Can estimate key in-plane effective properties such as E_1*, E_2*, v_{12}*, and G_{12}*.
- Capable of determining effective thermal properties, which other methods may not address.

Limitations:

- Unable to determine certain effective properties such as v_{23}* and G_{23}*.

2. Hill’s Concentration Factors Approach:



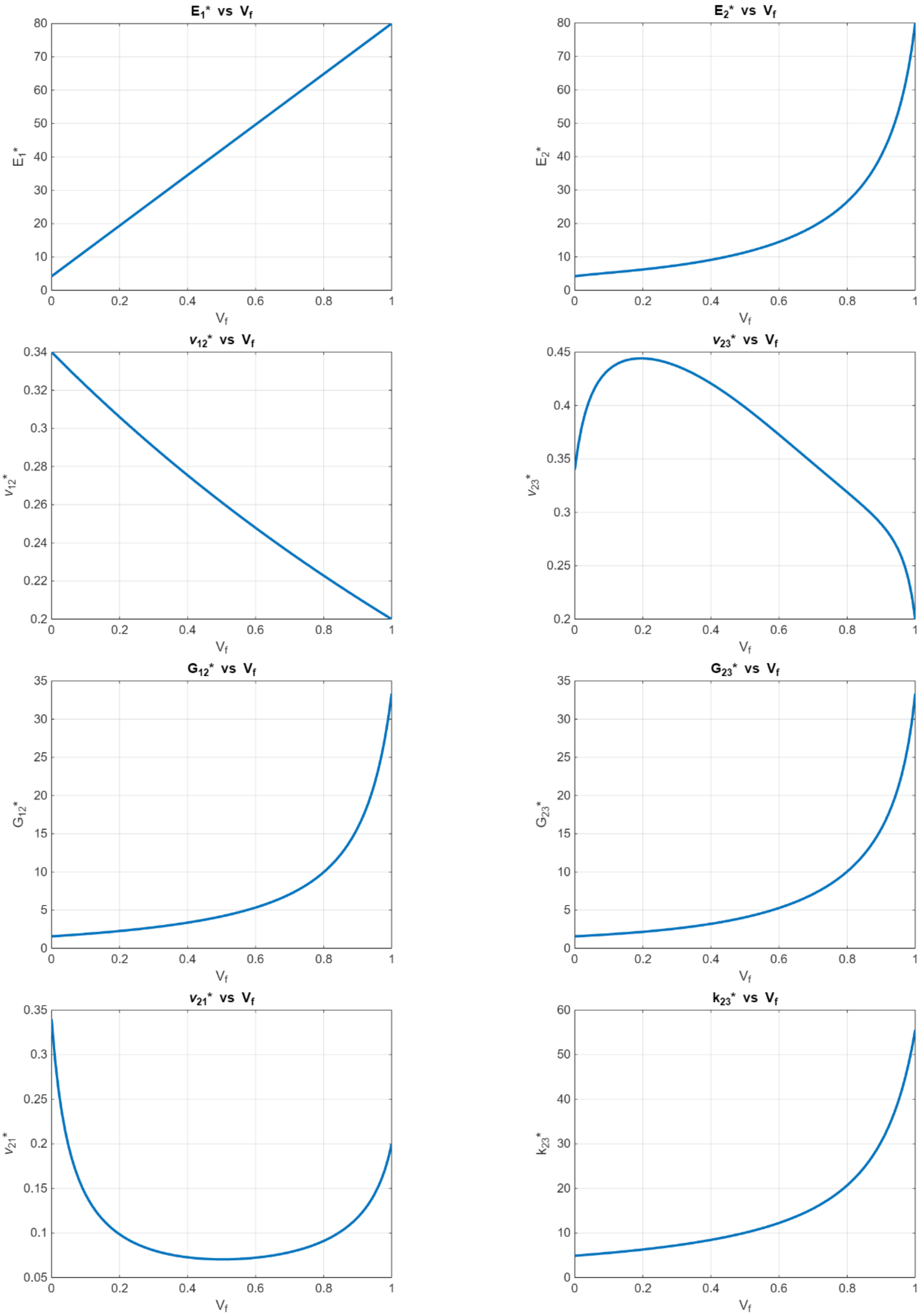
Advantages:

- Simple and straightforward implementation.
- We can find all effective properties E_1*, E_2*, v_{12}*, v_{23}*, G_{12}* and G_{23}*.
- Provides both upper (Voigt) and lower (Reuss) bounds, offering a useful range for property estimation.

Limitations:

- Lacks accuracy in some cases, as it assumes the composite behaves as an effective **isotropic** material when both fibre and matrix are isotropic. This is not realistic for unidirectional composites, which should exhibit **transversely isotropic** behaviour rather than fully isotropic behaviour.

3. Concentric Cylinder Assemblage Model:



Advantages:

- Capable of predicting all major effective properties of a composite, including typically difficult parameters such as ν_{21}^* and k_{23}^*.
- Provides a reasonably accurate representation of longitudinal and transverse elastic moduli E_1^*, E_2^* as well as Poisson's ratios ν_{12}^*, ν_{23}^*.

Limitations:

- The model often yields nearly equal values for G_{12}* and G_{23}*, which is not physically accurate for transversely isotropic composites where in-plane and out-of-plane shear moduli should differ. This reflects a limitation in capturing the anisotropic shear behaviour accurately.

Problem faced while implementation:

The direct formula given for G_{23}* was not giving accurate results as the shear modulus at V_f = 0, 1 were not matching to the values of matrix and fibre respectively.

$$\frac{G_{23}^*}{G_m} = 1 + \frac{V_f}{\frac{G_m}{\left(G_{23}^{(f)} - G_m\right)} + \frac{\left(k_m + \frac{7}{3}G_m\right)}{\left(2k_m + \frac{8}{3}G_m\right)}}$$

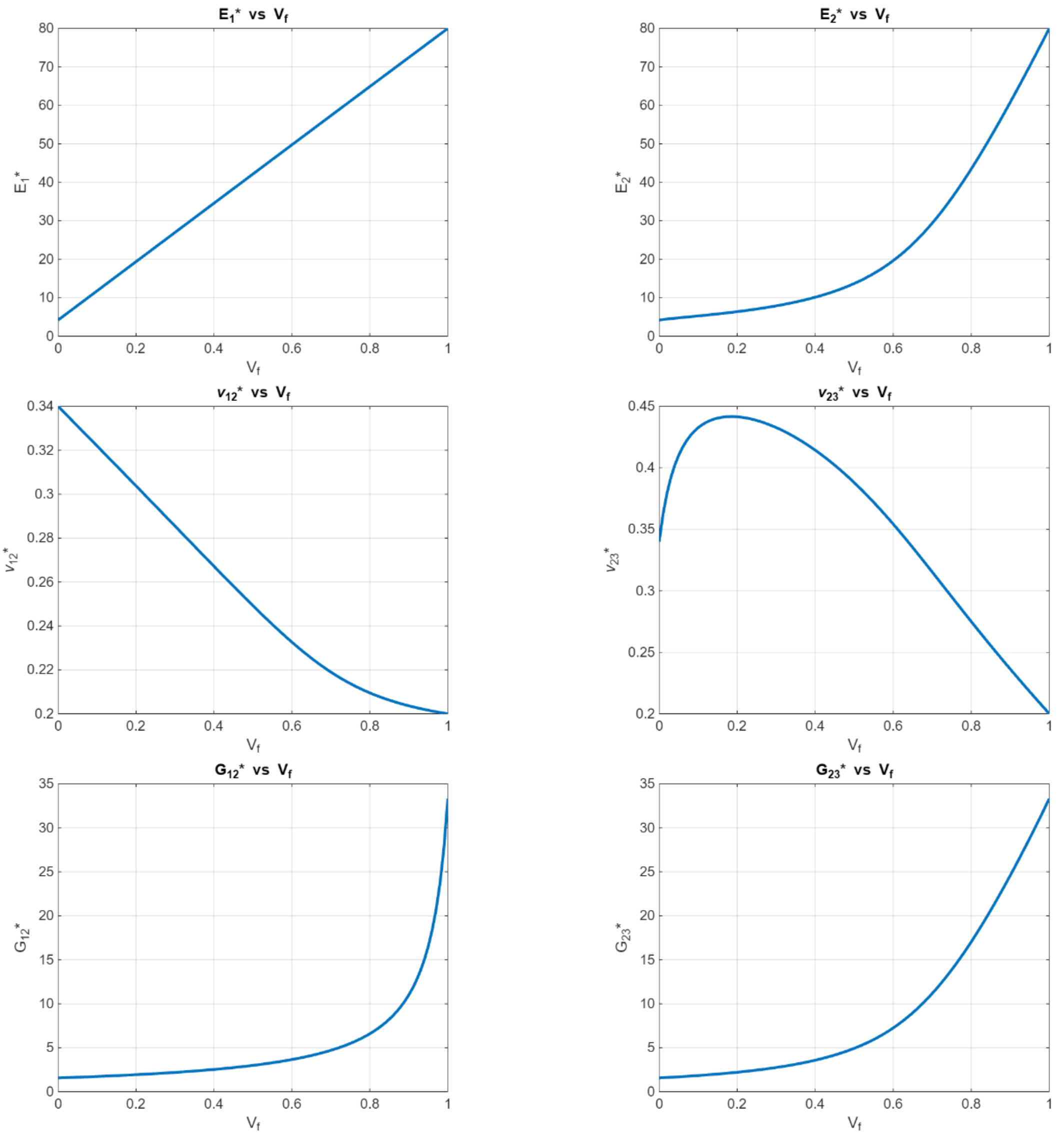
Fix:

To resolve this issue, we adopted a more general and accurate approach. Instead of relying on the direct formula, we used a quadratic equation of the form:

$$A\left(\frac{G_{23}^*}{G_{(m)}}\right)^2 + B\left(\frac{G_{23}^*}{G_{(m)}}\right) + D = 0$$

Solving this quadratic equation gave better and more accurate results for G_{23}*.

4. Self-consistent Method:



Advantages:

- Provides a highly accurate prediction of all effective composite properties.
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

Limitations:

- Computationally more involved, as it requires solving nonlinear equations, making implementation more complex compared to simpler analytical models.

Problem faced while implementation:

When solving the nonlinear equations in the self-consistent approach for composite materials, the solution array contains non-unique values, including physically unrealistic imaginary and negative solutions.

$$\frac{V_f k_f}{k_f + m^*} + \frac{V_m k_m}{k_m + m^*} = 2\left[\frac{V_f m_m}{m_m - m^*} + \frac{V_m m_f}{m_f - m^*}\right]$$

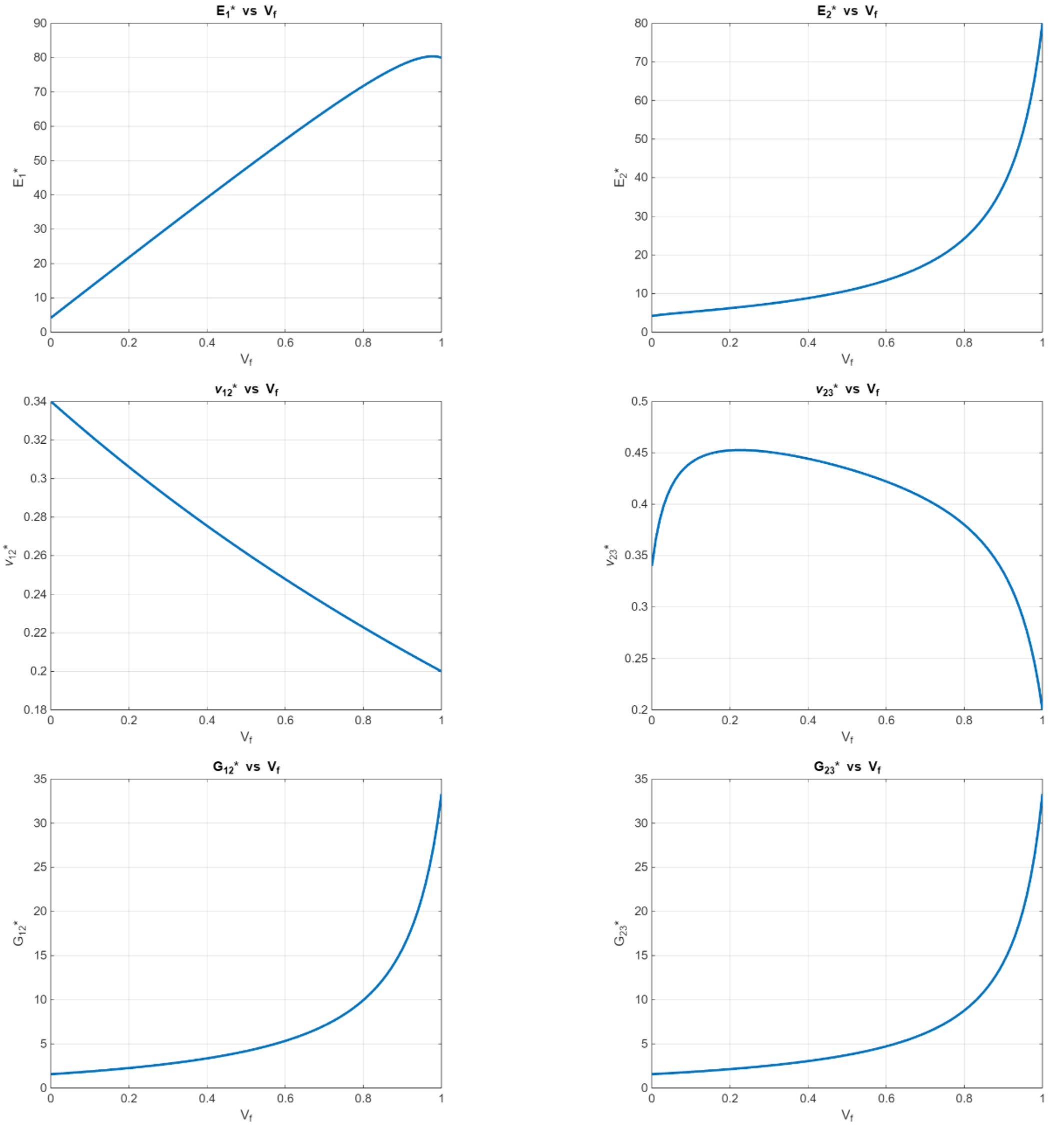
$$\frac{1}{2p^*} = \frac{V_f}{p^* - p_m} + \frac{V_m}{p^* - p_f}$$

$$\frac{1}{k^* + m^*} = \frac{V_f}{k_f + m^*} + \frac{V_m}{k_m + m^*}$$

Fix:

To address this issue, we should implement a filtering step that selects only the real and positive solutions from the computed 6×1 solution array. These physically meaningful solutions align with the underlying material properties of composites and ensure proper convergence of the self-consistent homogenization scheme.

5. Mori-Tanaka Method:



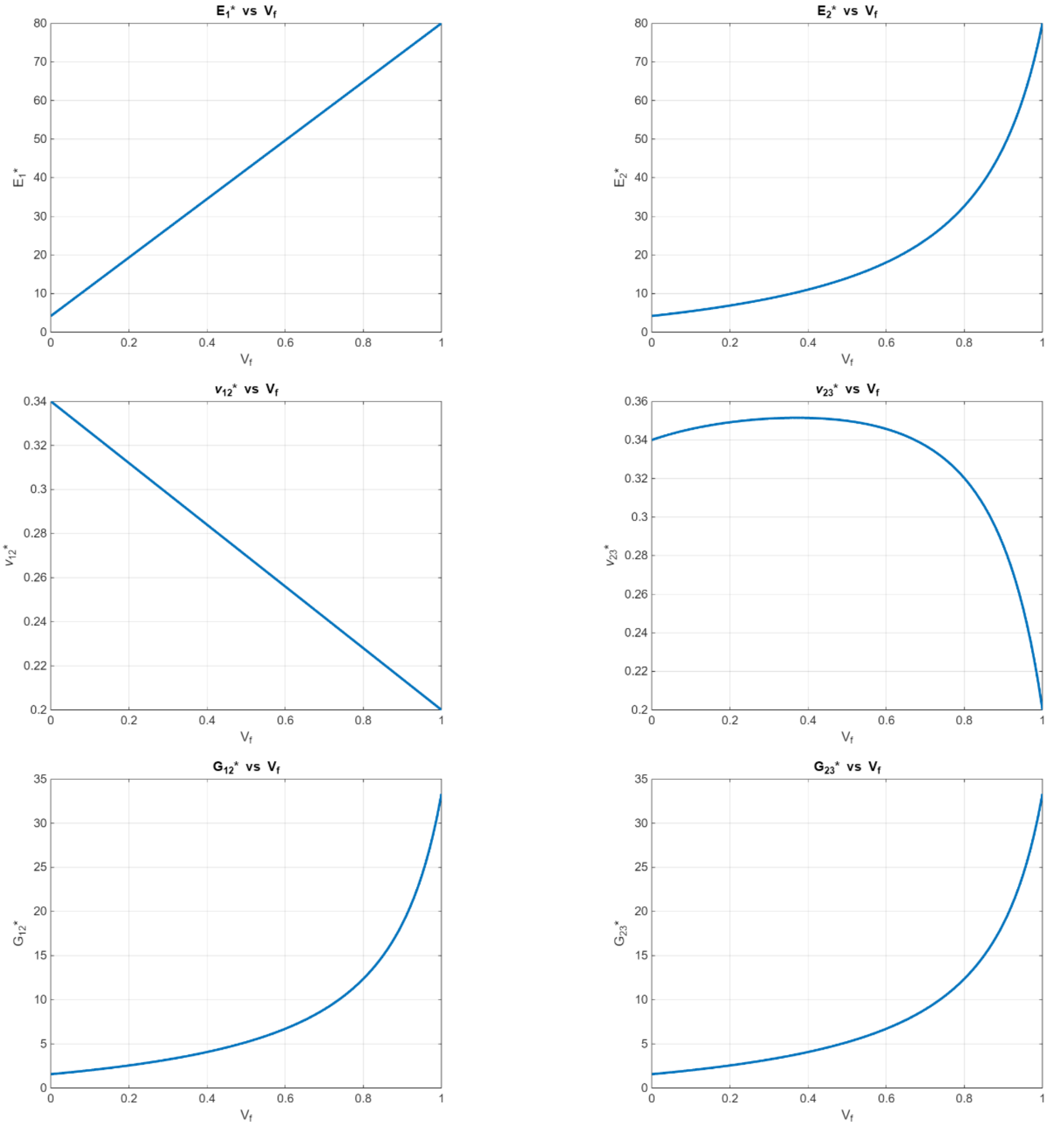
Advantages:

- We can find all effective properties E_1*, E_2*, v_{12}*, v_{23}*, G_{12}* and G_{23}*.
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

Limitations:

- Not much variation between G_{12}* and G_{23}*.

6. Halpin-Tsai method:



We assume $\zeta = 2$ which is widely accepted in literature for unidirectional composites with continuous circular fibres.

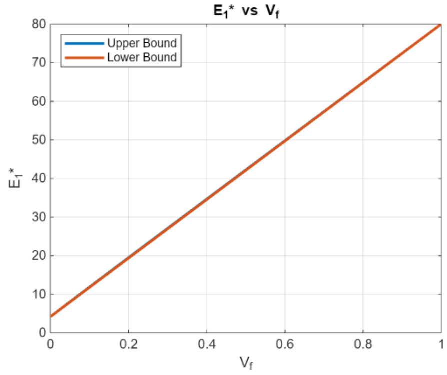
Advantages:

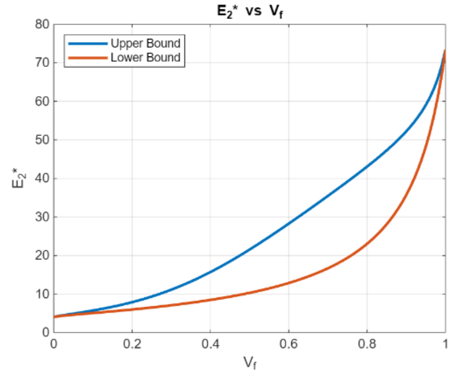
- Provides a reasonably accurate representation of longitudinal and transverse elastic moduli E_1^*, E_2^* as well as Poisson's ratios ν_{12}^*, ν_{23}^*.
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

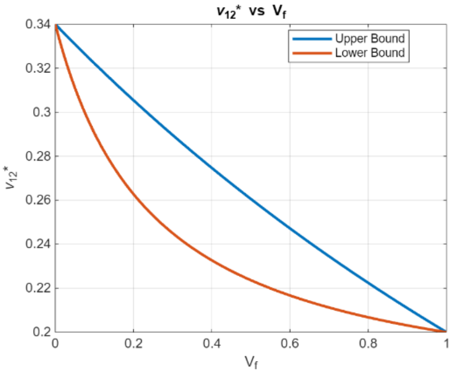
Limitations:

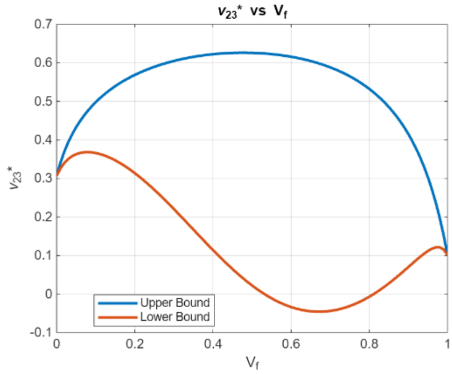
- Not much variation between G_{12}^* and G_{23}^*.

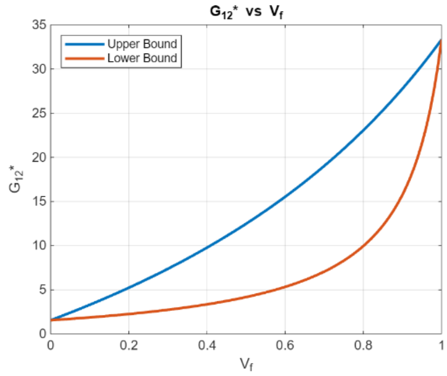
7. Hashin Shtrikman Bounds:

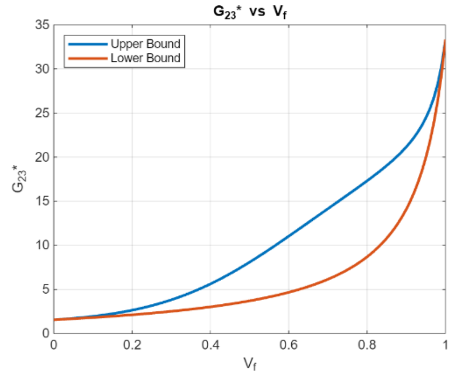


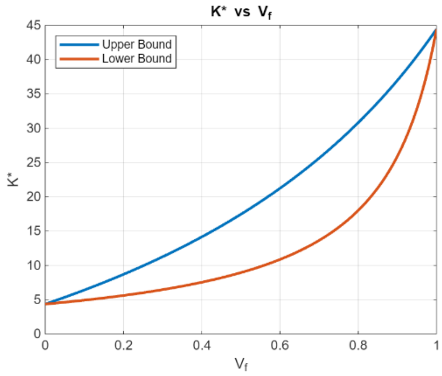












Advantages:

- Straightforward implementation requiring minimal computational resources.
- Provides rigorous upper and lower bounds on effective properties, establishing a clear range of possible values.

- Delivers accurate predictions for all effective mechanical properties.
- Explicitly determines the bulk modulus (K) along with other essential parameters.

Limitations:

- Bounds may become too wide to be practically useful.

Conclusion:

Method	Complexity	Accuracy
Strength of Materials Approach	Very Low	Moderate
Voigt Approximation	Low	Low
Reuss Approximation	Low	Low
Concentric Cylinder Assemblage (CCA)	Moderate	High for aligned continuous fibres
Self-consistent Method	High	High
Mori-Tanaka Method	Moderate	High
Halpin-Tsai Method	Low to Moderate	Moderate
Hashin-Shtrikman Bounds	Moderate	Moderate to High

For general composite systems, the Concentric Cylinder Assemblage (CCA) method, Self-consistent method, and Mori-Tanaka method typically provide the most accurate predictions of effective material properties.

However, in our specific case involving isotropic fibre and matrix materials, the Self-consistent method yields the most accurate results.