# Matrix-based approaches for pattern discovery

## Pattern discovery: typical dimensionality

#### Typical case: GAL genes

- q s 6 sequences
- <sub>q</sub> L size per sequence 800 bp
- q occ<sub>e</sub> expected pattern occurrences: 12

(2 sites per sequence)

g w matrix width = 25

#### Let us assume that

- A signal can be found on any strand
- Each sequence contains 0 or several occurrences
- Number of possible site positions

• 
$$T=2s(L-w+1)=9312$$

$$N_{alignments} = C_{2s(L-w+1)}^{occ_e} = 8.8 * 10^{38}$$

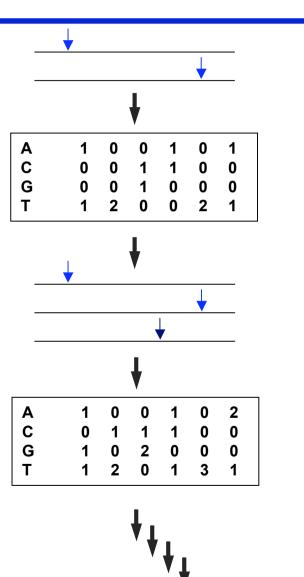
## Matrix-based pattern discovery

- Problem: the number of possible matrices is too large to be tractable
- Approaches: define heuristics to extract a matrix with highest possible information content (lowest probability to be due to random effect) → optimization techniques
- Two approaches working with regulatory sequences
  - greedy algorithm
  - gibbs sampling

## Greedy algorithm

## Pattern discovery: greedy algorithm (consensus, by Jerry Hertz)

- Create all possible matrices with two sequences
- Retain the most significant matrices only
- 3) Find best match in next sequence and incorporate it into the matrix
- 4) Iterate from (2) untill all sequences are incorporated
- 5) Return the most significant matrices



## Greedy algorithm: weaknesses

- Returns multiple matrices, but they are generally slight variants of the same pattern
- Time-consuming
- Sensitive to sequence ordering in the input data set
- Takes into account prior residue frequencies, but not oligonucleotide bias
- n References
  - Hertz et al. (1990). Comput Appl Biosci 6(2), 81-92.
  - Hertz, G. Z. & Stormo, G. D. (1999). Bioinformatics 15(7-8), 563-77.
  - Stormo, G. D. & Hartzell, G. W. d. (1989). Proc Natl Acad Sci U S A 86(4), 1183-7.

## Expectation- Maximization (EM)

# Gibbs sampling (stochastic Expectation - Maximization)

## Pattern discovery: The Gibbs sampler

(gibbs motif sampler, by Andrew Neuwald)

#### Pretend you know the motif, this might become true

#### 1) Initialization

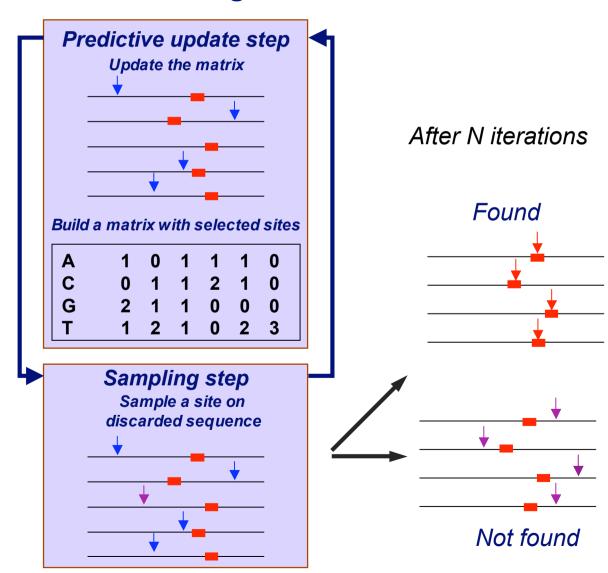
- o select a random set of sites in the sequence set
- Create a matrix with these sites

## 2) Sampling (Stochastic Expectation)

- Isolate one sequence from the set, and score each position (site) of the sequence.
- Select one "random" site, with a probability proportional to the score (A<sub>x</sub>, see next slide).

## 3) Predictive update (Maximization)

- Replace the old site with a new site, and update the matrix
- 4) **Iterate** steps 2 and 3 for a fixed number of cycles



## Gibbs sampling - scoring scheme

$$A_{x} = Q_{x}/P_{x}$$

- A<sub>x</sub> weight of segment x(used for random selection)
- $Q_x$  probability to generate segment x according to pattern probabilities  $q_{ij}$
- $P_x$  probability to generate segment x according to the background probabilities  $p_i$

$$q_{i,j} = \frac{c_{i,j} + b_j}{N - 1 + B}$$

- i index for the site
- j index for the residue
- $c_{i,j}$  counts for residue j at site i
- N number of sequences
- $b_i$  pseudo-count for residue j
- $\vec{B}$  sum of pseudo-counts

$$F = \sum_{i=1}^{W} \sum_{j=1}^{R} c_{i,j} \ln \left( \frac{q_{i,j}}{p_j} \right)$$

- W width of the matrix
- R number of distinct residues
- $p_i$  prior probability for residue j

#### Stochastic vs deterministic behaviour

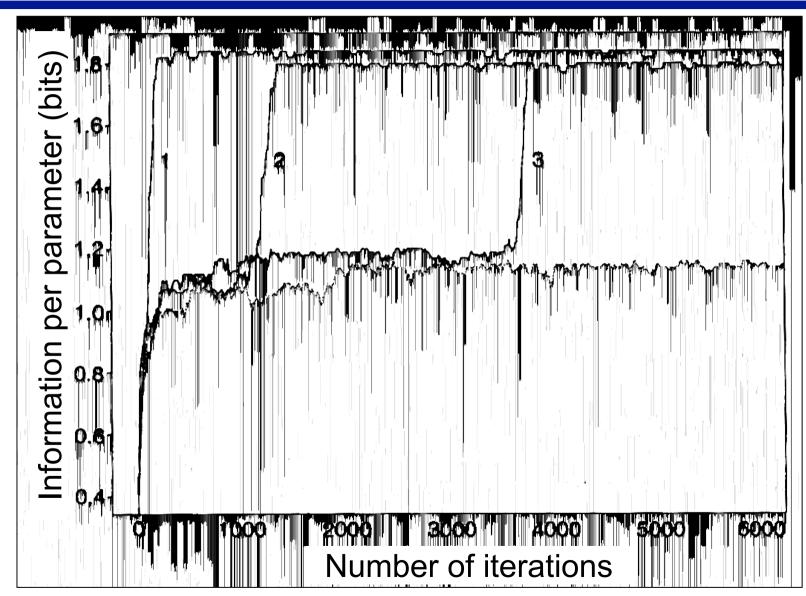
#### Why to select a random site?

- A deterministic behaviour would consist in selecting, at each iteration, the highest scoring site (the one which matches best the matrix)
- This would give poor results because the program is attracted too fast towards local optima.

#### Stochastic behaviour

- At each iteration, the next site is selected in a stochastic rather than deterministic way: the probability of each site to be selected is proportional to its scoring with the matrix
- This allows to avoid weak local optima, and converge towards better solutions.

### Gibbs sampling: optimization of information content



source: Lawrence et al.(1993). Science 262(5131), 208-14.

## Gibbs sampling: strength

- n Fast
- Probabilistic description of the patterns
- n Can run with proteins or DNA

## Gibbs sampling: weaknesses

- Returns a different result at each run
- Can be attracted by local maxima
  - g solution: run repeatedly and check which motifs come often
- The original Gibbs sampler takes into account prior residue frequencies, but not oligonucleotide bias
  - q → in yeast, often returns A/T-rich regions
  - This is however improved in some versions of the Gibbs samplers which use Markov chains for estimating the bacground probabilities (eg the MotifSampler developed by Gert Thijs)
- No threshold on pattern significance
  - q → frequent false positive

## Improvements of the gibbs sampler

- Neuwald 1993
  - Phase shifting
- Neuwald 1995
  - 0 or several matches per sequence
  - column sampling (spacings can be admitted between columns of the matrix)
- n Roth (1998): AlignACE
  - Specific implementation for DNA (double strand is treated)
  - post-filtering of motifs according to number of matches in the genome, in order to discard frequent motifs
- n Liu (2000), Thijs (2000)
  - Markov-chain based calculation of background probabilities

#### References

#### Original Gibbs sampler

- q Lawrence et al. (1993). Science 262(5131), 208-14.
- Neuwald et al. (1995). Protein Sci 4(8), 1618-32.
- q Neuwald et al. (1997). Nucleic Acids Res 25(9), 1665-77.

#### n MotifSampler

Thijs et al. (2002). J.Computational Biology 9:447-464.

## AlignACE, ScanACE and CompACE gibbs sampler tools for regulatory sequence analysis

- Single/both strands
- Return multiple matrices, with iterative masking preventing slight variants of the same pattern
- Matrix clustering
- A posteriori evaluation of pattern significance, by analysing the whole-genome frequency of the discovered matrix.
- n References
  - q Roth et al. (1998). Nat Biotechnol 16(10), 939-45.
  - q Tavazoie et al. (1999). Nat Genet 22(3), 281-5.
  - Hughes et al. (2000). J Mol Biol 296(5), 1205-14.
  - q McGuire et al. (2000). Genome Res 10(6), 744-57.

## Matrix-based pattern discovery: strengths

- More specific description of degeneracy than with stringbased approaches (frequency of each residue at each position).
- The resulting pattern is more accurate than a string for pattern matching (more sensitive scoring scheme)

## Matrix-based pattern discovery: weaknesses

- The results strongly depend on parameter setting. Two essential parameters have to be selected :
  - Matrix width
  - Expected number of sites
- The best parameter may change from gene family to gene family.
  Choosing the appropriate setting requires experience.
- Impossible to evaluate all possible alignments
- Does not take into account higher-order correlation between adjacent positions (oligonucleotide bias)