Preferential Reasoning

Beyond Propositional Logic and Argument Forms

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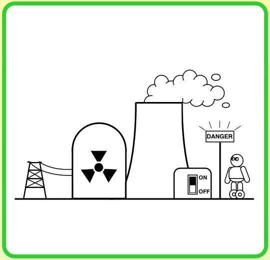
Motivation



Nuclear Power Plant

- Atomic pile, cooling system: On/Off
- Agent controls pile and cooler
- Hazardous situations
- Malfunctioning

- If *pile on*, then usually *not hazardous* ✓
- If *pile on* and *cooling off*, then usually *hazardous* ✓
- If *hazardous*, then usually after switching *not hazardous* ?
- If *pile on* and *cooling off*, then usually agent knows *hazardous* ?
- If *pile on*, then usually ought to be *not hazardous* ?
- An *atomic pile* usually has as component *some piece of Uranium* ?

Outline

Preferential and Rational Consequence [KLM90; LM92]

- *Defeasible* consequence relation $\mid\!\sim \subseteq \mathcal{L} \times \mathcal{L}$
- *pile* $\mid\!\sim$ $\neg$*hazardous*, *pile* $\wedge\neg$*cooling* $\mid\!\sim$*hazardous*
- *Preferential Consequence*: Satisfaction of properties

(Ref) $\alpha \mid\!\sim \alpha$ (LLE) $\dfrac{\alpha \equiv \beta, \quad \alpha \mid\!\sim \gamma}{\beta \mid\!\sim \gamma}$ (And) $\dfrac{\alpha \mid\!\sim \beta, \quad \alpha \mid\!\sim \gamma}{\alpha \mid\!\sim \beta \wedge \gamma}$

(Or) $\dfrac{\alpha \mid\!\sim \gamma, \quad \beta \mid\!\sim \gamma}{\alpha \vee \beta \mid\!\sim \gamma}$ (RW) $\dfrac{\alpha \mid\!\sim \beta, \quad \models \beta \rightarrow \gamma}{\alpha \mid\!\sim \gamma}$ (CM) $\dfrac{\alpha \mid\!\sim \beta, \quad \alpha \mid\!\sim \gamma}{\alpha \wedge \gamma \mid\!\sim \beta}$

- *Rational Consequence*: All of the above, plus

(RM) $\dfrac{\alpha \mid\!\sim \beta, \quad \alpha \not\mid\!\sim \neg\gamma}{\alpha \wedge \gamma \mid\!\sim \beta}$

- **Limitation**: propositional . . . We need **more**, but not **too much**

Modal Logic

What is it (briefly)?

- Modes of reasoning: necessity, possibility
- Philosophical origins; suitable to talk about relational structures
- Foundation for knowledge representation formalisms in AI

Nice features:

- Simplified syntax (links with DLs)
- Intuitive semantics
- Versatility: actions, epistemic reasoning, ontologies, ...
- Amenable to implementation (tableaux)
- Decidability

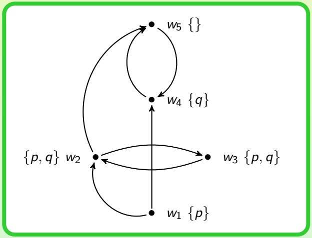
Modal Logic

Formulas

- $\alpha ::= p \mid \neg\alpha \mid \alpha \land \alpha \mid \alpha \lor \alpha \mid \alpha \to \alpha \mid \Box_i\alpha \mid \Diamond_i\alpha$

Kripke models: tuples $\mathscr{M} = \langle W, R, V \rangle$ where

- $W \neq \emptyset$ is a set of possible *worlds*
- $R = \langle R_1, \ldots, R_n \rangle$, where $R_i \subseteq W \times W$ is an *accessibility relation*
- $V : W \longrightarrow 2^{\mathcal{P}}$ is a *valuation* function



- Satisfaction, truth and validity: as usual
- Entailment: global and local
- We start with normal modal logic K

Modal Preferential and Rational Consequence [BMV11,BMV12]

Basic idea

- Interpret $\mid\!\sim$ as a relation on modal formulas
- Interpret $\models$ as local entailment

(Ref) $\alpha \mid\!\sim \alpha$

(LLE) $$\frac{\alpha \equiv \beta, \quad \alpha \mid\!\sim \gamma}{\beta \mid\!\sim \gamma}$$

(And) $$\frac{\alpha \mid\!\sim \beta, \quad \alpha \mid\!\sim \gamma}{\alpha \mid\!\sim \beta \wedge \gamma}$$

(Or) $$\frac{\alpha \mid\!\sim \gamma, \quad \beta \mid\!\sim \gamma}{\alpha \vee \beta \mid\!\sim \gamma}$$

(RW) $$\frac{\alpha \mid\!\sim \beta, \quad \models \beta \rightarrow \gamma}{\alpha \mid\!\sim \gamma}$$

(CM) $$\frac{\alpha \mid\!\sim \beta, \quad \alpha \mid\!\sim \gamma}{\alpha \wedge \gamma \mid\!\sim \beta}$$

(RM) $$\frac{\alpha \mid\!\sim \beta, \quad \alpha \not\mid\!\sim \neg\gamma}{\alpha \wedge \gamma \mid\!\sim \beta}$$

Definition

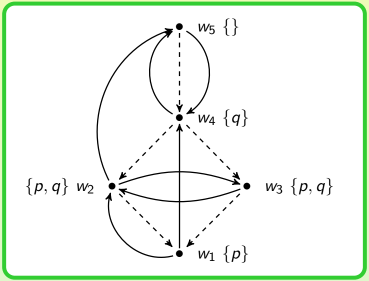
$\mid\!\sim$ is a *preferential modal conseq.* iff (Ref), (LLE), (And), (Or), (RW), (CM)

$\mid\!\sim$ is a *rational modal conseq.* iff $\mid\!\sim$ is preferential and satisfies (RM)

Preferential Reasoning in Modal Logic

Preferential Kripke models: tuples $\mathscr{P} = \langle W, R, V, \prec \rangle$ where

- W, R, V as before
- $\prec \subseteq W \times W$ is a (smooth) partial order on W



- $[\![\alpha]\!]$: worlds satisfying α
- α is *satisfiable* iff $[\![\alpha]\!] \neq \emptyset$
- α is *true* in $\mathscr{P}$ ($\mathscr{P} \Vdash \alpha$) iff $[\![\alpha]\!] = W$
- α is *valid* iff $\mathscr{P} \Vdash \alpha$ for every $\mathscr{P}$

Lemma

Let $\mathscr{P} = \langle W, R, V, \prec \rangle$ and $\mathscr{M} = \langle W, R, V \rangle$. Then $\mathscr{P} \Vdash \alpha$ iff $\mathscr{M} \Vdash \alpha$.

Preferential Reasoning in Modal Logic

$\mathscr{P} \Vdash \alpha \mid\!\sim \beta$ iff $\min_{\prec}[\![\alpha]\!] \subseteq [\![\beta]\!]$

Lemma (Soundness)

Let $\mathscr{P} = \langle W, R, V, \prec \rangle$ *and* $\mid\!\sim_{\mathscr{P}} := \{(\alpha, \beta) \mid \mathscr{P} \Vdash \alpha \mid\!\sim \beta\}$.

Then $\mid\!\sim_{\mathscr{P}}$ *satisfies all KLM preferential postulates plus:*

$$(\mathit{Cons}) \quad \top \not\mid\!\sim \bot \qquad (\mathit{Norm\ 1}) \ \frac{\alpha \mid\!\sim \bot}{\Diamond\alpha \mid\!\sim \bot} \qquad (\mathit{Norm\ 2}) \ \frac{\alpha \mid\!\sim \bot}{\Diamond^{-}\alpha \mid\!\sim \bot}$$

Lemma (Completeness)

Let $\mid\!\sim \subseteq \mathcal{L} \times \mathcal{L}$ *satisfy all KLM postulates plus Cons, Norm 1 and Norm 2. Then there exists* $\mathscr{P}$ *such that* $\mid\!\sim = \{(\alpha, \beta) \mid \mathscr{P} \Vdash \alpha \mid\!\sim \beta\}$.

Preferential Reasoning in Modal Logic

Ranked Kripke models $\mathscr{R} = \langle W, R, V, \prec \rangle$: $\prec$ is a modular order

Lemma (Soundness)

Let $\mathscr{R} = \langle W, R, V, \prec \rangle$ *and* $\mathrel{|\!\sim}_{\mathscr{R}} := \{(\alpha, \beta) \mid \mathscr{R} \Vdash \alpha \mathrel{|\!\sim} \beta\}$.

Then $\mathrel{|\!\sim}_{\mathscr{R}}$ *satisfies all KLM rational postulates plus:*

$$(\textit{Cons}) \quad \top \not\mathrel{|\!\sim} \bot \qquad (\textit{Norm 1}) \quad \frac{\alpha \mathrel{|\!\sim} \bot}{\Diamond\alpha \mathrel{|\!\sim} \bot} \qquad (\textit{Norm 2}) \quad \frac{\alpha \mathrel{|\!\sim} \bot}{\Diamond^{-}\alpha \mathrel{|\!\sim} \bot}$$

Lemma (Completeness)

Let $\mathrel{|\!\sim} \subseteq \mathcal{L} \times \mathcal{L}$ *satisfy all KLM postulates plus Cons, Norm 1 and Norm 2. Then there exists* $\mathscr{R}$ *such that* $\mathrel{|\!\sim} = \{(\alpha, \beta) \mid \mathscr{R} \Vdash \alpha \mathrel{|\!\sim} \beta\}$.

Beyond Defeasible Argument Forms

Things that we can say

- $\mathsf{c} \land \Diamond_{\mathsf{f}} \top \mathrel{|\!\sim} \Box_{\mathsf{f}} \mathsf{c}$, $\quad \top \mathrel{|\!\sim} \Box_{\mathsf{m}} \bot$
- $\mathsf{h} \mathrel{|\!\sim} K_{\mathsf{A}}(\mathsf{p} \land \neg \mathsf{c})$, $\quad K_{\mathsf{B}} \mathsf{p} \mathrel{|\!\sim} \mathsf{p} \land \mathsf{c}$

Things that we don't know how to say

- "Toggling the switch normally turns the light on"
- "We know that normally the speed of light is the fastest"
- "In soccer, it is my normal duty to be fair play"

$$\alpha \mathrel{|\!\sim} \beta$$

"The most normal α-worlds are β-worlds"

Defeasible Modes of Reasoning [BV12,BV13]

Defeasible versions of modalities: $\widetilde{\Box}$ ('flag') and $\widetilde{\Diamond}$ ('flame')

Extended language

- $\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \alpha \rightarrow \alpha \mid \Box_i\alpha \mid \Diamond_i\alpha \mid \widetilde{\Box}_i\alpha \mid \widetilde{\Diamond}_i\alpha$

Intuition

- $\widetilde{\Box}_i\alpha$: "all most normal i-successors are α"
- $\widetilde{\Diamond}_i\alpha$: "some most normal i-successors are α"

Semantics in terms of preferential models

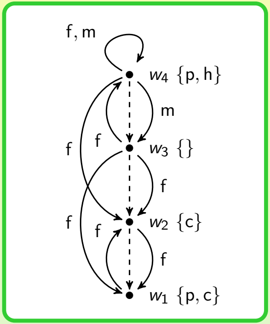
- $[\![\widetilde{\Box}_i\alpha]\!] := \{w \in W \mid \min_{\prec} R_i(w) \subseteq [\![\alpha]\!]\}$
- $[\![\widetilde{\Diamond}_i\alpha]\!] := \{w \in W \mid \min_{\prec} R_i(w) \cap [\![\alpha]\!] \neq \emptyset\}$

Satisfaction, truth and validity: as before

Defeasible Modes of Reasoning

Example in reasoning about actions

- Let $\mathcal{P} = \{\mathsf{p}, \mathsf{c}, \mathsf{h}\}$ and $\mathcal{A} = \{\mathsf{f}, \mathsf{m}\}$



- $\mathscr{P} \Vdash (\mathsf{p} \wedge \neg\mathsf{c}) \leftrightarrow \mathsf{h}$
- $w_4 \in [\![\mathsf{h} \wedge \widetilde{\Diamond}_{\mathsf{f}}\neg\mathsf{h}]\!]$
- $w_1 \in [\![\widetilde{\Box}_{\mathsf{m}}\bot]\!]$
- $\mathscr{P} \Vdash \neg\mathsf{p} \rightarrow \widetilde{\Box}_{\mathsf{f}}\mathsf{p}$ but $\mathscr{P} \nVdash \neg\mathsf{p} \rightarrow \Box_{\mathsf{f}}\mathsf{p}$
- $\mathscr{P} \Vdash \mathsf{c} \rightarrow \widetilde{\Box}_{\mathsf{f}}\neg\mathsf{h}$

Defeasible Modes of Reasoning

A few validities

- $\models \mathrel{\Box\!\!\!\sim}_i\alpha \leftrightarrow \neg\mathrel{\Diamond\!\!\!\sim}_i\neg\alpha$
- $\models \mathrel{\Box\!\!\!\sim}_i\bot \leftrightarrow \Box_i\bot$
- $\models \mathrel{\Box\!\!\!\sim}_i\top \leftrightarrow \top$
- $\models \mathrel{\Box\!\!\!\sim}_i(\alpha \rightarrow \beta) \rightarrow (\mathrel{\Box\!\!\!\sim}_i\alpha \rightarrow \mathrel{\Box\!\!\!\sim}_i\beta)$ (NK)
- $\models \mathrel{\Box\!\!\!\sim}_i(\alpha \wedge \beta) \leftrightarrow (\mathrel{\Box\!\!\!\sim}_i\alpha \wedge \mathrel{\Box\!\!\!\sim}_i\beta)$ (NR)
- $\models \Box_i\alpha \rightarrow \mathrel{\Box\!\!\!\sim}_i\alpha$ (N)

A few rules

$$(RNN)\ \frac{\alpha}{\mathrel{\Box\!\!\!\sim}_i\alpha} \qquad (NRK)\ \frac{(\alpha_1 \wedge \ldots \wedge \alpha_k) \rightarrow \beta}{(\mathrel{\Box\!\!\!\sim}_i\alpha_1 \wedge \ldots \wedge \mathrel{\Box\!\!\!\sim}_i\alpha_k) \rightarrow \mathrel{\Box\!\!\!\sim}_i\beta}$$

Defeasible Modes of Reasoning

Entailment

- *Knowledge base* $\mathcal{K}$: arbitrary set of $\mathcal{L}$-formulas
- $\mathscr{P} \Vdash \mathcal{K}$ iff $\mathscr{P} \Vdash \alpha$ for every $\alpha \in \mathcal{K}$

Definition

$\mathcal{K}$ *entails* α (denoted $\mathcal{K} \models \alpha$) iff for every $\mathscr{P}$, if $\mathscr{P} \Vdash \mathcal{K}$, then $\mathscr{P} \Vdash \alpha$

Theorem

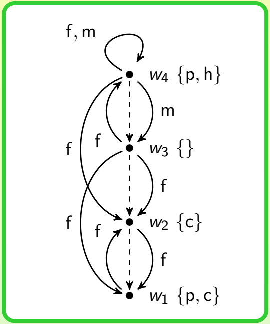
Let $\mathit{Cn}(\mathcal{K}) \equiv_{\mathit{def}} \{\alpha \mid \mathcal{K} \models \alpha\}$. *Then*

- $\mathcal{K} \subseteq \mathit{Cn}(\mathcal{K})$ *(Inclusion)*
- $\mathit{Cn}(\mathcal{K}) = \mathit{Cn}(\mathit{Cn}(\mathcal{K}))$ *(Idempotency)*
- *If* $\mathcal{K}_1 \subseteq \mathcal{K}_2$, *then* $\mathit{Cn}(\mathcal{K}_1) \subseteq \mathit{Cn}(\mathcal{K}_2)$ *(Monotonicity)*

Defeasible Modes of Reasoning

Example in reasoning about actions

- Let $\mathcal{P} = \{\mathsf{p}, \mathsf{c}, \mathsf{h}\}$ and $\mathcal{A} = \{\mathsf{f}, \mathsf{m}\}$



- $\mathscr{P} \Vdash (\mathsf{p} \wedge \neg \mathsf{c}) \leftrightarrow \mathsf{h}$
- $w_4 \in [\![\mathsf{h} \wedge \widetilde{\Diamond}_{\mathsf{f}} \neg \mathsf{h}]\!]$
- $w_1 \in [\![\widetilde{\Box}_{\mathsf{m}} \bot]\!]$
- $\mathscr{P} \Vdash \neg \mathsf{p} \rightarrow \widetilde{\Box}_{\mathsf{f}} \mathsf{p}$ but $\mathscr{P} \not\Vdash \neg \mathsf{p} \rightarrow \Box_{\mathsf{f}} \mathsf{p}$
- $\mathscr{P} \Vdash \mathsf{c} \rightarrow \widetilde{\Box}_{\mathsf{f}} \neg \mathsf{h}$

$$\mathcal{K} = \left\{ \begin{array}{c} (\mathsf{p} \wedge \neg \mathsf{c}) \leftrightarrow \mathsf{h}, \quad \mathsf{h} \rightarrow \widetilde{\Diamond}_{\mathsf{m}} \top, \\ \mathsf{p} \rightarrow \widetilde{\Box}_{\mathsf{f}} \neg \mathsf{p}, \quad \mathsf{c} \rightarrow \widetilde{\Box}_{\mathsf{f}} \mathsf{c}, \quad \Diamond_{\mathsf{f}} \neg \mathsf{h} \end{array} \right\}$$

- $\mathcal{K} \models \mathsf{p} \rightarrow \widetilde{\Box}_{\mathsf{f}} \neg \mathsf{h}$
- $\mathcal{K} \models \widetilde{\Box}_{\mathsf{m}} \bot \rightarrow (\neg \mathsf{p} \vee \mathsf{c})$
- $\mathcal{K} \models (\mathsf{p} \vee \mathsf{c}) \rightarrow \widetilde{\Box}_{\mathsf{f}} \neg \mathsf{h}$

Tableaux for Defeasible Modalities

$$(\bot)\ \frac{n :: \alpha,\ n :: \neg\alpha}{n :: \bot} \qquad (\neg)\ \frac{n :: \neg\neg\alpha}{n :: \alpha} \qquad (\wedge)\ \frac{n :: \alpha \wedge \beta}{n :: \alpha,\ n :: \beta} \qquad (\vee)\ \frac{n :: \neg(\alpha \wedge \beta)}{n :: \neg\alpha \mid n :: \neg\beta}$$

$$(\Box_i)\ \frac{n :: \Box_i\alpha\ ;\ n \xrightarrow{i} n'}{n' :: \alpha} \qquad (\Diamond_i)\ \frac{n :: \neg\Box_i\alpha}{n'^{\star} :: \neg\alpha\ ;\ \Gamma'_1 \mid n'^{\star} :: \neg\alpha\ ;\ \Gamma'_2}, \text{ where:}$$

$$\Gamma'_1 = \{n \xrightarrow{i} n'^{\star},\ n'^{\star} \in \min_{\prec} \Sigma_i(n)\}$$
$$\Gamma'_2 = \{n \xrightarrow{i} n'^{\star},\ n \xrightarrow{i} n''^{\star},\ n''^{\star} \prec n'^{\star},\ n''^{\star} \in \min_{\prec} \Sigma_i(n)\}$$

$$(\mathrel{\Box\!\!\!\sim}_i)\ \frac{n :: \mathrel{\Box\!\!\!\sim}_i\alpha\ ;\ n \xrightarrow{i} n',\ n' \in \min_{\prec} \Sigma_i(n)}{n' :: \alpha} \qquad (\mathrel{\Diamond\!\!\!\sim}_i)\ \frac{n :: \neg\mathrel{\Box\!\!\!\sim}_i\alpha}{n'^{\star} :: \neg\alpha\ ;\ n \xrightarrow{i} n'^{\star},\ n'^{\star} \in \min_{\prec} \Sigma_i(n)}$$

Theorem

The tableau calculus for defeasible modalities is sound and complete with respect to the modal preferential semantics

Adding Argument Forms

We define defeasible consequence on the more expressive language

- Now: $\mid\!\sim\ \subseteq \widetilde{\mathcal{L}} \times \widetilde{\mathcal{L}}$
- Semantics as before: $\mathscr{P} \Vdash \alpha \mid\!\sim \beta$ iff $\min_{\prec}[\![\alpha]\!] \subseteq [\![\beta]\!]$

Example

- $\mathsf{p} \mid\!\sim \approx\!\!\!\!\Box_{\mathsf{f}}\neg\mathsf{p}$
- $\Box_{\mathsf{f}}\neg\mathsf{p} \mid\!\sim \approx\!\!\!\!\Box_{\mathsf{f}}\neg\mathsf{h}$
- $\approx\!\!\!\!\Box_{\mathsf{m}}\bot \mid\!\sim \neg\mathsf{h}$
- $\top \mid\!\sim \approx\!\!\!\!\Box_{\mathsf{A}}(\mathsf{p} \rightarrow \mathsf{c})$

Important in getting to a coherent theory of defeasible reasoning

Adding Argument Forms

Given a set of statements $\alpha \mid\!\sim \beta$, what can we derive?

- *Defeasible knowledge bases* $\mathcal{K}^{\mid\sim}$
- $\mathscr{P} \Vdash \mathcal{K}^{\mid\sim}$ iff $\mathscr{P} \Vdash \alpha \mid\!\sim \beta$ for every $\alpha \mid\!\sim \beta \in \mathcal{K}$

Definition

$\alpha \mid\!\sim \beta$ is *preferentially entailed* by $\mathcal{K}^{\mid\sim}$ iff every preferential model $\mathscr{P}$ satisfying all statements in $\mathcal{K}^{\mid\sim}$ also satisfies $\alpha \mid\!\sim \beta$

Theorem

Let $Cn(\mathcal{K}^{\mid\sim}) \equiv_{def} \{\alpha \mid\!\sim \beta \mid \mathcal{K}^{\mid\sim} \models \alpha \mid\!\sim \beta\}$. *Then*

- $\mathcal{K}^{\mid\sim} \subseteq Cn(\mathcal{K}^{\mid\sim})$ *(Inclusion)*
- $Cn(\mathcal{K}^{\mid\sim}) = Cn(Cn(\mathcal{K}^{\mid\sim}))$ *(Idempotency)*
- *If* $\mathcal{K}_1^{\mid\sim} \subseteq \mathcal{K}_2^{\mid\sim}$, *then* $Cn(\mathcal{K}_1^{\mid\sim}) \subseteq Cn(\mathcal{K}_2^{\mid\sim})$ *(Monotonicity)*

Adding Argument Forms

Lemma

$\mathscr{P} \Vdash \alpha$ *if and only if* $\neg\alpha \mathrel{|\!\sim}_{\mathscr{P}} \bot$

Theorem

Let $\mathcal{K} \subseteq \widetilde{\mathcal{L}}$, *and let* $\mathcal{K}^{|\!\sim} = \{\neg\alpha \mathrel{|\!\sim} \bot \mid \alpha \in \mathcal{K}\}$. *Then*
$\mathcal{K} \models \alpha$ *if and only if* $\mathcal{K}^{|\!\sim} \models \neg\alpha \mathrel{|\!\sim} \bot$

Proof system for the more expressive $\mathrel{|\!\sim}$

- Soundness of all KLM-style rules ✓
- Completeness via Giordano et al.'s tableau for $\mathrel{|\!\sim}$ (to be done)

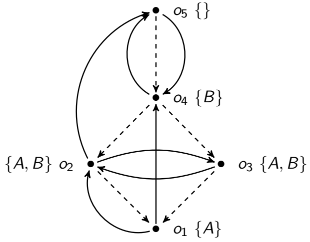
Defeasible Role Restrictions in DLs

Extend a DL with *defeasible quantifiers*

- $\tilde{\forall} r.C$ and $\tilde{\exists} r.C$

Preferential DL interpretations $\mathcal{I} := \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \prec \rangle$

- $(\tilde{\forall} r.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid \min_{\prec} r^{\mathcal{I}}(x) \subseteq C^{\mathcal{I}}\}$
- $(\tilde{\exists} r.C)^{\mathcal{I}} := \{x \in \Delta^{\mathcal{I}} \mid \min_{\prec} r^{\mathcal{I}}(x) \cap C^{\mathcal{I}} \neq \emptyset\}$



- $\mathcal{I} \Vdash A \sqsubseteq \tilde{\forall} r.(A \sqcap B)$
- $\mathcal{I} \not\Vdash A \sqsubseteq \forall r.(A \sqcap B)$

Conclusion

What we have done

- Moved beyond the (propositional) KLM approach
- Moved beyond defeasible argument forms
- Definition of 'weaker' versions of classical modalities
- Provision of a core formalism for further extensions
- Tableau method for defeasible modalities

To do list

- Generalization to a multi-preference setting
- Integration of our tableau into available tableaux for $\mid\!\sim$
- Study of specific modal systems and respective properties
- Rational case

References

- K. Britz, T. Meyer & I. Varzinczak. Preferential Reasoning for Modal Logic. In *Proc. of Methods for Modalities*. Osuna, Spain, 2011.
- R. Booth, T. Meyer & I. Varzinczak. PTL: A Propositional Typicality Logic. In *Proc. of JELIA*. Toulouse, France, 2012.
- K. Britz & I. Varzinczak. Defeasible Modalities. In *Proc. of TARK*. Chennai, India, to appear in 2013.

For more

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