ATHLONE INSTITUTE OF TECHNOLOGY
School of Engineering
Model Answers/Marking Scheme

Lecturer. Mark Daly	**Subject.** Mathematics 1
Course. BSc Software Design (Game/Web Dev) Year 1	

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(4) For descriptive questions please indicate clearly the type of answer required.

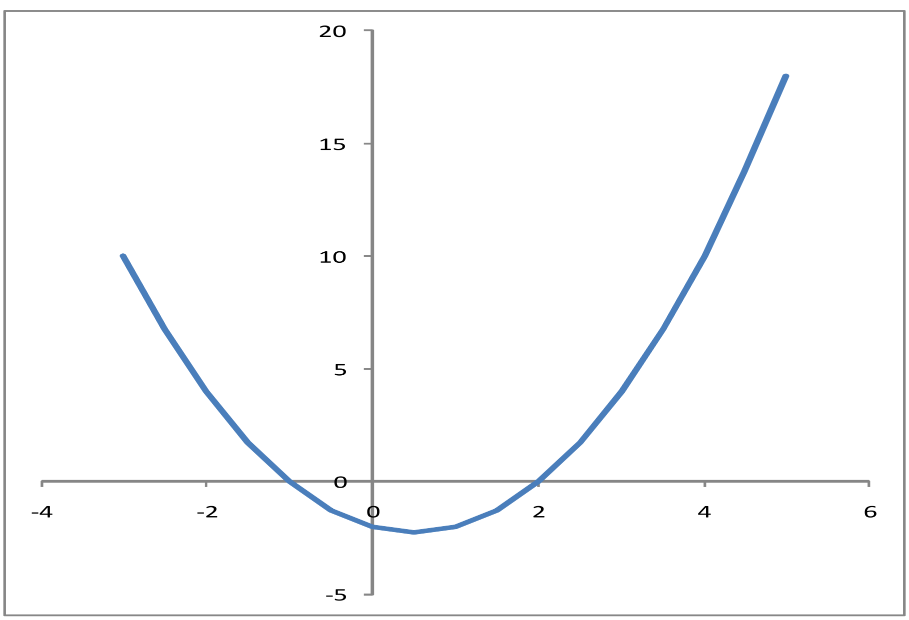
Ques No. **Marks.**

1(a) (i)

Determine the points (x,y) from the table below:

x	x^2	$-x$	-2	$f(x)$
-3	9	3	-2	10
-2.5	6.25	2.5	-2	6.75
-2	4	2	-2	4
-1.5	2.25	1.5	-2	1.75
-1	1	1	-2	0
-0.5	0.25	0.5	-2	-1.25
0	0	0	-2	-2
0.5	0.25	-0.5	-2	-2.25
1	1	-1	-2	-2
1.5	2.25	-1.5	-2	-1.25
2	4	-2	-2	0
2.5	6.25	-2.5	-2	1.75
3	9	-3	-2	4
3.5	12.25	-3.5	-2	6.75
4	16	-4	-2	10
4.5	20.25	-4.5	-2	13.75
5	25	-5	-2	18

Graph shown below



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1(a)

(ii) From the graph and the table above, the function crosses the x-axis (i.e. the function is zero) when x = -1 and 2

(iii) It is apparent from the graph and the table above that the function has a turning point in the vicinity of (0.5,-2.25); i.e. when x = 0.5. **10**

(b) Determine the points (x,y) from the table below:

x	x^3	$-3x^2$	$+4x$	-5	$f(x)$
-4	-64	-48	-16	-5	-133
-3.5	-42.88	-36.75	-14	-5	-98.63
-3	-27	-27	-12	-5	-71
-2.5	-15.63	-18.75	-10	-5	-49.38
-2	-8	-12	-8	-5	-33
-1.5	-6.75	-6.75	-6	-5	-21.13
-1	-1	-3	-4	-5	-13
-0.5	-0.13	-0.75	-2	-5	-7.88
0	0	0	0	-5	-5
0.5	0.13	-0.75	2	-5	-3.63
1	1	-3	4	-5	-3
1.5	3.38	-6.75	6	-5	-2.38
2	8	-12	8	-5	-1
2.5	15.63	-18.75	10	-5	1.88
3	27	-27	12	-5	7
3.5	42.88	-36.75	14	-5	15.13
4	64	-48	16	-5	27
4.5	91.13	-60.75	18	-5	43.38
5	125	-75	20	-5	65
5.5	166.38	-90.75	22	-5	92.63
6	216	-108	24	-5	127
6.5	274.63	-126.75	26	-5	168.88
7	343	-147	28	-5	219
7.5	421.88	-168.75	30	-5	278.13
8	512	-192	32	-5	347

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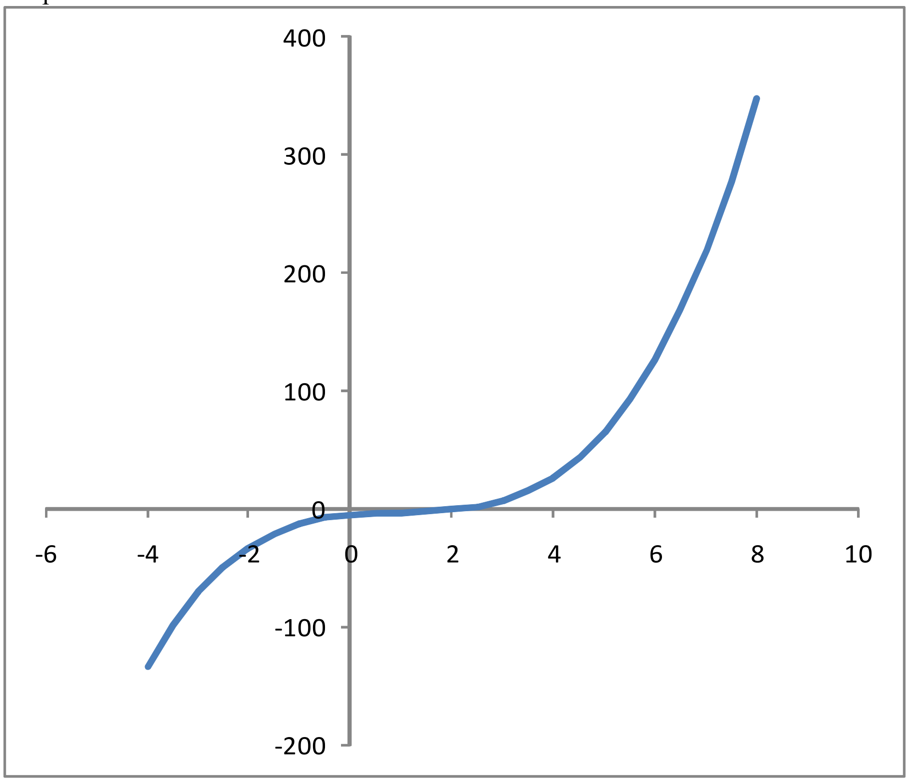
Ques No. | **Marks.**

1(a) (i)

Graph shown below:



(ii) From the graph and the table above, the function crosses the x-axis (i.e. the function is zero) when x is between 2 and 2.5

(iii) It is apparent from the graph above that this cubic equation doesn't exhibit the usual twin turning points but rather a flat region in the vicinity of the x-axis crossover. Therefore the determination of a turning point is immaterial.

10

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Ques No. | **Marks.**

2(a)

Determine the value(s) of x for which the determinant is zero. The complementary space is the set of values for which the matrix is non-singular.

(i)

$$\begin{vmatrix} 1 & 2 & -2 \\ x & -1 & 5 \\ 3 & 1-x & 4 \end{vmatrix} = (1)(-9+5x)+2(15-4x)-2(x-x^2+3)=0$$
$$= 2x^2-5x+15=0$$
$$\Rightarrow \ determinant \neq 0 \ \forall \ x\in\mathbb{R}$$

The matrix is non-singular $\forall \ x\in\mathbb{R}$ **4**

(ii)

$$\begin{vmatrix} 3-x & 4 & 6 \\ 0 & 1-x & 2 \\ 0 & 0 & 2+x \end{vmatrix} = (3-x)(1-x)(2+x)=0$$
$$\Rightarrow \ determinant \ = 0 \Leftrightarrow x=-2,1,3$$

The matrix is non-singular $\forall \ x\in\mathbb{R}\setminus\{-2,1,3\}$ **4**

(b)

Given the matrices

$$A=\begin{pmatrix} 2 & 1 & -1 \\ 5 & -2 & 1 \\ -1 & -4 & 3 \end{pmatrix} \text{ and } B=\begin{pmatrix} 2 & -7 & 0 \\ 4 & 0 & 1 \\ 5 & 9 & -2 \end{pmatrix}$$

we need to determine if the products $C=A\cdot B$ and $D=B\cdot A$ exist and if the resulting matrices, C and D, are equal. As A and B are both square matrices of the same dimensions, then both C and D exist and are 3x3 matrices.

First calculate $C=A\cdot B$:

$$C=A\cdot B=\begin{pmatrix} 2 & 1 & -1 \\ 5 & -2 & 1 \\ -1 & -4 & 3 \end{pmatrix}\cdot\begin{pmatrix} 2 & -7 & 0 \\ 4 & 0 & 1 \\ 5 & 9 & -2 \end{pmatrix}=\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

where

$$c_{11}=\sum_{k=1}^{3} a_{1k}b_{k1}=2\times2+1\times4+(-1)\times5=4+4-5=3$$
$$c_{12}=\sum_{k=1}^{3} a_{1k}b_{k2}=2\times(-7)+1\times0+(-1)\times9=-14+0-9=-23$$
$$c_{13}=\sum_{k=1}^{3} a_{1k}b_{k3}=2\times0+1\times1+(-1)\times(-2)=0+1+2=3$$
$$c_{21}=\sum_{k=1}^{3} a_{2k}b_{k1}=5\times2+(-2)\times4+1\times5=10-8+5=7$$
$$c_{22}=\sum_{k=1}^{3} a_{2k}b_{k2}=5\times(-7)+(-2)\times0+1\times9=-35+0+9=-26$$
$$c_{23}=\sum_{k=1}^{3} a_{2k}b_{k3}=5\times0+(-2)\times1+1\times(-2)=0-2-2=-4$$

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Ques No. **Marks.**

2(b)

$$c_{31}=\sum_{k=1}^{3} a_{3k} b_{k1}=(-1)\times 2+(-4)\times 4+3\times 5=-2-16+15=-3$$

$$c_{32}=\sum_{k=1}^{3} a_{3k} b_{k2}=(-1)\times(-7)+(-4)\times 0+3\times 9=7+0+27=34$$

$$c_{33}=\sum_{k=1}^{3} a_{3k} b_{k3}=(-1)\times 0+(-4)\times 1+3\times(-2)=0-4-6=-10$$

Now calculate $D=B\cdot A$:

$$D=B\cdot A=\begin{pmatrix} 2 & -7 & 0 \\ 4 & 0 & 1 \\ 5 & 9 & -2 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 & -1 \\ 5 & -2 & 1 \\ -1 & -4 & 3 \end{pmatrix}=\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

where

$$d_{11}=\sum_{k=1}^{3} b_{1k} a_{k1}=2\times 2+(-7)\times 5+0\times(-1)=4-35+0=-31$$

$$d_{12}=\sum_{k=1}^{3} b_{1k} a_{k2}=2\times 1+(-7)\times(-2)+0\times(-4)=2+14+0=16$$

$$d_{13}=\sum_{k=1}^{3} b_{1k} a_{k3}=2\times(-1)+(-7)\times 1+0\times 3=-2-7+0=-9$$

$$d_{21}=\sum_{k=1}^{3} b_{2k} a_{k1}=4\times 2+0\times 5+1\times(-1)=8+0-1=-7$$

$$d_{22}=\sum_{k=1}^{3} b_{2k} a_{k2}=4\times 1+0\times(-2)+1\times(-4)=4+0-4=0$$

$$d_{23}=\sum_{k=1}^{3} b_{2k} a_{k3}=4\times(-1)+0\times 1+1\times 3=-4+0+3=-1$$

$$d_{31}=\sum_{k=1}^{3} b_{3k} a_{k1}=5\times 2+9\times 5+(-2)\times(-1)=10+45+2=57$$

$$d_{32}=\sum_{k=1}^{3} b_{3k} a_{k2}=5\times 1+9\times(-2)+(-2)\times(-4)=5-18+8=-5$$

$$d_{33}=\sum_{k=1}^{3} b_{3k} a_{k3}=5\times(-1)+9\times 1+(-2)\times 3=-5+9-6=-2$$

Clearly $C=A\cdot B\neq D=B\cdot A$ **12**

3(a) (i)

Taylor series: $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

$f(x)=\cos(x) \qquad x_0=\pi/2$

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3(a) (i)

Then

$$f(\pi)=\cos(\pi/2)=0$$
$$f^{(1)}(x)=-\sin(x) \quad \Rightarrow f^{(1)}(\pi/2)=-1$$
$$f^{(2)}(x)=-\cos(x) \quad \Rightarrow f^{(2)}(\pi/2)=0$$
$$f^{(3)}(x)=\sin(x) \quad \Rightarrow f^{(3)}(\pi/2)=1$$
$$f^{(4)}(x)=\cos(x) \quad \Rightarrow f^{(4)}(\pi/2)=0$$
$$f^{(5)}(x)=-\sin(x) \quad \Rightarrow f^{(4)}(\pi/2)=-1$$

The first six terms in the Taylor series are defined:

$$T_5(x)=\sum_{n=0}^{4}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$
$$=f(\pi/2)+f^{(1)}(\pi/2)(x-x_0)+\frac{f^{(2)}(\pi/2)}{2!}(x-x_0)^2+\ldots+\frac{f^{(5)}(\pi/2)}{5!}(x-x_0)^5$$
$$=0+(-1)(x-\pi/2)+0+\frac{(x-\pi/2)^3}{3!}+0+(-1)\frac{(x-\pi/2)^5}{5!}$$

5

(ii)

$$f(x)=\ln|x| \qquad x_0=1$$

Then

$$f(0)=\ln|1|=0$$
$$f^{(1)}(x)=1/(x) \quad \Rightarrow f^{(1)}(1)=1$$
$$f^{(2)}(x)=-1/(x)^2 \quad \Rightarrow f^{(2)}(1)=-1$$
$$f^{(3)}(x)=2/(x)^3 \quad \Rightarrow f^{(3)}(1)=2$$
$$f^{(4)}(x)=-6/(x)^4 \quad \Rightarrow f^{(4)}(1)=-6$$
$$f^{(5)}(x)=24/(x)^5 \quad \Rightarrow f^{(5)}(1)=24$$

The first six terms in the Taylor series are defined:

$$T_5(x)=\sum_{n=0}^{5}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$
$$=f(1)+f^{(1)}(1)(x-x_0)+\frac{f^{(2)}(1)}{2!}(x-x_0)^2+\ldots+\frac{f^{(5)}(1)}{5!}(x-x_0)^5$$
$$=0+(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\frac{(x-1)^5}{5}$$

5

(b)

$$\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}=0.5$$

We've already determined that

$$T_5(x)=-(x-\pi/2)+\frac{(x-\pi/2)^3}{3!}-\frac{(x-\pi/2)^5}{5!}$$

for cos(x) about $x_0=\pi/2$.

Substitute $\pi/3$ for x in $T_5(x)$ to get the approximation for cos(x) at $\pi/3$.

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3(b)

$$T_5\left(\frac{\pi}{3}\right) = -(-\pi/6) + \frac{(-\pi/6)^3}{6} - \frac{(-\pi/6)^5}{120} \simeq 0.500002132$$

So the error is

$$|f(x) - T_5(x)| = |0.5 - 0.500002132| = 0.000002132 \simeq 0.000003$$

10

4(a)

$$f(x) = \left(\cos^2(x) + \sin^2(x)\right)\tan(2x)$$

As $\cos^2(x) + \sin^2(x) = 1$ then $f(x) = \tan(2x)$. Using the chain rule we get

$$\frac{d}{dx} f(x) = 2\sec^2(2\text{x})$$

5

(b)

$$f(x) = e^{\ln|3x^2-1|}$$

Here use the identity

$$e^{\ln|u(x)|} = u(x)$$

with

$$u(x) = 3x^2 - 1$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} 3x^2 - 1 = 6x$$

5

(c)

$$f(x) = \cos(\tan(\sin(2x)))$$

You use the chain rule 3 times to get

$$\frac{d}{dx} f(x) = \underbrace{-\sin(\tan(\sin(2x)))}_{1^{st}\,Chain\,Rule} \times \underbrace{\sec^2(\sin(2\text{x}))}_{2^{nd}\,Chain\,Rule} \times \underbrace{\cos(2x) \times 2}_{3^{rd}\,Chain\,Rule}$$

5

(d)

$$f(x) = \frac{x^7 - 1}{x^6 + x^5 + x^4 + x^3 + x^2 + x + 1}$$

From your lectures you know that

$$x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

Then and

$$\Rightarrow \frac{d}{dx} f(x) = \frac{d}{d\,x} \frac{x^7 - 1}{x^6 + x^5 + x^4 + x^3 + x^2 + x + 1} = \frac{d}{dx}(x-1) = 1$$

5