Digital Media

Conor Keighrey

Office: U214

Email: ckeighrey@ait.ie

What is a Number Base?

"A number base is a way of representing a numerical value…"

ckeighrey@ait.ie

Number Bases

- What is a number base?

- A number base is a way of representing a numerical value.

- Information in a computer is best visualized as a string of 1’s and 0’s. If the information is a number, it is natural to store the number using (perhaps a small modification of) base 2.

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Number Bases

- The most important number bases are:
  - **Base_2** - Binary Numeral System
  - **Base_8** - Octal Numeral System
  - **Base_{10}** - Decimal Numeral System
  - **Base_{16}** - Hexadecimal Numeral System

Number Bases

- You have to be able to use binary and decimal number bases along with hexadecimal, so it is necessary to have the knowledge required to change between these number bases.

Number Bases

- Where are these numbers used?
  - $Base_2$ Binary Numeral System
    - Used internally by all computers – the two digits in base 2 are “0” and “1” came from switches from on to off status!
  - $Base_8$ Octal Numeral System
    - Occasionally used by computers – often used to represent imaging

Number Bases

- Where are these numbers used?
  - $Base_{10}$ Decimal Numeral System
    - Used by humans daily!
  - $Base_{16}$ Hexadecimal Numeral System
    - Often used in addressing or in HTML used to identify colours (RRGGBB)

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Lets Examine Each Base

- The most important number bases are:
  - **$Base_2$** - Binary Numeral System
  - **$Base_8$** - Octal Numeral System
  - **$Base_{10}$** - Decimal Numeral System
  - **$Base_{16}$** - Hexadecimal Numeral System

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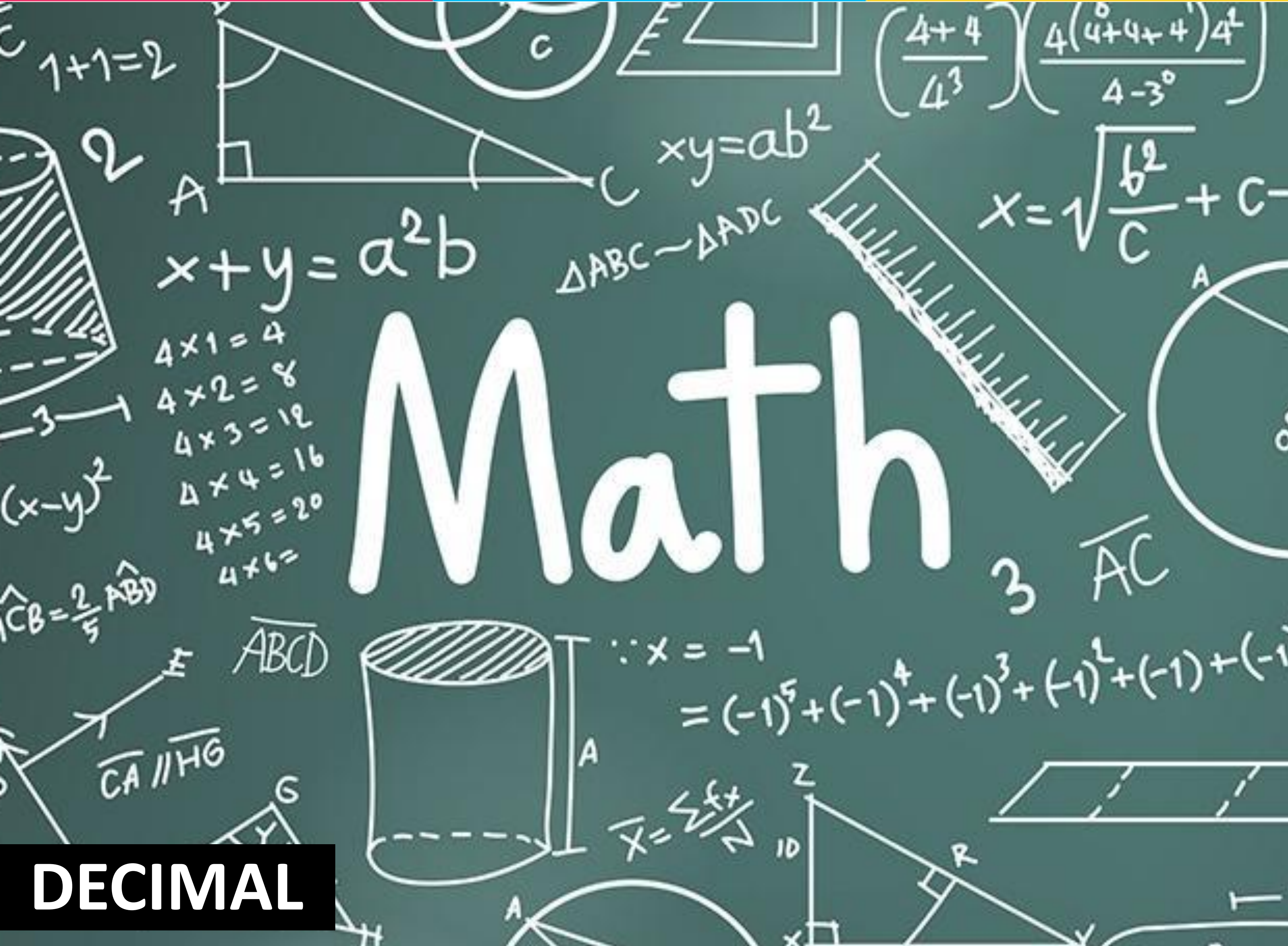
Math
1+1=2
xy=ab²
x+y=a²b
ΔABC~ΔADC
4×1=4
4×2=8
4×3=12
4×4=16
4×5=20
4×6=
(x−y)²
ABCD
CA // HG
AC
∵ x = −1
= (−1)⁵+(−1)⁴+(−1)³+(−1)²+(−1)+

DECIMAL

Decimal Numbers Base$_{10}$

- Decimal 10 is familiar to us all, we use it everyday.

- 10 possible digits
  - 0,1,2,3,4,5,6,7,8,9,

Decimal Numbers Base$_{10}$

	Thousand	hundred	ten	unit
			4	5
		2	3	4
	3	4	5	9

- We calculate place value automatically
- 45, 234, 3459

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Decimal Numbers Base$_{10}$

- Reminder re: exponents, power to which number is raised

exponent	0	1	2	3
10	1	10	100 (10 X10)	1000 (10X10X10)
2	1	2	4 (2x2)	8 (2x2x2)
8	1	8	64 (8x8)	512 (8x8x8)
16	1	16	256 (16x16)	4098 (16x16x16)

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Decimal Numbers Base_{10}

- Decimal numbers: place value of each digit expressed as power of 10

	10^3	10^2	10^1	10^0
			4	5
		2	3	4
	3	4	5	9

- Numbers: 45 $= 4 \times 10^1 + 5 \text{ X } 10^0$
  $= 4 \text{ X } 10 + 5 \text{ X}1$
  $= 40 + 5 = 45$

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Decimal Numbers $Base_{10}$

- 1234

$= (1 * 10^3) + (2 * 10^2) + (3 * 10^1) + (4 * 10^0)$

$= 1000 + 200 + 30 + 4 = 1234$

- Write the following decimal numbers in expanded form:
  - 1024, 20480, 10

BINARY

Grouping Binary Numbers

Name	Size	Visually
Bit	A “1” or “0”	0
Nibble	Four Bits	0000
Byte	Eight Bits	00000000
Word	Two Bytes	00000000 00000000
Long Word	Four Bytes	?

Binary: $Base_2$

- Computers do not use numbers in the form 2, 4, 6, 8, etc. to do mathematical calculations.
- Instead they use groups of 1’s or 0’s.
- So why is Binary $Base_2$?
- It uses just two values “0” or “1”
  - When reading binary numbers you must consider the entire number starting from the right.
  - A binary number is either proceeded by a % or followed by the letter “b” or in Java has prefix “0b”

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Binary: $Base_2$

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- Instead they use groups of 1's or 0's.
- So why is Binary $Base_2$?
- It uses just two values "0" or "1"
  - When reading binary numbers you must consider the entire number starting from the right.
  - A binary number is either proceeded by a "%" or followed by the letter "b" – **otherwise what is it**?

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Binary: $Base_2$

- In Binary, the use of the number “1” in a particular location represents a “value”, the use of the number “0” represents no value in this location.

- Place values in binary are powers of 2

- Consider: Table shown

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Binary: $Base_2$

- Consider the following 4 bits:
  - 1 1 1 1

2^3	2^2	2^1	2^0
1	1	1	1

$$[(1) \times 2^3] + [(\mathbf{1}) \times 2^2] + [(\mathbf{1}) \times 2^1] + [(\mathbf{1}) \times 2^0] =$$

$$[\mathbf{1} \times 8] + [\mathbf{1} \times 4] + [\mathbf{1} \times 2] + [\mathbf{1} \times 1] = 15$$

Binary: $Base_2$

- Consider the following 4 bits:
  - 1 0 1 0

2^3	2^2	2^1	2^0
1	1	1	1
8	4	2	1
8 + 4 + 2 + 1 = 15			

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Binary: $Base_2$

- Consider the following 8 bits:
  - 0 0 1 1 1 1 1 1

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	1	1	1	1	1

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Binary: $Base_2$

- Consider the following 8 bits:
  - 0 0 1 1 1 1 1 1

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	1	1	1	1	1
0	0	32	16	8	4	2	1
0+ 0 + 32 + 16 + 8 + 4 + 2 + 1 = 63							

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Binary: Base_2

- 100101 in Binary – what is this in decimal or how do we convert.

- $[(\mathbf{1}) \times 2^5] + [(\mathbf{0}) \times 2^4] + [(\mathbf{0}) \times 2^3] + [(\mathbf{1}) \times 2^2] + [(\mathbf{0}) \times 2^1] + [(\mathbf{1}) \times 2^0] =$

Binary: $Base_2$

- 100101 in Binary – what is this in decimal or how do we convert.
- $[(1) \times 2^5] + [(\mathbf{0}) \times 2^4] + [(\mathbf{0}) \times 2^3] + [(1) \times 2^2] + [(\mathbf{0}) \times 2^1] + [(1) \times 2^0] =$

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Binary: $Base_2$

- 100101 in Binary – what is this in decimal or how do we convert.

- $[(1) \times 2^5] + [(0) \times 2^4] + [(0) \times 2^3] + [(1) \times 2^2] + [(0) \times 2^1] + [(1) \times 2^0] =$

- $[1 \times 32] + [0 \times 16] + [0 \times 8] + [1 \times 4] + [0 \times 2] + [1 \times 1] = 37$

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Binary: $Base_2$

- Lets try some ourselves & do not use a calculator or go online for this – Do this on paper!
  - Determine 11101 in Decimal
  - Determine 0000000001 in Decimal
  - Determine 1110000010 in Decimal
  - Determine 120000111 in Decimal

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Binary Addition

- Binary Addition Rules: 4 rules!

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

2 in decimal

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Binary Addition

- Follow 4 rules!
- Add individual bits
- Carry "1" if applicable

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

```
 1       1
   10101          21
+  11001       +  25
 -------       -----
  101110          46
```


Connect & Discover

Binary Addition for 3 binary numbers!

- Follow 4 rules!
- Add individual bits
- Carry “1” if applicable

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

$$
\begin{array}{rrrrr}
 & {}^{1} & 1^{1} & 1^{1} & 1 \\
 & {}^{1} & 1^{0} & 1^{0} & 1_{0} \\
 & & +1^{1} & 0^{1} & 1 \\
\hline
10 & 0 & 1 & & 1
\end{array}
$$

ckeighrey@ait.ie

Binary Addition for 2 & 3 binary numbers!

- Les Try Some

```
   0  0  1
 +1  0  1
 ________
```

```
1  ¹1  0¹   1
   1⁰  1  1⁰
 +0  0⁰  1
 ________
   1  1   0  1
```

Binary Subtraction

- Binary Addition Rules: 4 rules!

A	B	A - B
0	0	0
0	1	1*
1	0	1
1	1	0

Binary Subtraction

- Binary Addition Rules: 4 rules!

A	B	A - B
0	0	0
0	1	1*
1	0	1
1	1	0

Borrow from the next more significant bit

Binary Subtraction

- Subtract individual bits!
- Follow 4 rules!
- Remember to borrow from the next most significant bit!

A	B	A - B
0	0	0
0	1	1*
1	0	1
1	1	0

$$\begin{array}{r} 1\ 1\ 0\ 1 \\ -\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

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Binary Subtraction

- Subtract individual bits!
- Follow 4 rules!
- Remember to borrow from the next most significant bit!

A	B	A - B
0	0	0
0	1	1*
1	0	1
1	1	0

```
    0 10
  1 1 0 1
- 1 0 1 1
---------
  0 0 1 0
```

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Decimal to Binary

- This involves a dividing by two, keeping track of the remainder.
- First remainder is “bit 0”
  - (LSB, least-significant bit)
- Second remainder is bit 1 and so on…..

Decimal to Binary

2	31		
2	15	1	
2	7	1	
2	3	1	
2	1	1	
	0	1	

$$31_{10} = 11111_2$$

Decimal to Binary

2	16	
2	8	0
2	4	0
2	2	0
2	1	0
	0	1

$$16_{10} = 10000_2$$

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Octal Numbers: $Base_8$

- Octal numbers are $Base_8$
- Considering why $Base_2$ is called $Base_2$, why $Base_{10}$ is called $Base_{10}$, it becomes obvious then that $Base_8$ is called $Base_8$ because there are how many values used?
- Generally a Octal number is either proceeded by a 0 (in Java) or followed by a o

ckeighrey@ait.ie

Octal Numbers: $Base_8$

- Octal numbers are $Base_8$
- Considering why $Base_2$ is called Base $_2$, why $Base_{10}$ is called $Base_{10}$, it becomes obvious then that $Base_8$ is called $Base_8$ because there are how many values used?
- Because there are 8 possible symbols used:
  - 0, 1, 2, 3, 4, 5, 6, 7

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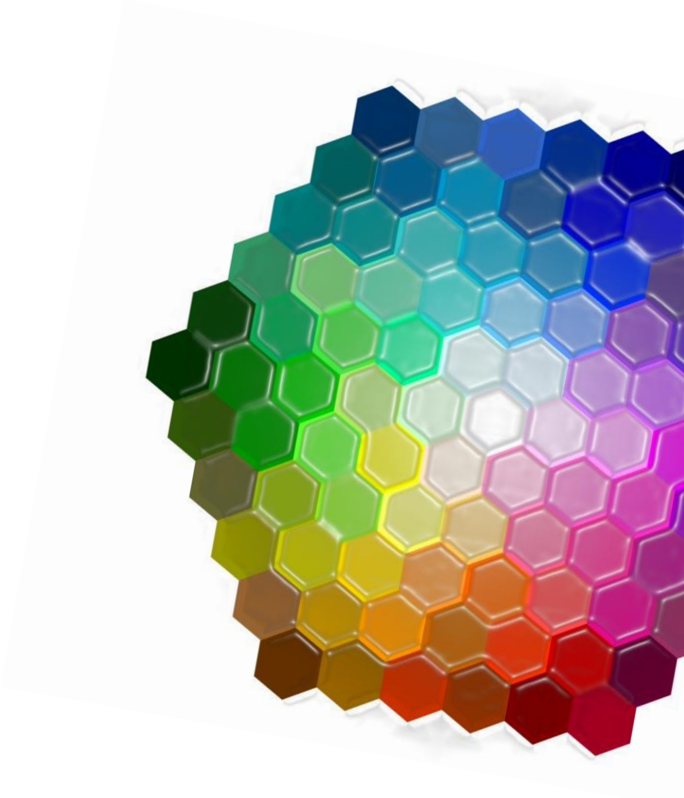
HEXADEXIMAL

Uses of Hexadecimal

- Computer operations are done by binary systems.
- One and Zero or on and off or yes and no.
- Because of the large nature of binary numbers, when talking computer code, they are normally represented in hexadecimal format.



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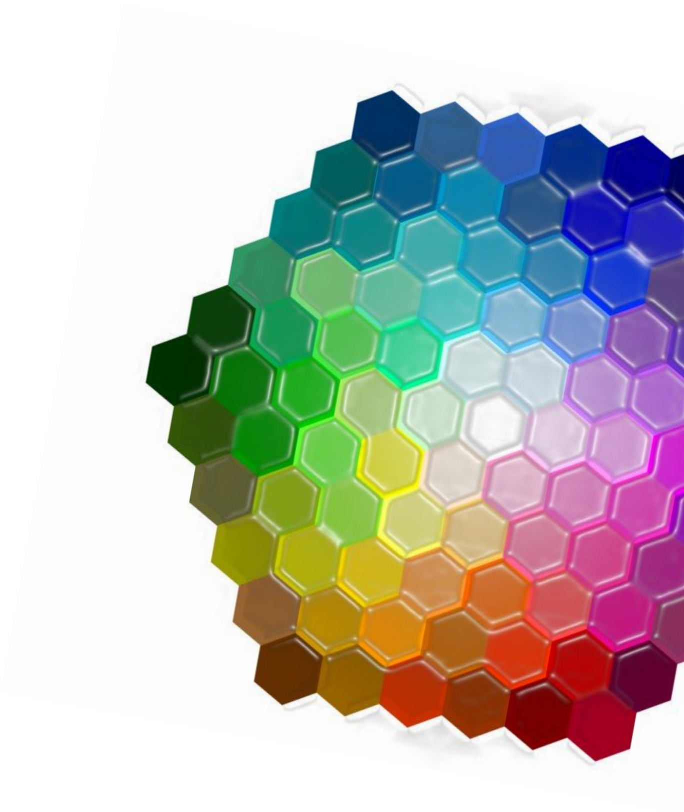
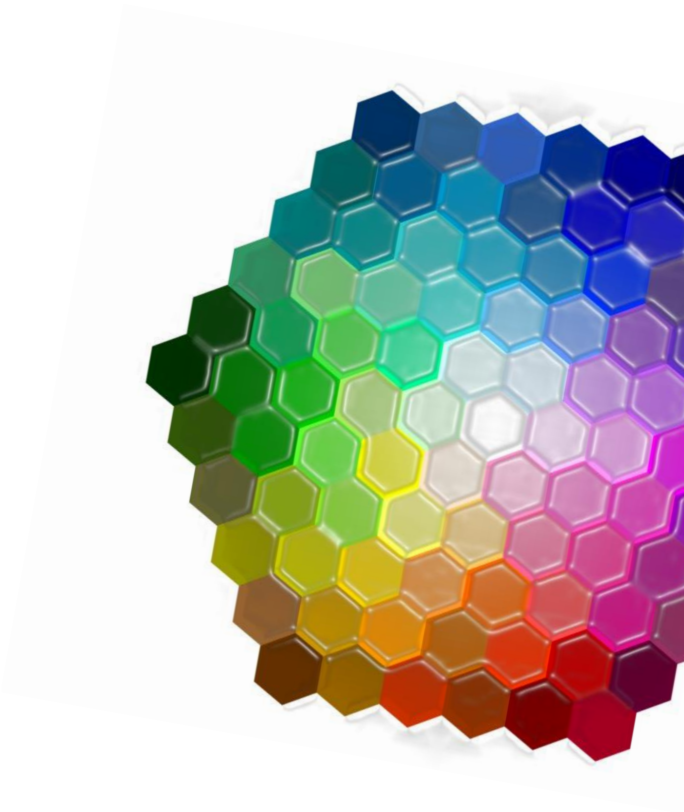
Uses of Hexadecimal

- These can be addresses of RAM.
- The hexadecimal notation is often used in error messages.
- Sometimes it is used in obscure web addresses.
- Also used in HTML code to convey the background colour required.



HTML Color

- The background colour of a webpage is made up of a combination of three colours:
  - Red, Green and Blue.
    - Note the background can also be a picture.
- Each colour is represented by two hexadecimal digits



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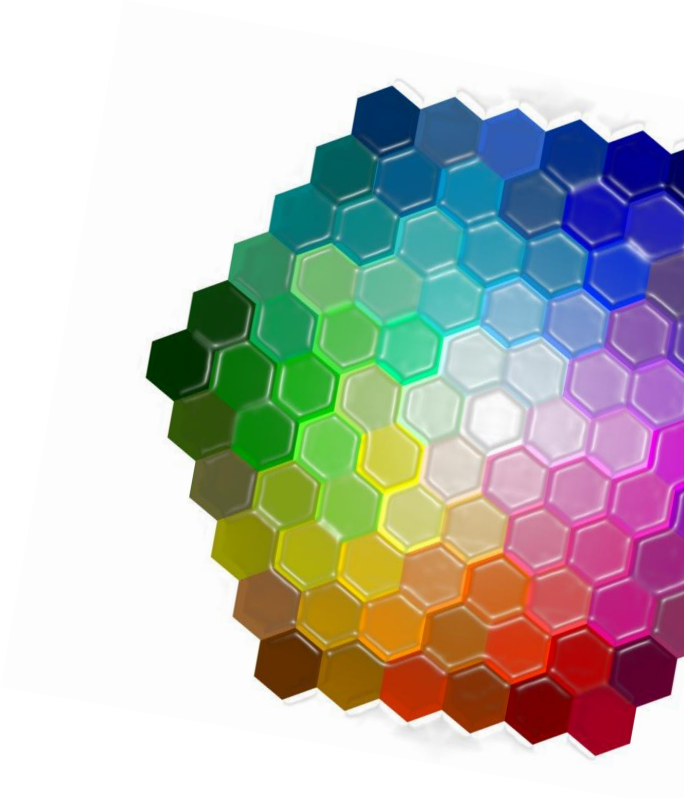
HTML Color

- Example: A4 5F 2D
  - This is known as the RGB color code.
- For each hexadecimal digit there is a choice of 16 values
- 1.6 million colours
  - 16^6 = 1.6 million colours



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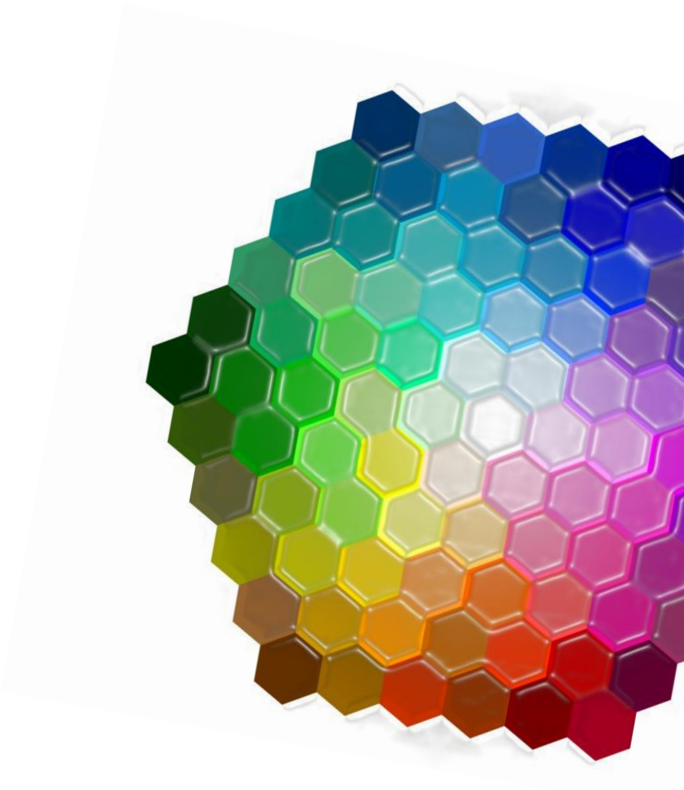
HTML Color

- Go to START, All Programs, Accessories and Notepad
- Type in the following code:
  - <BODY BGCOLOR=#A45F2D></BODY>
  - Save the file as color.html
- Open the file in a web browser
- Change the hex colour code in your Notepad program
- Save and refresh to see new colours.



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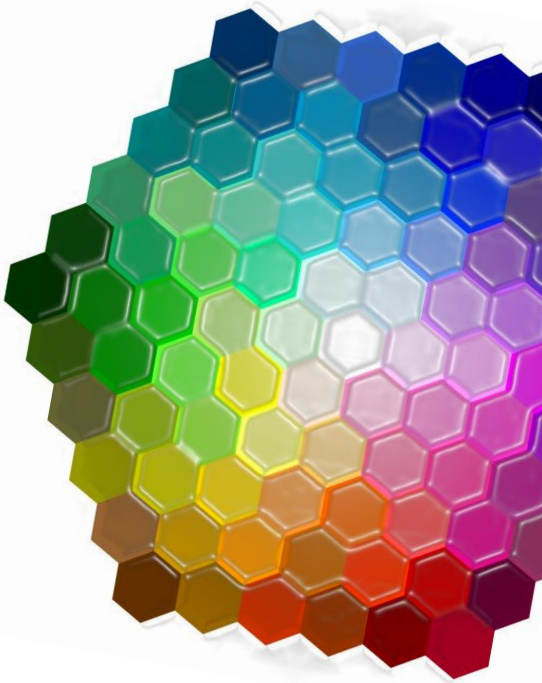
HTML Color

- Black 000000
- White FFFFFF
- Red FF0000
- Green 00FF00
- Blue 0000FF
- Gray 505050
  - i.e. the same amount of each colour.
- Yellow F0F0000
- Orange FF7000
- Brown 905030



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Hexadecimal Numbers: $Base_{16}$

- Hexadecimal numbers are $Base_{16}$

- Considering why $Base_2$ is called Base $_2$, why Base10 is called Base 10, why $Base_8$ is called $Base_8$ ……… because there are how many symbols used?

- Generally a hex number is either proceeded by a "$" or followed by a "h" or in Java proceeded by a: 0x

- Because there are 16 possible symbols used:
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

ckeighrey@ait.ie

Hexadecimal Numbers: $Base_{16}$

Dec	Hex
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

Dec	Hex
10	A
11	B
12	C
13	D
14	E
15	F
16	10
17	11
18	12
19	13

ckeighrey@ait.ie

Hexadecimal Numbers: $Base_{16}$

- Lets see how this is the case:
  - 2AF3 is a number in Hexadecimal format
  - Again each place, means a value!

- 2AF3

  $= (2 * 16^3) + (10 * 16^2) + (15 * 16^1) +(3 * 16^0)$

  $= (2 * 4096) + (10 * 256) + (15 * 16) +(3 * 1)$

  $= (8192) + (2560) + (240) + (3)$

  = 10995 in decimal!

ckeighrey@ait.ie

Hexadecimal Numbers: $Base_{16}$

- Lets try some ourselves & do not use a calculator or go online for this – Do this on paper!
  - Determine 2AF3h in decimal
  - Determine 107h in decimal
  - Determine 2BC3h in decimal
  - Determine 7F6h in decimal

Number Bases Summary

System	Base	Symbols	Used by humans?
Decimal	10	0, 1, … 9	Yes
Binary	2	0, 1	No
Octal	8	0, 1, … 7	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No

ckeighrey@ait.ie

Number Bases Summary

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Number Bases Summary

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

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Number Bases Summary

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

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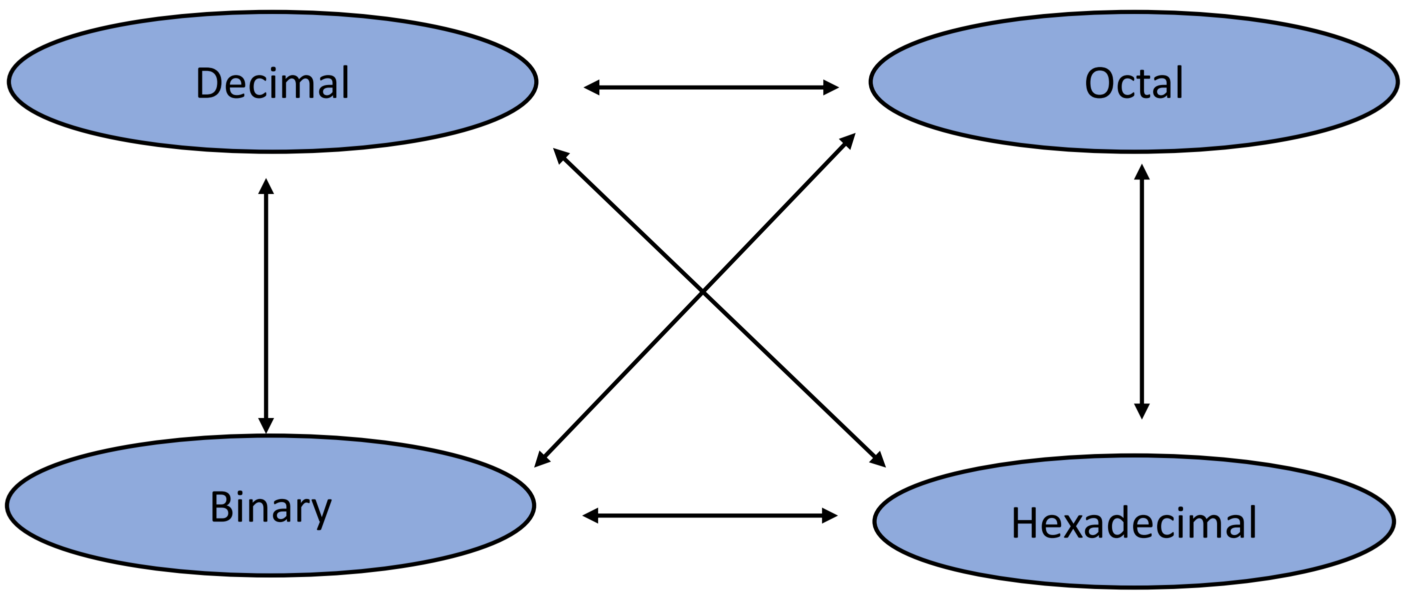
Number Base Conversion & Arithmetic

- Number Base Arithmetic:
  - Addition, Multiplication, Division
- Number Base Conversion



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Number Base Conversion

- $25_{10} = 11001_2 = 31_8 = 19_{16}$

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Decimal to Decimal

- $136_{10} = (1*10^2) + (3*10^1) + (6*10^0)$

  $= (100) + (30) + (6)$

  $= (136)$

ckeighrey@ait.ie

Binary to Hexadecimal

- Starting with LSB (the right most), each group of 4 binary digits is replaced by one hexadecimal digit, pad fill the left most bits with zeros if necessary.

- 1111_2

$$= (1X\ 2^3) + (1X\ 2^2) + (1X\ 2^1) + (1X\ 2^0)$$
$$= 8 + 4 + 2 + 1$$

= 15 in decimal = **F** in Hexadecimal

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Binary to Hexadecimal

binary	1	1	0	1	0	0	1	1	1	1	1	1
hex	D				3				F			

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Hexadecimal to Binary

- Convert each digit in the hexadecimal number into its 4-bit equivalent binary representation
- Convert $(10AB)_{16}$ into its binary equivalent.
- Solution:

1	0	A	B
↓	↓	↓	↓
0001	0000	1010	1011

$\Rightarrow (10AB)_{16} = 0001000010101011_2$

ckeighrey@ait.ie

Hexadecimal to Binary

hex	1				0				A				B			
B I N A R y	0	0	0	1	0	0	0	0	1	0	1	0	1	0	1	1

ckeighrey@ait.ie

Hexadecimal to Binary

- Lets try A0ABH in Binary?

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Decimal to Hexadecimal

- 2546852 → 26DCA4
  - Perform the following conversion:
    - Decimal to Binary
      - 2546852 → 1001101101110010100100
    - Binary to Hexadecimal
      - 1001101101110010100100 → 26DCA4

Conversion Table

- http://www.dewassoc.com/support/msdos/decimal_hexadecimal.htm

Dec	Hex	Bin	Dec	Hex	Bin	Dec	Hex	Bin
0	0	00000000	64	40	01000000	128	80	
1	1	00000001	65	41	01000001	129	81	
2	2	00000010	66	42	01000010	130	82	
3	3	00000011	67	43	01000011	131	83	
4	4	00000100	68	44	01000100	132	84	
5	5	00000101	69	45	01000101	133	85	
6	6	00000110	70	46	01000110	134	86	
7	7	00000111	71	47	01000111	135	87	
8	8	00001000	72	48	01001000	136	88	
9	9	00001001	73	49	01001001	137	89	
10	a	00001010	74	4a	01001010	138	8a	
11	b	00001011	75	4b	01001011	139	8b	
12	c	00001100	76	4c	01001100	140	8c	
13	d	00001101	77	4d	01001101	141	8d	
14	e	00001110	78	4e	01001110	142	8e	
15	f	00001111	79	4f	01001111	143	8f	
16	10	00010000	80	50	01010000	144	90	
17	11	00010001	81	51	01010001	145	9	
18	12	00010010	82	52	01010010	146	9	
19	13	00010011	83	53	01010011	147	9	
20	14	00010100	84	54	01010100	148	9	
21	15	00010101	85	55	01010101	149	9	
22	16	00010110	86	56	01010110	150	9	

Questions

ckeighrey@ait.ie