ATHLONE INSTITUTE OF TECHNOLOGY
School of Engineering
Model Answers/Marking Scheme

Lecturer. Mark Daly	**Subject.** Mathematics 1
Course. BSc Software Design (Game/Web Dev) Year 1	

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(4) For descriptive questions please indicate clearly the type of answer required.

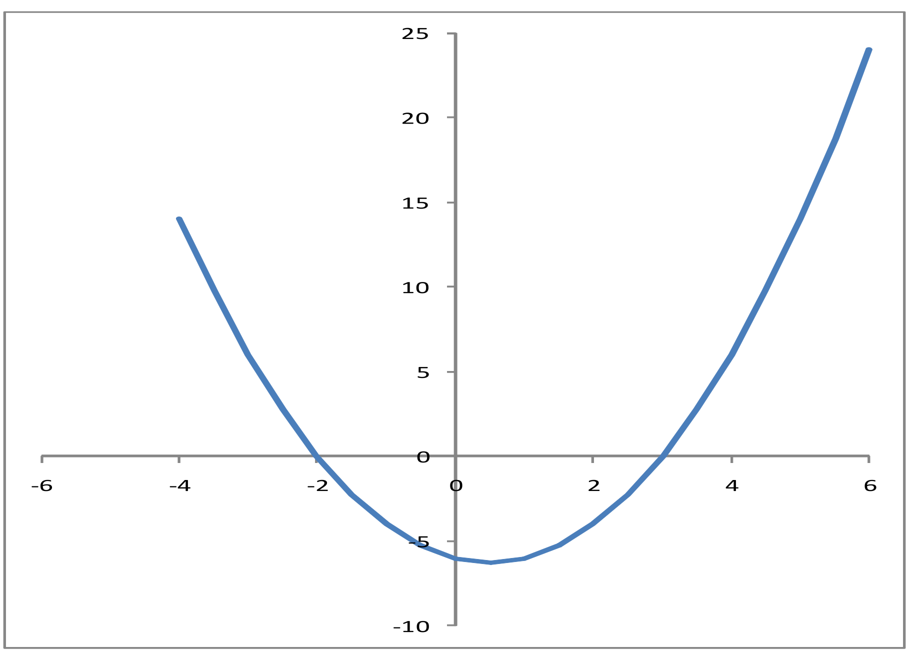
Ques No. **Marks.**

1(a) (i)

Determine the points (x,y) from the table below:

x	x^2	$-x$	-6	$f(x)$
-4	16	4	-6	14
-3	9	3	-6	6
-2	4	2	-6	0
-1.5	2.25	1.5	-6	-2.25
-1	1	1	-6	-4
-0.5	0.25	0.5	-6	-5.25
0	0	0	-6	-6
0.5	0.25	-0.5	-6	-6.25
1	1	-1	-6	-6
1.5	2.25	-1.5	-6	-5.25
2	4	-2	-6	-4
2.5	6.25	-2.5	-6	-2.25
3	9	-3	-6	0
4	16	-4	-6	6
5	25	-5	-6	14
6	36	-6	-6	24

Graph shown below



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Ques No. **Marks.**

1(a)

(ii) From the graph and the table above, the function crosses the x-axis (i.e. the function is zero) when x = -2 and 3

(iii) It is apparent from the graph and the table above that the function has a turning point in the vicinity of (0.5,-6.25); i.e. when x = 0.5. **10**

(b) Determine the points (x,y) from the table below:

x	x^3	$-8x^2$	$+2x$	10	$f(x)$
-3	-27	-72	-6	10	-95
-2.5	-15.63	-50	-5	10	-60.63
-2	-8	-32	-4	10	-34
-1.5	-6.75	-18	-3	10	-14.38
-1	-1	-8	-2	10	-1
-0.5	-0.13	-2	-1	10	6.88
0	0	0	0	10	10
0.5	0.13	-2	1	10	9.13
1	1	-8	2	10	5
1.5	3.38	-18	3	10	-1.63
2	8	-32	4	10	-10
2.5	15.63	-50	5	10	-19.38
3	27	-72	6	10	-29
3.5	42.88	-98	7	10	-38.13
4	64	-128	8	10	-46
4.5	91.13	-162	9	10	-51.88
5	125	-200	10	10	-55
5.5	166.38	-242	11	10	-54.63
6	216	-288	12	10	-50
6.5	274.63	-338	13	10	-40.38
7	343	-392	14	10	-25
7.5	421.88	-450	15	10	-3.13
8	512	-512	16	10	26
8.5	614.13	-578	17	10	63.13
9	729	-648	18	10	109

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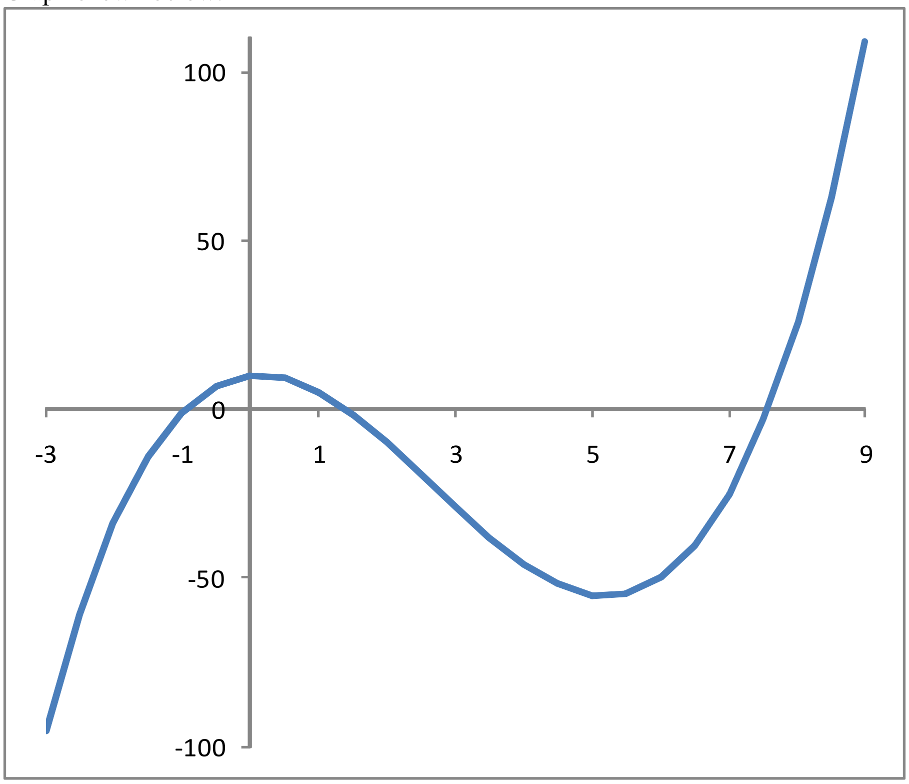
Ques No. | **Marks.**

1(a)

(i) Graph shown below:



(ii) From the graph and the table above, the function crosses the x-axis (i.e. the function is zero) when x is close to -1, 1.5 and 7.5

(iii) It is apparent from the graph above that this cubic equation exhibits the usual twin turning points. It has 2 turning points: the first is close to when x = 0 (i.e. close to the point (0,10)) and the second is close to when x = 5 (near the point (5,-55)).

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Ques No. **Marks.**

2(a) (i)

Determine the value(s) of x for which the determinant is zero. The complementary space is the set of values for which the matrix is non-singular.

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3x & -4 \\ 5 & 0 & x \end{vmatrix} \begin{array}{l} =(1)(x)(3x)=0 \\ \Rightarrow \textit{determinant} =0 \Leftrightarrow x=0. \end{array}$$

The matrix is non-singular $\forall \ x \in \mathbb{R} \setminus \{0\}$ **3**

(ii)

$$\begin{vmatrix} 2 & 1 & 6 \\ 3 & -x & 4 \\ -6 & 0 & 1-x \end{vmatrix} \begin{array}{l} = 2(x^2-x)+(3x-27)+6(-6x) \\ = 2x^2-35x-27=0 \\ \Rightarrow \textit{determinant} =0 \Leftrightarrow x=\frac{35}{4} \pm \frac{\sqrt{1441}}{4} \end{array}$$

The matrix is non-singular $\forall \ x \in \mathbb{R} \setminus \{\frac{1}{4}(35-\sqrt{1441}), \frac{1}{4}(35+\sqrt{1441})\}$ **3**

(b)

To calculate the inverse of the matrix $A=\begin{pmatrix} 1 & 1 & -2 \\ 2 & -1 & 3 \\ -1 & 1 & -3 \end{pmatrix}$ we need to calculate its determinant to determine if it is non-zero. If it is non-zero, then we calculate the adjoint.
First the determinant:

$$|A|=\begin{vmatrix} 1 & 1 & -2 \\ 2 & -1 & 3 \\ -1 & 1 & -3 \end{vmatrix}$$

$$=(-1\times(-3)-3\times1)+(3\times(-1)-2\times(-3))-2(2\times1-(-1)\times(-1))=1$$

As the determinant is non-zero, the inverse exists. Now calculate the adjoint:

$$A^*=\begin{pmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} \end{pmatrix} \text{ where } \Delta_{ij}=(-1)^{i+j}|\Delta(i,j)|$$

where $\Delta(i,j)$ is the 2x2 formed by removing the j[th] row and i[th] column of A.

$$\Delta_{11}=(-1)^2(-1\times(-3)-3\times1)=0$$
$$\Delta_{12}=(-1)^3(1\times(-3)-(-2)\times1)=1$$
$$\Delta_{13}=(-1)^4(1\times3-(-2)\times(-1))=1$$
$$\Delta_{21}=(-1)^3(2\times(-3)-3\times(-1))=3$$
$$\Delta_{22}=(-1)^4(1\times(-3)-(-2)\times(-1))=-5$$
$$\Delta_{23}=(-1)^5(1\times3-(-2)\times2)=-7$$

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Ques No. **Marks.**

2(b)

$$\Delta_{31}=(-1)^4(2\times1-(-1)\times(-1))=1$$
$$\Delta_{32}=(-1)^5(1\times1-1\times(-1))=-2$$
$$\Delta_{33}=(-1)^6(1\times(-1)-1\times2)=-3$$

The adjoint of A is thus

$$A^*=\begin{pmatrix}0 & 1 & 1\\ 3 & -5 & -7\\ 1 & -2 & -3\end{pmatrix}$$

with the inverse of A defined:

$$A^{-1}=\frac{1}{|A|}A^*=\frac{1}{1}\begin{pmatrix}0 & 1 & 1\\ 3 & -5 & -7\\ 1 & -2 & -3\end{pmatrix}$$

12

(c)

The system of equations can be expressed as AX=B where

$$A=\begin{pmatrix}1 & 1 & -2\\ 2 & -1 & 3\\ -1 & 1 & -3\end{pmatrix},\ X=\begin{pmatrix}x\\ y\\ z\end{pmatrix} \text{ and } B=\begin{pmatrix}0\\ 5\\ -4\end{pmatrix}$$

A is the matrix from (b). We can use the result from (b) to solve the system as follows:

$$AX=B\Rightarrow(A^{-1}A)X=I.X=A^{-1}B$$

where I is the identity matrix. Therefore the solution is obtained by multiplying B by A^{-1}.
Then

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{1}\begin{pmatrix}0 & 1 & 1\\ 3 & -5 & -7\\ 1 & -2 & -3\end{pmatrix}\cdot\begin{pmatrix}0\\ 5\\ -4\end{pmatrix}=\begin{pmatrix}1\\ 3\\ 2\end{pmatrix}$$

Therefore

$$x=1,\ y=3,\ z=2$$

is the solution of the system.

2

3(a)

Taylor series: $f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

(i)

$f(x)=\sin(x) \quad x_0=\pi/2$

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3(a) (i)

Then

$$f(\pi/2) = \sin(\pi/2) = 1$$
$$f^{(1)}(x) = \cos(x) \quad \Rightarrow f^{(1)}(\pi/2) = 0$$
$$f^{(2)}(x) = -\sin(x) \quad \Rightarrow f^{(2)}(\pi/2) = -1$$
$$f^{(3)}(x) = -\cos(x) \quad \Rightarrow f^{(3)}(\pi/2) = 0$$
$$f^{(4)}(x) = \sin(x) \quad \Rightarrow f^{(4)}(\pi/2) = 1$$
$$f^{(5)}(x) = \cos(x) \quad \Rightarrow f^{(5)}(\pi/2) = 0$$

The first six terms in the Taylor series are defined:

$$T_5(x) = \sum_{n=0}^{5} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$
$$= f(\pi/2) + f^{(1)}(\pi/2)(x-x_0) + \frac{f^{(2)}(\pi/2)}{2!}(x-x_0)^2 + \ldots + \frac{f^{(5)}(\pi/2)}{5!}(x-x_0)^5$$
$$= 1 + 0 + (-1)\frac{(x-\pi/2)^2}{2!} + 0 + \frac{(x-\pi/2)^4}{4!} + 0$$

5

(ii)

$$f(x) = e^x \qquad x_0 = 0$$

Then

$$f(0) = e^0 = 1$$
$$f^{(1)}(x) = f(x) = e^x \quad \Rightarrow f^{(1)}(0) = f(0) = 1$$
$$f^{(2)}(x) = f(x) = e^x \quad \Rightarrow f^{(2)}(0) = f(0) = 1$$
$$f^{(3)}(x) = f(x) = e^x \quad \Rightarrow f^{(3)}(0) = f(0) = 1$$
$$f^{(4)}(x) = f(x) = e^x \quad \Rightarrow f^{(4)}(0) = f(0) = 1$$
$$f^{(5)}(x) = f(x) = e^x \quad \Rightarrow f^{(5)}(0) = f(0) = 1$$

The first six terms in the Taylor series are defined:

$$T_5(x) = \sum_{n=0}^{5} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$
$$= f(0) + f^{(1)}(0)(x) + \frac{f^{(2)}(0)}{2!}(x)^2 + \ldots + \frac{f^{(5)}(0)}{5!}(x)^5$$
$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

5

(b)

$$e^2 \simeq 7.389056$$

We've already determined that

$$T_5(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

for exp(x) about x_0=0.
Substitute 2 for x in T_5(x) to get the approximation for exp(x) at 2.

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3(b)

$$T_5(2)=1+2+\frac{4}{2!}+\frac{8}{3!}+\frac{16}{4!}+\frac{32}{5!}\simeq 7.2667$$

So the error is

$$|f(x)-T_5(x)|=|7.389-7.27|=0.11\simeq 0.2$$

10

4(a)

$$f(x)=\left(\cos^2(x)+\sin^2(x)\right)e^{2x}$$

As $\cos^2(x)+\sin^2(x)=1$ then $f(x)=e^{2x}$. Using the chain rule we get

$$\frac{d}{dx}f(x)=2e^{2x}$$

5

(b)

$$f(x)=e^{\ln|e^x|}$$

Here use the identity

$$e^{\ln|u(x)|}=u(x)$$

with

$$u(x)=e^x$$

$$\frac{d}{dx}f(x)=\frac{d}{dx}e^x=e^x$$

5

(c)

$$f(x)=e^{\cos(2x)}$$

You use the chain rule 2 times to get

$$\frac{d}{dx}f(x)=\underbrace{e^{\cos(2x)}}_{1^{st}\,Chain\,Rule}\times\underbrace{\left(-\sin(2x)\right)\times 2}_{2^{nd}\,Chain\,Rule}$$

5

(d)

$$f(x)=\frac{x^8-1}{x^7+x^6+x^5+x^4+x^3+x^2+x+1}$$

From your lectures you know that

$$x^8-1=(x-1)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$$

Then and

$$\Rightarrow\frac{d}{dx}f(x)=\frac{d}{d\,x}\frac{x^8-1}{x^7+x^6+x^5+x^4+x^3+x^2+x+1}=\frac{d}{dx}(x-1)=1$$

5