EE610 – Image Processing

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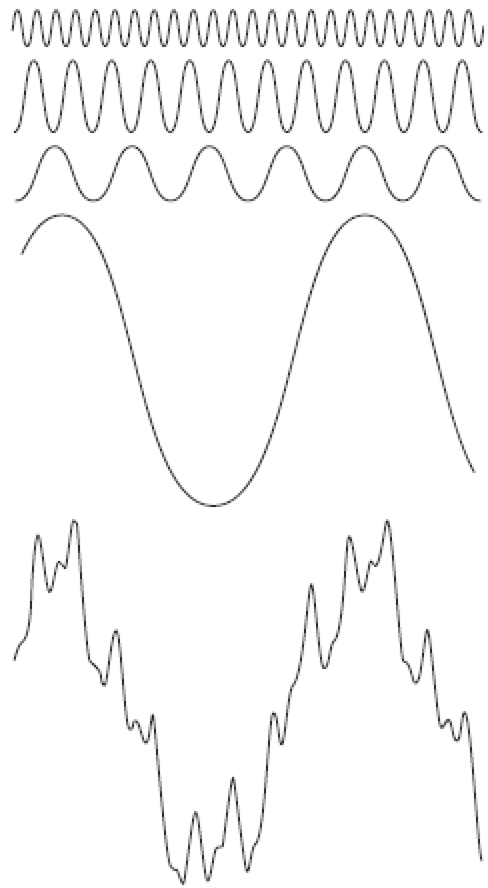
Periodic signal of arbitrary shape can be constructed by summing up sinusoids



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Revision of complex numbers

- Real and imaginary
- Complex conjugate
- Euler’s formula for $e^{j\theta}$
- Magnitude and phase

Revision of Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-j\frac{2\pi n}{T}t}\, dt \qquad \text{for } n = 0, \pm 1, \pm 2, \ldots$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

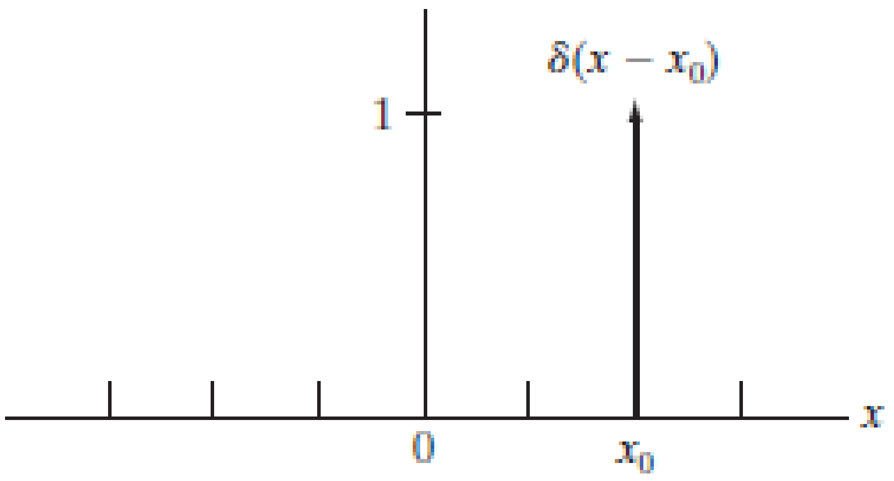
Fourier transform pair

$$F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{j2\pi\mu t}\, d\mu$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Impulse and shifted impulse



- Definition of an impulse function
- Sifting property of impulse and shifted impulse
- Same in discrete domain

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Fourier transform of Dirac delta

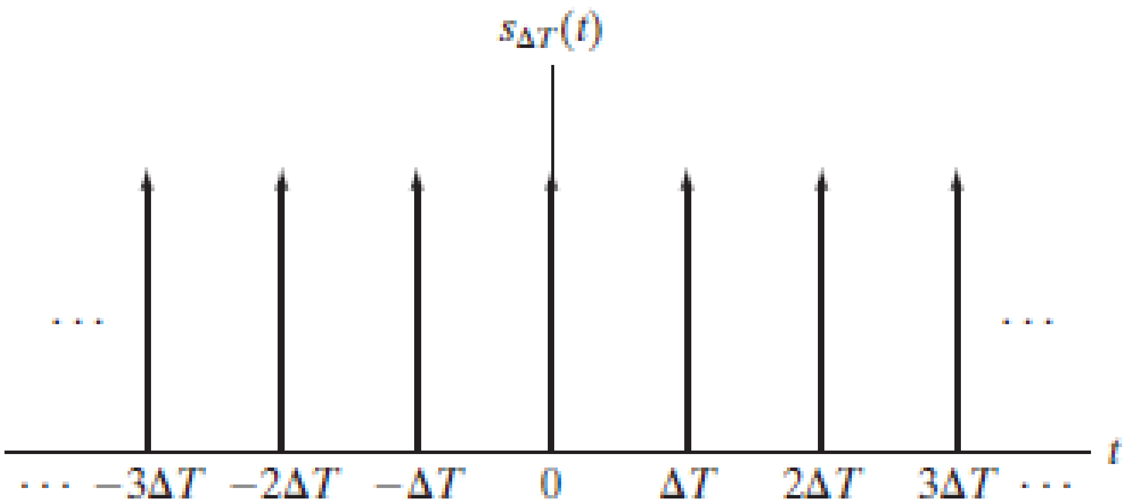
$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi\mu t}\, dt \\
&= \int_{-\infty}^{\infty} e^{-j2\pi\mu t}\, \delta(t)\, dt \\
&= e^{-j2\pi\mu 0} = e^{0} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{\infty} \delta(t - t_0)\, e^{-j2\pi\mu t}\, dt \\
&= \int_{-\infty}^{\infty} e^{-j2\pi\mu t}\, \delta(t - t_0)\, dt \\
&= e^{-j2\pi\mu t_0} \\
&= \cos(2\pi\mu t_0) - j\sin(2\pi\mu t_0)
\end{aligned}
$$

$$e^{-j2\pi\mu t_0};$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Impulse train



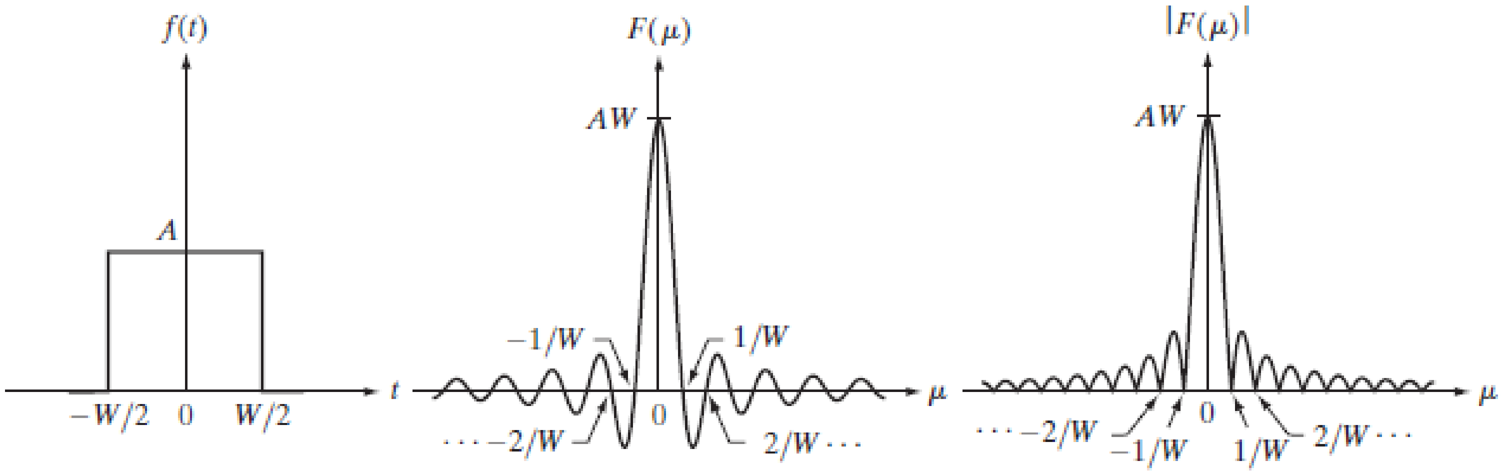
$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Fourier transform of an impulse train

$$
\begin{aligned}
S(\mu) &= \Im\left\{s_{\Delta T}(t)\right\} \\
&= \Im\left\{\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\} \\
&= \frac{1}{\Delta T}\Im\left\{\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi n}{\Delta T}t}\right\} \\
&= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta\left(\mu - \frac{n}{\Delta T}\right)
\end{aligned}
$$

Fourier transform of a box function



$$F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt = \int_{-W/2}^{W/2} A e^{-j2\pi\mu t}\, dt$$

$$= \frac{-A}{j2\pi\mu}\left[e^{-j2\pi\mu t}\right]_{-W/2}^{W/2} = \frac{-A}{j2\pi\mu}\left[e^{-j\pi\mu W} - e^{j\pi\mu W}\right]$$

$$\operatorname{sinc}(m) = \frac{\sin(\pi m)}{(\pi m)}$$

$$= \frac{A}{j2\pi\mu}\left[e^{j\pi\mu W} - e^{-j\pi\mu W}\right]$$

$$= AW\frac{\sin(\pi\mu W)}{(\pi\mu W)}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Convolution and multiplication

$$f(t) \star h(t) = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

$$\Im\{f(t) \star h(t)\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau \right] e^{-j2\pi\mu t}\, dt$$

$$= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\, e^{-j2\pi\mu t}\, dt \right] d\tau$$

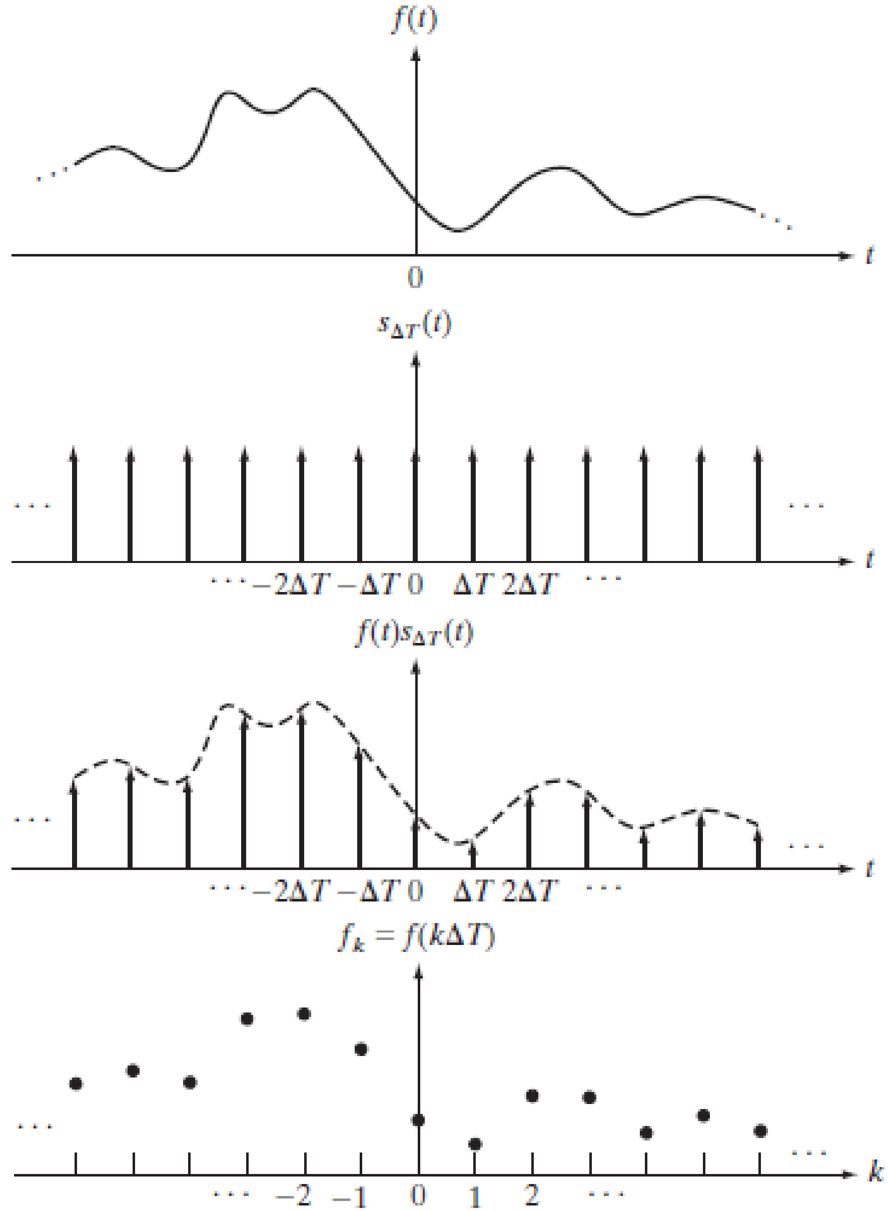
$$\Im\{f(t) \star h(t)\} = \int_{-\infty}^{\infty} f(\tau) \left[H(\mu)\, e^{-j2\pi\mu\tau} \right] d\tau$$

$$= H(\mu) \int_{-\infty}^{\infty} f(\tau)\, e^{-j2\pi\mu\tau}\, d\tau$$

$$= H(\mu)\, F(\mu)$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

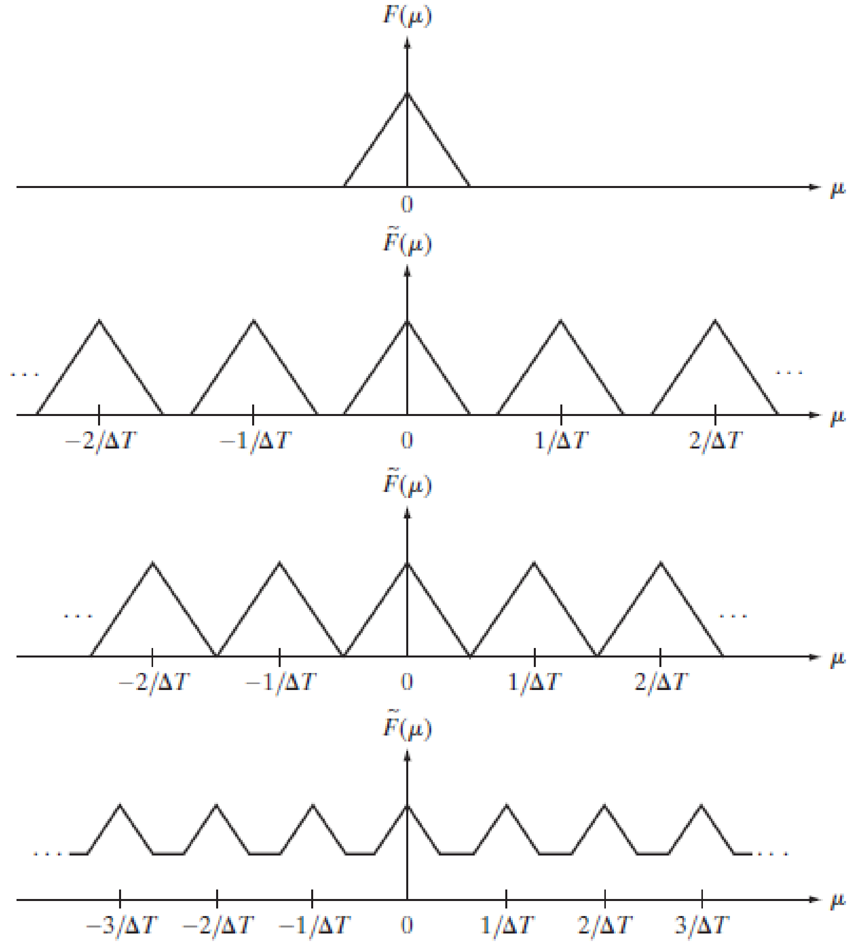
Sampling of a function



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

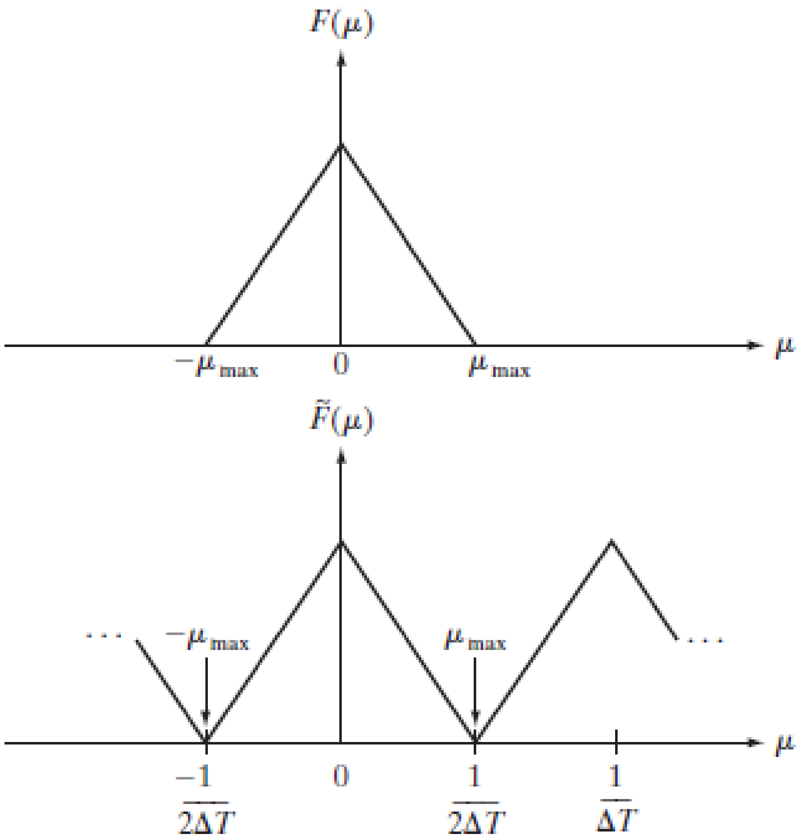
Frequency spectrum of sampled function

$$
\begin{aligned}
\tilde{F}(\mu) &= F(\mu) \star S(\mu) \\
&= \int_{-\infty}^{\infty} F(\tau) S(\mu - \tau)\, d\tau \\
&= \frac{1}{\Delta T} \int_{-\infty}^{\infty} F(\tau) \sum_{n=-\infty}^{\infty} \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau \\
&= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} F(\tau) \delta\left(\mu - \tau - \frac{n}{\Delta T}\right) d\tau \\
&= \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)
\end{aligned}
$$



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

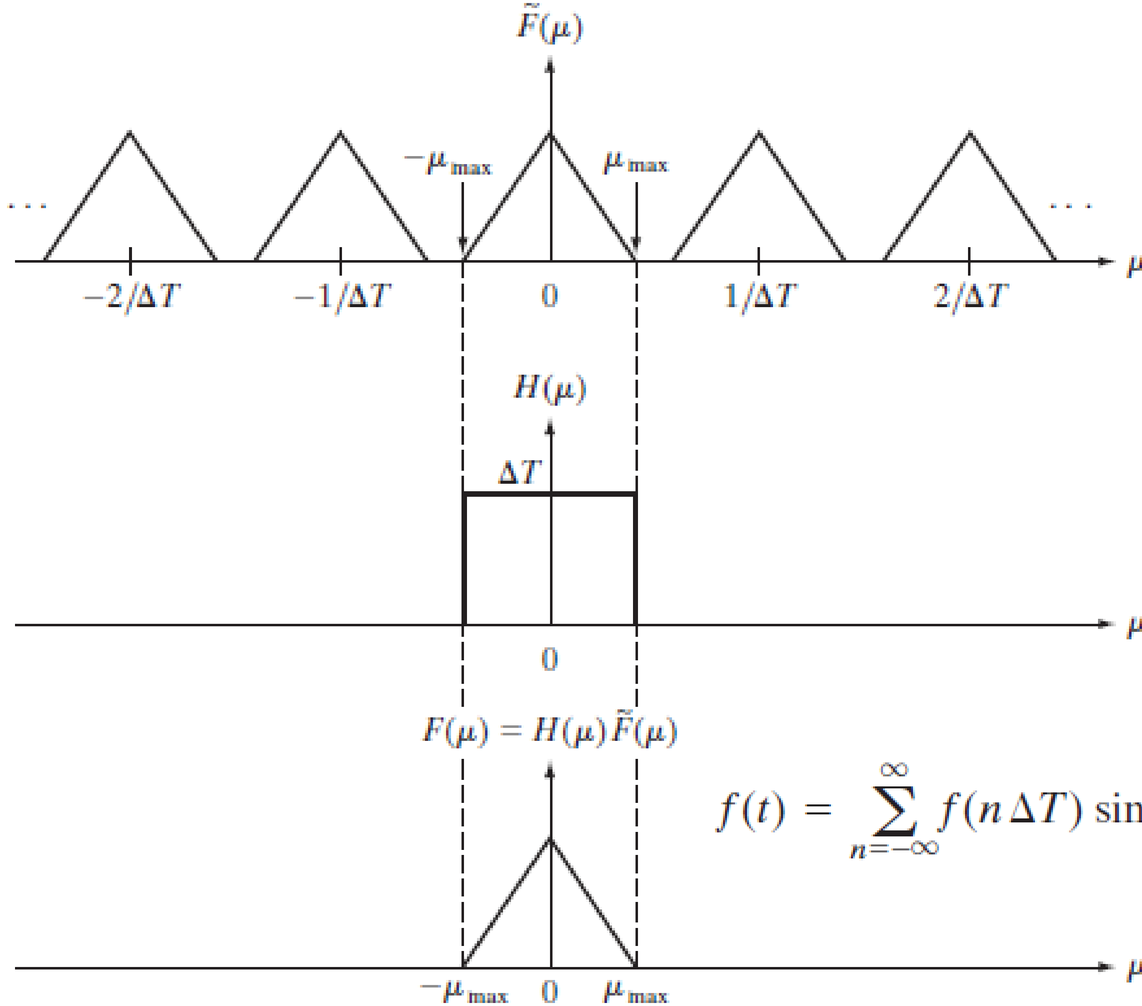
Nyquist criterion



$$\frac{1}{\Delta T} > 2\mu_{\max}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

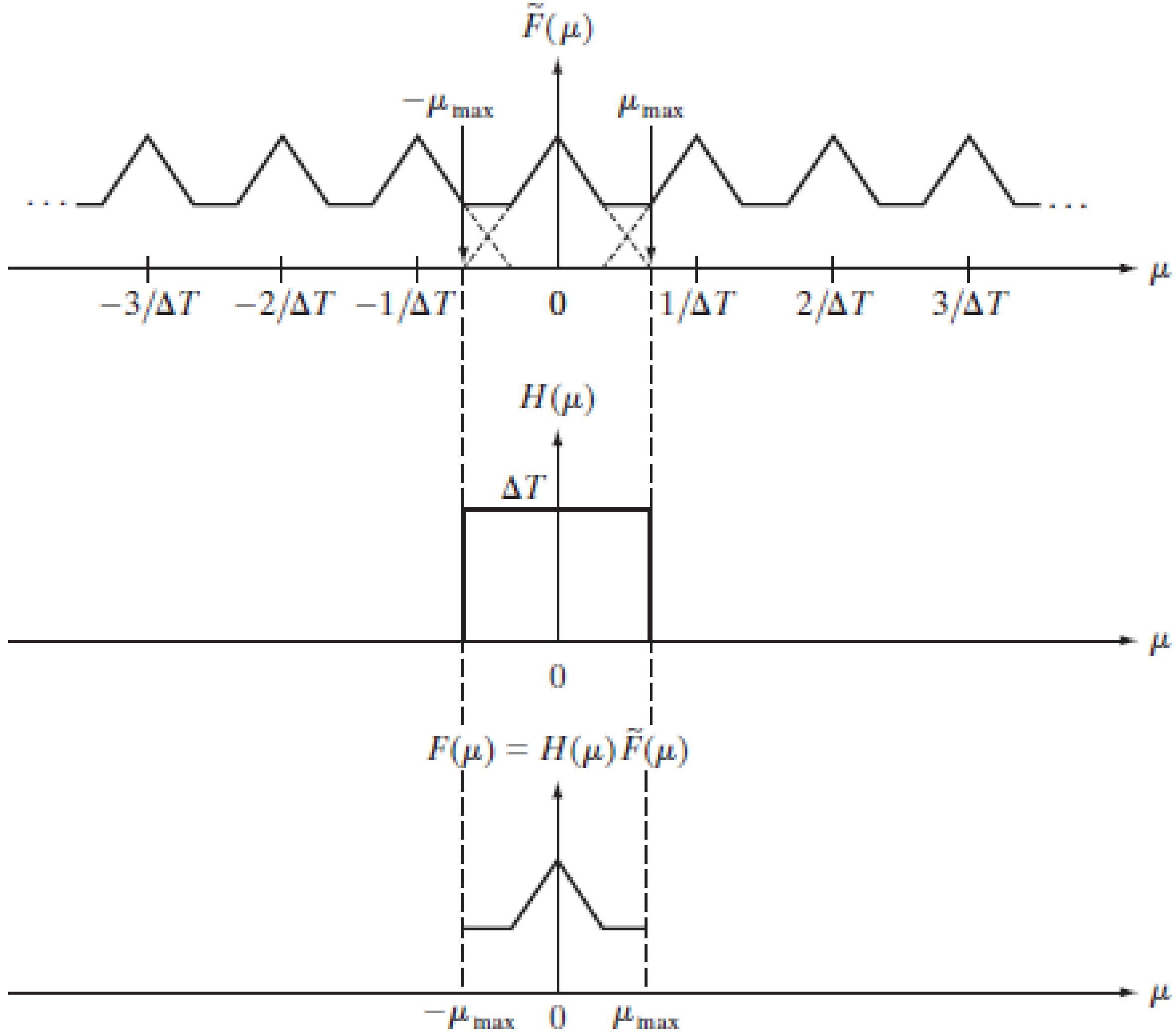
Convolving samples with a sinc function for interpolation



$$f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T)\,\mathrm{sinc}\big[(t - n\Delta T)/n\Delta T\big]$$

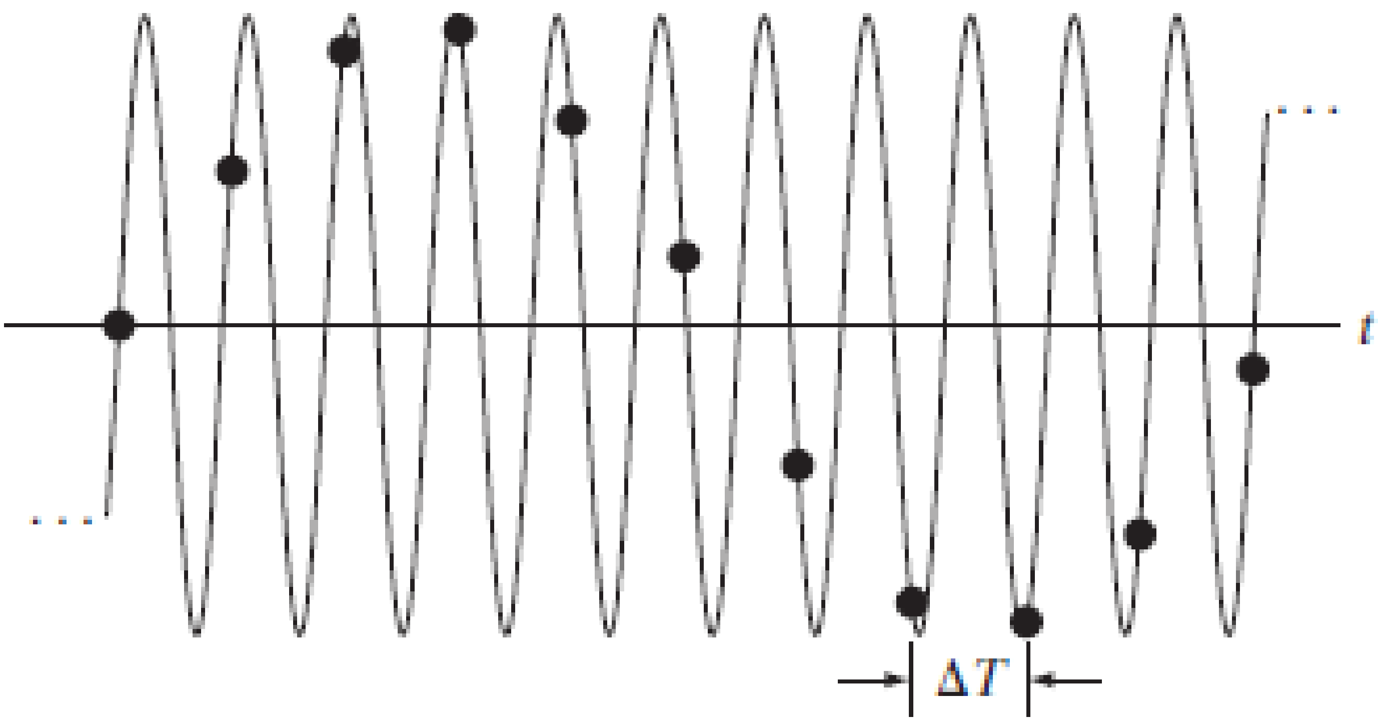
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Aliasing is a consequence of violating Nyquist limit during interpolation



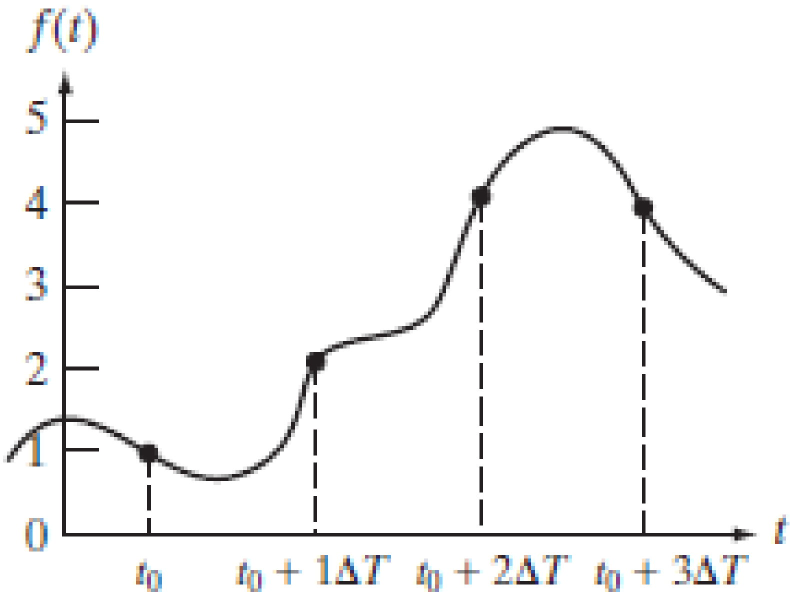
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

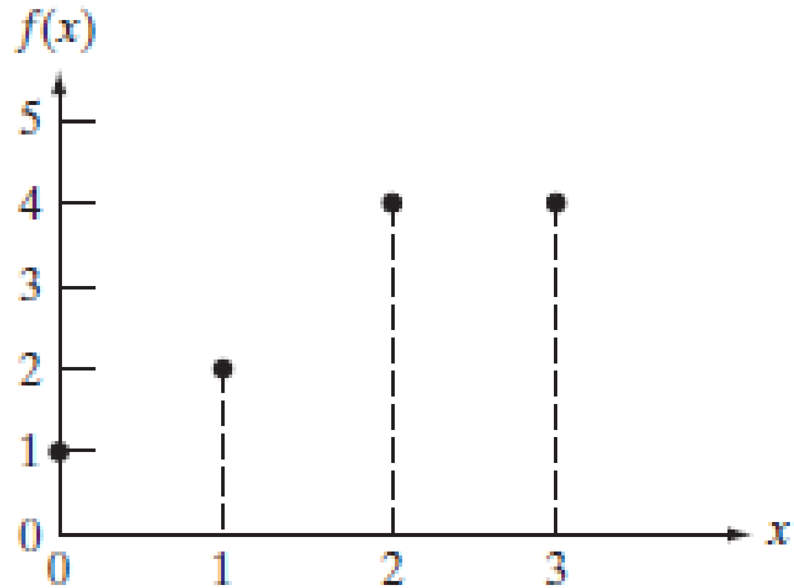
Aliasing in spatial (time) domain



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Making space (or time) discrete





Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Discrete Fourier Transform

$$\tilde{F}(\mu) = \int_{-\infty}^{\infty} \tilde{f}(t)\, e^{-j2\pi\mu t}\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(t)\,\delta(t - n\Delta T) e^{-j2\pi\mu t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\,\delta(t - n\Delta T) e^{-j2\pi\mu t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} f_n\, e^{-j2\pi\mu n\Delta T}$$

$$\mu = \frac{m}{M\Delta T}$$

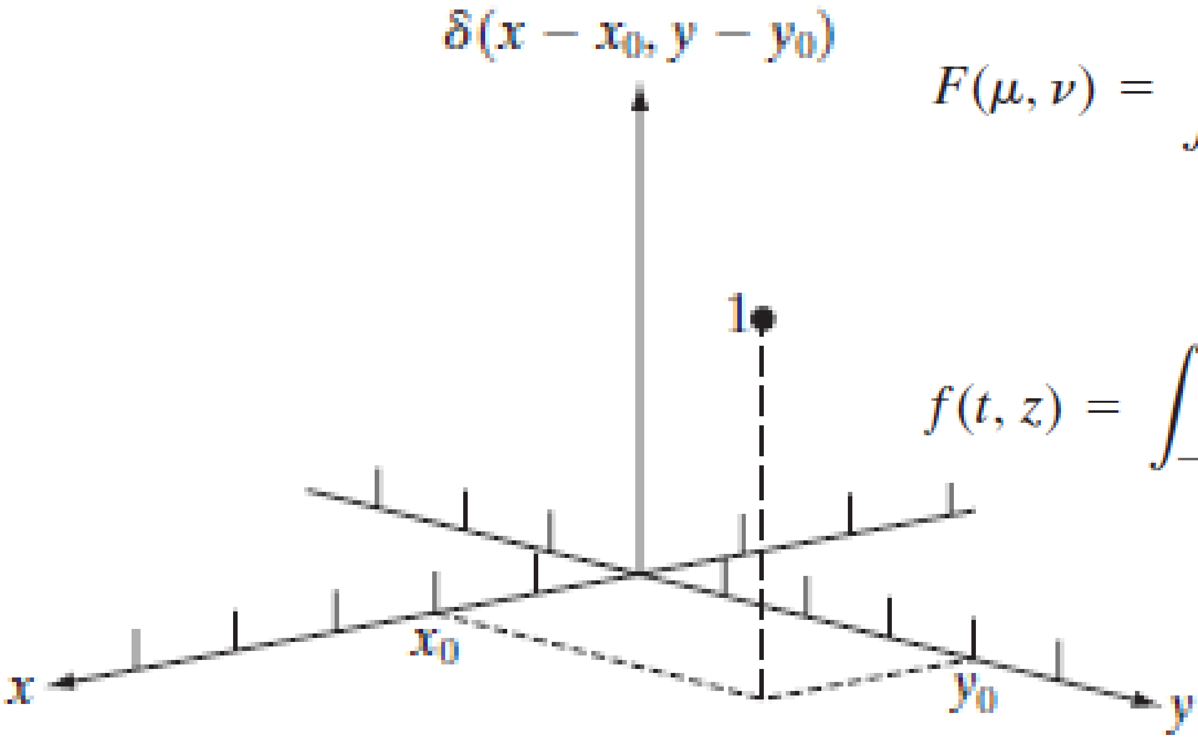
$$F_m = \sum_{n=0}^{M-1} f_n e^{-j2\pi mn/M}$$

$$f_n = \frac{1}{M}\sum_{m=0}^{M-1} F_m e^{j2\pi mn/M}$$

$$f(x) \star h(x) = \sum_{m=0}^{M-1} f(m) h(x-m)$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Extension to two variables

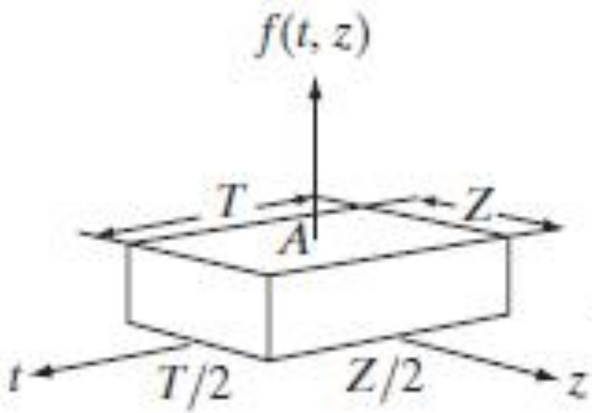


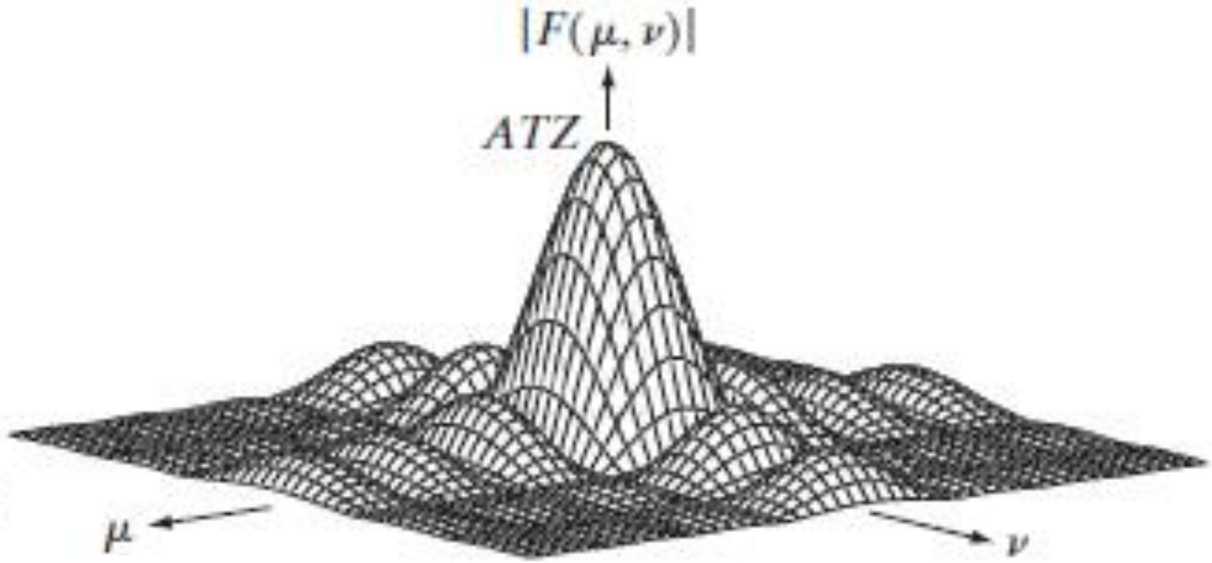
$$F(\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z)\, e^{-j2\pi(\mu t + \nu z)}\, dt\, dz$$

$$f(t, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\mu, \nu)\, e^{j2\pi(\mu t + \nu z)}\, d\mu\, d\nu$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Box and Sinc in 2-D





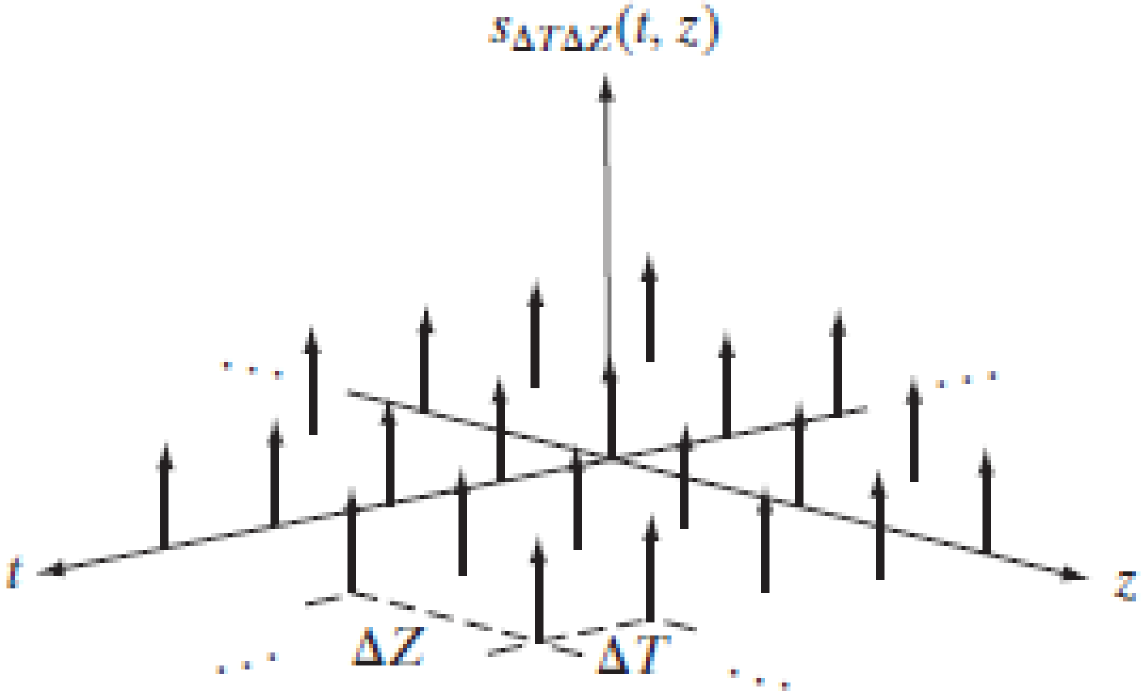
$$F(\mu, \nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t, z)\, e^{-j2\pi(\mu t + \nu z)}\, dt\, dz$$

$$= \int_{-T/2}^{T/2}\int_{-Z/2}^{Z/2} A\, e^{-j2\pi(\mu t + \nu z)}\, dt\, dz$$

$$= ATZ\left[\frac{\sin(\pi\mu T)}{(\pi\mu T)}\right]\left[\frac{\sin(\pi\nu Z)}{(\pi\nu Z)}\right]$$

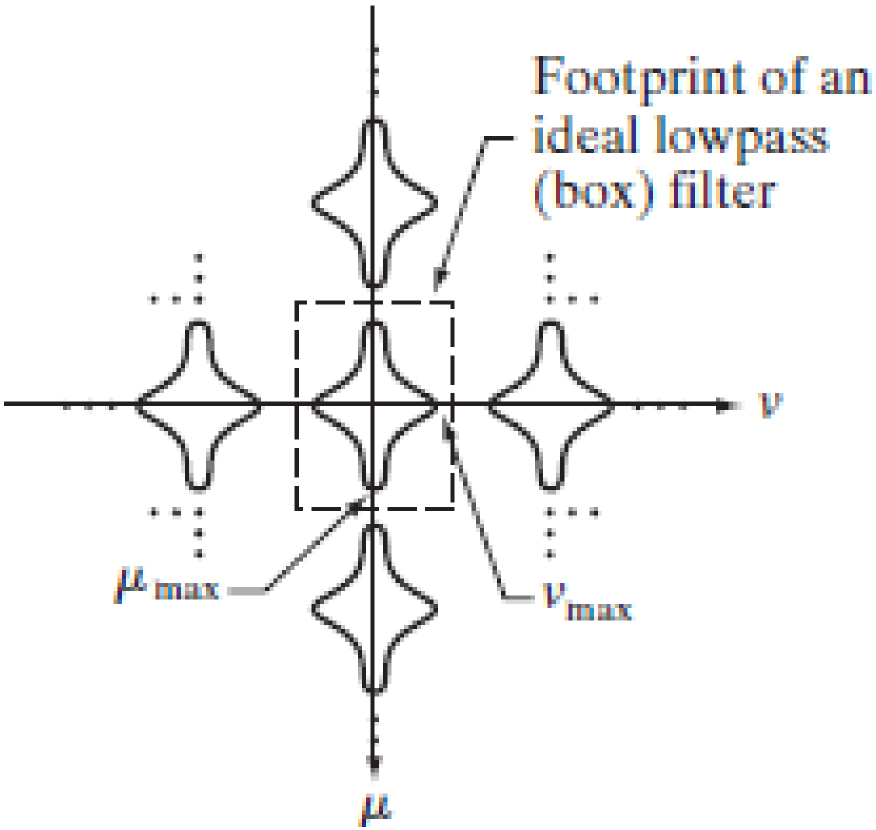
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

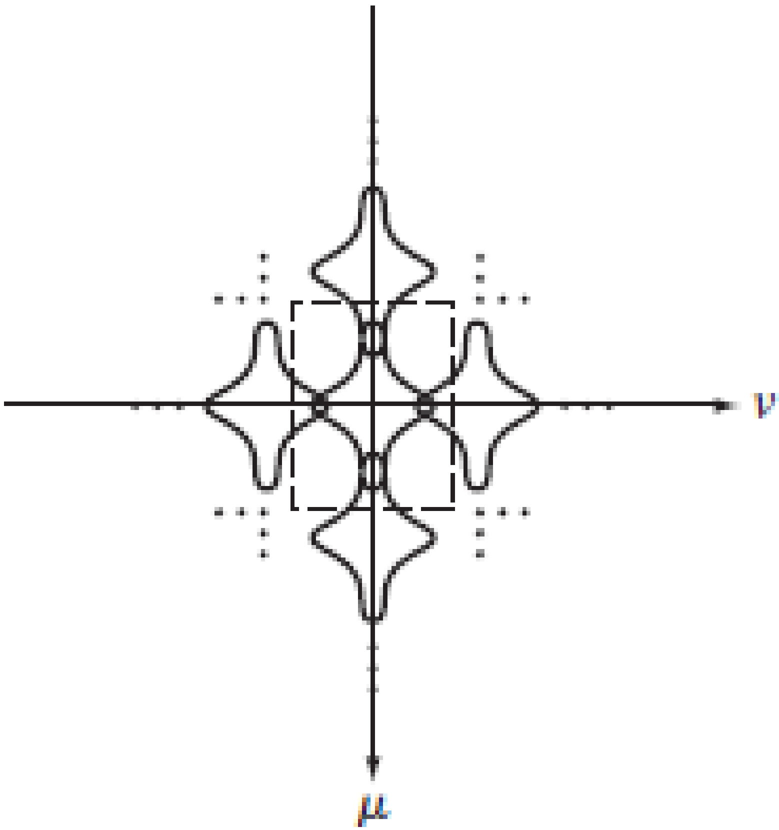
Impulse train in 2-D



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

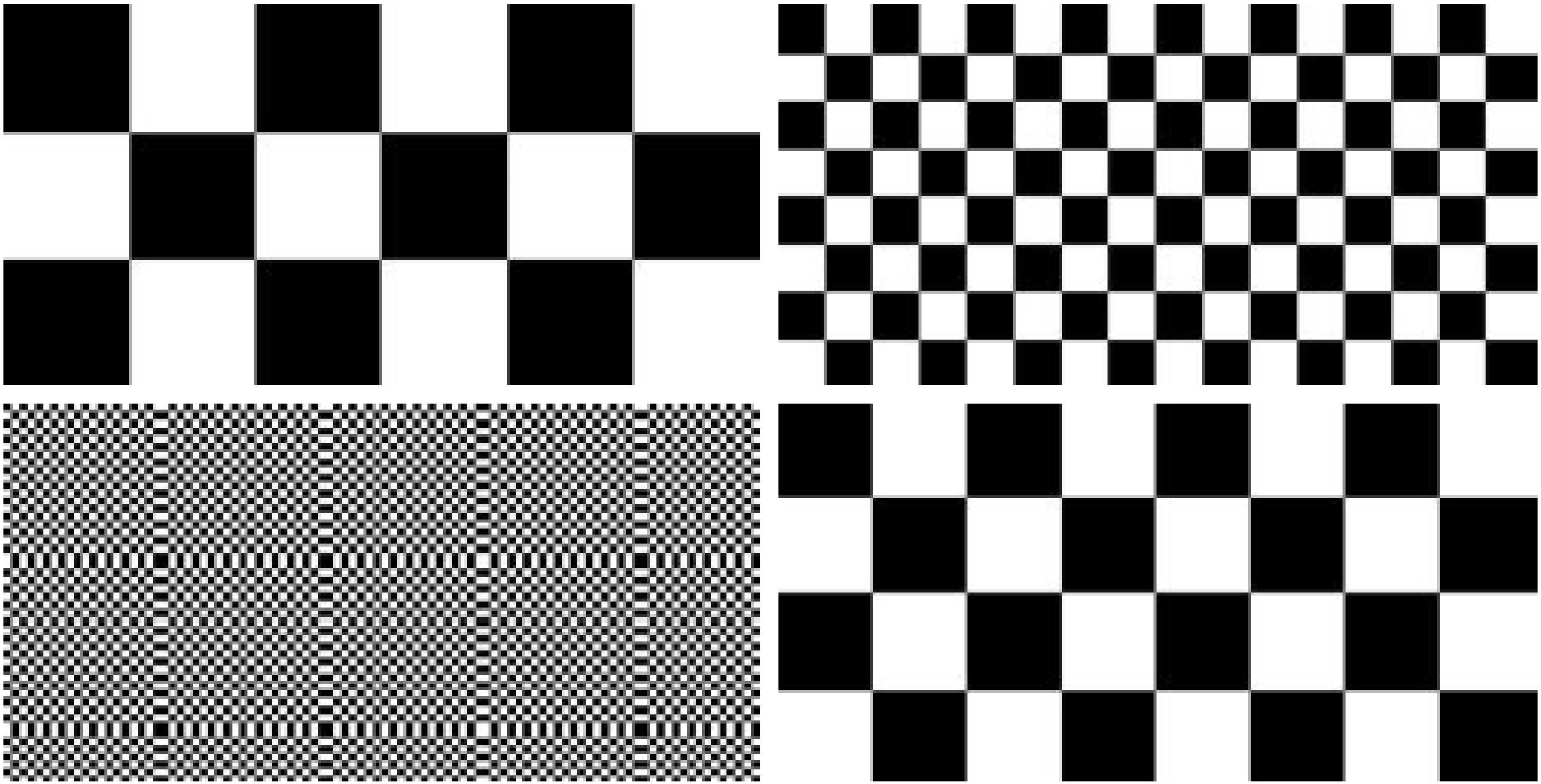
Aliasing in 2-D





Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Spatial examples of aliasing

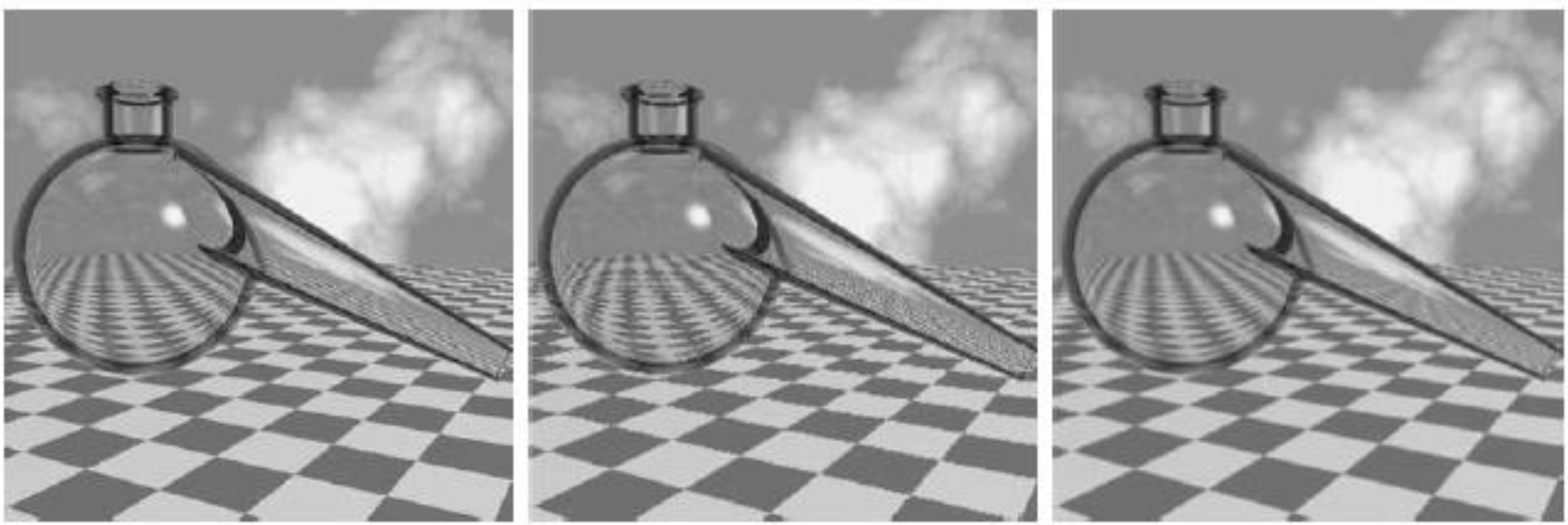


Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Aliasing due to reduction of image resolution and anti-aliasing using smoothing

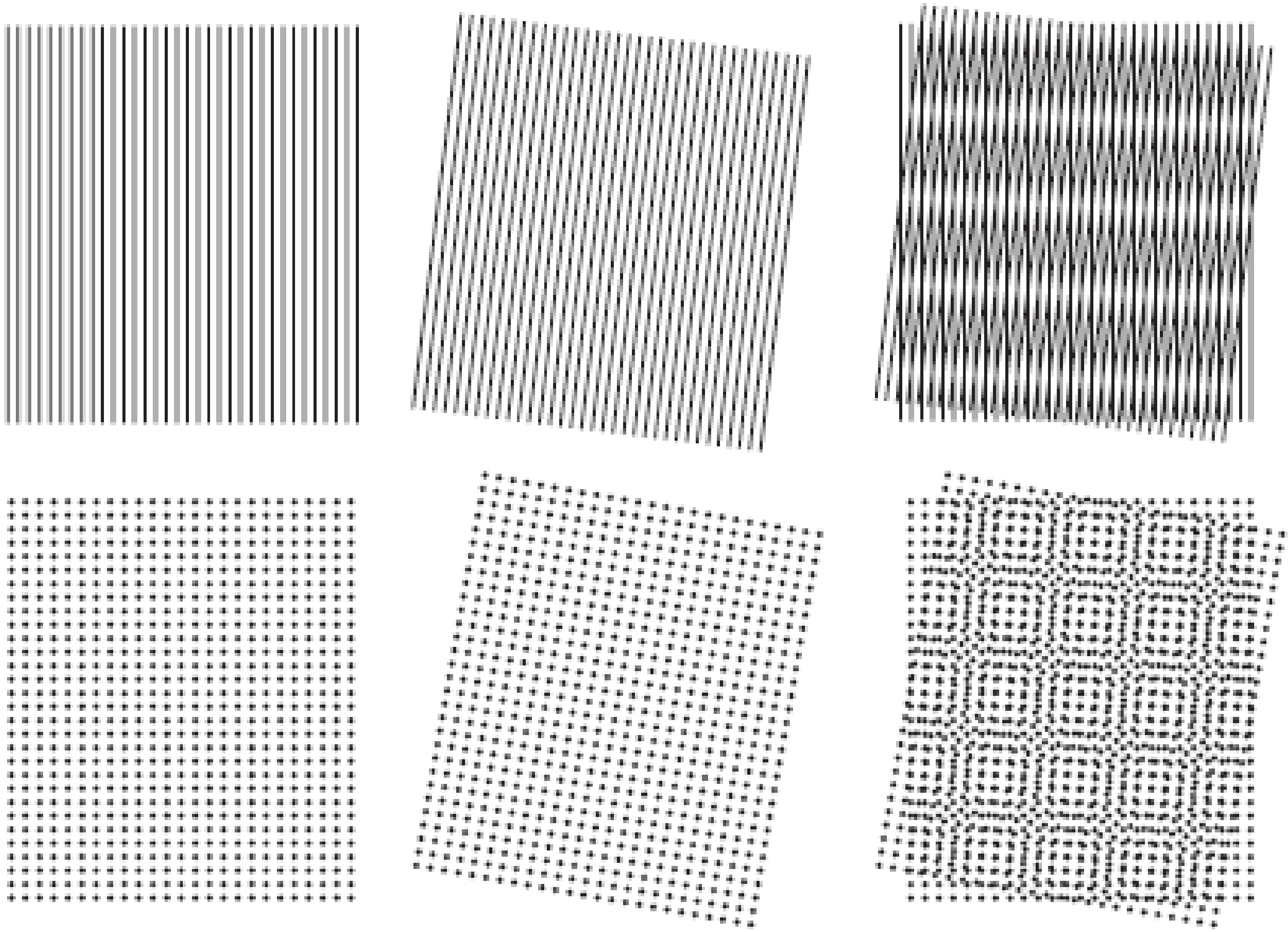
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Jaggies due to aliasing, and anti-aliasing using smoothing



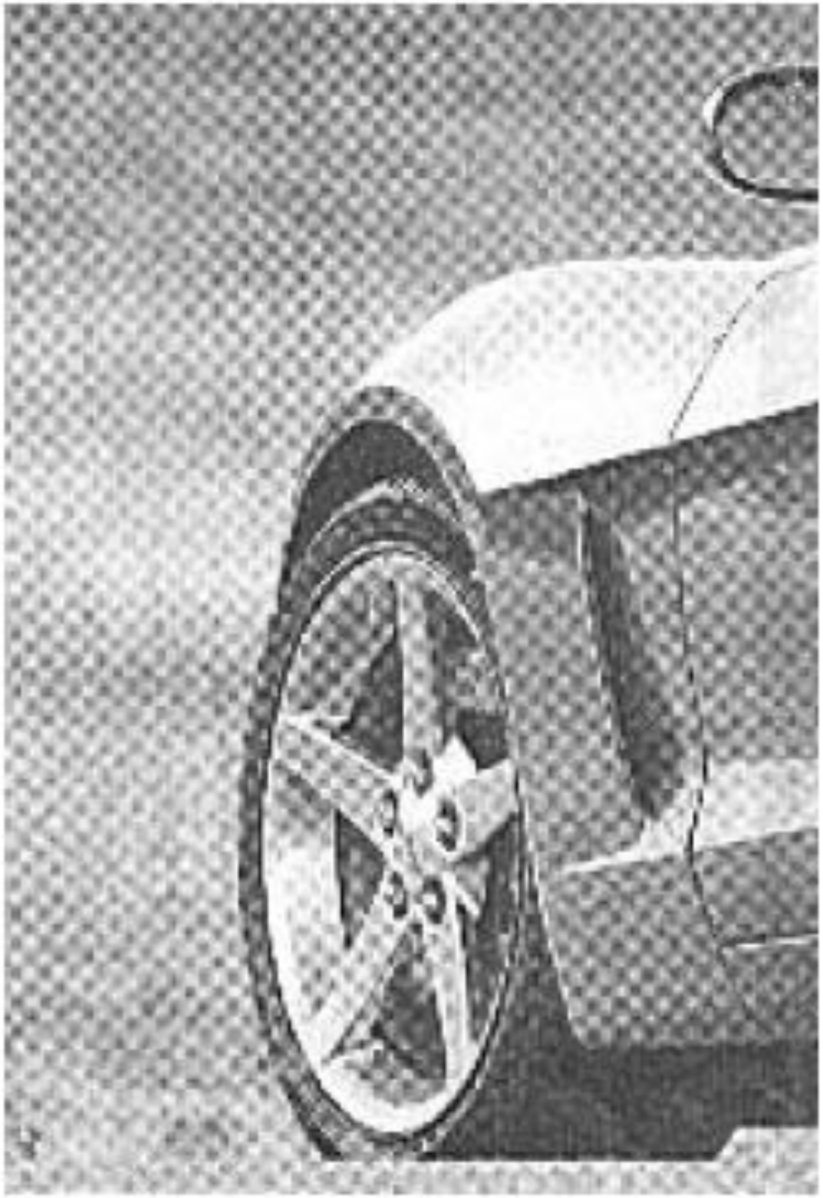
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Moiré effect



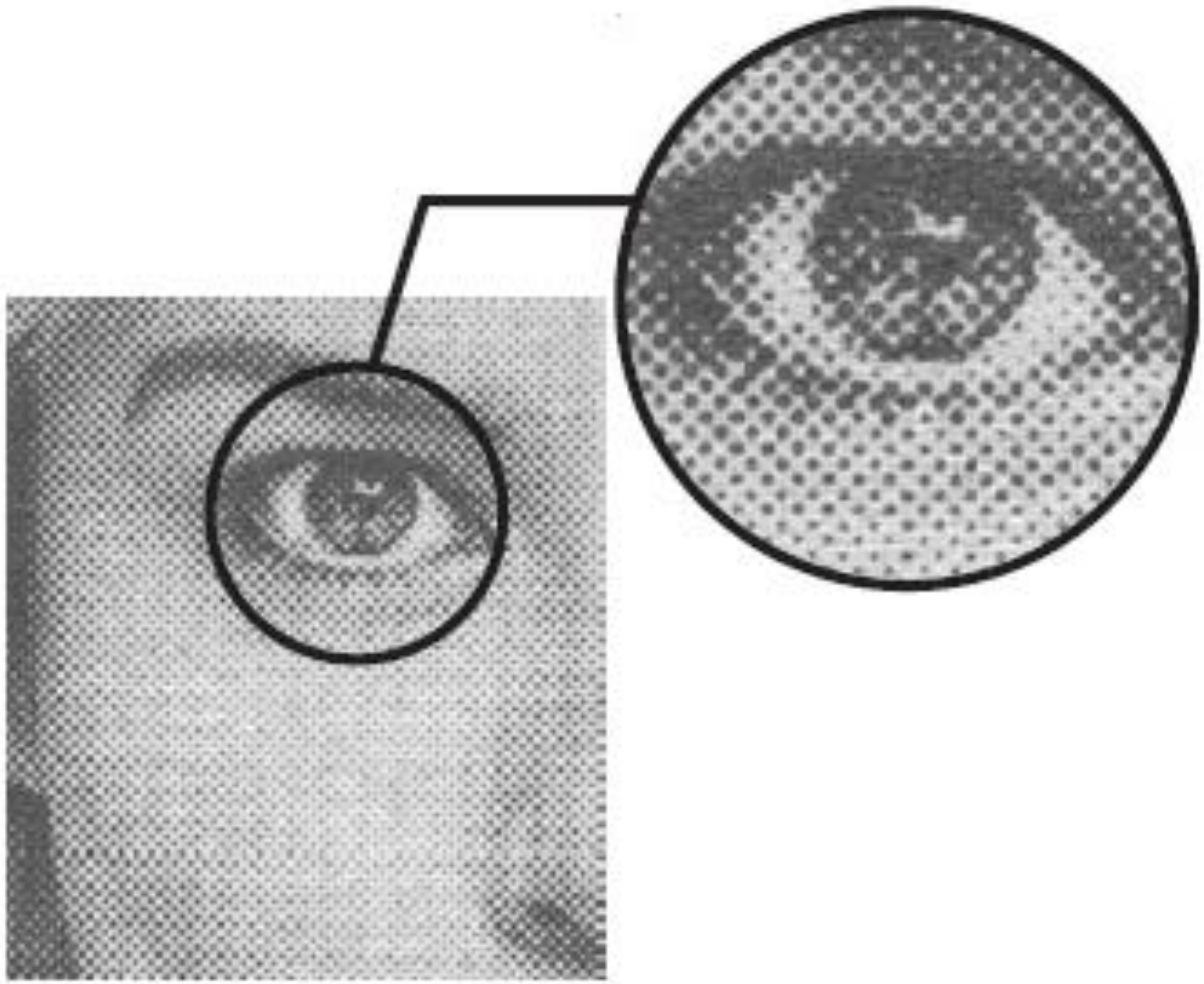
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Oriented dots and Cartesian grid create Moiré patterns



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Close up of newspaper printing



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

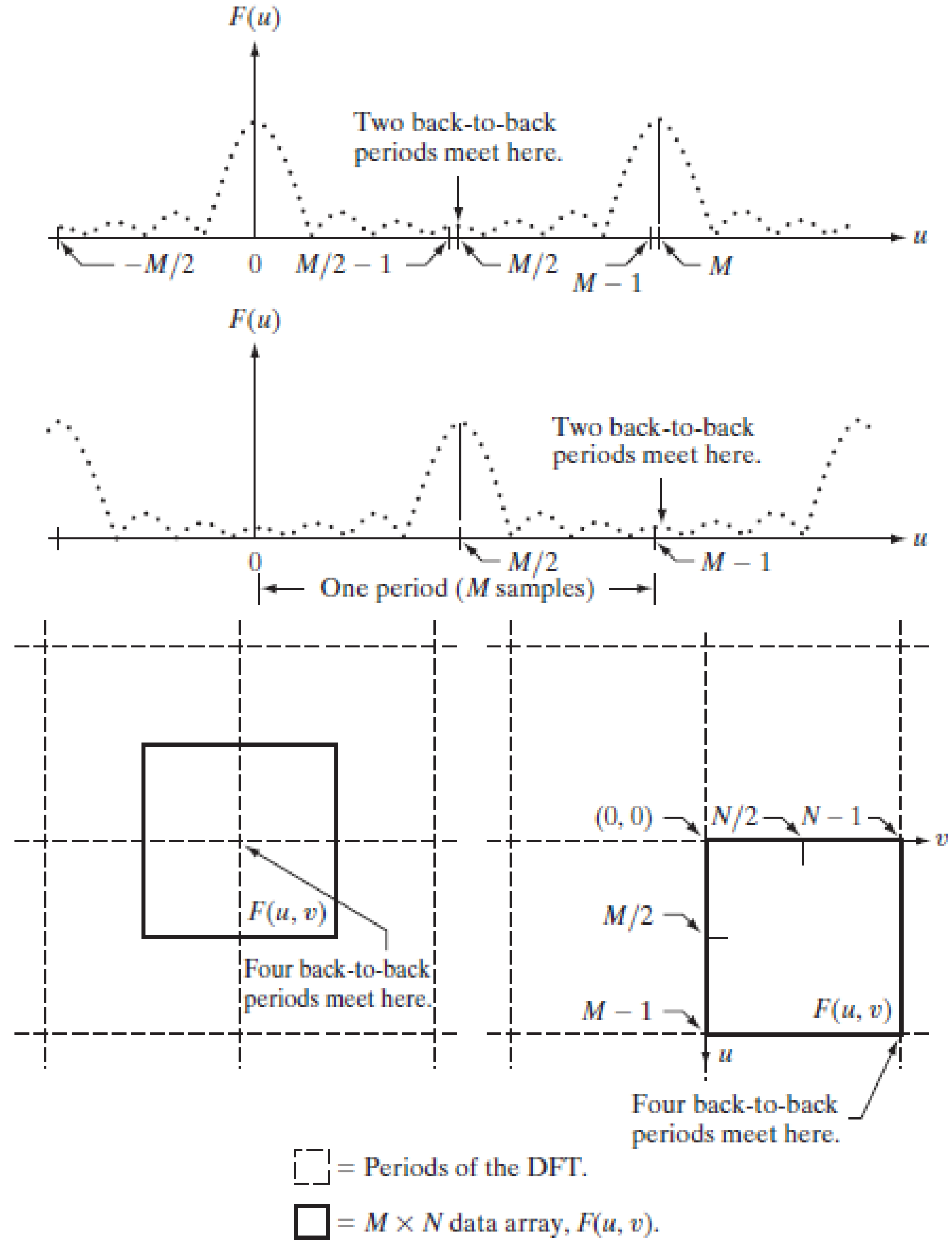
DFT in 2-D

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Representation of DFT

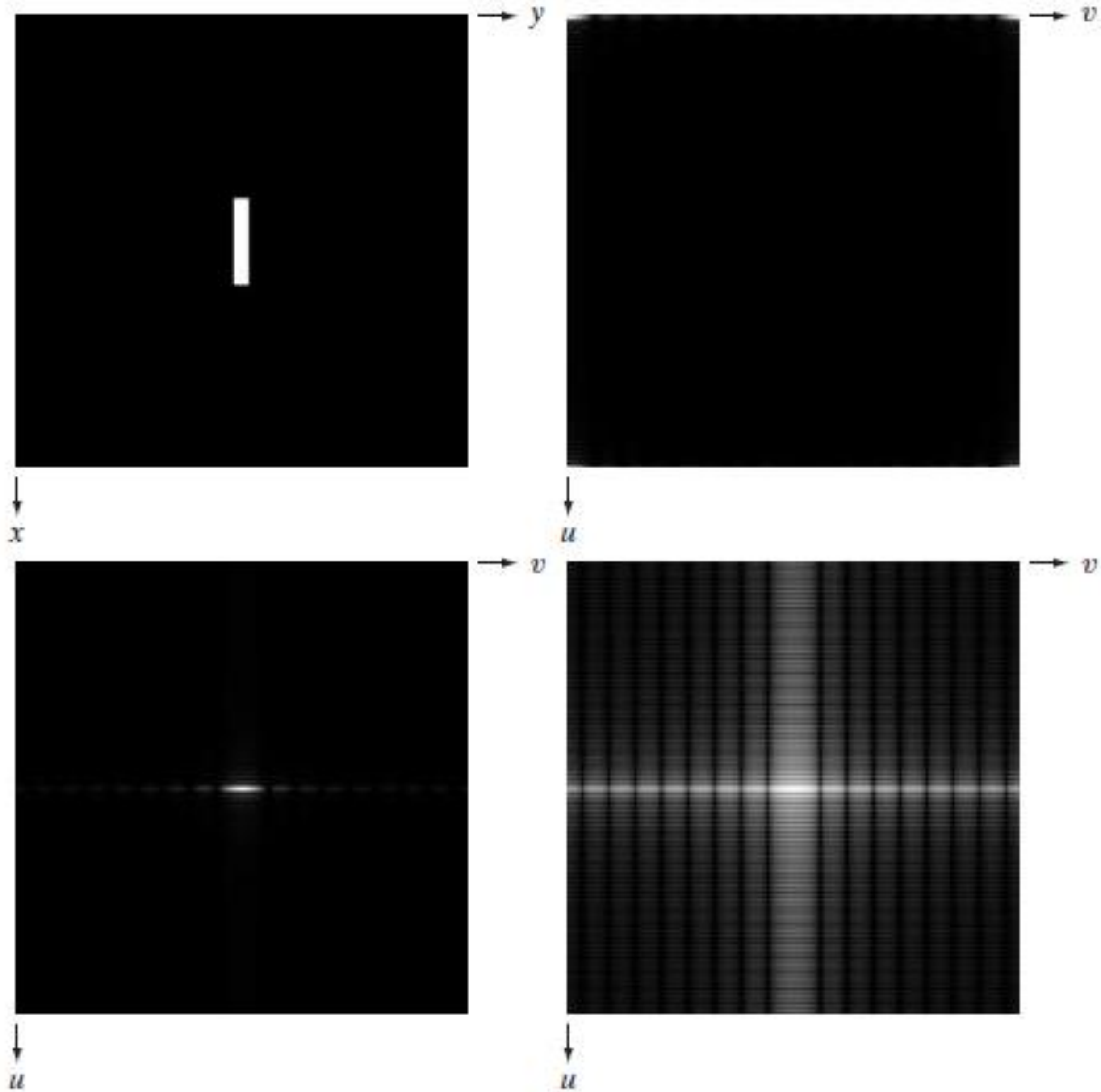


Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

	Spatial Domain†		Frequency Domain†
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

DFT of a bar: Original, DFT not centered, centered, log Xform



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

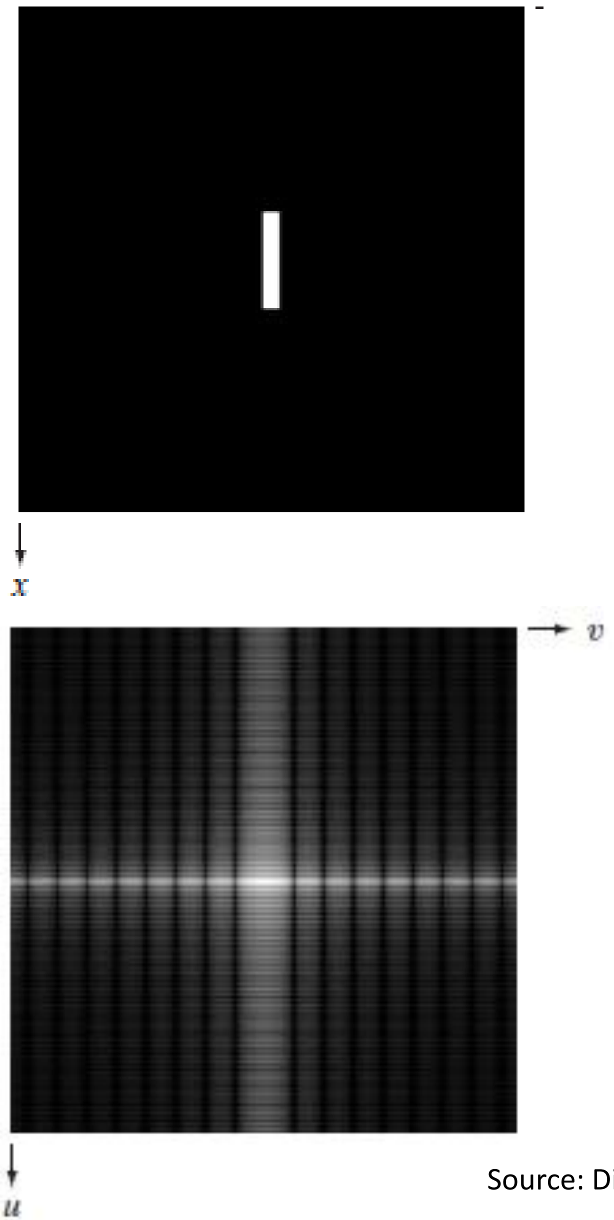
Rotation and phase shift

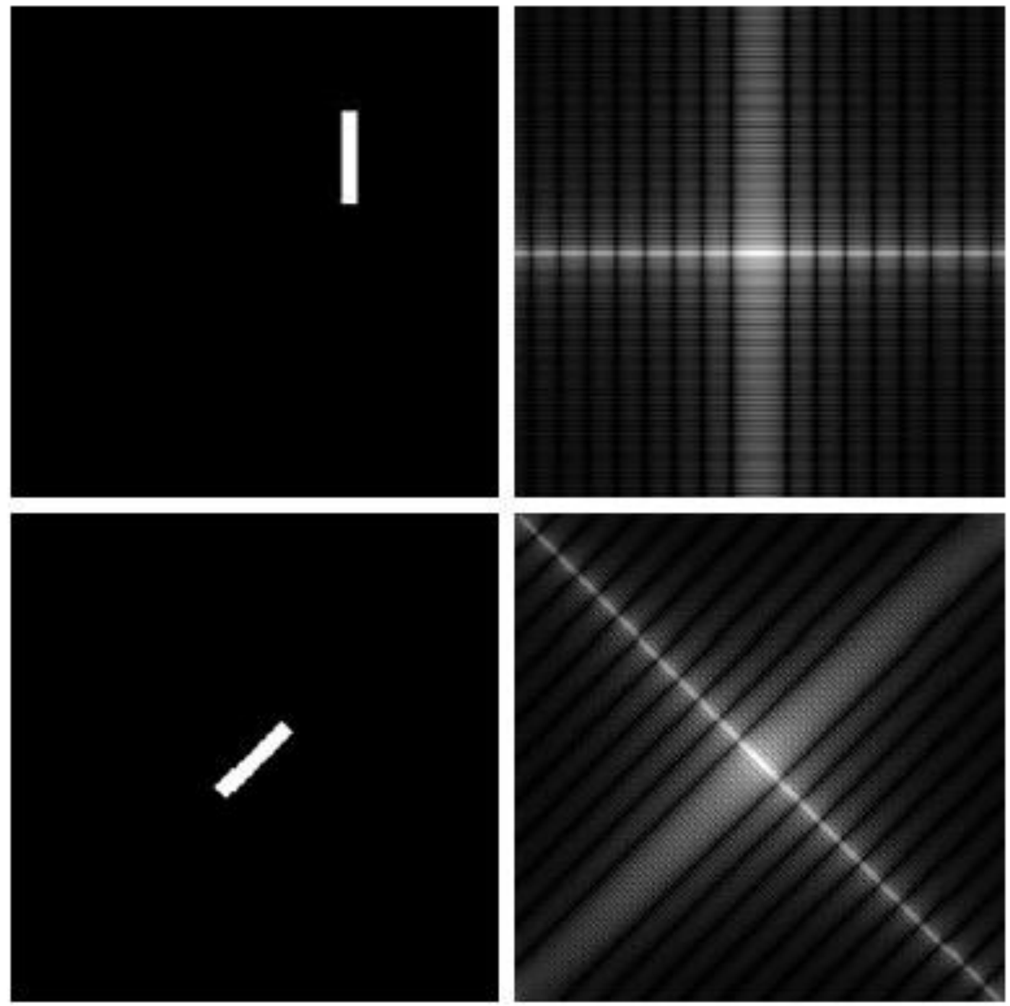
$$f(x, y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$$

$$f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(x_0u/M+y_0v/N)}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

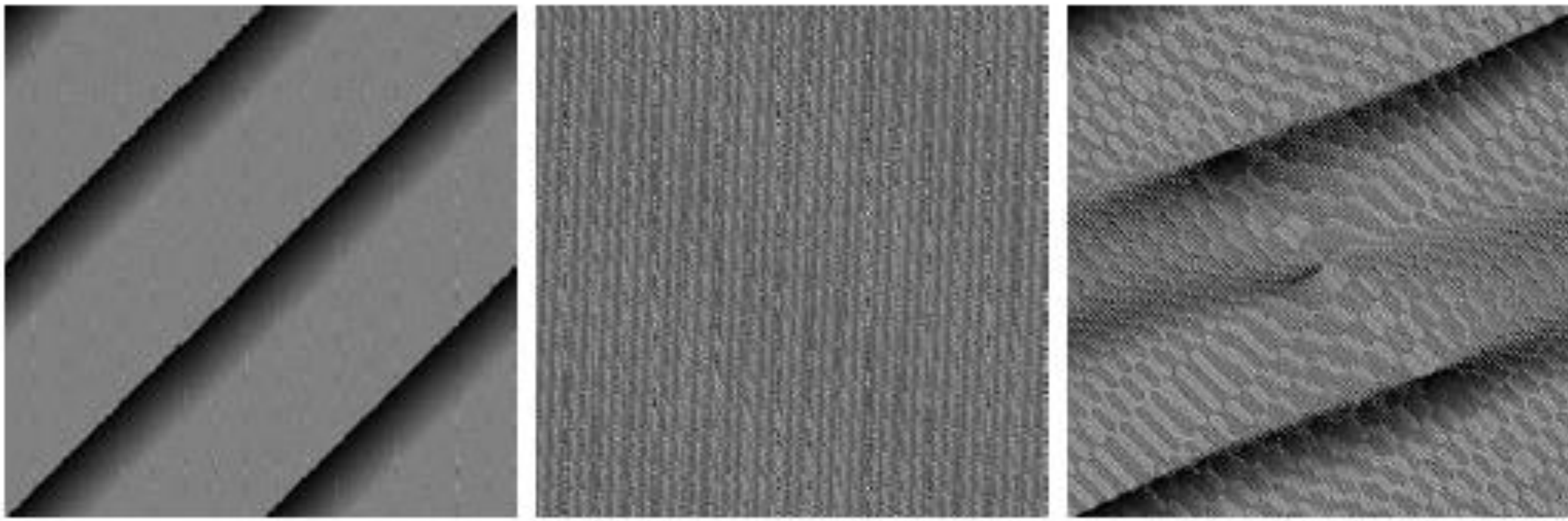
Visualizing rotation and translation





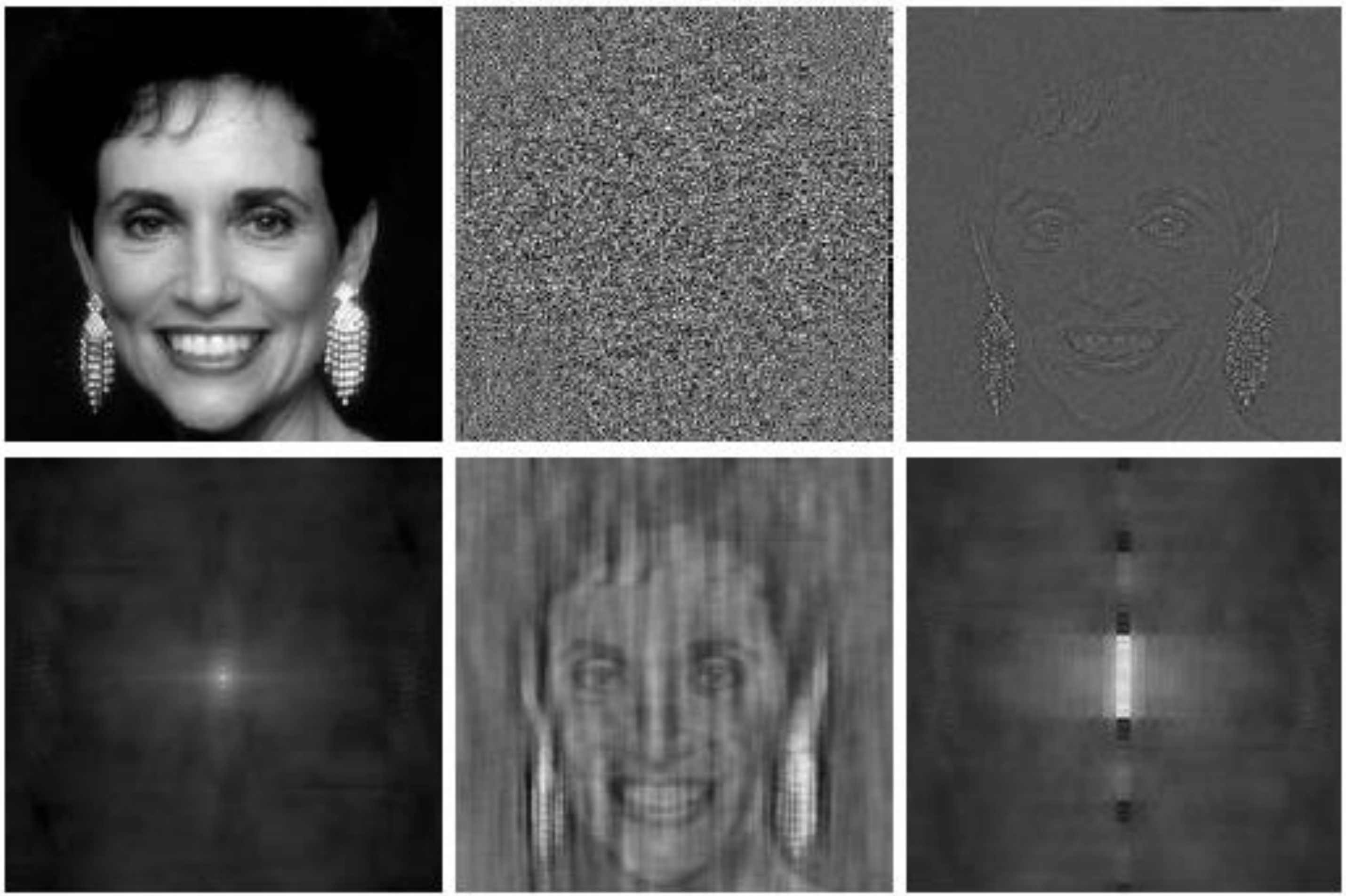
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Phase of original bar, translated, rotated



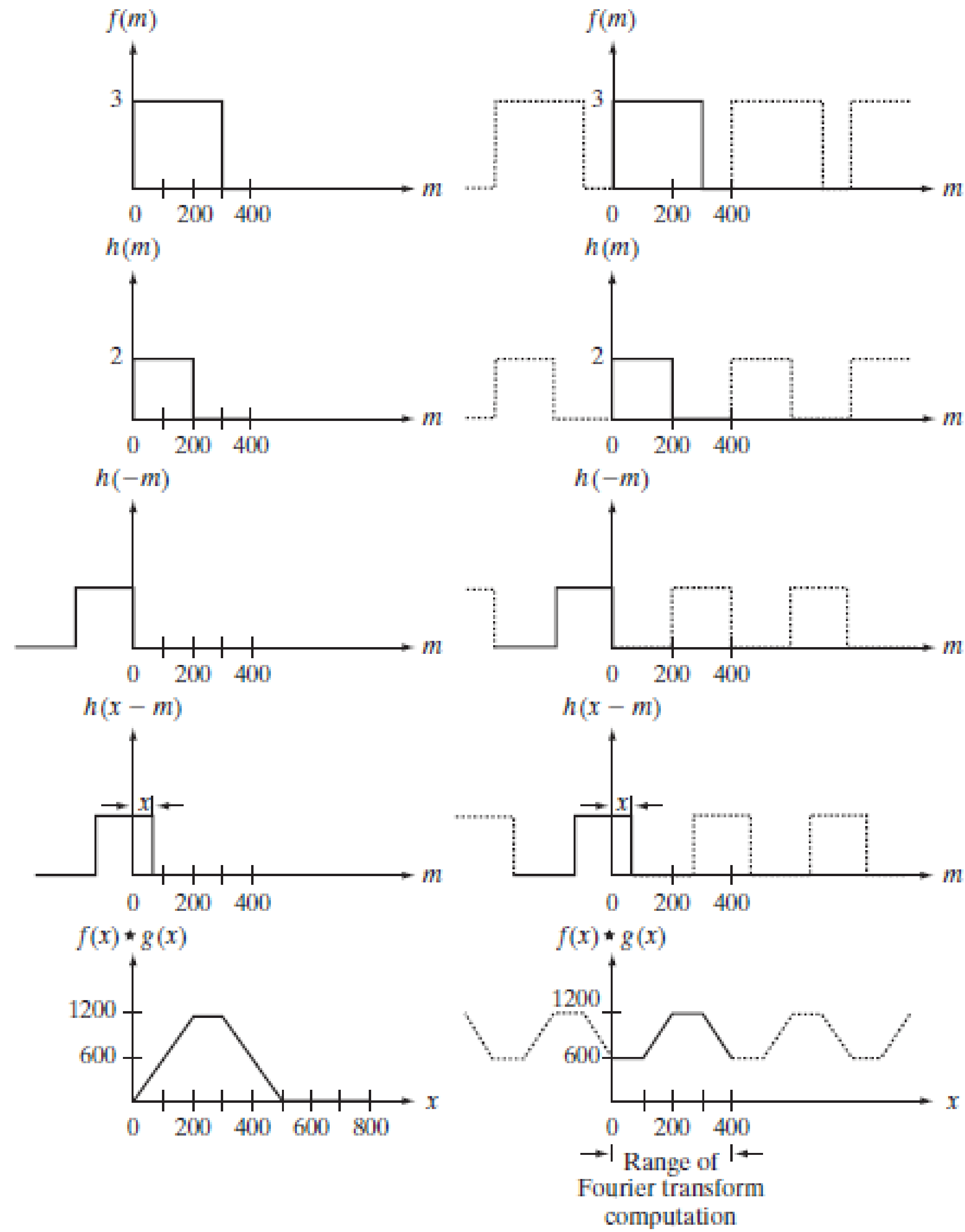
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Importance of phase: Original, phase, reconstruction based on: phase, magnitude, some other image’s magnitude, some other image’s phase



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Implicit periodicity leads to wraparound



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$
3) Polar representation	$F(u, v) = \lvert F(u, v) \rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u, v) \rvert = \left[R^2(u, v) + I^2(u, v) \right]^{1/2}$ $R = \text{Real}(F); \quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)} \right]$
6) Power spectrum	$P(u, v) = \lvert F(u, v) \rvert^2$
7) Average value	$\overline{f}(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN} F(0, 0)$

(Continued)

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Name	Expression(s)
8) Periodicity (k_1 and k_2 are integers)	$F(u, v) = F(u + k_1M, v) = F(u, v + k_2N)$ $= F(u + k_1M, v + k_2N)$ $f(x, y) = f(x + k_1M, y) = f(x, y + k_2N)$ $= f(x + k_1M, y + k_2N)$
9) Convolution	$f(x, y) \star h(x, y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m, n)h(x - m, y - n)$
10) Correlation	$f(x, y) \,☆\, h(x, y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f^*(m, n)h(x + m, y + n)$
11) Separability	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x, y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F^*(u, v)e^{-j2\pi(ux/M + vy/N)}$ This equation indicates that inputting $F^*(u, v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x, y)$. Taking the complex conjugate and dividing by MN gives the desired inverse. See Section 4.11.2.

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$
6) Convolution theorem[†]	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

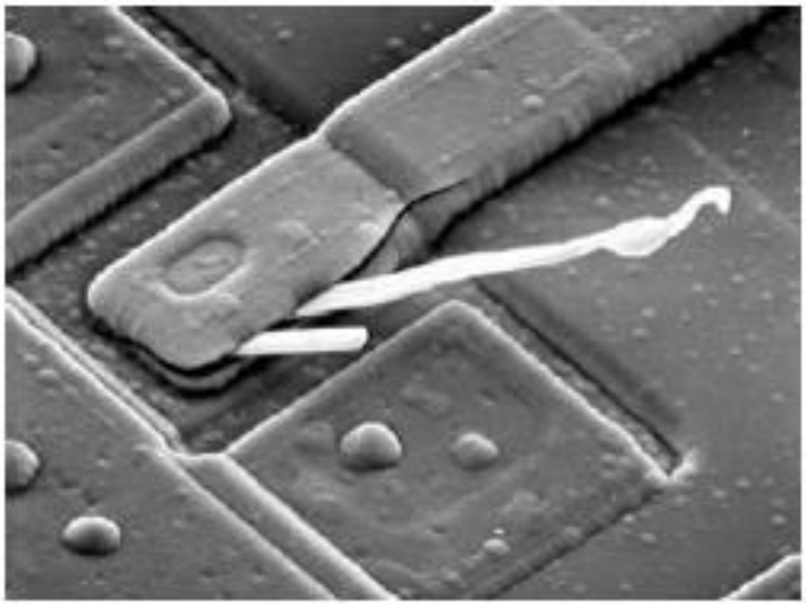
(Continued)

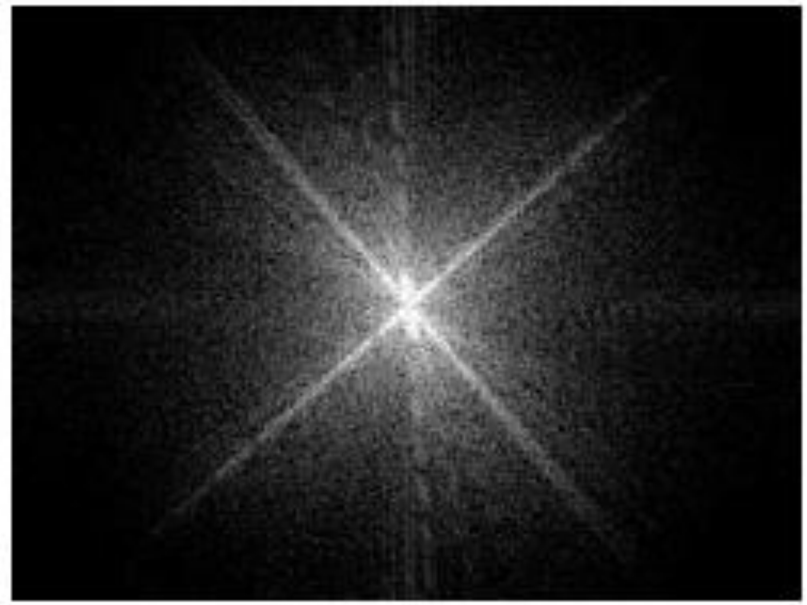
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Name	DFT Pairs
7) Correlation theorem[†]	$f(x, y) \star h(x, y) \Leftrightarrow F^*(u, v)\, H(u, v)$ $f^*(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$
8) Discrete unit impulse	$\delta(x, y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a, b] \Leftrightarrow ab\dfrac{\sin(\pi ua)}{(\pi ua)}\dfrac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j\dfrac{1}{2}\big[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\big]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\dfrac{1}{2}\big[\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)\big]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) *Differentiation* (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\dfrac{\partial}{\partial t}\right)^m\left(\dfrac{\partial}{\partial z}\right)^n f(t, z) \Leftrightarrow (j2\pi\mu)^m(j2\pi\nu)^n F(\mu, \nu)$ $\dfrac{\partial^m f(t, z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu);\ \dfrac{\partial^n f(t, z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) *Gaussian*	$A2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow Ae^{-(\mu^2+\nu^2)/2\sigma^2}$ (A is a constant)

[†]Assumes that the functions have been extended by zero padding. Convolution and correlation are associative, commutative, and distributive.

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

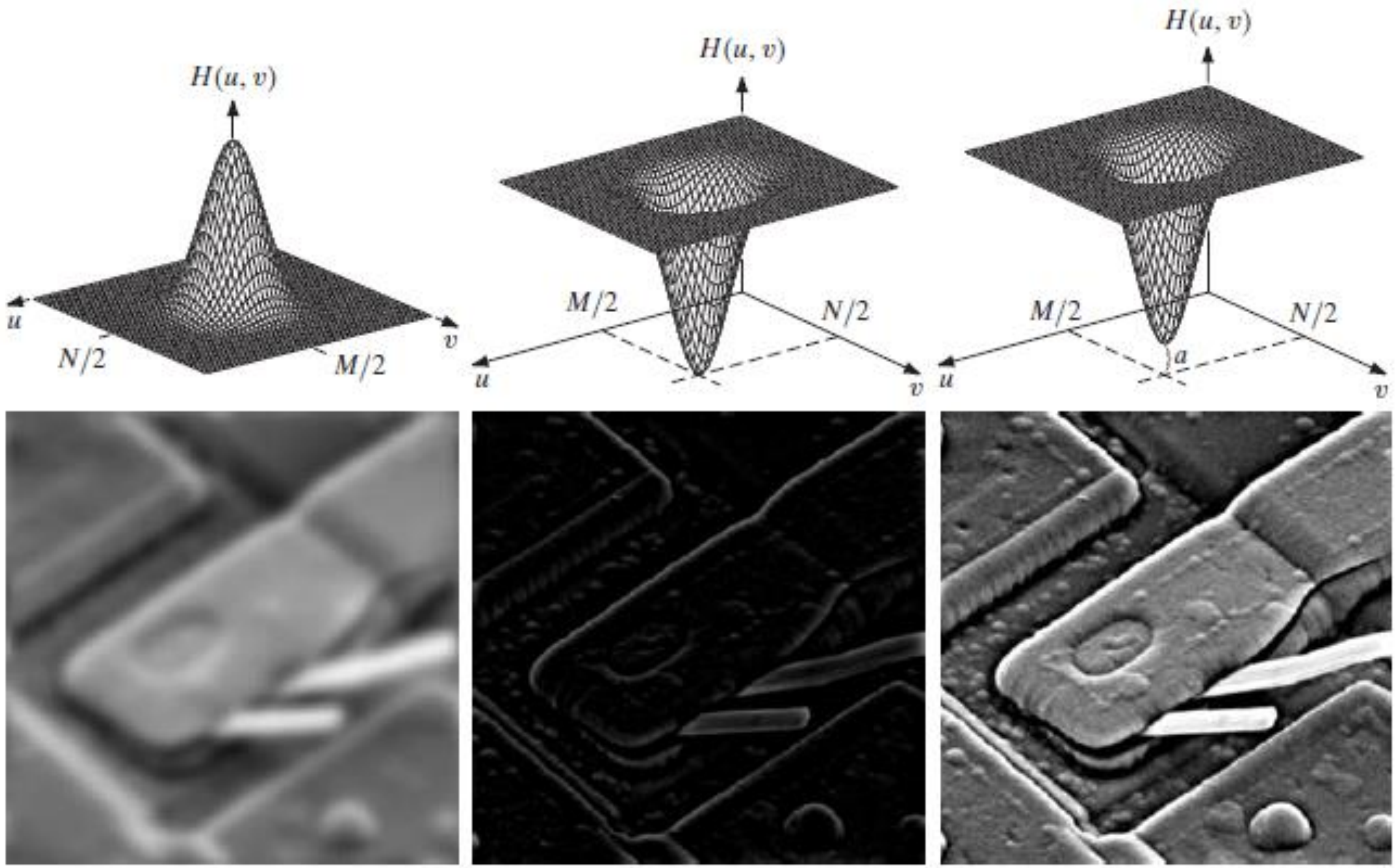




Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Filtering in frequency domain

$$g(x, y) = \Im^{-1}[H(u, v)F(u, v)]$$

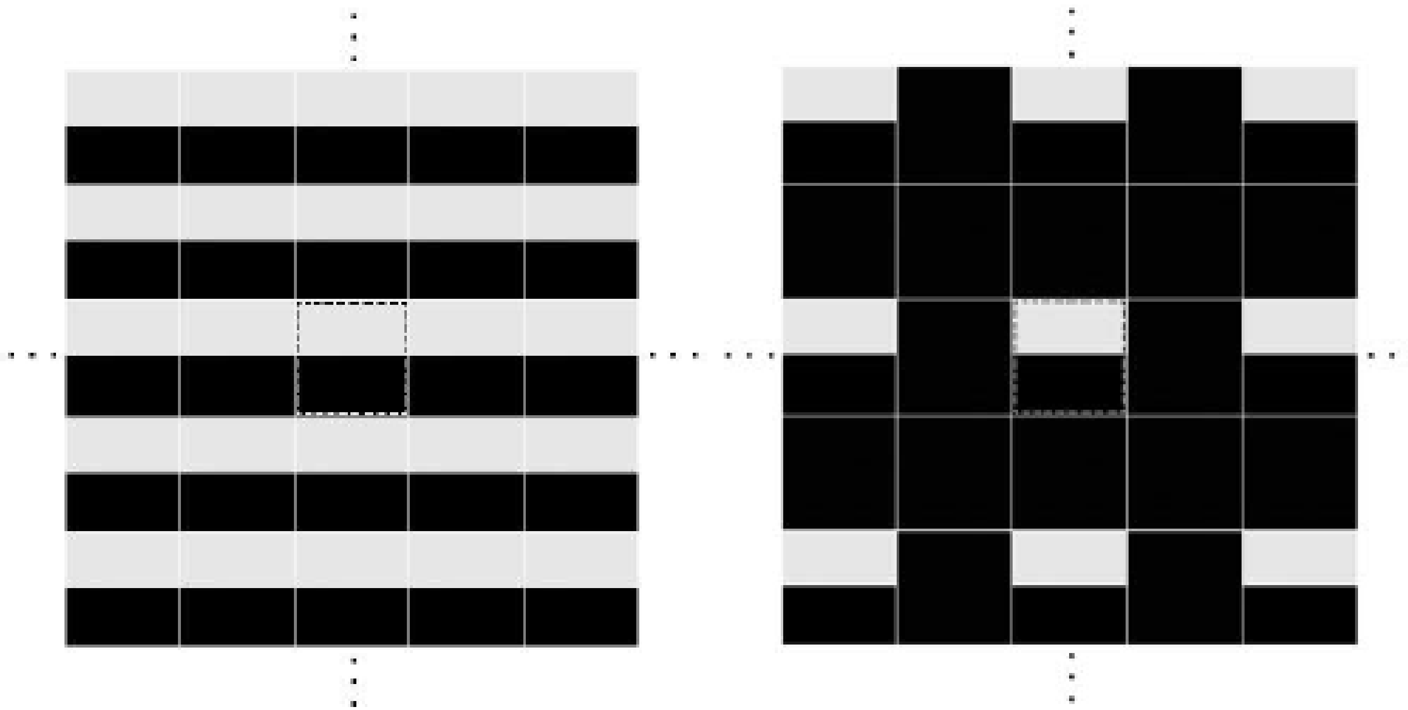


Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Effect of wraparound and zero padding

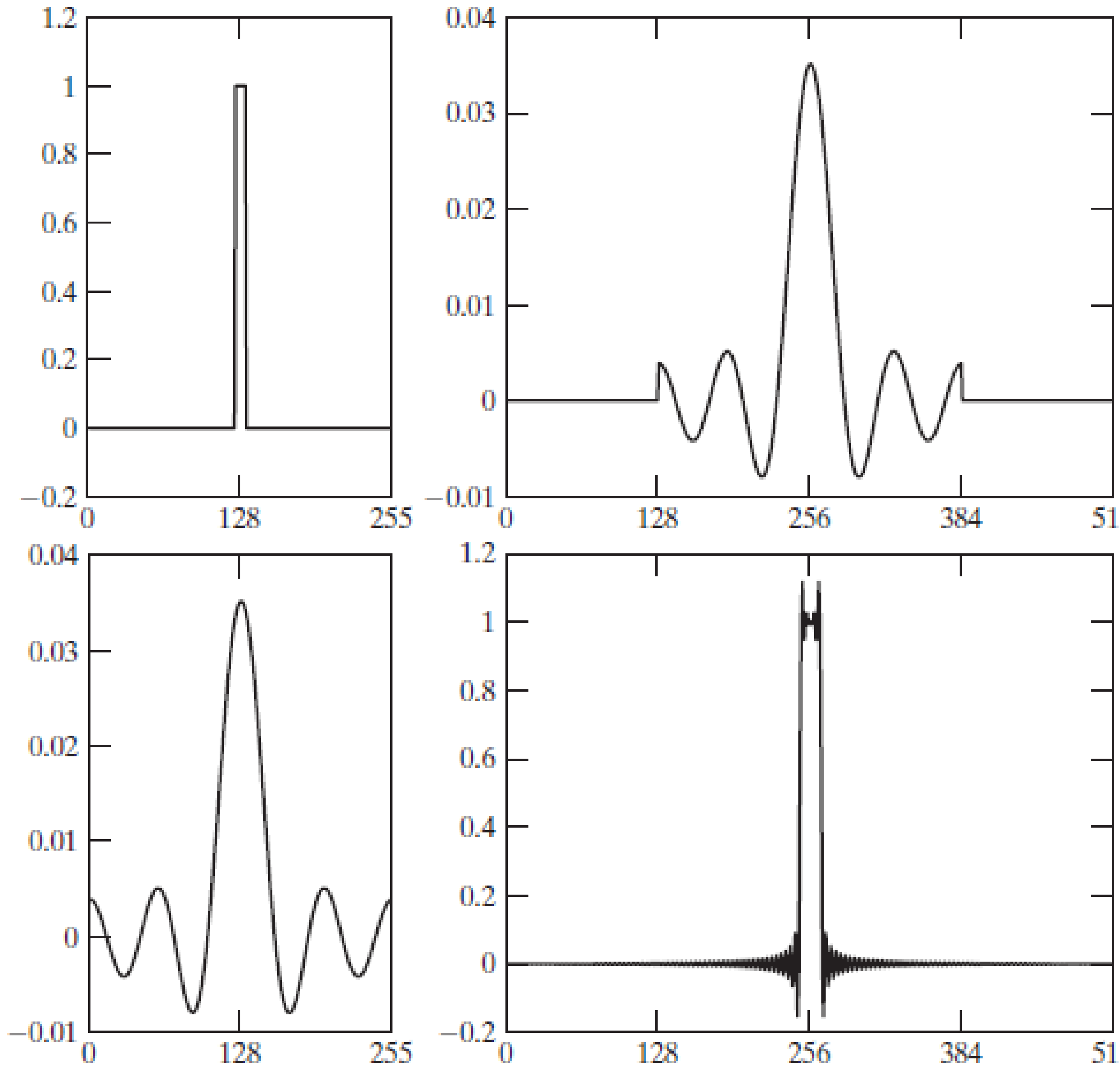
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Implicit periodicity without and with zero padding



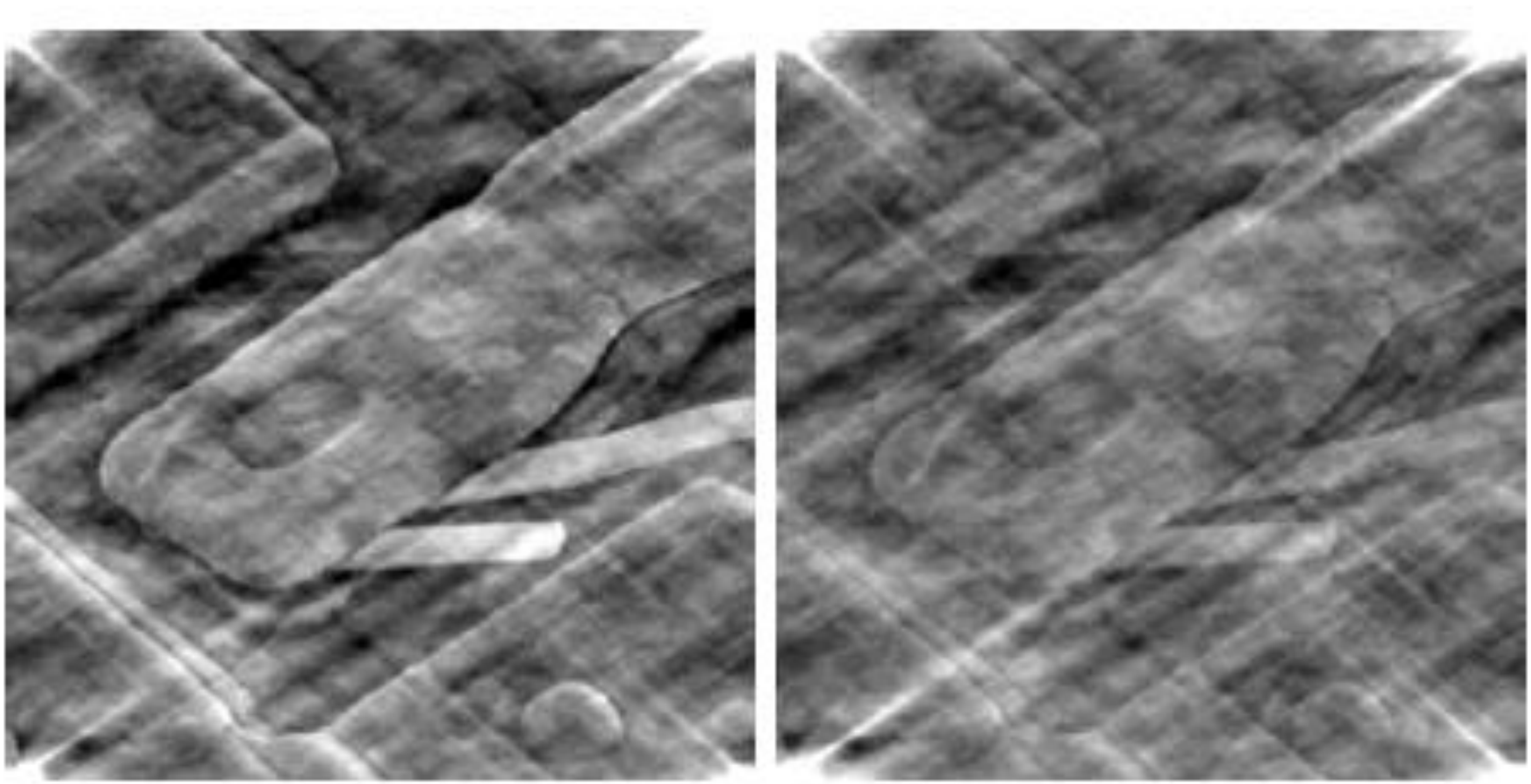
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

One cannot work with ideal frequency cut-offs and avoiding wraparound errors



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Importance of phase – Multiplying phase with 0.5 and 0.25; and zero phase shift filters



$$g(x, y) = \Im^{-1}\left[H(u, v)R(u, v) + jH(u, v)I(u, v)\right]$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Summary of frequency filtering

1. Given an input image $f(x, y)$ of size $M \times N$, obtain the padding parameters P and Q from Eqs. (4.6-31) and (4.6-32). Typically, we select $P = 2M$ and $Q = 2N$.
2. Form a padded image, $f_p(x, y)$, of size $P \times Q$ by appending the necessary number of zeros to $f(x, y)$.
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center its transform.
4. Compute the DFT, $F(u, v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u, v)$, of size $P \times Q$ with center at coordinates $(P/2, Q/2)$.† Form the product $G(u, v) = H(u, v)F(u, v)$ using array multiplication; that is, $G(i, k) = H(i, k)F(i, k)$.
6. Obtain the processed image:

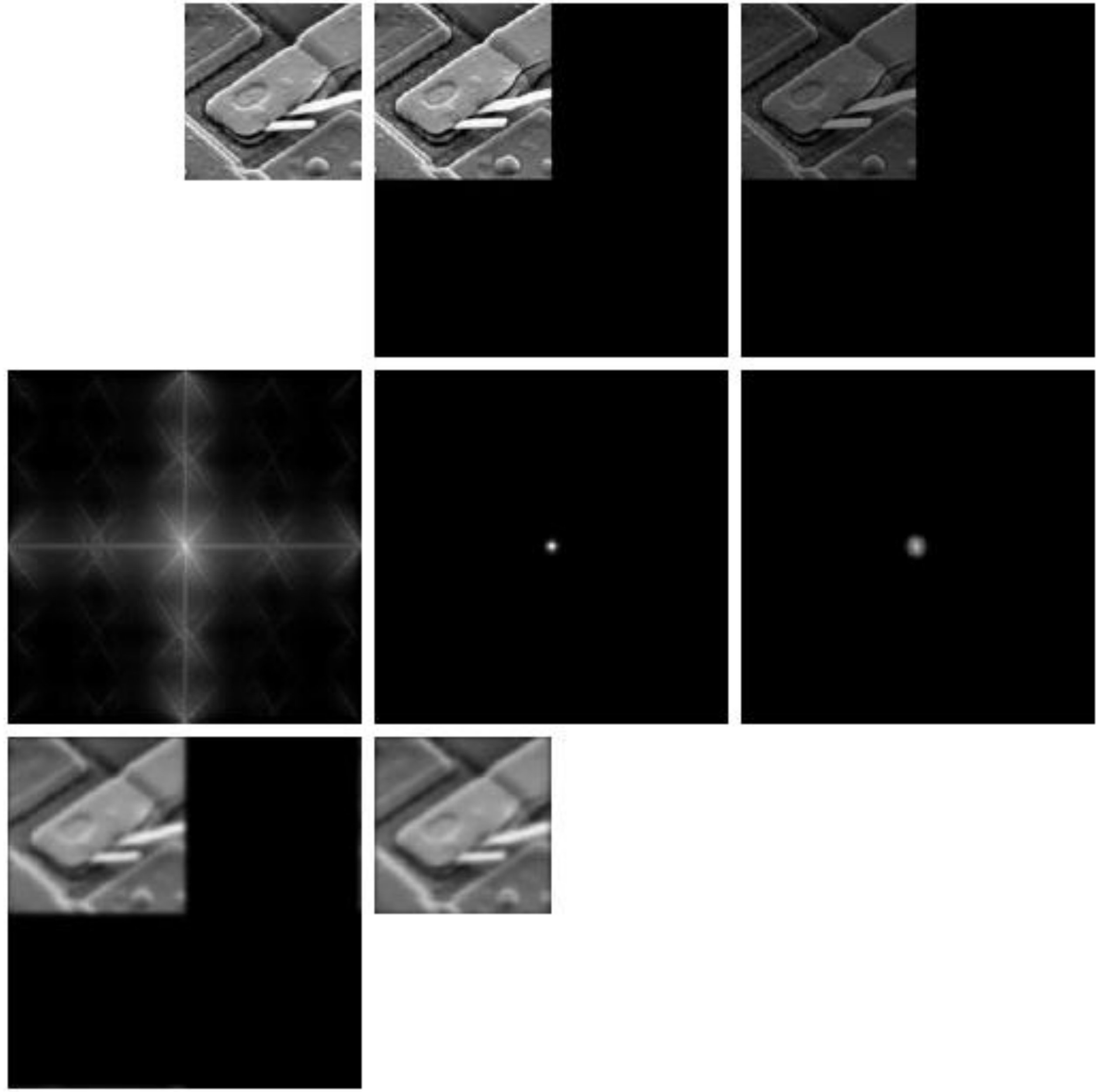
$$g_p(x, y) = \left\{\text{real}\left[\Im^{-1}[G(u, v)]\right]\right\}(-1)^{x+y}$$

   where the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies, and the subscript p indicates that we are dealing with padded arrays.
7. Obtain the final processed result, $g(x, y)$, by extracting the $M \times N$ region from the top, left quadrant of $g_p(x, y)$.

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Example

FIGURE 4.36
(a) An $M \times N$ image, f.
(b) Padded image, f_p of size $P \times Q$.
(c) Result of multiplying f_p by $(-1)^{x+y}$.
(d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$.
(f) Spectrum of the product HF_p.
(g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p.
(h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

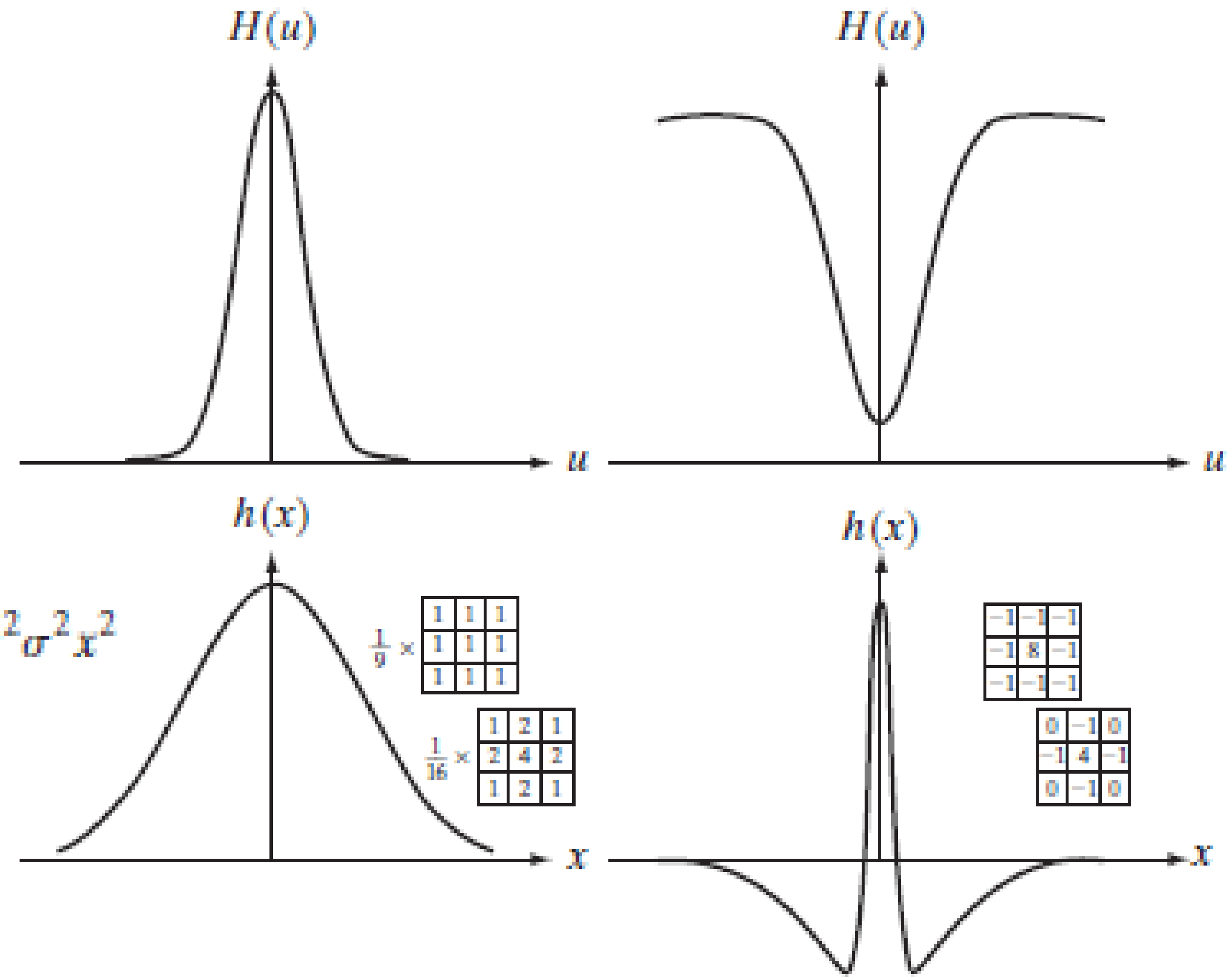


Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Filtering using Gaussian filters

$$H(u) = A e^{-u^2/2\sigma^2}$$

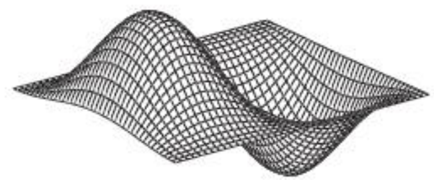
$$h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2 x^2}$$

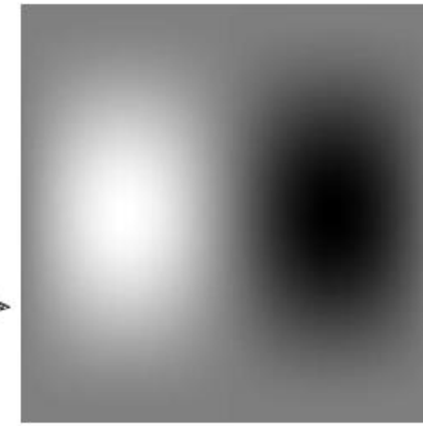


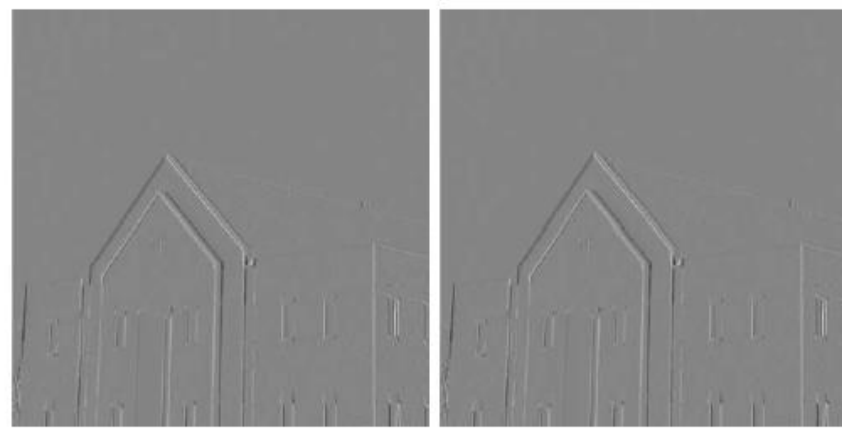
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Equivalence of spatial and frequency filtering

−1	0	1
−2	0	2
−1	0	1

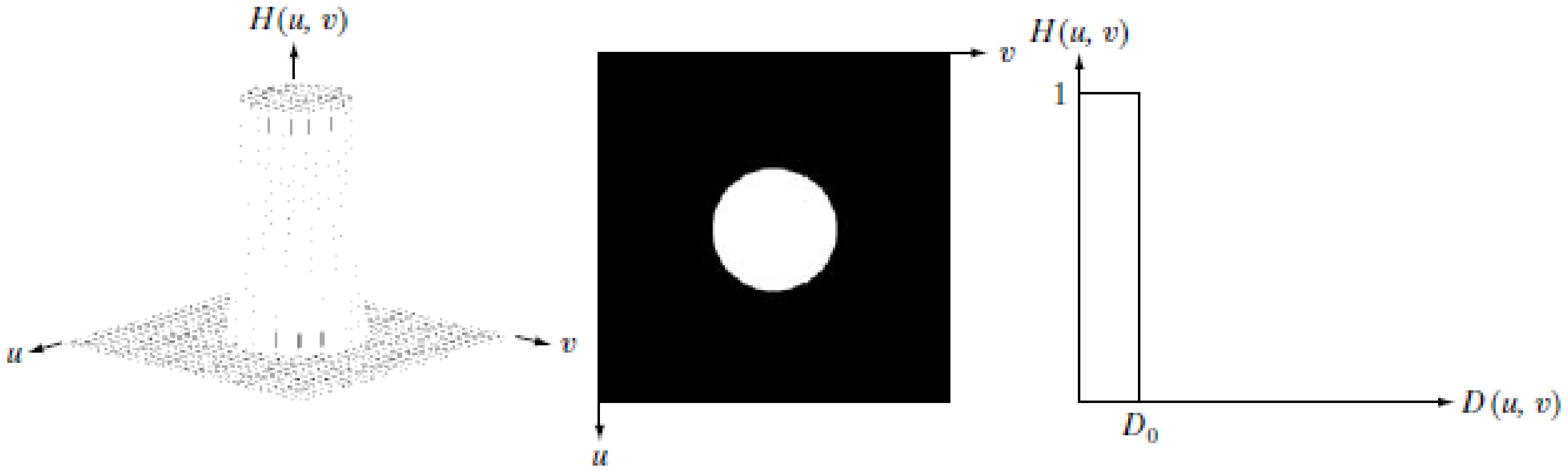






Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

An ideal low pass filter

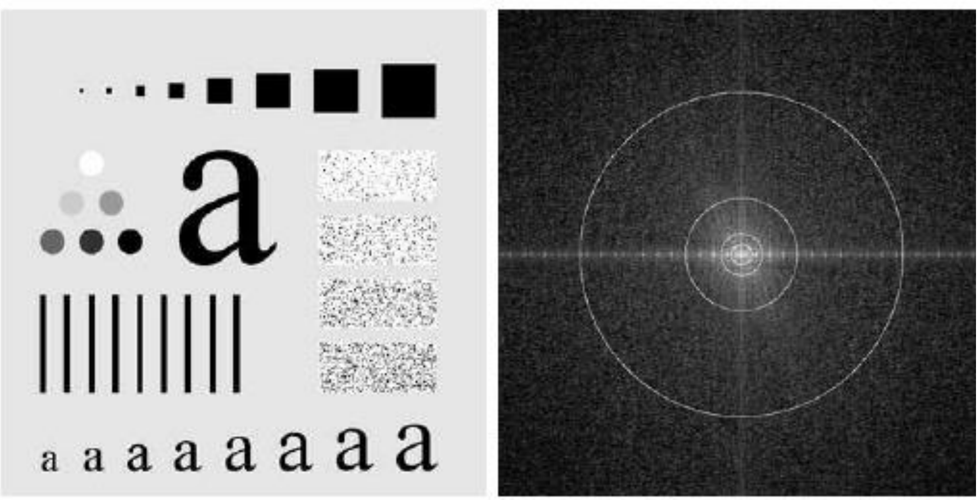


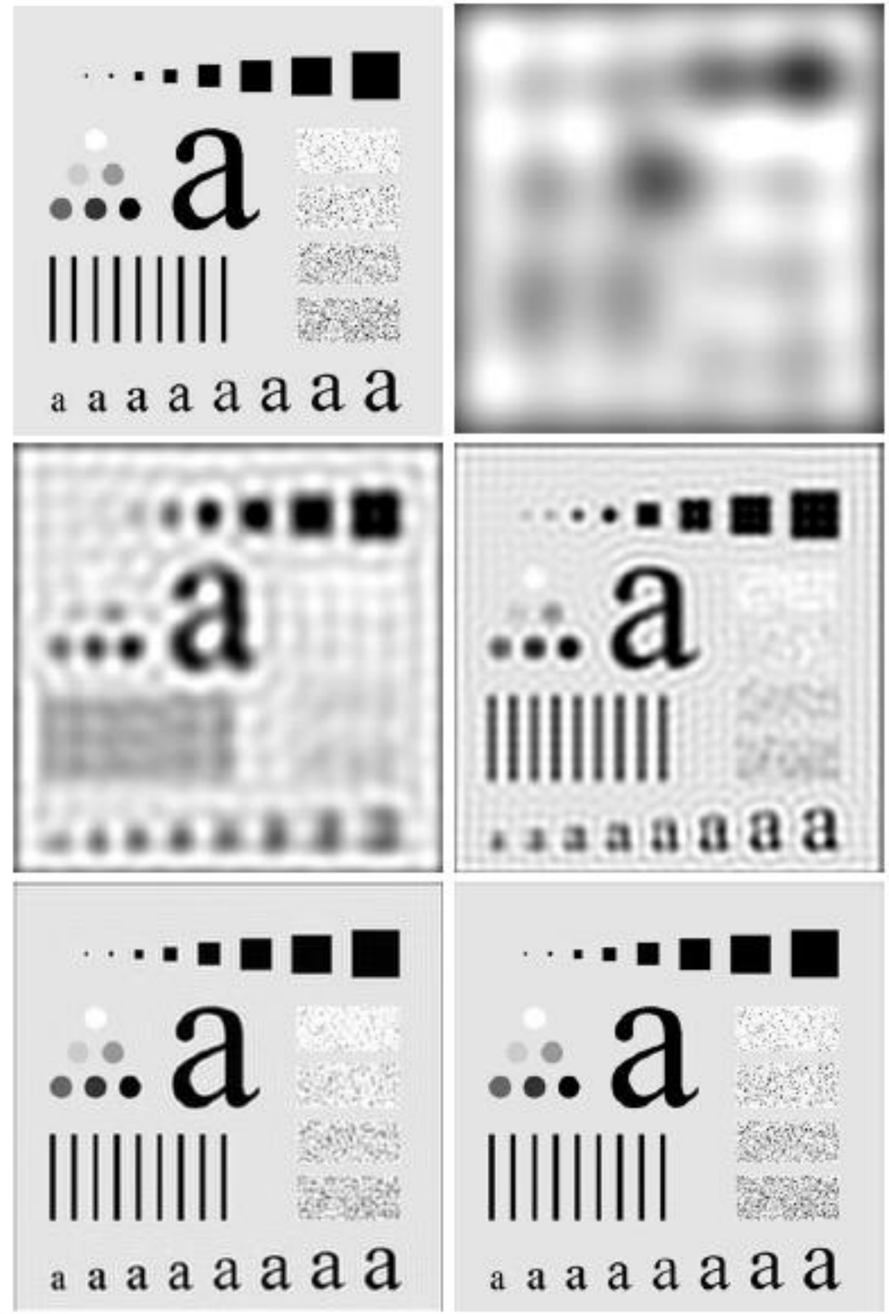
$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2\right]^{1/2}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

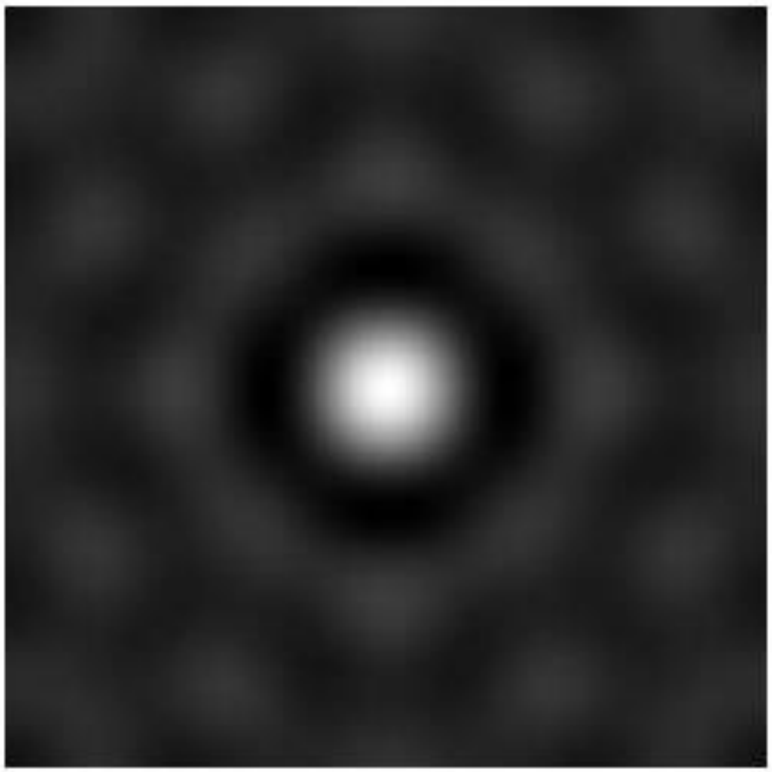
Results of ILPF; notice the ringing

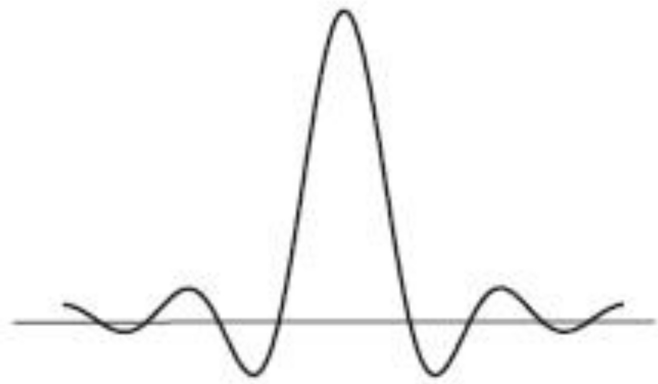




Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

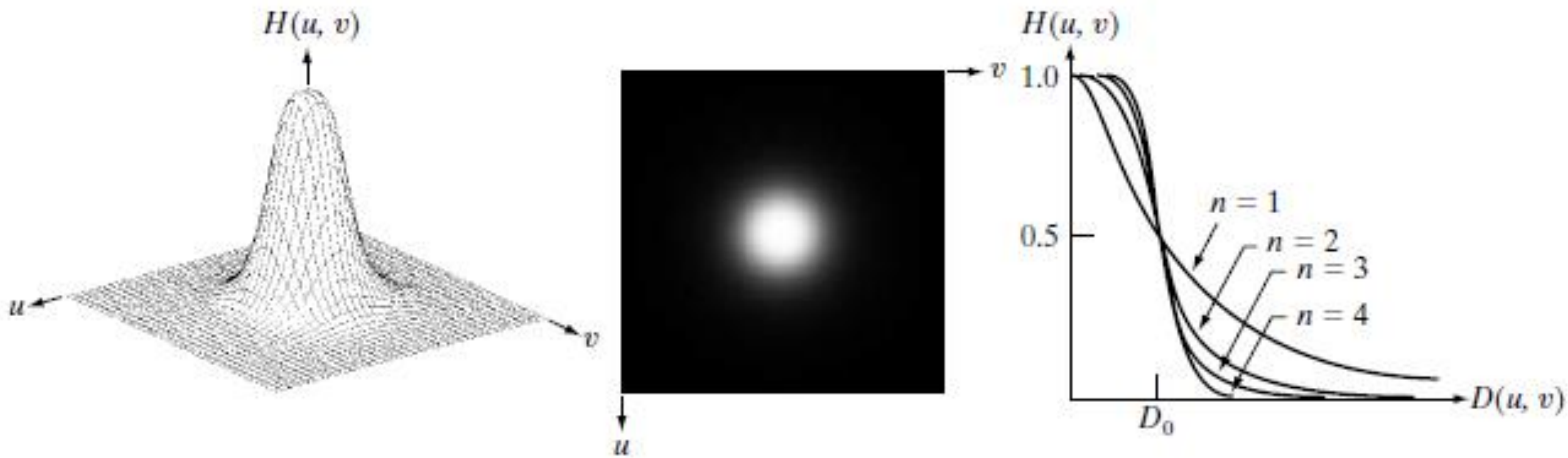
Spatial domain ILPF shows where ringing comes from





Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

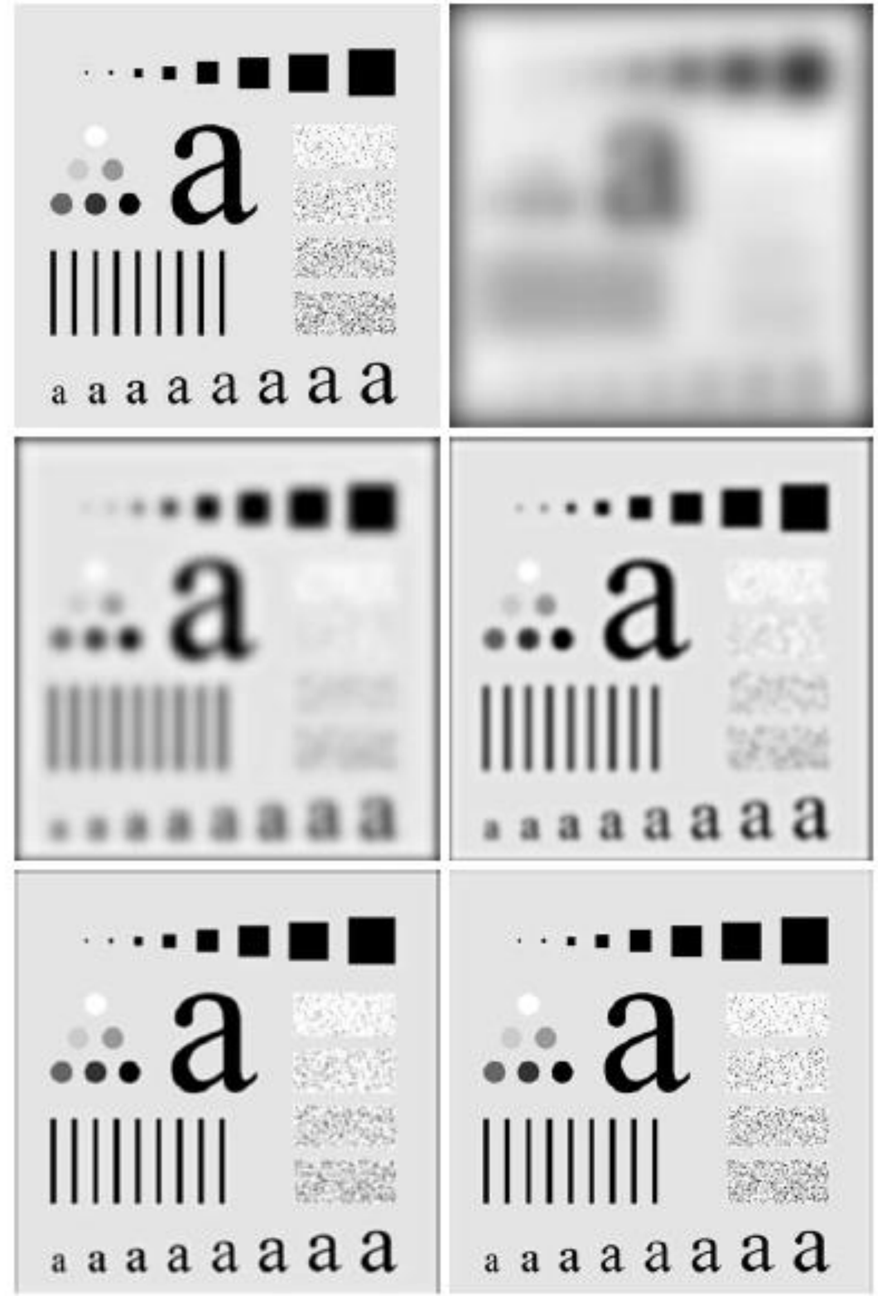
Butterworth filters reduce ringing



$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

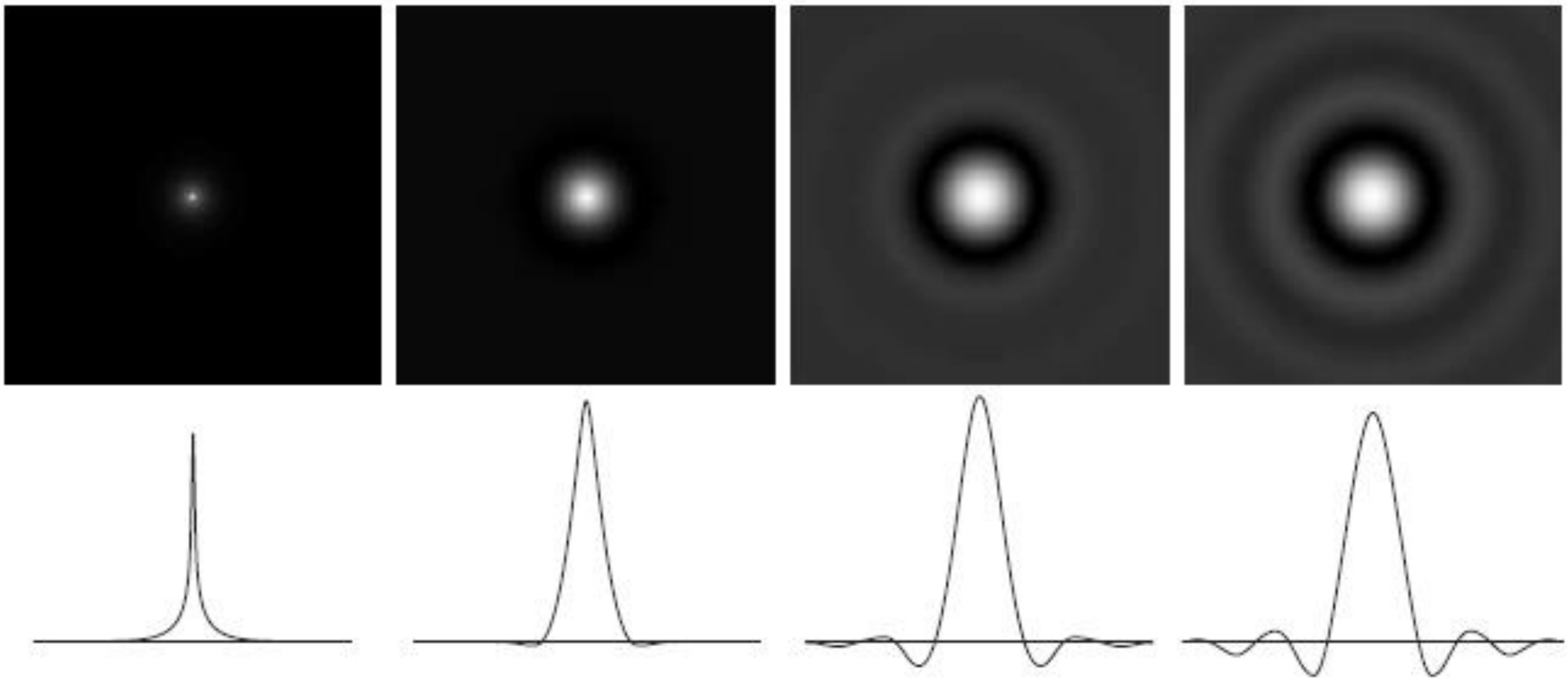
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Ringing is imperceptible in Butterworth filters of order 2 of various D_0



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

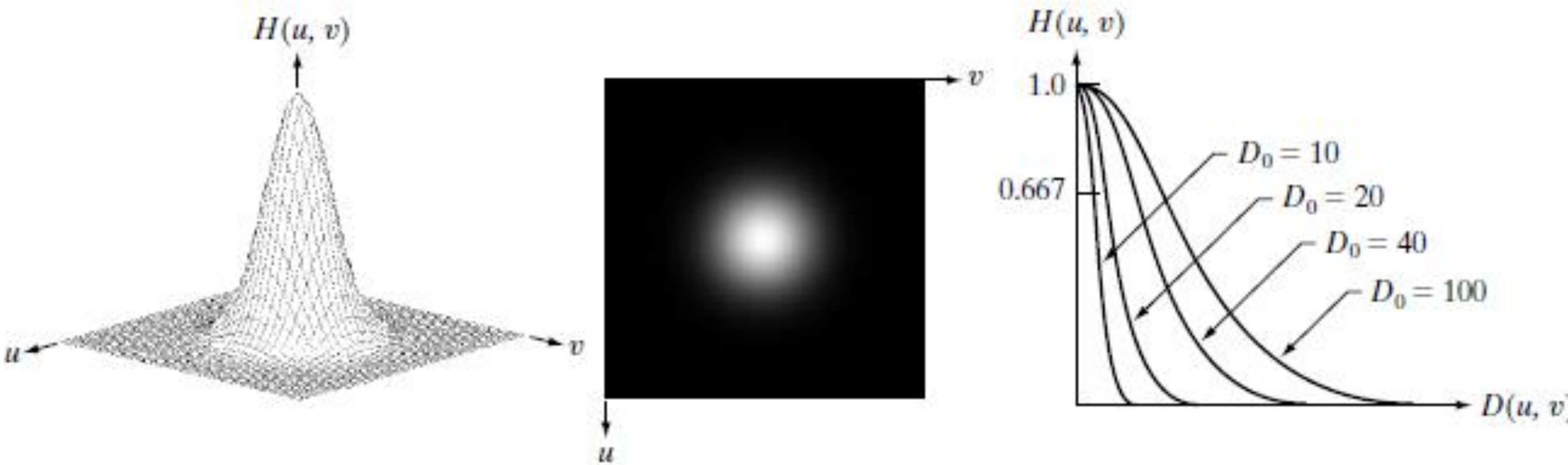
Ringing increases in BLPFs with order (1, 2, 5, 20)



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

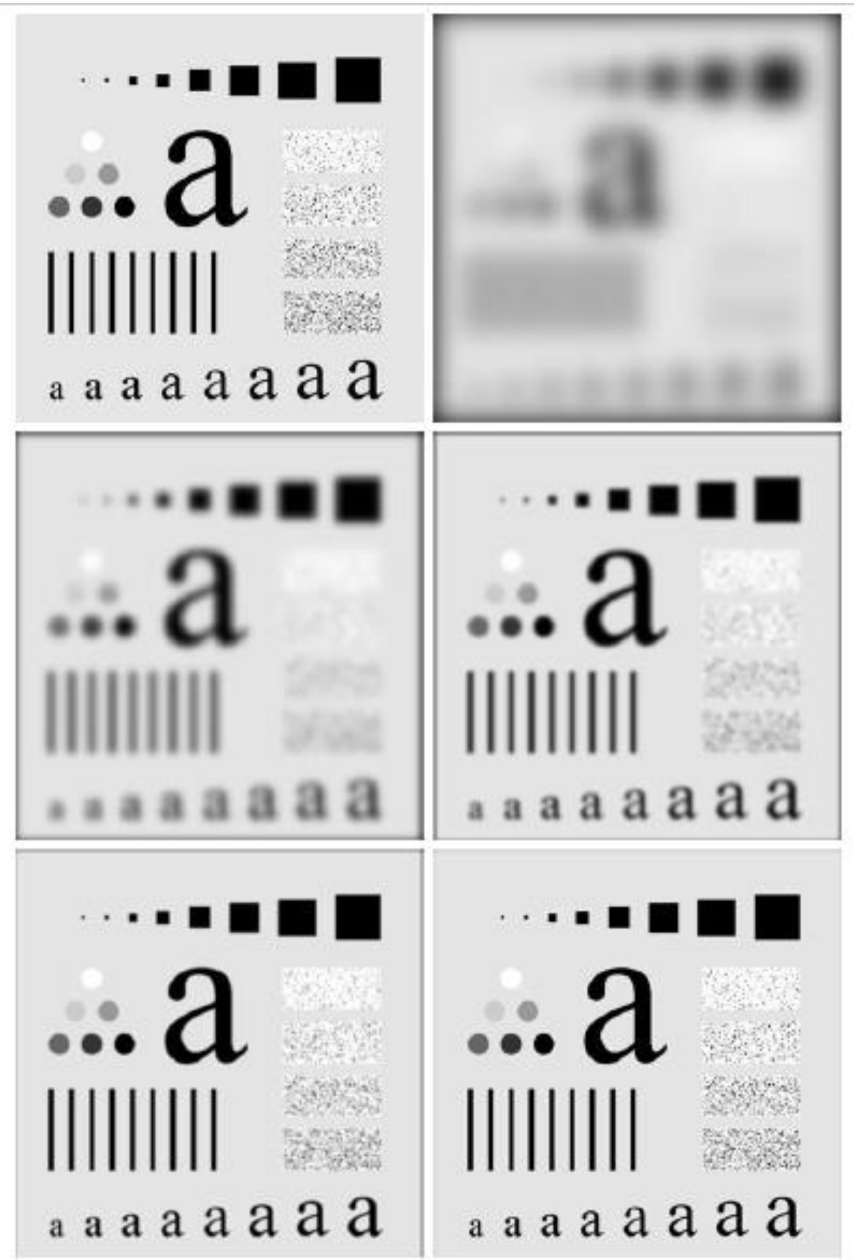
Another option is GLPF

$$H(u, v) = e^{-D^2(u,v)/2D_0^2}$$



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Use of different cut-offs in GLPFs



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

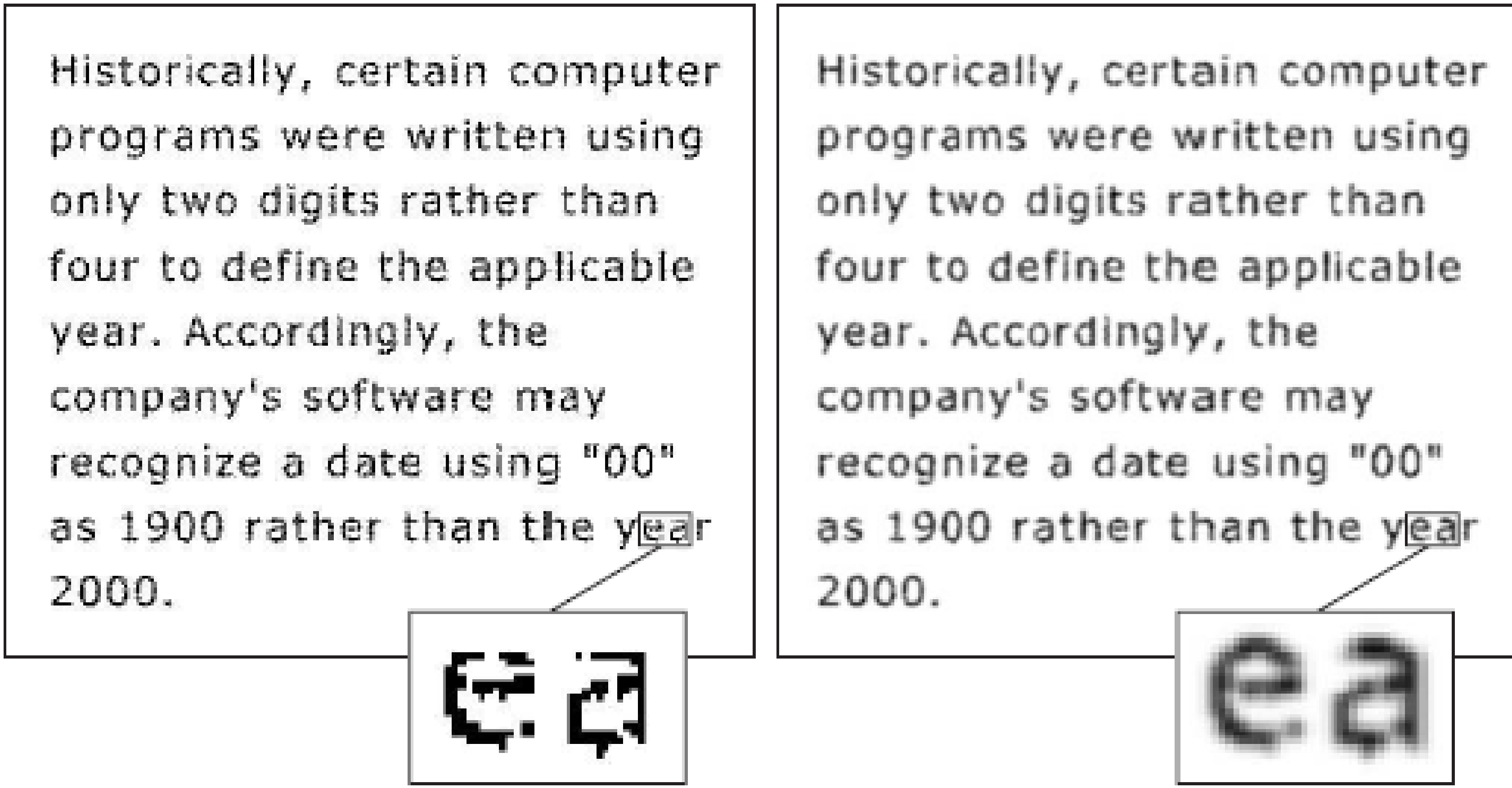
Summary of the three LPFs

Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Blurring with GLPF for continuity

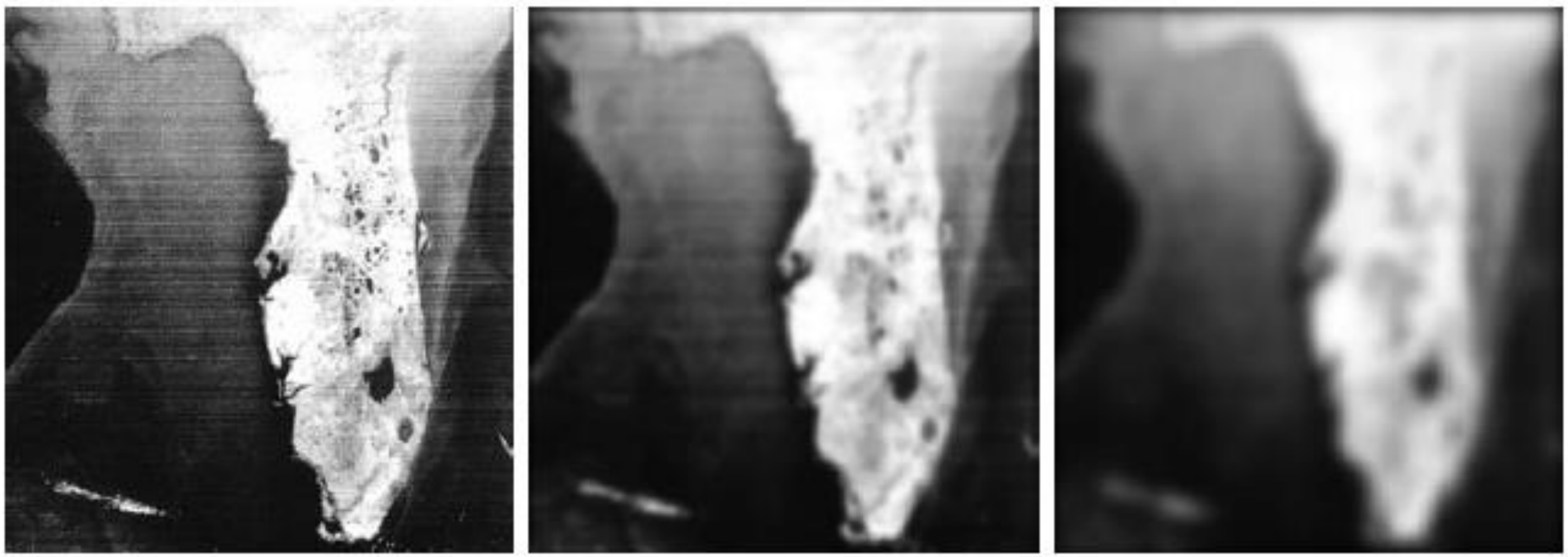


Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Anti-wrinkle cream or smoothening with GLPFs

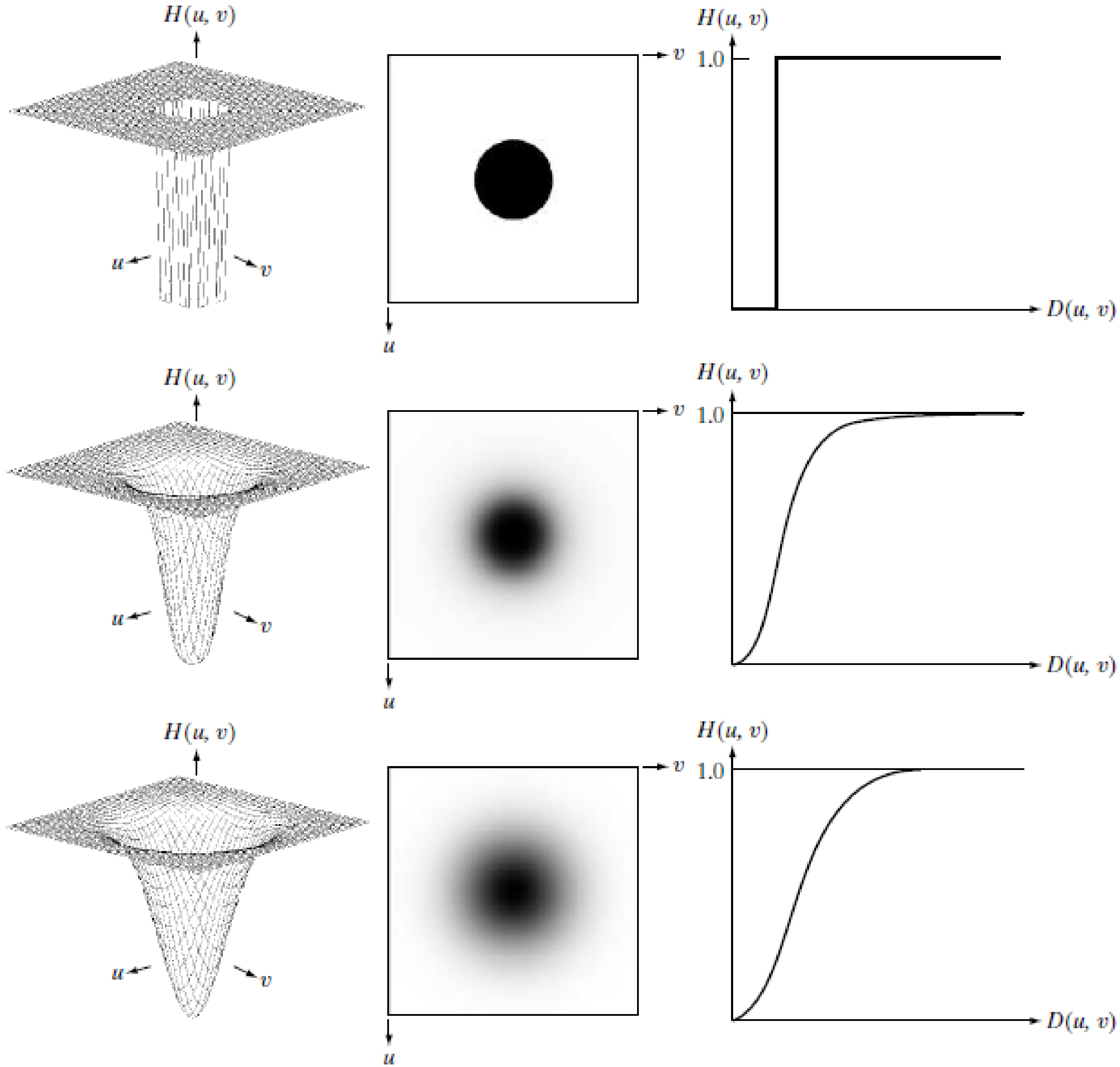
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Removal of scan lines using GLPF



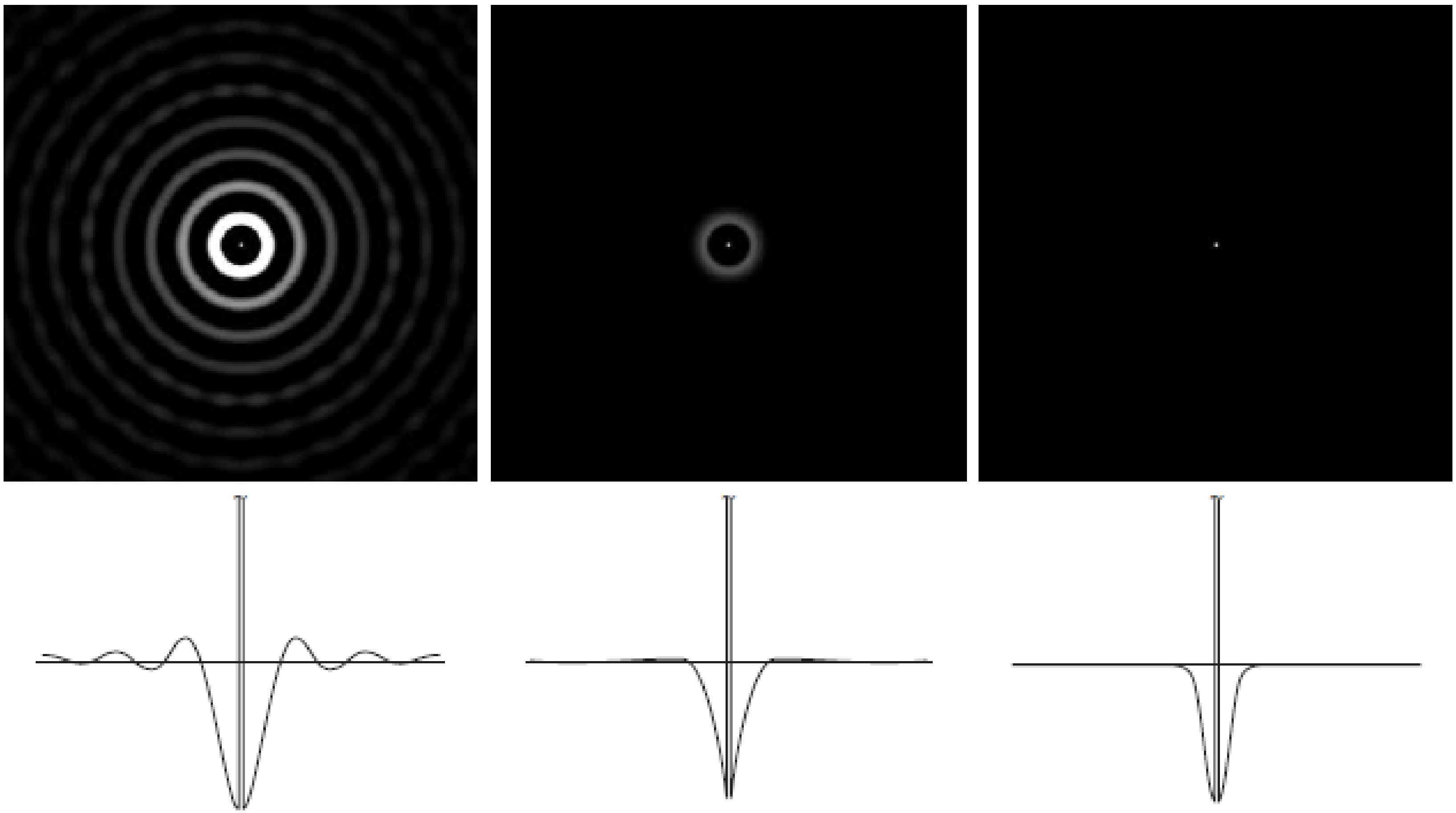
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

High pass filters



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Ideal, Butterworth, and Gaussian HPFs in spatial domain



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

IHPF with different radii

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

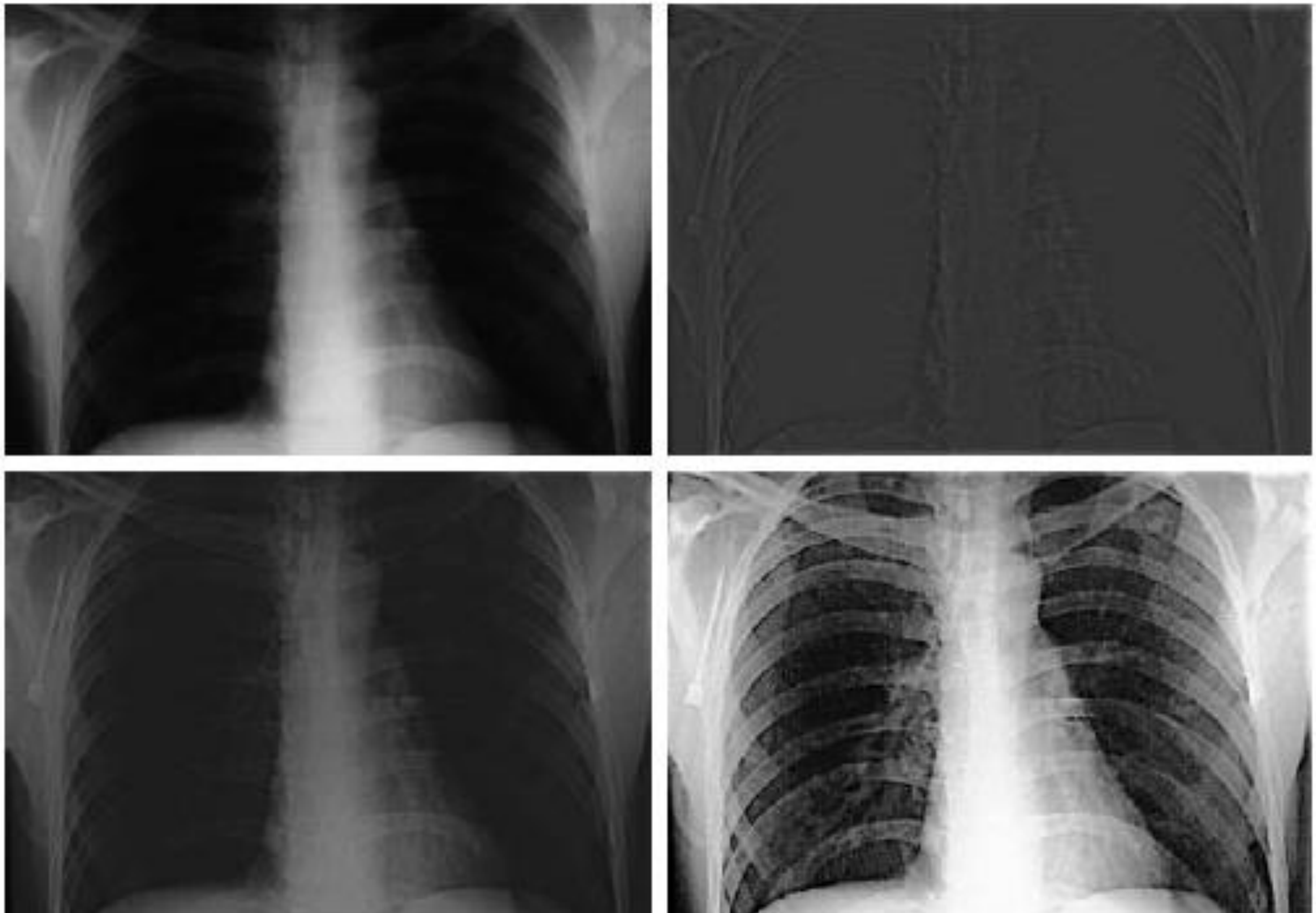
HPF and thresholding

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Enhancement using Laplacian

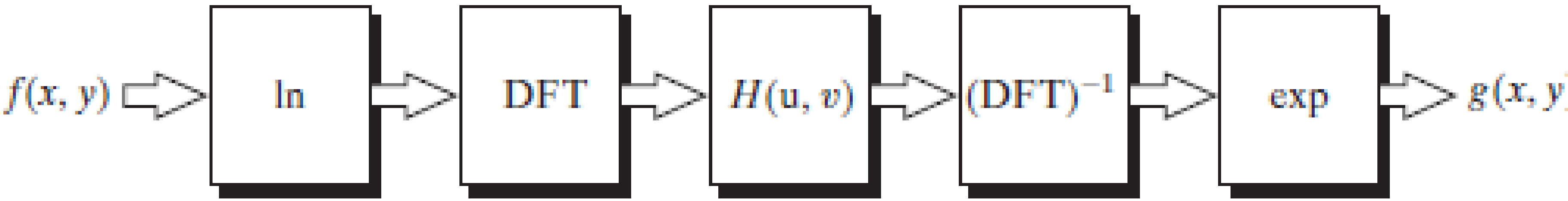
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Original, GHPF, Adding the two, equalizing histogram



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Homomorphic filtering

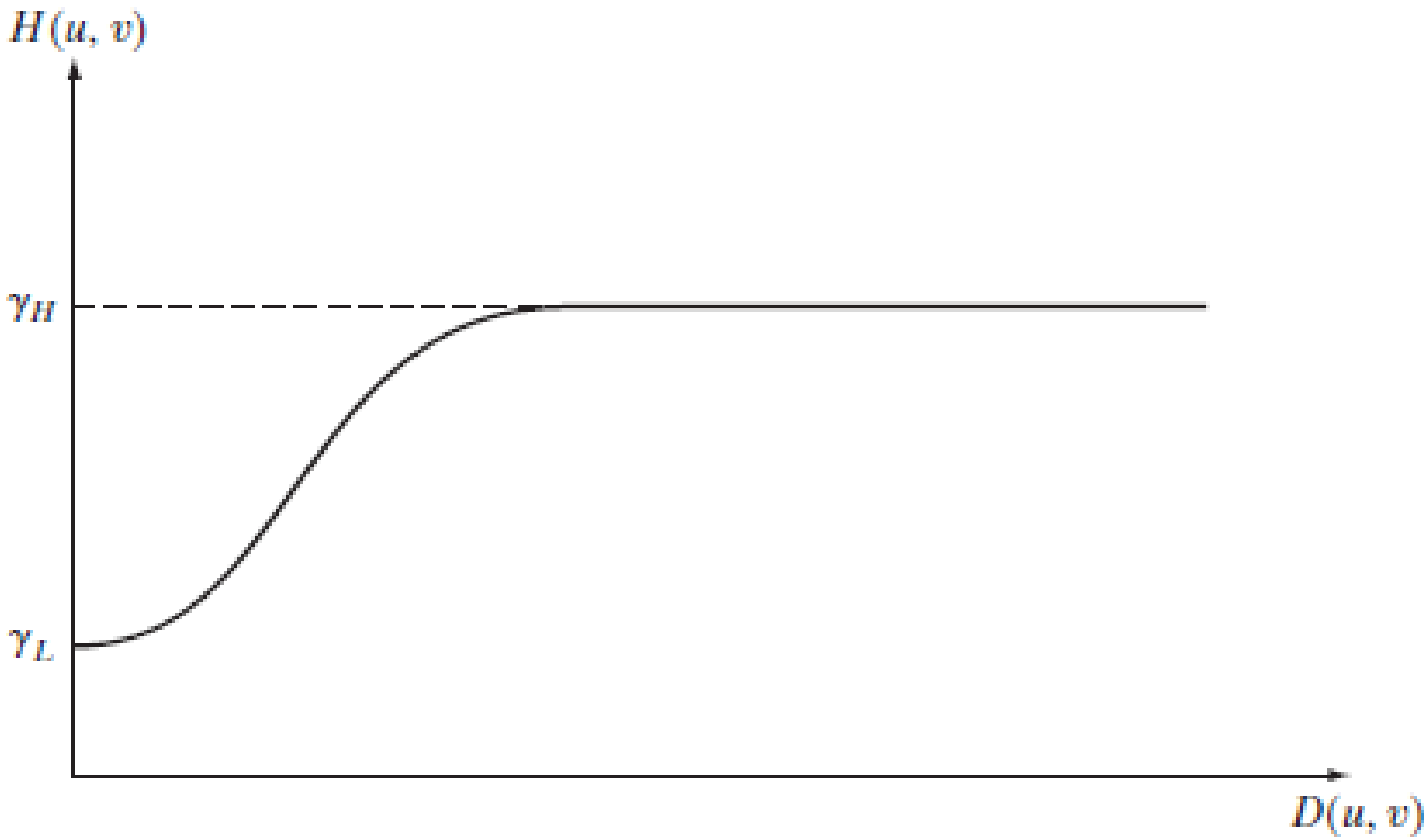


- Taking log of illumination × reflectance
- Filter is precisely designed to affect illumination and reflectance separately, assuming low frequency illumination and high frequency reflectance

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

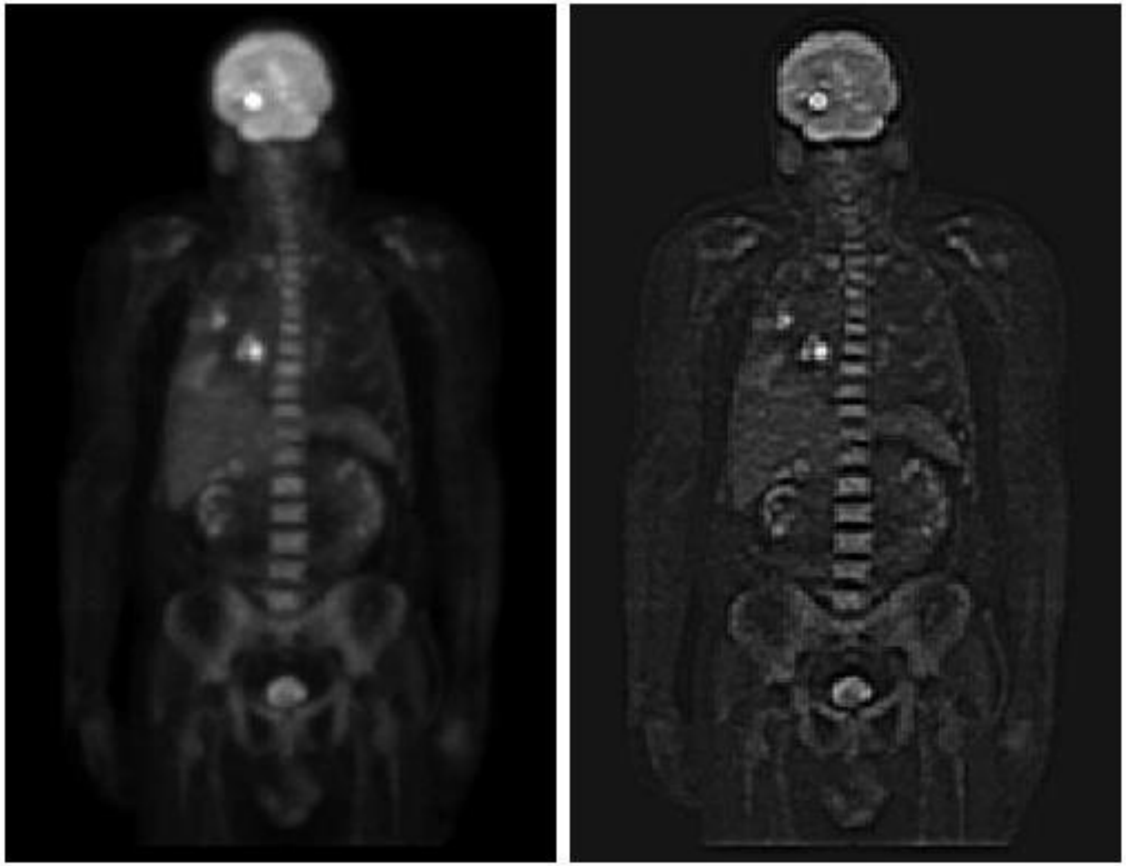
Example radial cross section of a filter

$$H(u, v) = (\gamma_H - \gamma_L)\left[1 - e^{-c[D^2(u,v)/D_0^2]}\right] + \gamma_L \qquad (4.9\text{-}29)$$



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Example enhancement of tumors in PET scan using homomorphic filtering



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

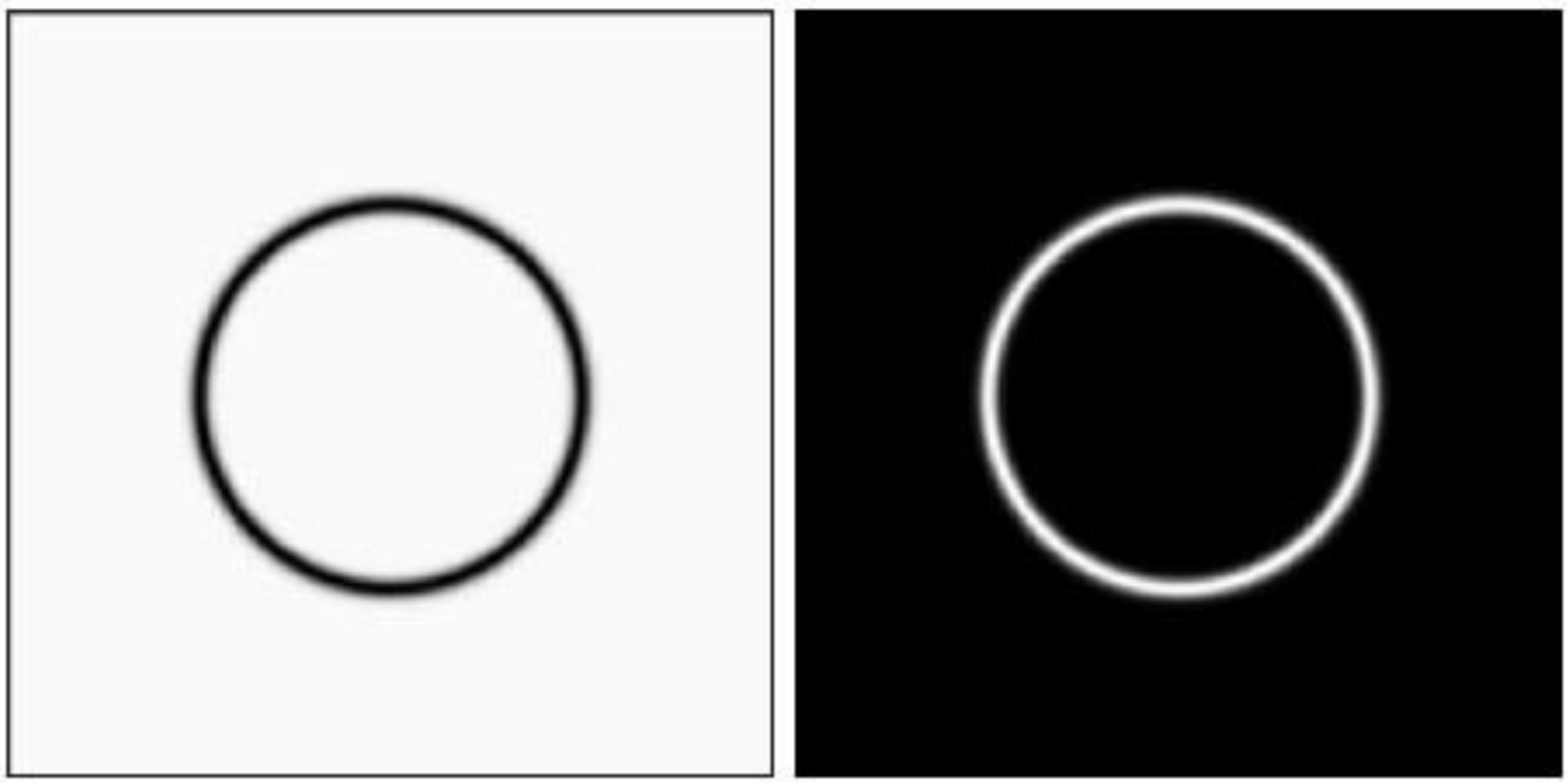
Examples of band reject filters

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Similarly, bandpass filters can be defined

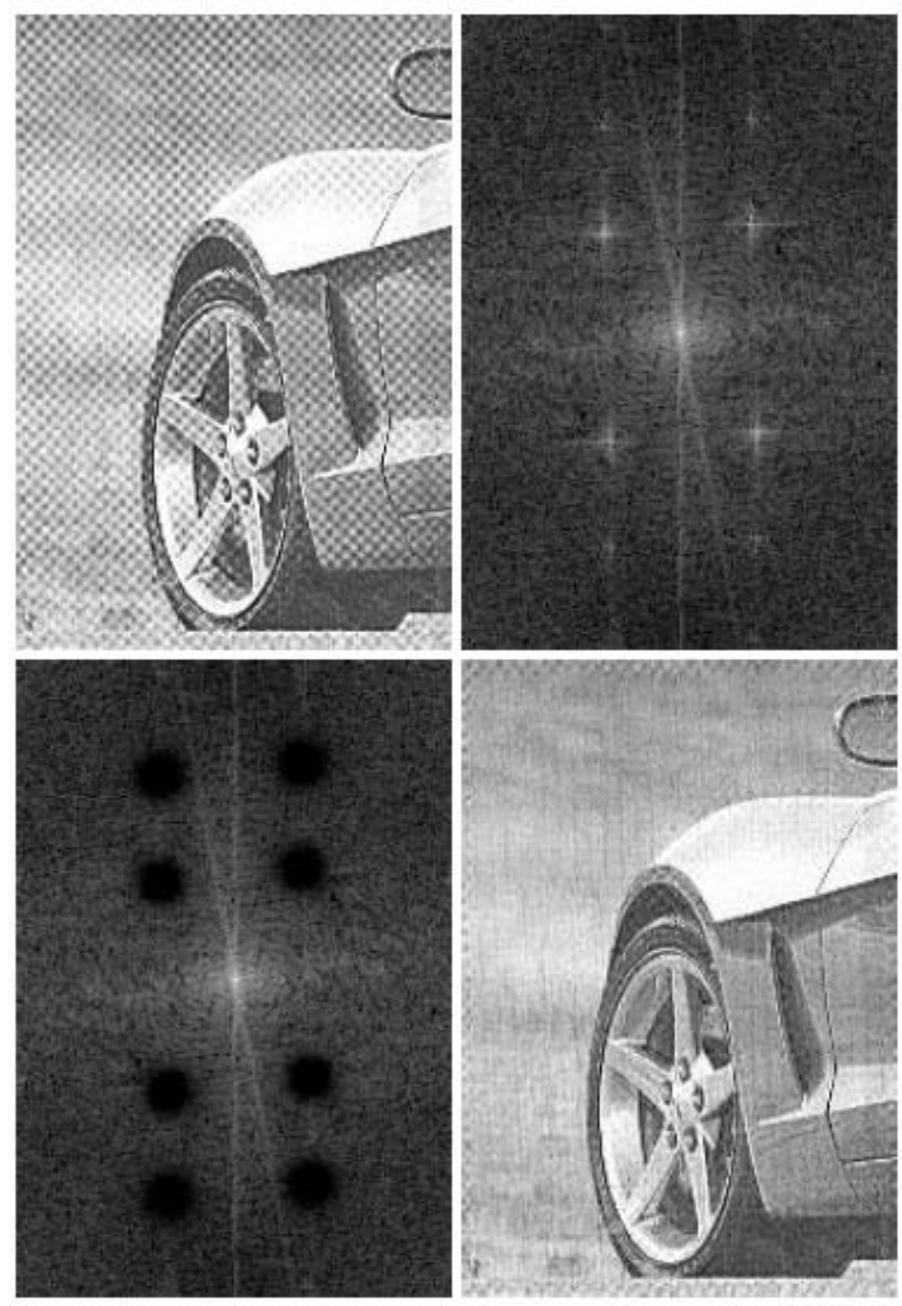
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Band reject vs. bandpass



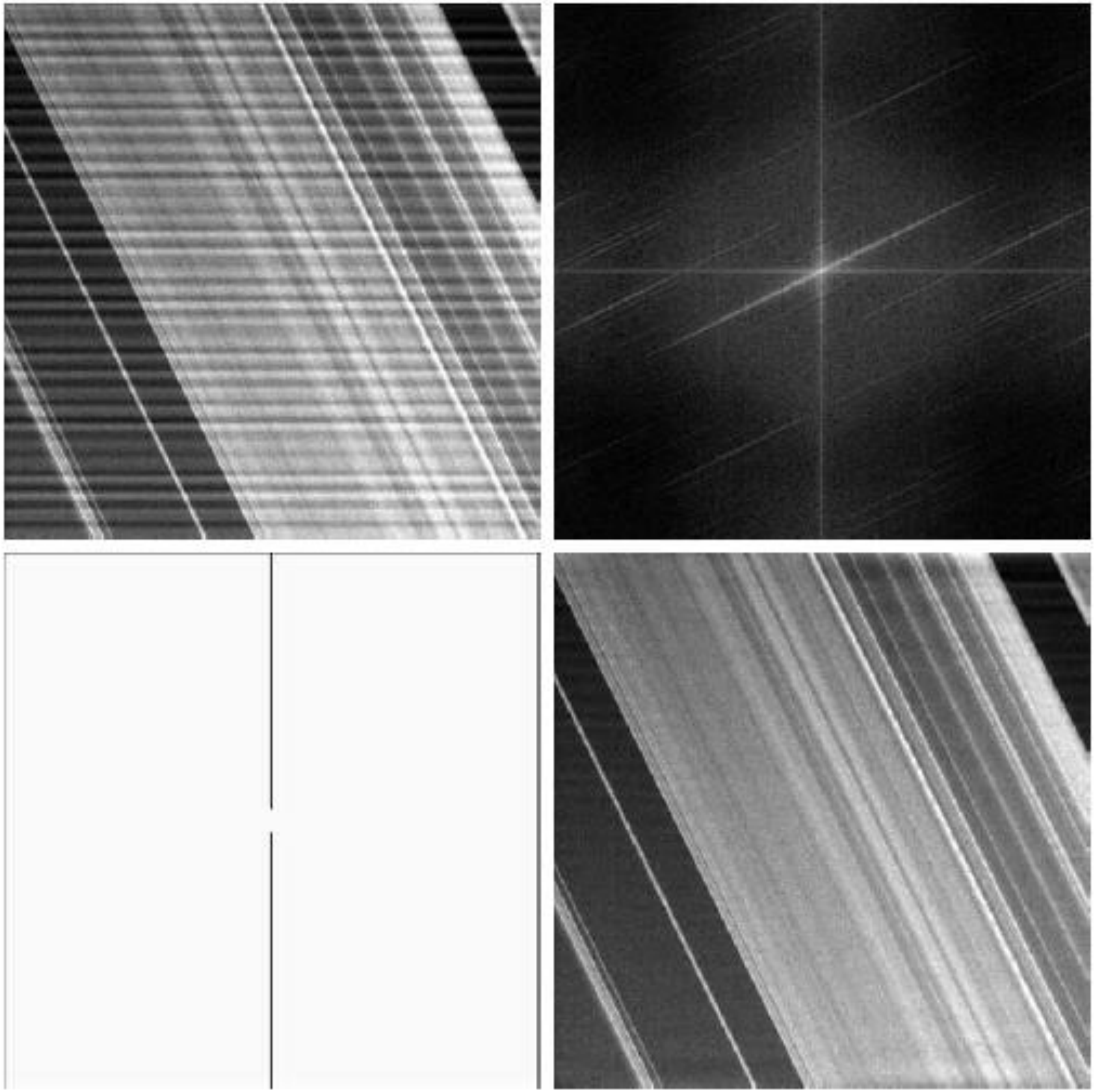
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Original, DFT Butterworth notch-reject multiplied, result



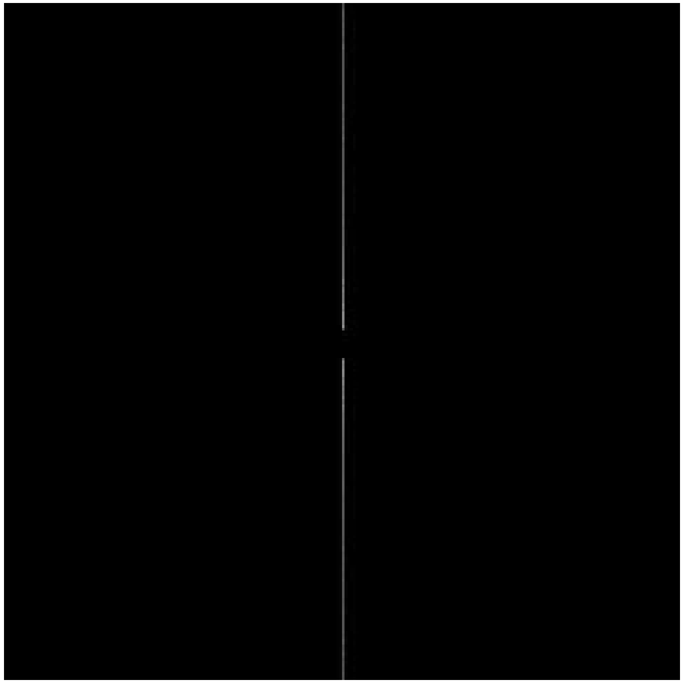
Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

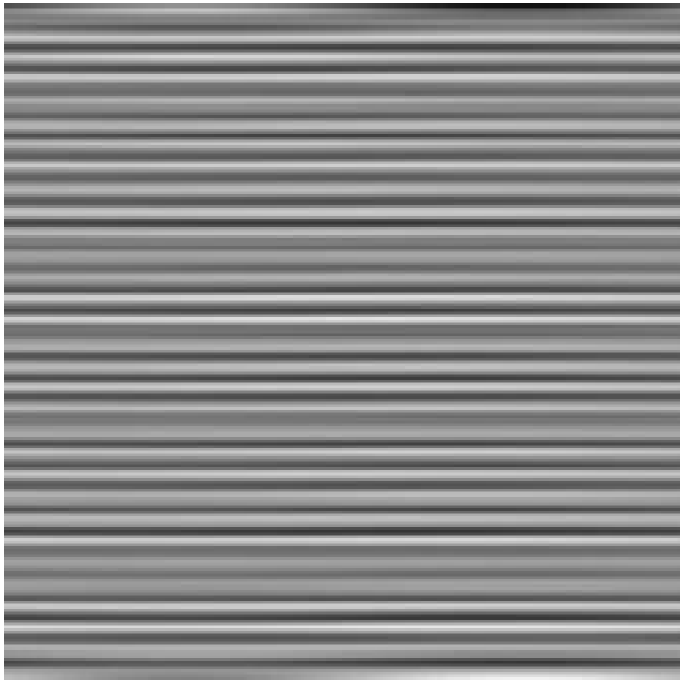
Saturn rings, DFT, Vertical notch-reject, result



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Notch-pass filter shows interference pattern





Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Separability of 2-D DFT and IDFT

$$F(u, v) = \sum_{x=0}^{M-1} e^{-j2\pi ux/M} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi vy/N}$$

$$= \sum_{x=0}^{M-1} F(x, v) e^{-j2\pi ux/M}$$

$$m(n) = \frac{1}{2} M \log_2 M$$

$$a(n) = M \log_2 M$$

$$F(x, v) = \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi vy/N}$$

$$MNf^*(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F^*(u, v) e^{-j2\pi(ux/M + vy/N)}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

FFT algorithm for DFT

$$F(u) = \sum_{x=0}^{M-1} f(x) W_M^{ux} \qquad M = 2^n$$

$$W_M = e^{-j2\pi/M} \qquad M = 2K$$

$$F(u) = \sum_{x=0}^{2K-1} f(x) W_{2K}^{ux}$$

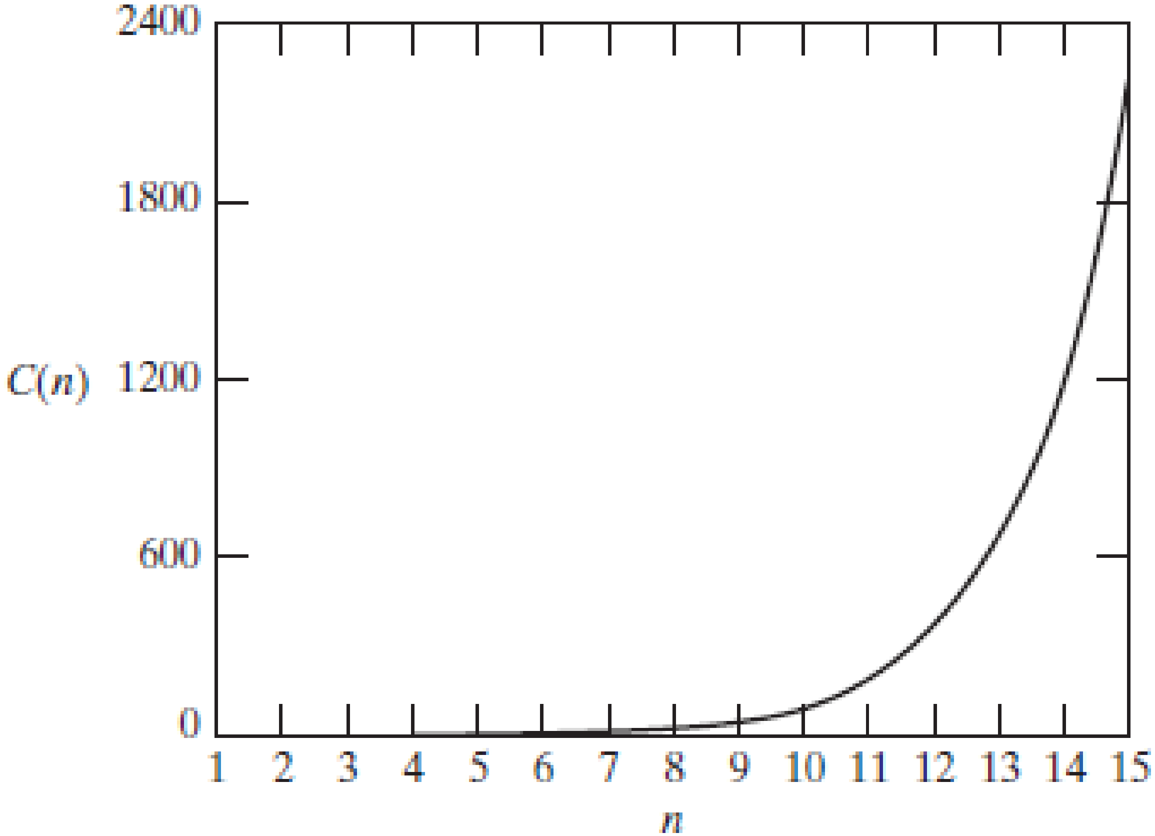
$$= \sum_{x=0}^{K-1} f(2x) W_{2K}^{u(2x)} + \sum_{x=0}^{K-1} f(2x+1) W_{2K}^{u(2x+1)}$$

$$F(u) = \sum_{x=0}^{K-1} f(2x) W_K^{ux} + \sum_{x=0}^{K-1} f(2x+1) W_K^{ux} W_{2K}^{u}$$

$$F(u) = F_{\text{even}}(u) + F_{\text{odd}}(u) W_{2K}^{u}$$

Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall

Savings of FFT over DFT $2^n/n$



Source: Digital Image Processing, Gonzalez and Woods, Prentice Hall