Team Number:	1284

Problem Chosen:	B

2017 APMCM summary sheet

As a new method to improve the automation of ceramic production process, robot glazing has a certain effect on improving glaze efficiency. Therefore, it is important to explore the automatic trajectory planning of robot glaze under different surface conditions of the workpiece to improve the level of modernization of ceramic technology.

Plane glaze automatic trajectory planning: First, the method of calculus and the elliptic double β distribution model is combined to establish a plane glaze trajectory optimization model with glaze thickness uniformity as the target. Then MATLAB is used to find the optimal solution 116.30mm for the overlap interval d. Finally, the simulation analysis of the glaze film thickness models with different cross sections verified the correctness of the model.

Trajectory planning of glaze spraying on curved surface (Perpendicular to the horizontal direction): First, the ellipse double β distribution model is corrected by using the projection method, and the glaze film thickness distribution model of the curved surface when the direction of the glaze spray is perpendicular to the horizontal direction is established. Then, the optimization model of curved track was set up to optimize the minimum glaze thickness difference. Finally, it is proved that the overlapping interval in question 1 is not applicable to the surface of question 2, and the optimal solution of the overlapping interval d of the problem 2 is 89.36~95.05mm.

Trajectory planning of glaze spraying on curved surface (Along the normal direction of the spray point)： Firstly, the plane ellipse double β distribution model is corrected by projection method, and the glaze film thickness distribution model is established when the spray glaze direction is the normal direction of the spray point of Fog Cone Center. Then, based on the slice algorithm, the optimization model of surface spray glaze trajectory is established with the coating uniformity as the optimization target. Finally, it is proved that the optimal solution of the overlapping interval d of the curved surface is 80.26~90.53mm.

Arbitrary surface glaze trajectory planning: Firstly, the differences between different curved surfaces are described by using parameters such as β angle, θ angle and height of the spray gun. By changing the parameters of different surface observation, the parameters of the launcher are repeated. Finally, the golden section iteration method is used to find the value of d, and a program of any surface glaze trajectory planning is established. The correctness of the model is proved by MATLAB simulation and the result accords with the standard.

Keywords: Spray glaze automatic trajectory planning; Glaze film thickness distribution model; Slicing algorithm; Projection method; Golden section method

Contents

1 Restatement of The Problem

1.1 Problem Background

Spray glaze is an important part of the ceramic production process, due to uneven glaze in the firing process will be cracking, resulting in scrapped parts, so the spraying process requires the spray glaze as thick as possible, but also reduces the efficiency [1].

The emergence of robotic glazing provides a new way to improve glaze efficiency and is of great importance in improving the automation of ceramic production processes.

1.2 Our work

- Analyze the distribution of plane glaze thickness of the robot spray glaze, and design the automatic trajectory optimization scheme of plane glaze.
- The distribution of the glaze film thickness on the curved surface when the robot enamel glazing direction is perpendicular to the horizontal direction is explored, and the automatic trajectory optimization scheme of the surface glazing under the condition is established.
- The distribution of the glaze film thickness on the curved surface of the cone sprayed with the normal direction of the cone was studied. The optimization scheme of the automatic trajectory of the surface glazing under the condition was established.
- Explore the surface of the workpiece is an arbitrary surface, whether there is a universal automatic robot spray glaze optimization program to solve the spray path planning.

2 Assumption

- The thickness of the edge of the spray coating thickness distribution model has no effect.
- Spraying robot spraying a certain height, does not change.
- Robot in the coating process of the speed constant, there is no sudden change.

3 Symbol and Definitions

Symbols	Definitions	Symbols	Definitions
$z_{\max}$	Glaze film maximum thickness	d	Overlap interval
d_l	Slice thickness	$h_{p'}$	Spray glaze distance
n	Normal vector	θ_p	Spray glaze spray point of view
θ	The angle of the axis direction	σ	Spray glaze thickness difference percentage

4 Establishment of the Model

4.1 Plane glaze automatic trajectory planning

4.1.1 Plane glaze trajectory model

In the planning and optimization of the trajectory of the spray gun, a design of the spraying trajectory with the most uniform thickness distribution on the surface of the workpiece is studied, which reduces the workload and difficulty of optimization. After spraying the workpiece, there is a difference between the actual coating thickness of each point on the workpiece surface and the ideal average coating thickness, and based on this, the paper chooses to optimize the spraying trajectory by the variance and minimum of the actual film thickness and the ideal average paint film thickness on the plane [3,4].

The thickness distribution model of the workpiece surface is the basis of the trajectory planning and optimization of the spray gun, which is closely related to the establishment and solution of the optimization objective function. In this paper, the cumulative rate model of the coating film on the surface of single channel spraying--elliptic double β distribution model is presented [5]:

$$Z(x,y)=z_{\max}\left(1-\frac{x^2}{a^2}\right)^{\beta_1-1}\left[1-\frac{y^2}{b^2\left(1-\frac{x^2}{a^2}\right)}\right]^{\beta_2-1} \tag{1}$$

In the formula : a -amplitude oval length half axis (mm); b -spray amplitude ellipse short half axis (mm); $z_{\max}$ -the maximum thickness of paint film; β_1 -exponent of β distribution on x direction section; β_2 -exponent of β distribution on y direction section[6].

In addition, it is shown that the atomization pressure P_1, the pressure of diaphragm pump P_2 and the distance h are the main factors affecting the above parameters, and the relationship is:

$$\begin{bmatrix} 129.8665 & -55.2435 & 1.7436 & -297.3908 \\ 52.5130 & -5.7480 & 0.7394 & -128.6368 \\ 59.7245 & 393.9655 & -0.1244 & 150.0184 \\ -7.0125 & 34.5045 & 0.0284 & -9.5229 \\ -4.6130 & 18.3620 & 0.0113 & -0.3924 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ h \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ z_{\max} \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{2}$$

The following parameters are obtained through MATLAB software:

Table 1 Gun model parameters

Long axis a(mm)	Short axis b(mm)	z_{max}(mm)	β_1	β_2
116.303	47.081	212.767	2.365	4.899

For the Ellipse double β distribution model the spraying trajectory can be planned in X and Y two directions, if the spray width direction is the X direction of the spraying trajectory planning, then in the same time, the spray gun through the same spraying point of time than the Y-direction, easy to cause the workpiece surface thickness of paint film, not only reduce the spraying efficiency, and it is not conducive to improve the quality of spraying. Therefore, this paper chooses the spraying trajectory planning in the y direction of the spray gun [7].

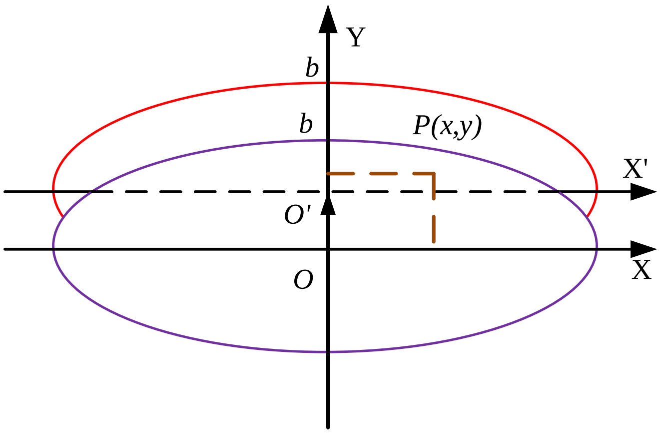


Fig. 1 Plane glaze trajectory diagram

As shown in the picture, when spraying speed is constant v, the spraying range of the spray gun is required by the point P on the plane is $t = 2b(1 - x_p^2/a^2)^{1/2}/v$, substituting$x = x_p, y = y_p = b(1 - x_p^2/a^2)^{1/2} - vt$ for Formula 1,The thickness of the glaze film with any point p on the plane in the process of single spraying is:

$$Z\left(x_p, y_p\right) = \int_0^t z_{\max} \left(1 - \frac{x_p{}^2}{a^2}\right)^{\beta_1 - 1} \left[1 - \frac{\left[b(1 - x_p^2/a^2)^{1/2} - vt\right]^2}{b^2\left(1 - \frac{x_p{}^2}{a^2}\right)}\right]^{\beta_2 - 1} \tag{3}$$

Among them:$-a \le x_p \le a, \frac{x_p^2}{a^2} + v^2t^2/4b^2 \le 1$.

The formula (3) Describes the thickness distribution profile of the paint film on the spraying plane along any section of the spraying direction after spraying the spray gun uniform straight line. According to some experimental data and the parameters of A, B, and β_1 β_2, using MATLAB to simulate and analyze the thickness distribution model of the spray gun, the results of the analysis are shown in Fig.2.

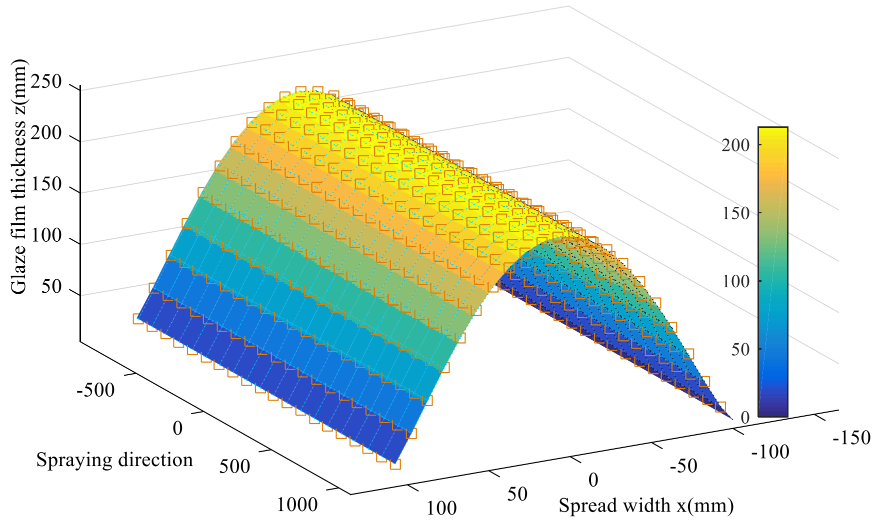


Fig.2 Single-trip glaze film thickness distribution

It is observed from Fig. 2 that the thickness of the paint film at each point in a single spray is obtained through accumulation, the thickness of the film in the middle is the largest, the value of the thickness of both sides is getting smaller; in the process of multi-channel spraying, the thickness of the paint film in the superimposed area of adjacent two tracks is also obtained by accumulation, so the trajectory planning and optimization of the plane is must plan the overlap width between adjacent trajectories *d*[8].

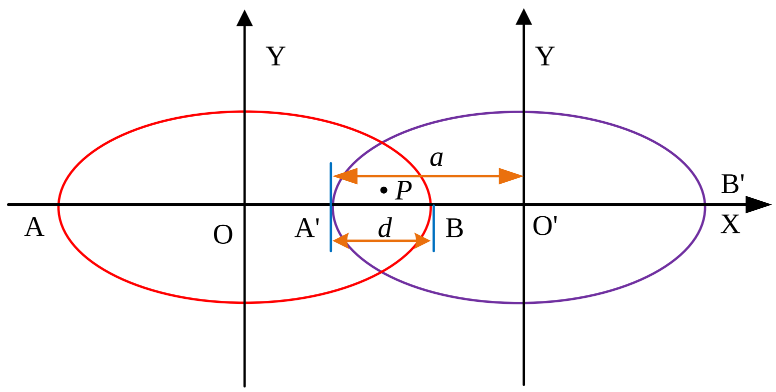


Fig.3 Plane double spray glaze process

As shown in Fig. 3, d indicates that thickness of paint film is two times the width of overlay area. In the actual spraying operation, in order to maintain the uniformity of the film thickness, there is a certain overlap of the film overlay width of the two spraying tracks, and the width D of the paint film is decided by the spray breadth of the spray gun and the ellipse long axis a value. It is assumed that some process parameters of the spray gun are unchanged during the multiple spraying stroke (spraying pressure, spraying range, paint concentration, etc., then in the overlapping part of the workpiece surface of the film thickness of *P* can be expressed as $Z_P = Z_{p3} + Z_{p2}$, wherein $Z_{p,1}$ and $Z_{p,2}$ respectively indicate the cumulative thickness of the paint film on the surface point *P* at two times of spraying stroke, then the thickness distribution of the surface point *P* on the two-channel spraying stroke can be obtained as a piecewise function[9]:

$$Z_p(\mathrm{x},\mathrm{d}) = \begin{cases} Z_{p,1}(\mathrm{x}) & 0 \le y \le a-d \\ Z_{p,1}(x) + Z_{p,2}(x,d) & a-d \le y \le a \\ Z_{p,2}(x,d) & a \le y \le 2a-d \end{cases} \tag{4}$$

Generally, in the spraying process, the distance between the spray gun and the workpiece surface *h* is valued at the experiential value, and the *h* value remains unchanged, therefore, in the spraying operation to obtain the uniform distribution of the thickness of the paint film, the key is to plan a reasonable overlap width *d*. To solve this problem, the variance and minimum of the actual film thickness Z_p and the average thickness Z_A of the ideal paint film is to establish the optimization function for the optimization objective[10]:

$$\min_{d\in(0,a),v} E(d,v) = \int_0^{2a-d} (S_p - S_A)^2 dx \tag{5}$$

Formula 5 is used as fitness function to optimize *d*, and the optimal solution of *d* is obtained. By summarizing the actual spraying experience, the range of *d* value is 2a/3~a, the spraying distance is $h = 225mm$.

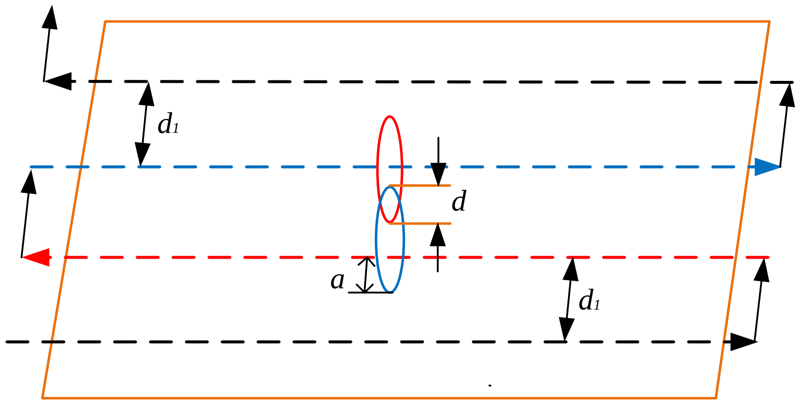
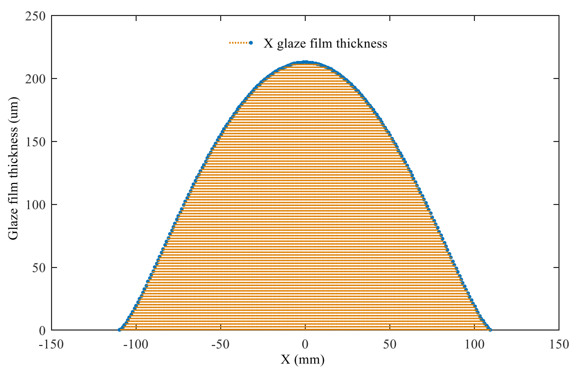


Fig. 4 Plane glaze trajectory planning

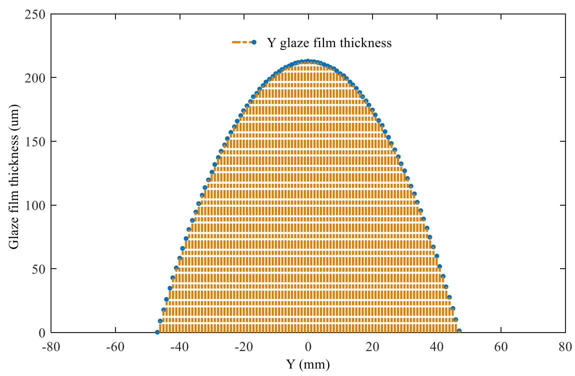
According to the above data, the optimum d=116.30mm is obtained by using the least squares calculation method by MATLAB. The effect of spraying trajectory on plane is shown in Fig. 4.

4.1.2 Simulation of planar spraying trajectory

Based on the model of the thickness distribution of the paint film, the ellipse double β distribution model, MATLAB programming is used to simulate the thickness distribution of the film X and Y to verify the correctness of the model, and the thickness distribution of the film cross-section is plotted according to the parameters of the Lance model and the experimental data in the 4.1.1.



a) when y=40, the thickness of the Glaze Film Section profile



b) When x=40, the thickness of the Glaze Film Section profile

Fig. 5 Thickness distribution of paint film in different sections of X and Y

It can be seen from Fig. 5 that the thickness of the paint film in any section obeys the elliptic double β distribution model, which accords with the actual spraying effect, thus verifying the applicability of the film thickness distribution model established in this paper.

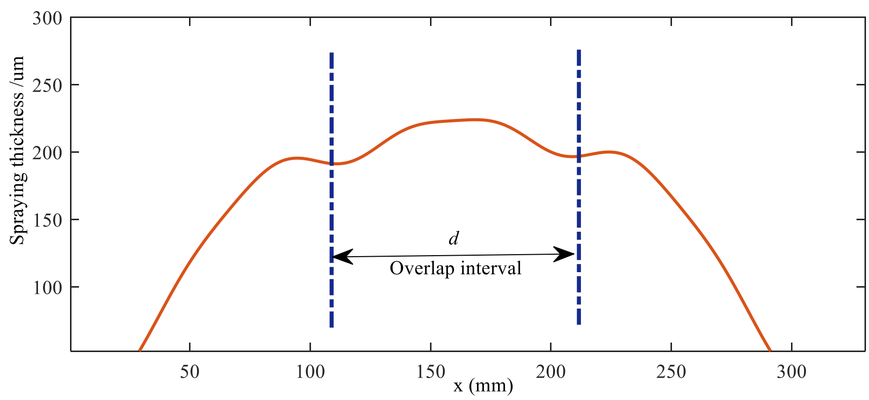


Fig.6 Dual trajectory glaze spray optimized simulation effect

The list of parameters into the formula in (4), using the least squares method in the MATLAB programming environment to function type (5) as the fitness function iterative algorithm to calculate the optimal solution, obtained when the uniform film thickness by d =116.30mm, the double track of coating film thickness distribution results as shown in Fig.6 show.

4.2 Surface spray glaze trajectory planning (perpendicular to horizontal direction)

4.2.1 Thickness distribution model of surface spray glaze

When the surface of the workpiece is curved, the thickness distribution model of the spray glaze on the surface is deduced according to the thickness distribution model of the spray glaze on the plane. However, there are fewer factors in the glazing model, only considering the thickness distribution of the spray gun. However, the surface model needs to consider more factors [11,12].

The following illustration shows a schematic of the spray glaze on the surface of the spray gun:

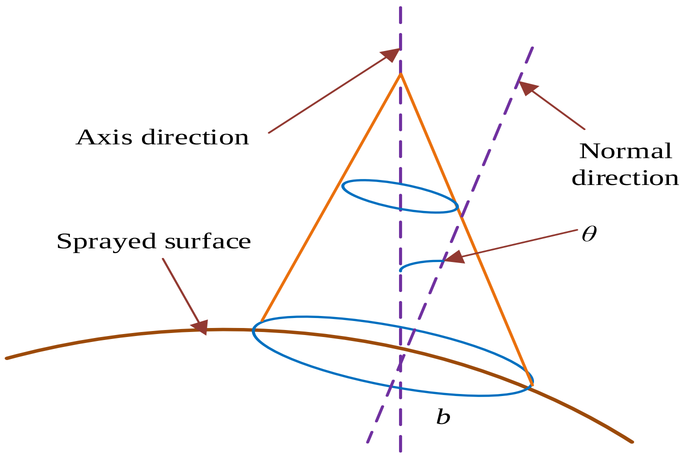


Fig. 7 Schematic diagram of spray nozzle on curvature surface

As shown in Fig. 7, in the case of the movement of the lance, the angle between the axial direction of the lance and the normal direction of an infinitesimal region is determined, but as the position changes, the angle of the two is gradually changed, and the angle between the direction of the lance Axis and the normal direction of an infinitesimal region is the function of the position of the area on the surface, that is[13]:

$$\theta = f(x, y) \tag{6}$$

Take the surface given in question 2 as an example, draw the surface image and represent the normal vector:

$$z = -x^2 + x - xy\left(-10 \le x \le 10, -10 \le y \le 10\right) \tag{7}$$

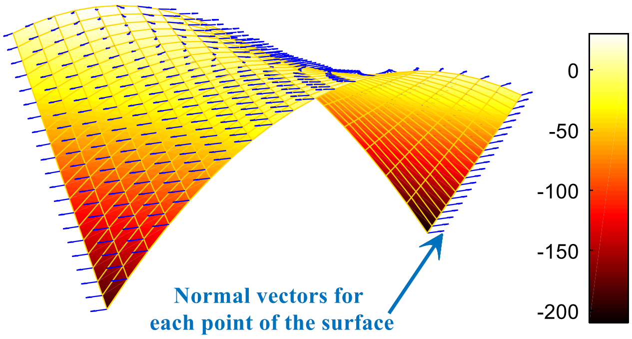


Fig. 8 Surface normal vector schematic diagram in problem 2

By observing Fig. 8, It is the surface that in question 2 is a figure similar to that of hyperbolic paraboloid, which curvature is larger than that of the plane in question 1, so it is necessary to modify the thickness distribution model of the spray glaze in the plane of problem 1.

The above analysis shows that the angle between the axial direction of the lance and the normal direction of an infinitesimal region is the most prominent difference between the surface model and the plane model. Then in this paper, the projection method, the surface under the same area projection, the actual area is larger than the projected area, there is a $\cos\theta$ relationship between the surface model and the plane model, that is [14]:

$$S_c \cdot \cos\theta = S_p \tag{8}$$

By definition, the angle is the angle between the normal direction of the infinitesimal area and the lance, and the position of the lance is always upright, so the angle can be calculated by calculating the normal direction of the surface.

For the normal direction of the curvature surface, it can be written as an implicit function:

$$x^2 - x + xy + z = 0 \tag{9}$$

Then the implicit function is obtained by a biased derivative:

$$\begin{cases} F_x = 2x + y - 1 \\ F_y = x \\ F_z = 1 \end{cases} \tag{10}$$

The method vector n is $(2x_0 + y_0 - 1, x_0, 1)$, where (x_0, y_0, z_0) is the coordinate of any point on the surface, so the normal equation of the surface is:

$$\frac{x - x_0}{2x_0 + y_0 - 1} = \frac{y - y_0}{x_0} = z - z_0 \tag{11}$$

The *x, y, z* is brought in separately, which can be obtained:

$$\theta = \arctan(\sqrt{(2x_0 + y_0 - 1)^2 + x_0^{\ 2}}) \tag{12}$$

Through the above analysis, this angle is related to the area of spraying glaze. The model assumes that the velocity of the lance is constant, that is, the spray glaze of the spray gun must be in a certain time, the larger the angle of θ, the larger the area, the thinner the thickness of the spray glaze. As shown in the Fig.9:

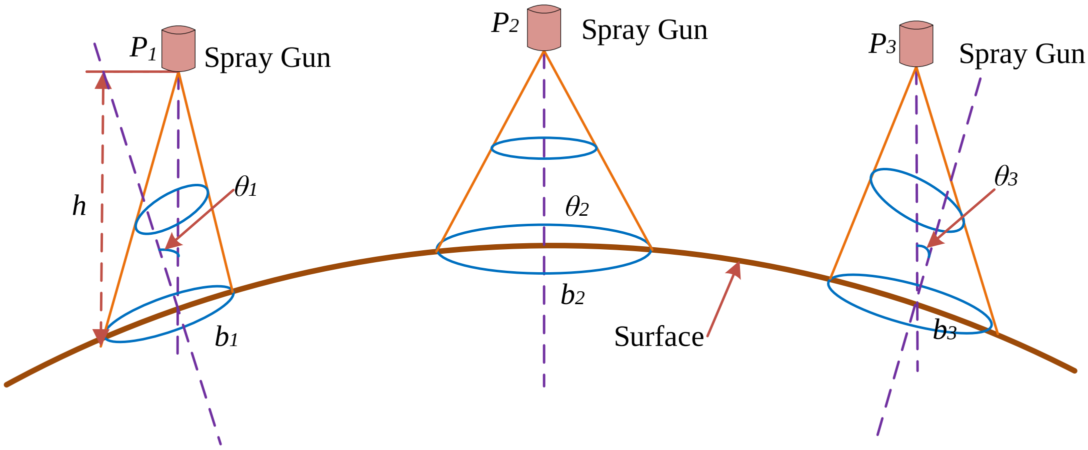


Fig. 9 Schematic diagram of different angles of spraying glaze

That is, the thickness of the spray glaze is proportional to the numerical value of $\cos\theta$, then the spray glaze thickness model in question 1 can be corrected by the numerical method of the $\cos\theta$;

$$Z_p(x, d) = \begin{cases} Z_{p,1}(x)\cos\theta & 0 \le y \le a - d \\ Z_{p,1}(x) + Z_{p,2}(x, d)\cos\theta & a - d \le y \le a \\ Z_{p,2}(x, d)\cos\theta & a \le y \le 2a - d \end{cases} \tag{13}$$

The above formula is the surface spray glaze thickness distribution model.

4.2.2 Optimization of surface spray glaze trajectory

For the problem of whether the *d* value in the planar model is suitable, selecting the highest point and the minimum thickness difference on the surface after the spray glazing process is not more than 10% as the criterion, if the thickness difference is within 10%, if more than 10% is considered unsuitable [15].

$$Z_c(x,d)=\begin{cases}\int_{-b}^{d-b} Z_{p,1}(x)\cos\theta\, dx & 0\le y\le a-d \\ \int_{-b}^{d-b}\left[Z_{p,1}(x)+Z_{p,2}(x,d)\right]\cos\theta dx & a-d\le y\le a \\ \int_{-b}^{d-b} Z_{p,2}(x,d)\cos\theta dx & a\le y\le 2a-d\end{cases} \tag{14}$$

And define: The calculation formula of the percentage of glaze thickness difference between the highest point and the lowest level on the surface is

$$\sigma=\frac{h_{\max}-h_{\min}}{h_{\max}}\times 100\% \tag{15}$$

The d=116.30mm of the spray glaze interval value obtained in question 1 is brought in, then the above-type integrals are evaluated:

$$\begin{cases}\min\left[Z_c(x,d)\right] \\ \max\left[Z_c(x,d)\right]\end{cases} \tag{16}$$

Substituting the formula (15) To verify that the value of σ is greater than 10%, the interval value is not within an acceptable range, that is, the interval value in the problem 1 model does not apply to the problem 2 surface.

Then, the formula (14) and the formula (15) are used to reverse the spray glaze interval value. By making $\sigma\le 10\%$, the scope of d is obtained:

$$\sigma=\frac{h_{\max}-h_{\min}}{h_{\max}}\times 100\%\le 10\%$$

Then: d= 89.36~95.05mm.

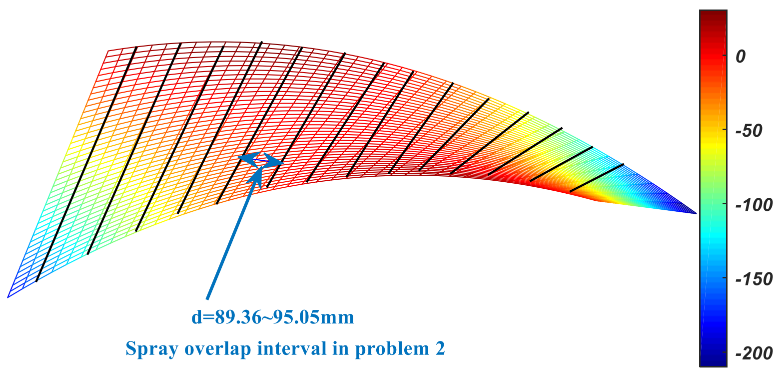


Fig. 10 Simulation results of surface trajectory planning

The curved surface of the d=89.36~95.05mm is optimized after the trajectory in question 2. Surface trajectory planning simulation results as shown in Fig. 10, the intersection line of parallel tangent plane and curved surface is a black intersection.

4.3 Surface spray trajectory optimization (normal direction)

4.3.1 Surface spray glaze thickness distribution model

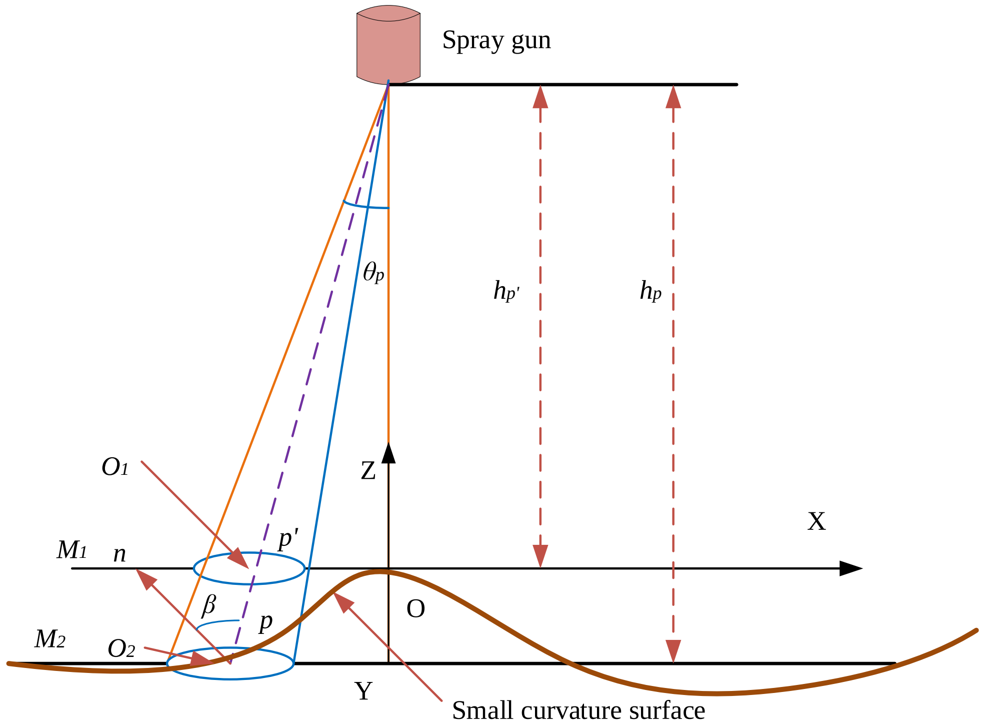


Fig.11 Small curvature surface glaze film growth rate model

As shown in Fig.11, space coordinate system XOY is established with point O on the surface. Similarly, the surface spraying thickness distribution model (FIG. 12) is established when spraying direction is always the normal direction of the spray point of fog cone. Cross point O for tangent plane M1; A point p on the surface is parallel to the plane $M_1 // M_2$; θ_p is the spray Angle of the spray point; $h_{p'}$ is for spraying distance. h_p is the distance from the spray gun to the plane M_2 ; n is the normal vector of the ellipse O_2 ; The Angle between the point p and the nozzle and the normal vector n; l_p is the distance from the spray gun to the spray point p . Based on the theory of the distribution model of spraying thickness in the horizontal direction in 4.2, the distribution function of the surface spraying thickness of the normal surface is established[16]:

$$Z(x,y)=\begin{cases} Z_{p'}\left(\frac{h_{p'}}{h_p}\right)^2 \frac{\cos\beta}{\cos\theta_p} & \beta<90^\circ \\ 0 & \beta<90^\circ \end{cases}$$

$$Z(x,y)=z_{\max}\left(1-\frac{x_{p'}^{2}}{a^2}\right)^{\beta_1-1}\left[1-\frac{y^2}{b^2\left(1-x_{p'}/a^2\right)}\right]^{\beta_2-1}\left(\frac{h_{p'}}{h_p}\right)^2\frac{\cos\beta}{\cos\theta_p} \tag{17}$$

Among them: $-a\le x_{p'}\le a-b\left(1-x_{p'}^{2}/a^2\right)^{\frac{1}{2}}\le y\le b\left(1-x_{p'}^{2}/a^2\right)^{\frac{1}{2}}$

$x_{p'}=h_{p'}\cdot\tan\theta_p=h_{p'}\cdot x_p/h_p$

$h_p=l_p\cos\theta$

In equation(17),the parameters a, b, spraying distance h_p and β distribution parameters β_1 and β_2 are all obtained by the plane spraying experiment of the article 4.1.

After the model of surface spraying thickness was established, the method of the joint point cloud slicing algorithm and the surface spraying thickness distribution model were studied.

4.3.2 Spray trajectory planning of surface

(1) Determination of slice direction and thickness

According to the principle of slice can determine small curvature surface belong to the type of surface is not sensitive to slice direction, therefore this chapter according to the method of point cloud surface change rule to determine the direction of tangent plane and section, as shown in Fig.12.

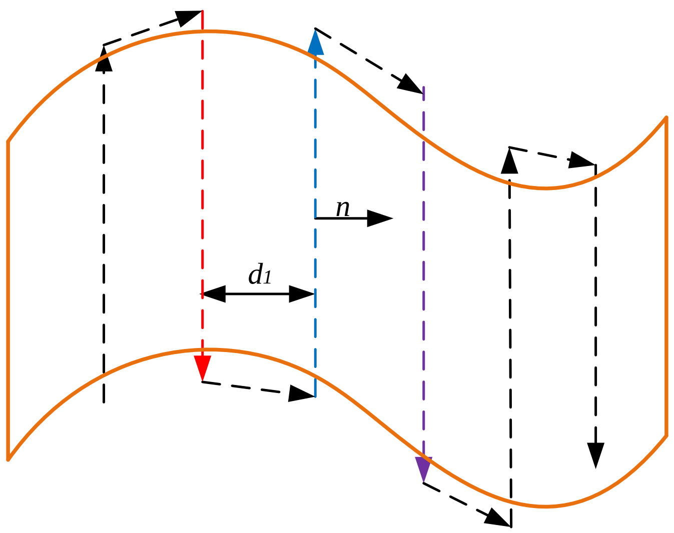


Fig. 12 Slice direction and thickness of surface

The surface section is selected as d_1 set of parallel planes. For the intersection of the slice plane and the surface point cloud, the calculation point cloud method is to determine the slice direction of the slice direction to the variance. As shown in figure 13, the section direction is the method of tangent plane. It is also possible to determine the curvature change of surface by observing the surface curvature. According to the method obtained above, the direction of spray gun attitude is not reduced, and the planning efficiency and spray quality are improved [17].

(2) the construction of slice contour data

Like to RDB for cylindrical surface profile data, for small curvature point cloud surface intersection distance threshold (on a bit with the curved tangent plane distance is less than a certain value), the definition of the tangent plane to Y Z plane, section for the X axis direction, establish a cartesian coordinate system.

The point cloud band that defines the JTH tangent plane is represented as $\Omega_1 = \{p_{li}(x_{li}, y_{li}, z_{li}), d_{li} \le \varepsilon, i = 0,1,\ldots,n\}$ in the form of space set, which is represented by $\Omega_2 = \{p_{rj}(x_{rj}, y_{rj}, z_{rj}), d_{rj} \le \varepsilon, j = 0,1,\ldots,n\}$ space set in the other side. d_{li} and d_{ri} respectively represent the distance between plane $E_{i,j}$, E_{rj} and tangent plane E, and n is the number of points in the threshold. The difference is to establish spherical search space with $r = \varepsilon / 2$ radius of step. All of the matching point is obtained by traversing the calculation of P_{li} and P_{ri}, calculated two attachment and tangent plane intersection point coordinates $P_{i,j} = (x_{i,j}, y_{i,j}, z_{i,j})$, P_{li} and P_{ri} form of spatial straight line can be represented as:

$$\frac{x_{i,j} - x_{ri}}{x_{ri} - x_{li}} = \frac{y_{i,j} - y_{ri}}{y_{ri} - y_{li}} = \frac{z_{i,j} - z_{ri}}{z_{ri} - z_{li}} = C \tag{18}$$

According to equation (18), the coordinates of intersection point $P_{i,j}$ are obtained:

$$\begin{cases} \dfrac{x_{i,j} - x_{ri}}{x_{ri} - x_{li}} = C(x_{ri} - x_{li}) + x_{ri} \\ \dfrac{y_{i,j} - y_{ri}}{y_{ri} - y_{li}} = C(y_{ri} - y_{li}) + y_{ri} \\ \dfrac{z_{i,j} - z_{ri}}{z_{ri} - z_{li}} = C(z_{ri} - z_{li}) + z_{ri} \end{cases} \tag{19}$$

The calculated point $p_{i,j}$ is stored in Arr, and the contour data points of the section of the contour obtained by the intersection method are listed in the

$Arr = \{p_{i,j}(x_{i,j}, y_{i,j}, z_{i,j})\}$, which can be used to generate the spray trajectory.

(3) Generation of spray path

By sorting the contour data of the small curvature surface, the data of adjacent slices are arranged in the opposite direction. The complete and smooth spray trajectory can be obtained by using interpolation algorithm to connect the contour of adjacent sections.

Spraying trajectory is the intersection of a surface tangent plane with small curvature, by calculating RDB for surface spraying trajectory can be simple, but the coating quality cannot meet the ideal requirements, so we must optimize the trajectory of guarantee the quality of spraying.

4.3.3 Optimization of surface spraying trajectory

The section of this paper is selected as parallel plane group, and the coating thickness is optimized by the uniformity of spraying thickness. Firstly, the coating thickness distribution model is established in the two-channel spraying process, as shown in Fig.13.

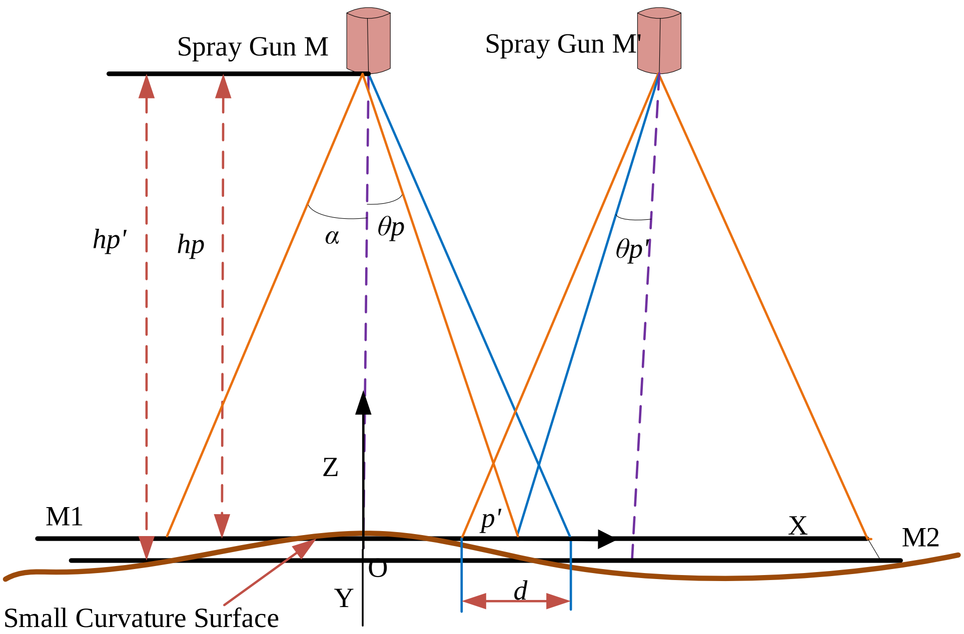


Fig. 13 thickness distribution of the paint film of small curvature surface

Fig. 13 α indicates the jet range critical point and the Angle between the line of the spray gun and the axis of the nozzle, and θ_p means spray point injection Angle; $\theta_{p'}$ said spraying point p in the name of the second spraying trajectory jet Angle, in the process of spray, spray gun axis perpendicular to the surface, surface point for point O, O point tangential as X axis, method to establish the cartesian coordinate system as the

Z axis. The spraying velocity of the spray nozzle is v, and the total time of the spray nozzle is $t = 2b\left(1-x_{p'}^{2}/a^{2}\right)^{\sqrt{2}}/v$, $y = y_{p'} = b\left(1-x_{p'}^{2}/a^{2}\right)^{\sqrt{2}} - vt$, and according to equation (17), the distribution function of the spraying thickness of p is obtained as follows[18]:

$$Z_p\left(x_{p'}, v\right) = \int_0^{l_{\text{总}}} z_{\max}\left(1-\frac{x_{p'}^{2}}{a^2}\right)^{\beta_1-1}\left[1-\frac{\left(b\left(1-x_{p'}^{2}\right)\right)}{b^2\left(1-x_{p'}^{2}/a^2\right)}\right]^{\beta_1-1}\left(\frac{h_{p'}}{h_p}\right)^2\frac{\cos\beta}{\cos\theta_p}\,dt \quad (20)$$

According to the formula (17), the distribution function of the spraying thickness of P in the two-channel spraying process is:

$$Z_p\left(x_p.d\right) = \begin{cases} z_{p,1}\left(x_p\right) & 0 \le x_p \le ah_p/h_{p'} - d \\ z_{p,1}\left(x_p\right) + s_{p,2}\left(x_p, d\right) & ah_p/h_{p'} - d < x_p < ah_p/h_{p'} \\ z_{p,2}\left(x_p, d\right) & ah_p/h_{p'} \le x_p \le 2ah_p/h_{p'} - d \end{cases} \quad (21)$$

When $0 \le x_p \le ah/h_{p'} - d$, there are:

$$Z_{p,1}\left(x_p, v\right) = \int_0^{l_{\text{总}}} z_{\max}\left(1-\frac{x_{p'}^{2}}{a^2}\right)^{\beta_1-1}\left[1-\frac{\left(b\left(1-x_{p'}^{2}/a^2\right)^{1/2}-vt\right)^2}{b^2\left(1-x_{p'}^{2}/a^2\right)}\right]^{\beta_1-1}\left(\frac{h_{p'}}{h_p}\right)^2\frac{\cos\beta}{\cos\theta_p}\,dt \quad (22)$$

When $ah_p/h_{p'} \le x_p \le 2ah_p/h_{p'} - d$, there are:

$$Z_{p,2}\left(x_p, v, d\right) = \int_0^{l_{\text{总}}} z_{\max}\left(1-\frac{\left(2a-d-x_{p'}\right)^2}{a^2}\right)^{\beta_1-1}\left[1-\frac{\left(b\left(1-\left(\left(2a-d-x_{p'}\right)^2/a^2\right)^{\frac{1}{2}}\right)-vt\right)^2}{b^2\left(1-\left(2a-d-x_{p'}\right)^2/a^2\right)}\right]^{\beta_2-1}\left(\frac{h_{p'}}{h_p}\right)^2\frac{\cos\beta}{\cos\theta_p}\,dt \quad (23)$$

Among them:

$$-a \le x_{p'} \le a - b\left(1-x_{p'}^{2}/a^{2}\right)^{\frac{1}{2}} \le y \le b\left(1-x_{p'}^{2}/a^{2}\right)^{\frac{1}{2}}$$

$$x_{p'} = h_{p'}\cdot\tan\theta_p = x_p\frac{h_{p'}}{h_p}$$

Formula (21), (22), and (23) can obtain a segmented function containing variable x_p, d. $Z_{p,1(x_p,v)}$ and $Z_{p,2(x_p,d,v)}$ represent the cumulative thickness of the spraying of the surface point P. Also for surface coating thickness uniformity with a target trajectory optimization, assumes that the spraying distance h = 400 mm, take the actual coating thickness of P is Z_p an ideal coating average thickness variance between $Z_A = 50um$ and minimum as the objective function of optimization:

$$\min_{d\in(0,a),v} E(d) = \int_0^{2ah_p/h_{p'} - d}\left(Z_p - Z_A\right)^2 dx_{p'} \quad (24)$$

Using formula (24) as the fitness function, the parameter d was optimized in

MATLAB, and the optimal solution d was obtained by calculating equation (21). Then slice thickness $d_1 = 2ah_p / h_{p'} - d$ .In the case of surface $z = -x^2 + x - xy(-10 \le x \le 10, -10 \le y \le 10)$ in question 2, the distance between the spraying distance of the curved surface after optimized trajectory $d_1 = 80.26 \sim 90.53$ mm. The simulation results of curved trajectory planning are shown in Fig.14. The intersection of parallel tangent plane and surface is the spray trajectory.

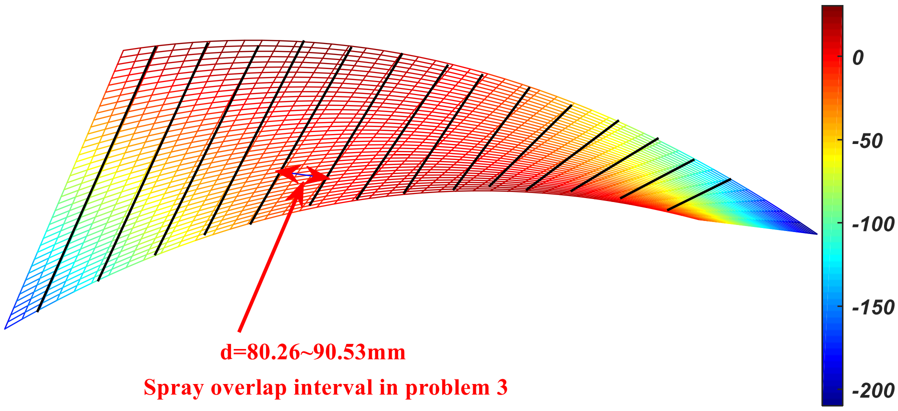


Fig. 14 Simulation results of surface trajectory planning

4.4 Arbitrary surface automatic trajectory planning

4.4.1 Arbitrary surface trajectory planning model

In this section, the method of trajectory optimization of spray gun on general surface is studied. Firstly, according to the common ground of the free surface, the relative parameters are selected to represent the plane, according to the change of the surface, the change trend and the result of the parameters are studied. By surface model in the process of looking for d value and the change of the parameters, the combination of the verification results of consistency: if do not agree, that no relevant system to solve the general surface spray gun trajectory model to solve the problem; If consistent, this method can be used to generalize the model to the general plane. Finally, the feasibility of mathematical model and algorithm is verified by computer simulation[19].

Through the above analysis, we can get the distance between the motion track of the specific surface, but whether the application of all surfaces should be analyzed. The biggest difference between surfaces is that the curvature of each point is different, and the form of the analytic form is different between the surfaces. These two problems are difficult to optimize the trajectory.

For different surfaces, we assume that the equation is:

$$z = f(x, y) \tag{25}$$

Different from the equation, the values of β and θ_p in the surface model are different, and the above study shows that the two values are also related to the space location, so they can be obtained:

$$\begin{cases} \beta = f(f(x, y), x, y) \\ \theta_p = f(f(x, y), x, y) \end{cases} \tag{26}$$

By definition:

$$\theta_p = \arctan(\frac{x_p}{h_p}) \tag{27}$$

Including the scope of x_p is（$-b, b$）, h_p as parameters, can the value scope of θ_p, for different surface, h_p value judgment by experience, the different surface has different values, but through the available values are relative literatures were similar, which can be regarded as a constant. In this way, the value range is only related to, for the same spray gun, the main influence factors are. In the same way, you can find the range of values by geometrical relationships, and it's also a function of the function, and the cumulative amount of each point can be related to these two angles by the third question model. It is considered that the method of surface fixed analytic model is applicable to the general surface. The model will be generalized and studied below[20].

The cumulative spray quantity of each point on the surface can be obtained by the integral formula in the model of the third question:

$$Z_{p,2}(x_p, v, d) = \int_0^{l_{总}} z_{\max} \left(1 - \frac{(2a-d-x_{p'})^2}{a^2}\right)^{\beta_1 - 1} \left[1 - \frac{\left(b\left(1-\left((2a-d-x_{p'})^2/a^2\right)^2\right) - vt\right)^2}{b^2\left(1-(2a-d-x_{p'})^2/a^2\right)}\right]^{\beta_2 - 1} \left(\frac{h_{p'}}{h_p}\right)^2 \frac{\cos\beta}{\cos\theta_p} dt \tag{28}$$

So, the change in the surface of the surface is not very important, the size is only related to the position of the gun.

In order to make the surface coating thickness of the workpiece as uniform as possible, the variance between the actual coating thickness of a certain point and the ideal coating thickness is optimized for the objective function:

$$\min_{d \in [0,b], v} z(d) = \int_0^{2b-d} (q_d - q_s(x, d, v))^2 dx \tag{29}$$

The Q_d in the formula is ideal coating thickness. Fig. 3.5 shows the process of calculating *d*. The value of *d* can be computed by the Golden section in the optimization algorithm. When the deviation of *d* in the program is less than the given value, the loop stops.

The *d* value is solved by a simple iterative method, as shown above, the condition of the final termination cycle is that the thickness difference is less than 10%, then the qualified condition is considered.

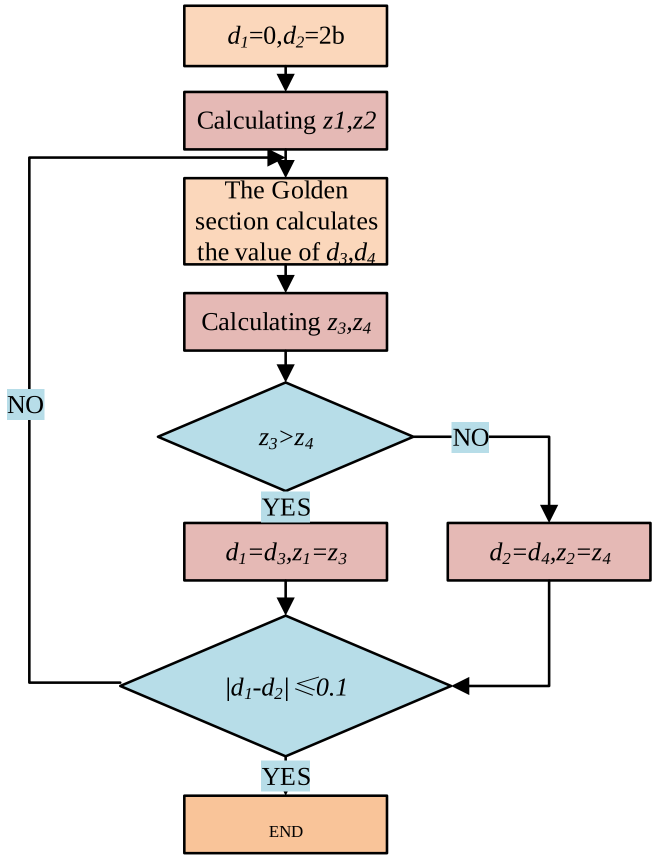


Fig.15 Two spraying stroke coating overlapping area width *d* optimization flowchart

4.4.2 Arbitrary surface trajectory optimization simulation

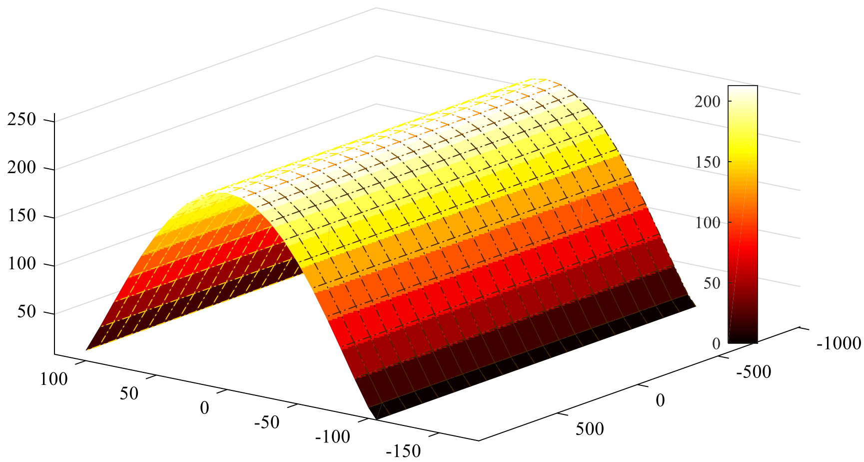


Fig.16 Three-dimensional graph of instance surface

The model is simulated and the function surface $z = -x^2 + x - \frac{xy}{75}$ is validated. The height of the h=225mm is set, the half-length axis of the ellipse is a=116.30mm, and the short half axis is b=47.081mm. The results are obtained by the Simulink simulation Toolbox in MATLAB. The three-dimensional image of the surface is shown in the picture

The specific parameters are as follows:

Table 2 Specific parameters of the instance

Parameter	Simulation result
Average thickness （μm）	29.09
Maximum thickness （μm）	30.47
Minimum thickness （μm）	27.72
Trajectory overlap value d（mm）	103.6

The simulation results show that the difference between the maximum and minimum thickness is 9.03%, which accords with the expected result. It is indicated that this method can be used as a theory to solve the overlapping spacing of general surfaces. Consistent with the above proofs, the model can be popularized

5 Model evaluation and improvement

5.1 Evaluation of the model

In this paper, in order to get a better result of robot spray glaze and improve glaze spraying efficiency, established for the surface and the curvature of surface spray gun model under different direction, and point cloud slicing model, slicing algorithm, projection method is used to spray glaze trajectory planning and optimization work. Specifically, the main innovation points are as follows:

- The correctness of the model elliptical dual-distribution model is verified by MATLAB simulation through the known double-distribution model and parameter relations. An ellipse dual distribution model for the surface in different directions is established by modifying the plane spray glazing model.
- By using slicing algorithm and combining slice algorithm, the optimization model of surface spray glaze trajectory was established based on the uniformity of the coating, and the overlap interval of glaze was optimized.

5.2 Improvement of the model

There are some deficiencies in the content of this research. Some problems need to be further studied.

- This article is through the correct glaze in the plane of the film thickness distribution model is established in different directions under the glaze surface film thickness distribution model, so we need a large number of experiments on the characteristics of the distribution rules of directly spray glaze surface was simulated, more in line with the actual spray glaze model is set up.
- In this paper, the spray gun trajectory optimization is coating uniformity optimization principle, the spray glaze, glaze film overlapping width optimized speed, not considering the robot dynamics, utilization rate of coating and spray gun gesture to glaze the influence of film thickness. Therefore, it is necessary to carry out research work on multi-objective optimization.
- This article is based on small curvature surface as the research object, but also encountered in the actual production large curvature of complex curved surface, so it is necessary for large complex surface curvature trajectory planning studies, and the characteristics of the surface at the junction of trajectory optimization.

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