Team Number :	2577

Problem Chosen :	B

2017 APMCM summary sheet

In the ceramic production process, it's important to keep its thickness uniform in the glaze spray process. Based on this background, this paper studies the trajectory planning of jet trajectory to achieve it. The research shows that when the spray gun sprayed on different surface, the distribution function of the coating thickness formed is different. So we try to find out the suitable overlap interval of the spray gun trajectory to keep the thickness uniform, and draw several conclusions as following:

1）For the given P_1、P_2 and h, the spraying direction of the spray gun always keeps unchanged. We obtain the cumulative situation of spraying in the plane which obeys function: $Z_1(x,y)=\int_{-\infty}^{y}\int_{-\infty}^{x}Z(u,v)dudv$ by the cumulative density function of the cross section along long axis and short axis direction of the ellipse. And get out of the overlap interval in each direction: $d_1=107.9115mm,\ d_2=41.2128mm.$

2) It's unsuitable to study the spray gun trajectory in curved surface $z=-x^2+x-xy$ likes plane. We found that the mainly difference is jet distance change. Therefore, we establish a new thickness function $Z_3(x,y)$, and calculate the overlap interval d_1^*, d_2^* to keep the glaze thickness difference less than 10%.

3) When the spray gun spray along the normal direction of the cone center, the model in the issue 2 is unsuitable. We re-establish the model Z_f by the relationship between the thickness of the plane and surface. Here we use the relationship between radius of curvature and surface, getting overlap interval functions: $d_1^*(x,y), d_2^*(x,y)$.

4) The starting point of this problem lies in discussing the nature of the curve formed by the intersection of any surface and the plane. In this problem, we introduced the curvature and radius of curvature and by analyzing the relationship between them drew to conclusion that the model established in issue 3 can popularize to any surface.

Spray Trajectory Planning model

Team:#2577

November, 2017

Catalog

1. Introduction

1.1 Restatement of The Problem

The sprayer produces a cone of mist during injection, which finally attached to the spray surface. The thickness shows thinner on both sides of the middle thickness. Therefore, it’s necessary to use the trajectory of the lance and the lap interval as appropriate to maintain the same spray thickness in the spraying process. Also we need specific analysis of different surface, given the applicable solution.

According to materials, if the spraying direction of the spray gun always keeps unchanged, please calculate the cumulative situation of spraying in the plane and find out the suitable overlap interval of the spray gun trajectory (P_1 and P_2 takes $0.2Mpa$, h takes $225mm$) .

1) Determine whether the solution to issue 1 is still suitable for curved surface $z=-x^2+x-xy(-10\le x\le 10,-10\le y\le 10)$ in this issue. If not, please plan the spray gun trajectory again, and calculate the overlap interval, so that the glaze thickness difference is less than 10%. (Interval for different trajectories can be different, P_1 and P_2 takes $0.2Mpa$, h can be selected according to actual needs).

2) If the spray gun spray direction is always the normal direction of the spraying point of the mist cone center during spraying. And other conditions remain unchange, please recalculate the result of issue 2.

3) Determine whether the result of issue 3 applies to curved surface $z=f(x,y)$. If not, give a general solution to spray path planning.

1.2. Assumptions and Justifications

1) Paint will not fall, due to the external conditions such as gravity and adhesion, also paint adhesion is always the same, and very strong

2) In this paper, the angle of the fan plane is unchanged.

3) It is assumed that the smoothness of the spray surface does not affect the spray thickness.

4) In this paper， atomization pressure , diaphragm pump pressure is unchanged.

1.3. Notations

semi-major axis of spray ellipse	a
semi-minor axis of spray ellipse	b
the index of β distribution in the section in the x direction	β_1
the index of β distribution in the section in the y direction	β_2
maximum thickness of paint film	$Z_{\max}$
atomization pressure	P_1
diaphragm pump pressure	P_2
spray distance	h

Among them,the unit of a , b , Z and h are millimeter;
The unit of atomization pressure and diaphragm pump pressure are Mpa.

2. Model for the problem

2.1 Analysis and Solution of The First Problem

2.1.1 Problem Presentation

Mist cone, out of Spray gun, falling on the plane will form an oval, in the spraying process. However, the thickness of the coating is not uniform, showing thicker in the middle than those of each side.

2.1.2 Problem Analysis

Cumulative conditions can be integrated with the thickness function $Z(x, y)$ over the entire plane. For spray paths and lap spacing, we need to find a suitable spray path and lap spacing to make our spray-painted coatings uniform in thickness.

As the spray surface is an ellipse, the thickness can be divided into uniform thickness of the long axis direction and the short axis direction. In each direction, the thickness function is actually the section of thickness function $Z(x, y)$.

There are many such sections, we only need to meet the maximum cross-section in this direction can be overlapped with each other, the rest of the section also overlap evenly. So in the design of the path, as long as the movement of spray gun can make the overlap thickness uniform.

Through the above ideas, we can build a model about the overlap interval according to the relationship between the cumulative thickness and the uniform thickness, and then take the parameters into account to get d .

2.1.3 Problem Solution

1）Calculate the cumulative situation of spraying in the plane

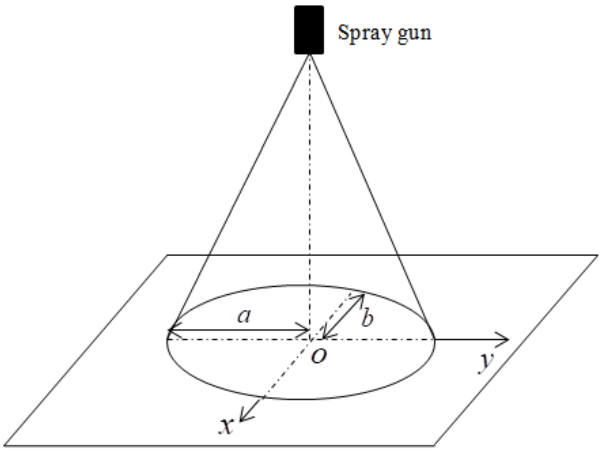


Fig.1 The Diagram of Static Spray on Plane

First, take the midpoint of the region on the plane covered by the spray cone as the coordinate system’s origin. Then, take the semi-minor axis of the paint ellipse as the x axial direction. Finally, take the semi-major axis of the paint ellipse as the y axial direction and the coordinate system is established. We can get the accurately position of any point on the plane and then according to the following relationship between the atomization pressure P_1, diaphragm pump pressure P_2 and spray distance d

$$\begin{bmatrix} 129.8665 & -55.2435 & 1.7436 & -297.3908 \\ 52.5130 & -5.7480 & 0.7394 & -128.6368 \\ 59.7245 & 393.9655 & -0.1244 & 150.0184 \\ -7.0125 & 34.5045 & 0.0284 & -9.5229 \\ -4.6130 & 18.3620 & 0.0113 & -0.3924 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ h \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \\ Z_{\max} \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

With $P_1 = P_2 = 0.2Mpa$, $h = 225mm$, we have:

$$a = 109.8438,\ b = 47.0812,\ Z_{\max} = 212.7664,\ \beta_1 = 2.3655,\ \beta_2 = 4.8999.$$

Then substituting those parameters into the elliptic double β distribution model in the elliptic distribution region[1]

$$Z(x,y) = Z_{\max}(1-\frac{x^2}{a^2})^{\beta_1-1}\left[1-\frac{y^2}{b^2(1-\frac{x^2}{a^2})}\right]^{\beta_2-1} \tag{2.1}$$

We have

$$Z(x,y) = 212.7664\times(1-\frac{x^2}{12065.6604})^{2.3655-1}\left[1-\frac{y^2}{2216.6393(1-\frac{x^2}{12065.6604})}\right]^{4.8999-1} \tag{2.2}$$

Therefore, the cumulative situation of spraying in the plane can be regarded as a cumulative function, defining as $Z_1(x,y)$. We can get it by

$$Z_1(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} Z(u,v)dudv \tag{2.3}$$

2）Plan the spray gun trajectory and calculate the overlap interval

In the spraying process, the region on the plane covered by the spray cone formed by the paint mist is an ellipse. Now we choose the spraying direction from left to right and spray the straight line, as shown in Fig.2

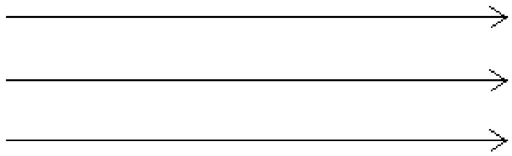
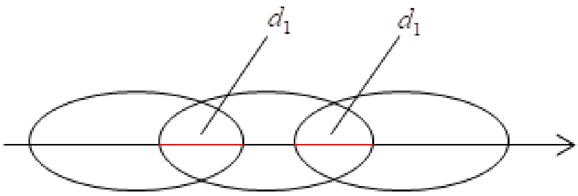


Fig.2 Spray Gun Trajectory

However, the trajectory of spray gun needs to be divided into two directions, defining the one along the semi-major axis as d_1 and the other along the semi-minor axis as d_2 shown in Fig.3 and Fig.4



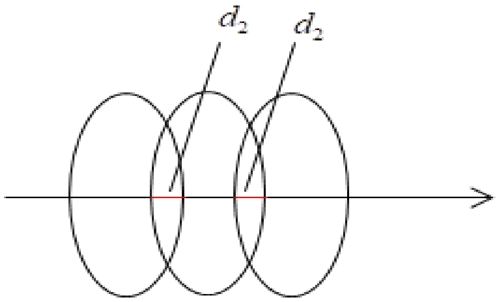


Fig.3 Top View of The Spray Cone While Spraying along y

Fig.4 Top View of The Spray Cone While Spraying along x

①Calculate d_1. While spraying along the semi-major axis, as shown in Fig.5, the overlap interval between the spray gun trajectories is the overlap interval along the semi-minor axis d_2.

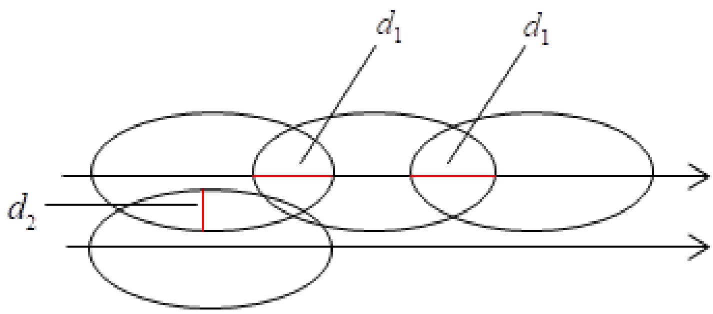


Fig.5 Top View of Spraying

Now take $x=0$ and then intercept the sectional view of the elliptic double β distribution model of the nearly twice spraying, shown as Fig.6. According to the figure, OD is the semi-major axis of ellipses and at the point D, the function $Z(0,y)$ takes its minimum value.

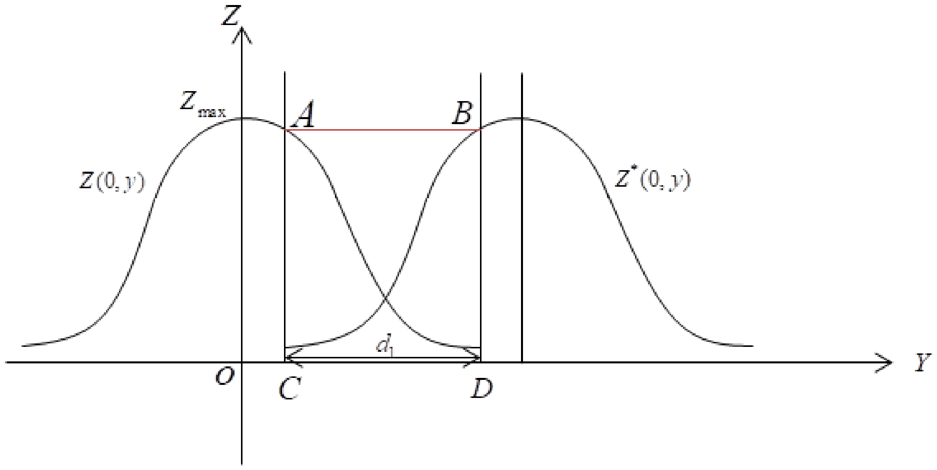


Fig.6 The Sectional View of $Z(0,y)$ of the nearly twice spraying

While $x=0$, we have the function

$$Z(0,y)=\mathrm{Z}_{\max}(1-\frac{y^2}{b^2})^{\beta_2-1}=212.7664(1-\frac{y^2}{2216.6393})^{3.8999}) \tag{2.4}$$

As the spraying gun moving, it's able to get a new function after moving, defined as $Z^*(0,y)$. We want the thickness to become uniform after the nearly twice spraying as far as uniform and be equal to the uniform thickness $\mathrm{Z}_{\max}=212.7664$ as much as possible. According to the cumulative thickness in the overlap interval d_1 is equal to the uniform thickness $\mathrm{Z}_{\max}$. In Fig.6, that is the area s_1 of the surface surrounded by line AC, axis Y and function $Z(0,y)$ add the area s_2 of the surface surrounded by line BD, axis Y and function $Z^*(0,y)$ is equal to the area s of the rectangle $ABDC$. That is $s=s_1+s_2$ and $s_1=s_2$. Then we have

$$2\int_{a-d_1}^{a} Z(0,y)dy = Z_{\max}\cdot d \tag{2.5}$$

Thus we can get $d_1=107.9115$.

② Calculate d_2. While spraying along the semi-minor axis, as shown in Fig.7, the overlap interval between the spray gun trajectories is the overlap interval along the semi-minor axis d_1.

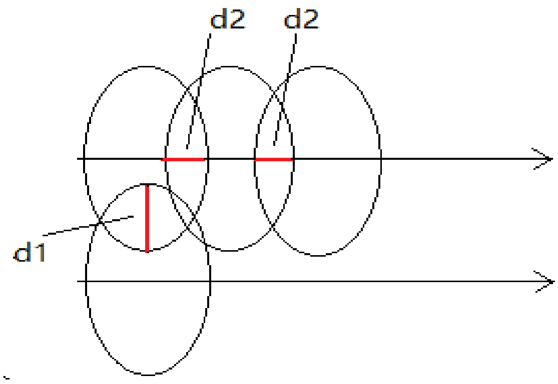


Fig.7 Top View of Spraying

Now take $y=0$ and then intercept the sectional view of the elliptic double β distribution model of the nearly twice spraying, shown as Fig.8. According to the figure, OH is the semi-minor axis of ellipses and at the point H, the function $Z(x,0)$ takes its minimum value.

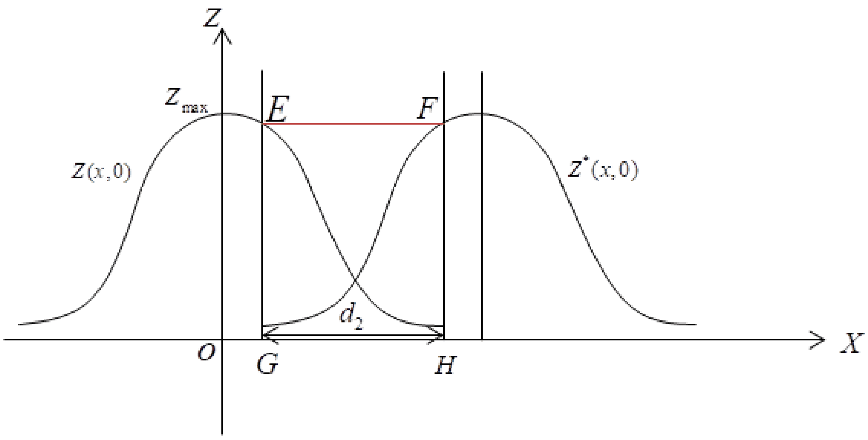


Fig.8 The Sectional View of $Z(x,0)$ of the nearly twice spraying

While $y=0$, we have the function :

$$Z(x,0)=\mathrm{Z}_{\max}(1-\frac{x^2}{a^2})^{\beta_1-1}=212.7664(1-\frac{x^2}{12065.6604})^{1.3655}) \tag{2.6}$$

As the spraying gun moving, it is able to get a new function after moving, defined as $Z^*(x,0)$. We want the thickness to become uniform after the nearly twice spraying as far as uniform and be equal to the uniform thickness $\mathrm{Z}^*_{\max}=212.7664$ as much as possible. According to the cumulative thickness in the overlap interval d_2 is equal to the uniform thickness $Z^*_{\max}$. In Fig.8, that is the area s_3 of the surface surrounded by line EG, axis X and function $Z(x,0)$ add the area s_4 of the surface surrounded by line FH, axis X and function $Z^*(x,0)$ is equal to the area s^* of the rectangle $EFGH$. That is $s^* = s_3 + s_4$ and $s_3 = s_4$. Then we have

$$2\int_{b-d_2}^{b} Z(x,0)dx = Z_{\max} \cdot d_2 \tag{2.7}$$

Thus we can get $d_2 = 41.2128$.

3）Planning the spray gun trajectory

According to 2）, we get the overlap interval $d_1 = 107.9115$ and $d_2 = 41.2128$, choosing the trajectory remains the same as Fig.2.

While spraying along the semi-major axis, as shown in Fig.5, the overlap interval d_1 along the spraying direction is equal to 107.9115 and the overlap interval d_2 between the spray gun trajectories is equal to 41.2128.

While spraying along the semi-minor axis, as shown in Fig.7, the overlap interval d_2 along the spraying direction is equal to 41.2128 and the overlap interval d_1 between the spray gun trajectories is equal to 107.9115.

2.2 Analysis and Solution of Second Problem

2.2.1 Problem Presentation

Paint on the plane will form an oval. However, if it falls on the surface, determine whether the path in issue 1 and the overlap interval still apply. If not, give a new path and lap spacing.

2.2.2 Problem Analysis

When the spray surface turns into a curved surface, Spray distance has changed. So the thickness function has also changed. Therefore, the ejection solution of issue 1 is not suitable for issue 2. According to the relationship between the surface and the plane spray distance, we can calculate the new spray distance. Then take them into the relationship matrix to new parameter of the original thickness function to get new thickness function. Finally, we can finish this issue by the ideas in last issue.

2.2.3 Problem Solution

1) Study whether the solution is suitable for the curved surface

First, put the curved surface $z = -x^2 + x - xy$ and the plane $z = 0$ on the same coordinate system, as shown in Figure 9

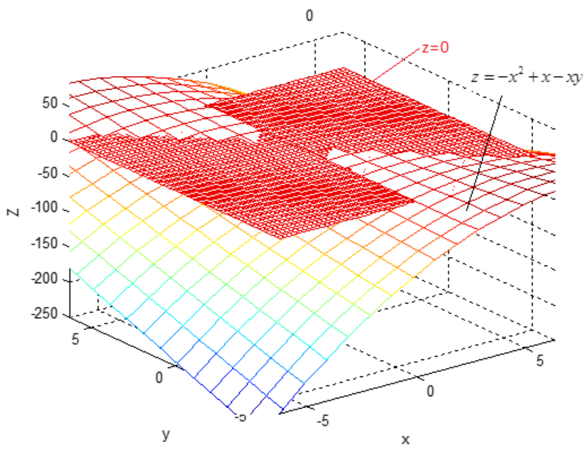


Fig. 9 Surface $z = -x^2 + x - xy$ and plane $z = 0$

In Fig.9, take an intersection as fowling Fig.10, and take point O as the origin of coordinates to establish a new coordinate system. So O's coordinates are $(0,0,0)$, take O as the center of the oval for a spray area.

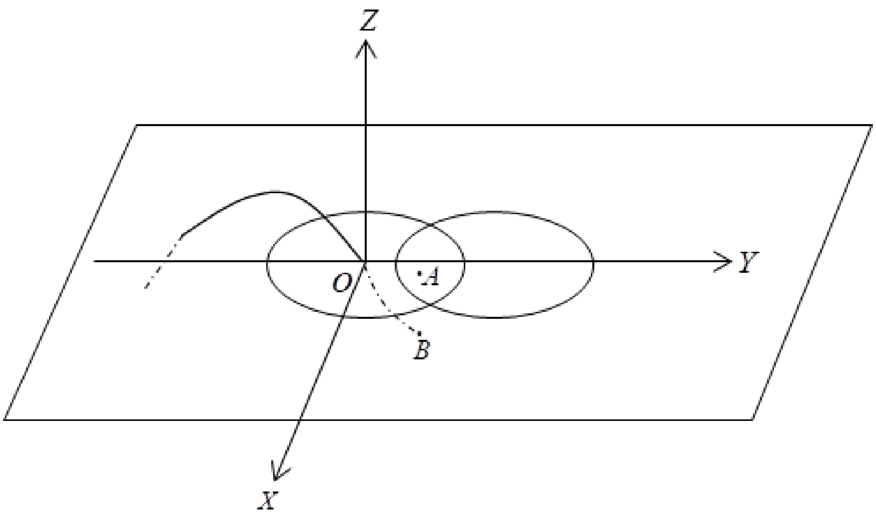


Fig.10 Intersection graph of curve and plane

Then move O by one unit along the x axis and $a-d_1+1$ units along the y axis to get point $A(1, a-d_1+1)$, that is $(1, 2.9323, 0)$. Take A as the center of the oval for a spray area which is equal to oval O. When the spray surface falls on the surface, the point A is equal to $B(1, 2.9323, -2.9353)$.

As we all know the paint on ellipse A is equal to the paint that falls on B, we only need to take the two points' coordinates into $Z(x_A, y_A) = Z(x_B, y_B)$ and judge whether it is true.

After calculation, we get $Z(x_A, y_A) = 212.7664, Z(x_B, y_B) = 209.4323$.

It's obvious that they are not equal. So we draw a conclusion that the solution in issue 1 is not suitable for this one.

2) Determine the new ballistic design for the curved surface.

① Calculate the new thickness function

If the curved surface is placed in the coordinate system of issue 2, the relationship between the surface and the plane is shown in Fig.11, we can get a new jet distance:

$$h' = h - (-x^2 + x - x) y \geq 2\,2\,5 + x^2 x - x \tag{2.8}$$

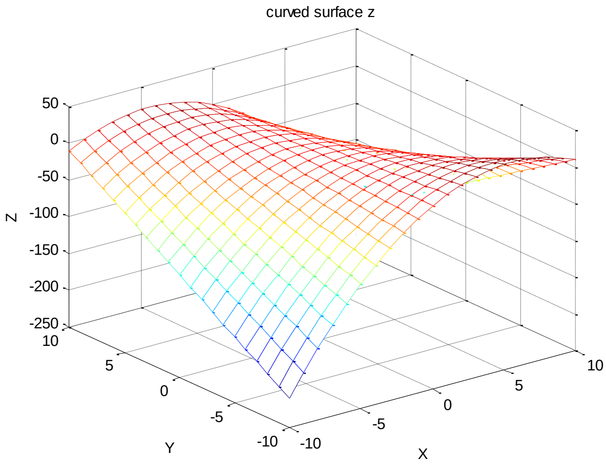


Fig.11 the image of curved surface $z = -x^2 + x - xy$

Take $h^{'}$ into the relationship matrix to get several parameters, as following

$$\begin{bmatrix} 129.8665 & -55.2435 & 1.7436 & -297.3908 \\ 52.5130 & -5.7480 & 0.7394 & -128.6368 \\ 59.7245 & 393.9655 & -0.1244 & 150.0184 \\ -7.0125 & 34.5045 & 0.0284 & -9.5229 \\ -4.6130 & 18.3620 & 0.0113 & -0.3924 \end{bmatrix} \times \begin{bmatrix} 0.2 \\ 0.2 \\ 225-(-x^2+x-xy) \\ 1 \end{bmatrix} = \begin{bmatrix} 109.8438-1.7436(-x^2+x-xy) \\ 47.0812-0.7394(-x^2+x-xy) \\ 212.7644+0.1244(-x^2+x-xy) \\ 2.3655-0.0284(-x^2+x-xy) \\ 4.8999-0.0113(-x^2+x-xy) \end{bmatrix}$$

$a^* = 109.8438 - 1.7436(-x^2 + x - xy)$ $\qquad$ $b^* = 47.0812 - 0.7394(-x^2 + x - xy)$

$Z_{\max}^* = 212.7664 - 0.1244(-x^2 + x - xy)$ $\qquad$ $\beta_1^* = 2.3655 - 0.0284(-x^2 + x - xy)$

$\beta_2^* = 4.8999 - 0.0113(-x^2 + x - xy)$

Substitute the parameters into $Z(x, y)$ to get the new thickness function:

$$Z_3(x,y) = Z_{\max}^*\left(1-\frac{x^2}{(a^*)^2}\right)^{\beta_1^*-1}\left[1-\frac{y^2}{(b^*)^2\left(1-\frac{x^2}{(a^*)^2}\right)}\right]^{\beta_2^*-1} \tag{2.9}$$

② find the overlap interval

In issue 1, we respectively studied the overlap interval along the long axis and the short axis. Similarly spray gun sprayed on the curved surface, we still need to classify this discussion. Spray path is still the same direction of a straight line, just get out of two overlap interval.

When the gun is sprayed along the long axis direction, such as Fig.12. At this point, we take the nearly twice cumulative cross section, showing as Fig.12. At point

D, the thickness is the thinnest, that is to say the length of OD is equal to the ellipse semi-major axis.

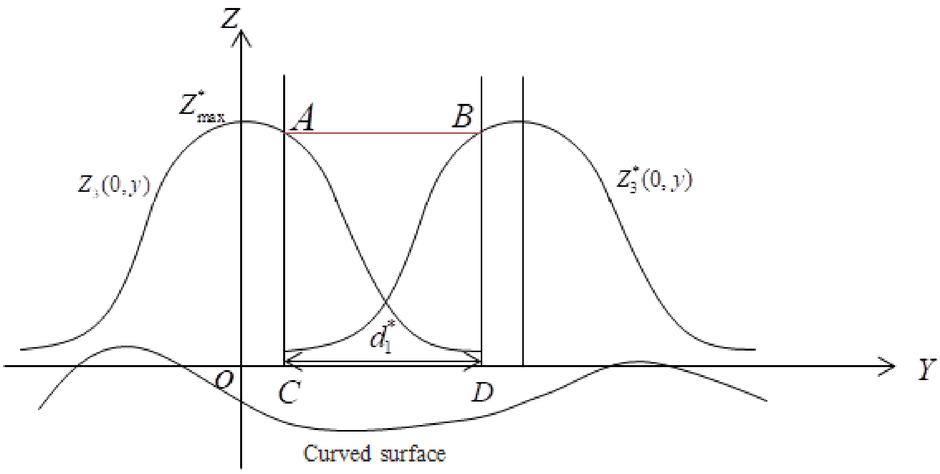


Fig.12 Cumulative cross section

We can get the function $Z_3(0, \mathrm{y}) = \mathrm{Z}^*_{\max}(1-\frac{y^2}{b^2})^{\beta_2^*-1}$ at $x=0$. The gun moves one step and we can get the moved function $Z_3^*(0, \mathrm{y})$ .In order to make the thickness of the superposition as much as possible to achieve a uniform thickness Z^*_{max}, The cumulative thickness between overlap interval d_1^* is almost equal to the area of the rectangular $ABDC$. In Fig.12, that is the area s_1 of the surface surrounded by line AC, axis Y and function $Z(0,y)$ add the area s_2 of the surface surrounded by line BD, axis Y and function $Z^*(0,y)$ is almost equal to the area s of the rectangle $ABDC$. That is $\frac{s_1+s_2}{s} \geq 90\%$

$$\frac{\int_{a^*-d_1^*}^{a^*} z_3(0,\mathrm{y})dy + \int_{a^*-d_1^*}^{a^*} z_3^*(0,\mathrm{y})dy}{z^*_{\max}\cdot d_1^*} \geq 90\% \qquad (2.10)$$

Thus we can get the function $d_1^*(x,y)$.

While spraying along the semi-major axis, the overlap interval between the spray gun trajectories is equal to the overlap interval along the semi-minor axis d_1^*.

Now take $x=0$ and then intercept the sectional view of the elliptic double β distribution model of the nearly twice spraying, shown as Fig.13. According to the figure, OH is the semi-minor axis of ellipses and at the point H.

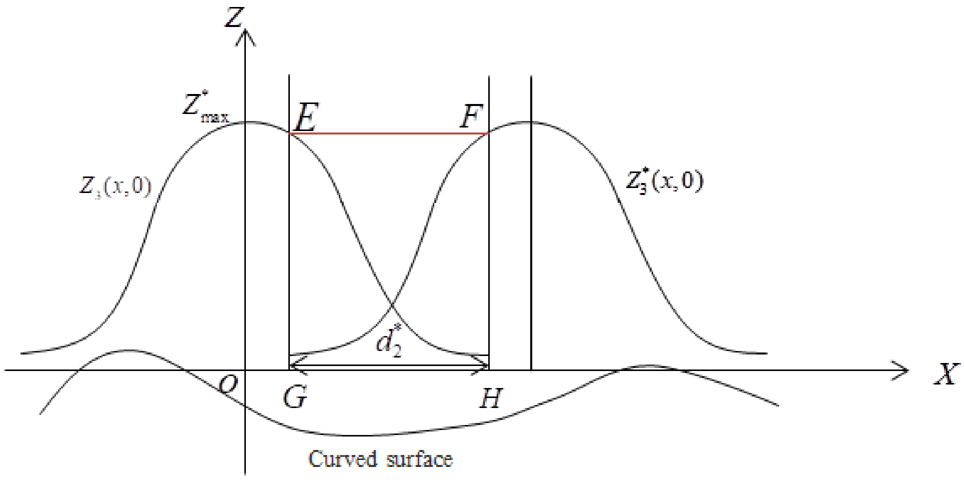


Fig.13 The Sectional View of $Z_3(x,0)$ of the nearly twice spraying

While $y=0$, we have the function

$$Z_3(x,0)=Z^*_{\max}(1-\frac{x^2}{(a^*)^2})^{\beta_1^*-1} \tag{2.11}$$

As the spraying gun moving a step, it is able to get a new function after moving, defined as $Z^*_3(x,0)$. We want the thickness to become uniform after the nearly twice spraying as far as uniform and be equal to the uniform thickness $Z^*_{\max}$ as much as possible. The cumulative thickness between overlap interval d^*_2 is almost equal to the area of the rectangular $EFHG$. In Fig.13, that is the area s_3 of the surface surrounded by line EG, axis X and function $Z_3(x,0)$ add the area s_4 of the surface surrounded by line FH, axis X and function $Z^*_3(x,0)$ is almost equal to the area s^* of the rectangle $EFHG$. That is $\frac{s_3+s_4}{s^*}\geq 90\%$.

$$\frac{\int_{b^*-d_2^*}^{b^*} z_3(x,0)dx+\int_{b^*-d_2^*}^{b^*} z^*_3(x,0)dx}{z^*_{\max}\cdot d^*_2}\geq 90\% \tag{2.12}$$

Thus we can get the function $d^*_2(x,y)$.

③ Research on overlap interval

According to the overlap interval d^*_1 and d^*_2 obtained by ②, for each point of the coordinates (x, y) have different value ranges. That is, in this issue, our overlap interval are functions of the unknowns x and y. We can get the certain values of the overlap interval according to the specific (x, y).

2.3 Analysis and Solution of Third Problem

2.3.1 Problem Presentation

In issue 2, we have discussed and established a model when the painted surface is curved surface. In the spraying process, spray gun spray direction remains unchanged. However, when the spray gun's direction changes, the situation will be different. Thus, we turn to study this question. Keeping other conditions unchanged, re-explore the issue 2.

When the spray surface changes from flat to curved, the corresponding model of cumulative thickness changes also changes. The model established in issue 2 no longer applies. Here we create a new model to solve. First, take a curved surface above the surface as the test surface. Then find out the relationship between the thickness of the coating on the surface and that on the curved surface. Finally, build a new function model about the thickness of the surface.

2.3.2 problem Analysis

When the spray surface changes from flat to curved, the corresponding model of cumulative thickness changes also changes. The model established in issue 2 no longer applies. Here we create a new model to solve. First, take a curved surface above the surface as the test surface. Then find out the relationship between the thickness of the coating on the surface and that on the curved surface. Finally, build a new function model about the thickness of the surface.

2.3.3 Problem Solution

1）Select the appropriate test plane[2]

When the spray surface changes from plane to curved surface, take a plane above the curved surface as our test plane, as shown:

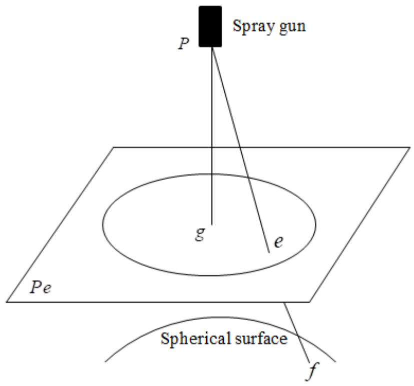


Fig.14 The Diagram of Static spray on Spherical Surface

In the figures: f - a point in the area of the sprayed surface;

Pe - The test plane;

g - Spray point on the plane of the spray gun center point;

e - Intersection of line pf and plane Pe .

After taking a plane Pe as the test surface, since we have solved the spray pattern of the spray gun on the plane in issue one, we can use the model in issue one to analyze.

2）Analysis the thickness of coating in the test plane and modeling

To see more clearly the derivation of the cumulative coating rate, we performed the analysis by making the following figure:

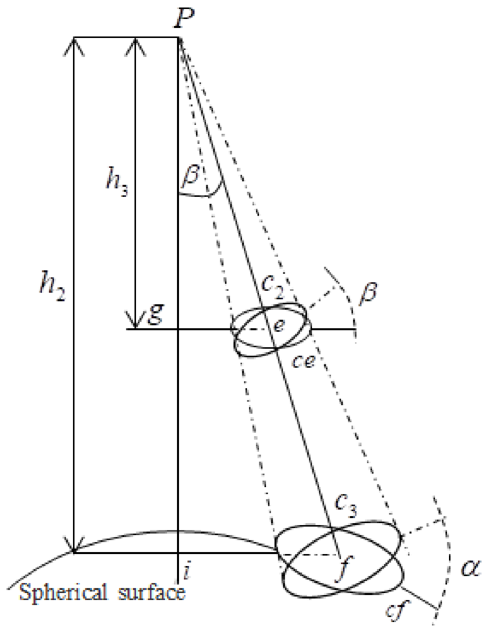


Fig.15 Relationship Between Each patches

In the figure: β - the angle between the axis of point f and the center point p of the gun. h_3, h_2 - the height of point p to point e and point f .

Assumptions: The spray gun is sprayed onto the test surface and the surface of the coating amount equal.

Spraying the paint onto an elliptical area ce formed on the test plane and then project a circle c_2 perpendicular to the line pf , let the radius of this circle be Δr , the angle of projection is also β . In this case, the minor axis of the ellipse can be regarded as equal to the radius of the circle, the long axis is half of the line, in order to calculate half of the long axis, we simplified the Fig.15 to the following one:

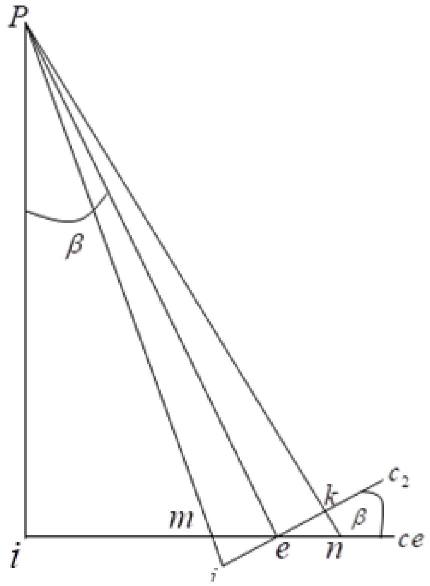


Fig.16 The Plane Figure of Fig2.3.2

Based on current knowledge, we can draw $\Delta mje \cong \Delta nke$ and $S_{\Delta nke} = S_{\Delta mje}$

In obtuse triangle pmn , $S_{\Delta pmn} = \frac{1}{2} mn \cdot pi = \frac{1}{2} mn \cdot pe \cdot \cos\beta$;

In Δpjk , $S_{\Delta pjk} = \frac{1}{2} jk \cdot pe = \Delta r \cdot pe$;

Since $S_{\Delta pmn} = S_{\Delta pjk}$, therefore , $\frac{1}{2} mn \cdot pe \cdot \cos\beta = \Delta r \cdot pe$,which is $\frac{1}{2} mn = \frac{\Delta r}{\cos\beta}$.

The ratio of the area of the circle c_2 to the area of the ellipse ce is:

$$\frac{S_{c_2}}{S_{ce}} = \frac{\pi \Delta r^2}{\frac{1}{2}\pi \Delta r \cdot mn} = \frac{\Delta r^2}{\frac{\Delta r^2}{\cos\beta}} = \cos\beta \tag{2.13}$$

Since it is assumed that the amount of paint sprayed onto the experimental plane and sphere is equal, there is $V_{c_2} = V_{ce}$, that is $S_{c_2} \cdot Z_2 = S_{ce} \cdot Z_{ce}$

Thus

$$\frac{S_{c_2}}{S_{ce}} = \frac{Z_{ce}}{Z_2} = \cos\beta \tag{2.14}$$

So the relationship between the thickness of the coating Z_2 in the circle c_2 and the thickness Z_{ce} in the ellipse ce is:

$$Z_2 = \frac{Z_{ce}}{\cos\beta} \tag{2.15}$$

3） Based on the model has been established on experimental plane to establish the surface thickness of the coating model

Using the bit-like principle, the circular surface c_3 and the circular surface c_2 parallel and in the same conical coating opening angle, according to the geometric relationship we can see the relationship between the area of the two circular surface is as follows:

$$S_{c_3} = \left(\frac{h_2}{h_3}\right)^2 S_{c_2} \tag{2.16}$$

The relationship about the thickness of the coating on the circle as follows:

$$Z_3 = \left(\frac{h_3}{h_2}\right)^2 Z_2 \tag{2.17}$$

The oval surface is at a point f and tangent to the surface, And at the same conical coating with c_3, The angle between cf and c_3 is denoted as α ,according to $Z_2 = \frac{Z_{ce}}{\cos\beta}$,we can drown the relationship between the thickness of coating in oval surface cf and in circular surface c_3 are ：

$$Z_f = Z_3 \cdot \cos\alpha \tag{2.18}$$

4）Decreasing the dimension of the surface to obtain the curve, introducing the round surface to analyze and establish the model

For the surface $z = -x^2 + x - xy(-10 \le x \le 10, -10 \le y \le 10)$， using the plane $y = c(-10 \le c \le 10)$ to cut it, in the intersection of two surface to form a curve, which is $\tilde{z} = -x^2 + x - cx$.

For the curve $\tilde{z}$, at every point on $\tilde{z}$ can use curvature to characterize its degree of bending[3], each point on the curve $\tilde{z}$ can make a radius of curvature[4], which is the reciprocal of the curvature. So this point as the center of the gun along the direction of the spray gun on the surface of the projection point, get the following figure for the next analysis:

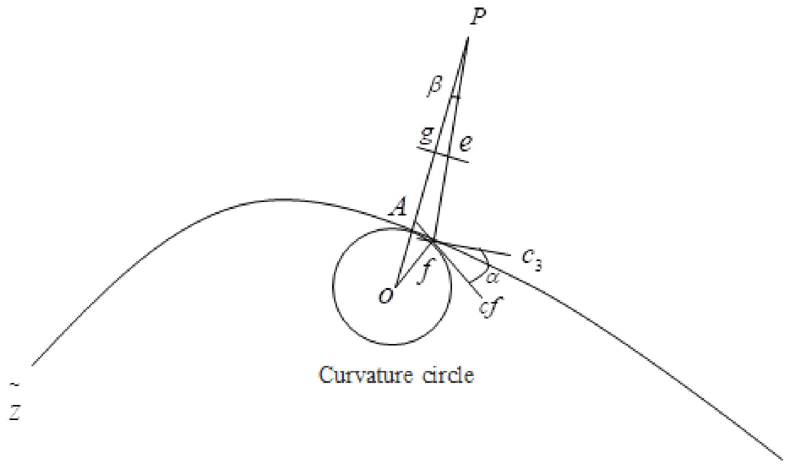


Fig.17 The Figure of Adding Curvature Circle

In the figure: A - Projection point of the center of the gun on the curved surface in the direction of the gun.

Set the radius of curvature as r , then $r = 1/K$.

Since $K = 2/[1+(2x-1+c)^2]^{\frac{3}{2}}$, thus $r = [1+(2x-1+c)^2]^{\frac{3}{2}}/2$.

Set $pA = h_A, pf = l$, then $h_2 = l\cos\beta$, by cosine theorem:

$$\alpha = \pi - \arccos\frac{l^2 + r^2 - po^2}{2lr} \tag{2.19}$$

Then:

$$\cos\alpha = \frac{(h_A + r)^2 - l^2 - r^2}{2lr} \tag{2.20}$$

$$\cos\beta = \frac{(h_A + r)^2 + l^2 - r^2}{2l(h_A + r)} \tag{2.21}$$

When $\alpha \geq 90^\circ$, paint can't spray to this point.

Combined (2.15) ,(2.17) and (2.18) we can conclude that:

$$Z_f=\begin{cases} Z_e\cdot\dfrac{4h_3^{\,2}(h_A+\frac{[1+(2x-1+c)^2]^{\frac{3}{2}}}{2})^3[(h_A+\frac{[1+(2x-1+c)^2]^{\frac{3}{2}}}{2})^2-l^2-\frac{[1+(2x-1+c)^2]^3}{2}]}{\frac{[1+(2x-1+c)^2]^{\frac{3}{2}}}{2}[(h_A+\frac{[1+(2x-1+c)^2]^{\frac{3}{2}}}{2})^2+l^2-\frac{[1+(2x-1+c)^2]^3}{2}]^3} & \alpha<90^{\circ} \\ 0 \quad \alpha\ge 90^{\circ} & \end{cases}$$

(2.22)

5) Based on the thickness of the model established in the previous step to calculate the interval

According to the research methods of issue 1 and issue 2, two overlapping areas are obtained by integral, and the addition can approximate a rectangular area, and the relationship between the thickness function and the thickness is established to further obtain the interval d .

When the long axis of the spray pattern is consistent with the spraying direction, the lap spacing between the paths at this time is the lap spacing $d_2^{'}$ along the short axis. According to the sum of the two overlapping areas, the area of the rectangle can be obtained as follows:

$$\frac{\int_{y_{z\min}-d_1^{'}}^{y_{z\min}} z_f(0,y)dy+\int_{y_{z\min}-d_1^{'}}^{y_{z\min}} z_f^*(0,y)dy}{z'_{\max}\cdot d'_1}\ge 90\% \tag{2.23}$$

Where $z'_{\max}$ is the uniform thickness and $y_{z\min}$ is the semi-major axis of spray ellipse cf .

When the short axis of the spray pattern is consistent with the spraying direction, the lap spacing between the paths at this time is the lap spacing $d_1^{'}$ along the long axis. According to the sum of the two overlapping areas, the area of the rectangle can be obtained as follows:

$$\frac{\int_{x_{z\min}-d_2^{'}}^{x_{z\min}} z_f(x,0)dx+\int_{x_{z\min}-d_2^{'}}^{x_{z\min}} z_f^*(x,0)dx}{z'_{\max}\cdot d'_2}\ge 90\% \tag{2.24}$$

Where $z'_{\max}$ is the uniform thickness and $x_{z\min}$ is the semi-minor axis of ellipse cf .

According to two integrals, we can find that $d_1^{'}$ and $d_2^{'}$ are two functions about unknown x and y.

2.4 Analysis and Solution of Third Problem

2.4.1 Problem Presentation

In issue 3, we successfully established the surface thickness model to find the overlap interval. If you change the surface of the issue 3 to any surface $z = f(x, y)$. We need to further explore whether the established model and the corresponding lap spacing still apply.

2.4.2 Problem Analysis

In the previous question, the most important step was to introduce the curvature and radius of curvature in the curve. For any surface, you can intersect with a plane to form a curve, but also any point of the degree of bending on the curve can be portrayed. Therefore, the problem of whether the model constructed in issue 3 is applicable to arbitrary surface is transformed into the problem of whether curvature and radius of a curve exist.

2.4.3 Problem Solution

1) Where the curvature and curvature radii of arbitrary curves exist.

Starting from the definition of curvature, the curvature of the curve describes the degree of curvature of the curve, which is the inverse of the curvature at a point on the curve [5]. In short, as long as the curvature of the curve exists, the radius of curvature exists. Then we only need to discuss whether the curvature of the curve exists. In real life, it can be seen that the curve of a curve exists. However, the curvature of the curve at some point is calculated as: $K = |y^{''} / (1 + y^{'})^{\frac{3}{2}}|$ [6] .To calculate the curvature, we need find the first and second-derivative of the curve. But not all first-derivative and second-derivative of the curve must exist.

2) Process low-order derivative does not exist in the curve

Since there is a radius of curvature at every point in the model built in issue 3. So we can cut the curve into many small breaks, converted to the curve, which has the first-derivative and the second-derivative. Then find the required curvature and curvature radius. Finally, we can get the conclusion that Model Three is applicable to any surface.

In summary, the results in question three apply to any surface $z = f(x, y)$. The common solution to spray path planning is the one planned in issue 3.

3. Conclusion

We have successfully established a " Spray Trajectory Planning model" to study the factors that affect spray path and lap spacing when spray gun spraying different surface.

First, when spraying the plane, we take the idea that the cumulative thickness of the thickness is different in the long axis and in the short axis into count. Establish the relationship between overlap thickness and uniform thickness, in order to obtain the long axis and short axis lap spacing d_1 and d_2, by which Plan out the spray path, making the spray thickness uniform.

When the spray surface into curved surface with the spray direction unchanged, we found that the jet distance h has changed compared with spraying on the plane. The thickness function on the surface can be found through the relationship among the atomization pressure P_1, diaphragm pressure P_2 and jet distance h in the matrix. We can establish the relationship between the overlap interval and the uniform thickness in two directions to obtain the overlap interval according to the research method of issue 1.

If the gun center is always perpendicular to the surface in the spraying process.

We consider each small surface as a spherical fan, then build the relationship between surface and radius by curvature of the surface in order to find the new thickness function. And get a function of overlap interval through the previous method. Combined with the actual situation, we conclude that the solution of issue 3 suits all surface, because our painted ceramic area is generally curved surface.

4. Model Evaluation

4.1 Sensitivity Analysis

1) This article focuses on the spray of uniform thickness, so the path planning is relatively simple, which is sprayed along the two vertical directions, as long as the cumulative thickness in both directions can be uniform, but we did not take the optimization of the spray path into account. Therefore, we need study in depth and supplement it.

2) The main idea of this paper is to solve the overlap interval according to the cumulative function of the thickness of the cone on the curved surface, but in the process of integration, the calculation becomes complicated because of the complexity of the cumulative thickness function. Therefore, the treatment of the cumulative function needs to be improved.

3) In this paper, we only consider the effect of injection distance on the cumulative thickness, considering the atomization pressure and the diaphragm pressure as constant. Exactly speaking, you can further design experiments to find the best match of the three factors.

4.2 Model Generalization and Evaluation

1) In practical problems, this model can solve the problem of uniform thickness in the spraying process. However, the accuracy is not high enough to further study and improve the accuracy of the thickness difference.

2) In this model, we only consider whether the thickness of spray coating is

uniform or not. The change of unknowns makes the manual operation difficult in practical problems and can be operated in combination with mechanical work.

3) The factors that affect the thickness function of this model include atomization pressure, diaphragm pressure and jet distance. However, in practical problems, in addition to the above three factors, the ejection angle and the gun speed are important factors affecting the thickness. So we can further discuss the impact of these factors on the cumulative thickness on the basis of this model.

5. Quote

[1]Yonggui Zhang,Yumei Huang,Feng Gao,Wei Wang, A new model of spray gun robot air gun[J],Chinese Journal of Mechanical Engineering, 42(11):228-232,2006(11)，

[2]Hao Feng,Qiu Wu,Xiaoping Wang, Spherical Spray Trajectory Optimization Based on Oval Double β Model[J],Machine Design & Manufacture,4:249-257,2016(4)

[3]Xiangming Mei,Jingzhi Huang,Differential Geometry[M].(Fourth edition)Beijing: Higher Education Press,2008(5).

[4] https://baike.baidu.com/item/%E6%9B%B2%E7%8E%87%E5%8D%8A%E5%BE%84

[5] https://baike.baidu.com/item/%E6%9B%B2%E7%8E%87/9985286?fr=aladdin

[6] Guangzhong Ouyang,Xueyan Zhu,fulin Jin,Chuanzhang Chen, Mathematical Analysis[M].(Third Edition) Beijing: Higher Education Press,2007(4).

6. Appendix

1. Programming Software: MATLAB
2. Program Code:

1) The figure of $z=-x^2+x-xy(-10\le x\le 10,-10\le y\le 10)$

```
[x,y]=meshgrid(-10:1:10);
z=-x.^2+x-x.*y;
mesh(x,y,z);
title('curved surface z')
xlabel('X');
ylabel('Y');
zlabel('Z');
```

2) The figure of $z=-x^2+x-xy(-10\le x\le 10,-10\le y\le 10)$ and $z=0$

```
[x,y]=meshgrid(-10:1:10);
z=-x.^2+x-x.*y;
mesh(x,y,z);
title('curved surface z')
xlabel('X');
ylabel('Y');
zlabel('Z');
hold on
ezmesh('0')
```

3) The issue 1

```
A=[129.8665 -55.2435 1.7436 -297.3908;
   52.5130 -5.7480 0.7394 -128.6368;
   59.7245 393.9655 -0.1244 150.0184;
   -7.0125 34.5045 0.0284 -9.5229;
   -4.6130 18.3620 0.0113 -0.3924];
B=[0.2 0.2 225 1];
C=A*B'
a=C(1);
b=C(2);
Zmax=C(3);
beta1=C(4);
beta2=C(5);
Zmin1=Zmax*(1-(a^2/b^2))^(beta2-1);
y1=sqrt((b^2)*(1-exp((1/(beta2-1))*log((Zmax-Zmin1)/Zmax))));
d1=abs(a-y1);
Zmin2=Zmax*(1-b^2/a^2)^(beta1-1);
x2=sqrt((a^2)*(1-exp((1/(beta1-1))*log((Zmax-Zmin2)/Zmax))));
```

```
d2=abs(b-x2);
```

4) The issue 2

```
A=[129.8665 -55.2435 1.7436 -297.3908;
    52.5130 -5.7480 0.7394 -128.6368;
    59.7245 393.9655 -0.1244 150.0184;
    -7.0125 34.5045 0.0284 -9.5229;
    -4.6130 18.3620 0.0113 -0.3924];
B=[0.2 0.2 225 1];
C=A*B';
a=C(1);
b=C(2);
Zmax=C(3);
beta1=C(4);
beta2=C(5);
Zmin1=Zmax*(1-(a^2/b^2))^(beta2-1);
y1=sqrt((b^2)*(1-exp((1/(beta2-1))*log((Zmax-Zmin1)/Zmax))));
d1=abs(a-y1);
Zmax=C(3);
Z1=Zmax
z=-1+1-(a-d1+1);
h1=0-z;
h=B(3)+h1;
B1=[0.2 0.2 h 1];
C1=A*B1';
a1=C1(1);
b1=C1(2);
Zmax=C1(3);
beta1=C1(4);
beta2=C1(5);
Z2=Zmax*(1-1/a1^2)^(beta1-1)*(1-(a-d1+1)^2/(b1^2*(1-1/a1^2)))^(beta2-
1);
if Z1==Z2
    fprintf('the overlap interval is suitable')
else
    fprintf('the overlap interval isn’t suitable')
end
```