K-Nearest Neighbors

Simple Analogy..

- Tell me about your friends(*who your neighbours are*) and ?
- *I will tell you who you are*.

KNN – Different names

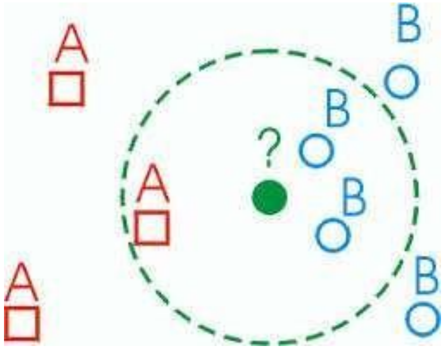
- K-Nearest Neighbours
- Memory-Based Reasoning
- Example-Based Reasoning
- Instance-Based Learning
- Lazy Learning

What is KNN?

- A powerful classification algorithm used in pattern recognition.
- K nearest neighbours stores all available cases and classifies new cases based on a similarity measure(e.g. **distance function**)
- One of the top data mining algorithms used today.
- A non-parametric **lazy learning** algorithm (An Instance-based Learning method).

KNN: Classification Approach

- An object (a new instance) is classified by a majority votes for its neighbour classes.
- The object is assigned to the most common class amongst its K nearest neighbours.(measured by a distant function)



Distance measure for Continuous Variables

Distance functions

Euclidean

$$\sqrt{\sum_{i=1}^{k}(x_i - y_i)^2}$$

Manhattan

$$\sum_{i=1}^{k}|x_i - y_i|$$

Minkowski

$$\left(\sum_{i=1}^{k}(|x_i - y_i|)^q\right)^{1/q}$$

Distance Between Neighbors

- Calculate the distance between new example
- (E) and all examples in the training set.

- *Euclidean* distance between two examples.
- – $X=[x_1,x_2,x_3,...,x_n]$
- – $Y=[x`_1,x`_2,x`_3,....,x`_n]$

- – The Euclidean distance between *X* and *X`* is defined

$$d(x,x') = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + \ldots + (x_n - x'_n)^2}$$

K-Nearest Neighbor Algorithm

- All the instances correspond to points in an n-dimensional feature space.
- Each instance is represented with a set of numerical attributes.
- Each of the training data consists of a set of vectors and a class label associated with each vector.
- Classification is done by comparing feature vectors of different **K nearest** points.
- Select the K-nearest examples in the training set.
- Assign to the most common class among its **K-nearest neighbors**.

3-KNN: Example(1)

Customer	Age	Income	No. credit cards	Class
George	35	35K	3	No
Rachel	22	50K	2	Yes
Steve	63	200K	1	No
Tom	59	170K	1	No
Anne	25	40K	4	Yes
John	37	50K	2	???

3-KNN: Example(1)

Customer	Age	Income	No. credit cards	Class	Distance from John
George	35	35K	3	No	sqrt $[(35-37)^2+(35-50)^2+(3-2)^2]$=15.16
Rachel	22	50K	2	Yes	sqrt $[(22-37)^2+(50-50)^2+(2-2)^2]$=15
Steve	63	200K	1	No	sqrt $[(63-37)^2+(200-50)^2+(1-2)^2]$=152.23
Tom	59	170K	1	No	sqrt $[(59-37)^2+(170-50)^2+(1-2)^2]$=122
Anne	25	40K	4	Yes	sqrt $[(25-37)^2+(40-50)^2+(4-2)^2]$=15.74
John	37	50K	2	???	

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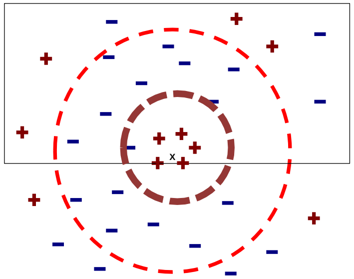
How to choose K?

- If K is too small it is sensitive to noise points.
- Larger K works well. But too large K may include majority points from other classes.



- Rule of thumb is **K < sqrt(n)**, n is number of examples.

Feature Normalization

- Distance between neighbors could be dominated by some attributes with relatively large numbers.
- e.g., income of customers in our previous example.

$$a_i = \frac{v_i - \min v_i}{\max v_i - \min v_i}$$

- Arises when two features are in different scales.
- Important to normalize those features.
- Mapping values to numbers between 0 – 1.

Nominal/Categorical Data

- Distance works naturally with numerical attributes.

- Binary value categorical data attributes can be regarded as 1 or 0.

Hamming Distance

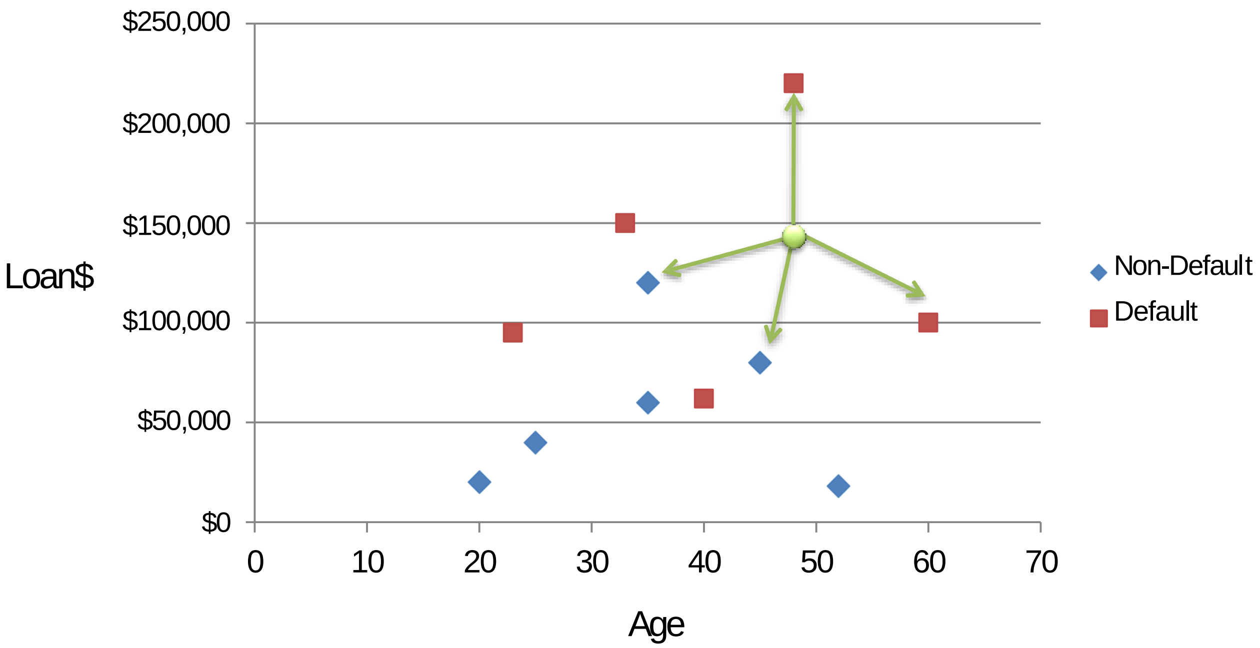
$$D_H = \sum_{i=1}^{k} |x_i - y_i|$$

$$x = y \Rightarrow D = 0$$

$$x \neq y \Rightarrow D = 1$$

X	Y	Distance
Male	Male	0
Male	Female	1

KNN Classification



KNN Classification – Distance

Age	Loan	Default	Distance
25	$40,000	N	102000
35	$60,000	N	82000
45	$80,000	N	62000
20	$20,000	N	122000
35	$120,000	N	22000
52	$18,000	N	124000
23	$95,000	Y	47000
40	$62,000	Y	80000
60	$100,000	Y	42000
48	$220,000	Y	78000
33	$150,000	Y	8000
48	**$142,000**	**?**	

Euclidean Distance

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

KNN Classification – Standardized Distance

Age	Loan	Default	Distance
0.125	0.11	N	0.7652
0.375	0.21	N	0.5200
0.625	0.31	N	0.3160
0	0.01	N	0.9245
0.375	0.50	N	0.3428
0.8	0.00	N	0.6220
0.075	0.38	Y	0.6669
0.5	0.22	Y	0.4437
1	0.41	Y	0.3650
0.7	1.00	Y	0.3861
0.325	0.65	Y	0.3771
0.7	**0.61**	**?**	

Standardized Variable

$$X_s = \frac{X - Min}{Max - Min}$$

We have data from survey (to ask people opinion) and objective testing with two attributes(acid durability and strength) to classify whether a special paper tissue is good or not. Here is four training samples

X1(Acid) in seconds	**X2(Strength) in kg/square meter**	**Y = Classification**
7	7	Bad
7	4	Bad
3	4	Good
1	4	Good

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Now the factory produces a new paper tissue that pass laboratory test with X1 = 3 and X2 = 7.

Without another expensive survey, can we guess what the classification of this new tissue is?

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We have 2 good and 1 bad, since 2>1 then we conclude that a new paper tissue that pass laboratory test with X1 = 3 and X2 =7 is included in Good category

Step 1

```
import numpy as np
from sklearn.preprocessing import Imputer
from sklearn.cross_validation import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from sklearn.metrics import accuracy_score
```

Step 2 - Import Data

Step 3

```
X_train, X_test, y_train, y_test = train_test_split(
 X, Y, test_size = 0.3, random_state = 100)
y_train = y_train.ravel()
y_test = y_test.ravel()
```

Step 4

```
for K in range(25):
 K_value = K+1
 neigh = KNeighborsClassifier(n_neighbors = K_value, weights='uniform', algorithm='auto')
 neigh.fit(X_train, y_train)
 y_pred = neigh.predict(X_test)
 print "Accuracy is ", accuracy_score(y_test,y_pred)*100,"% for K-Value:",K_value
```

Strengths and Weakness of KNN

- **Strengths of KNN**
- Very simple and intuitive.
- Can be applied to the data from any distribution.
- Good classification if the number of samples is large enough.

- **Weakness of KNN**
- Takes more time to classify a new example.
- Need to calculate and compare distance from new example to all other examples.
- Choosing k may be tricky.
- Need large number of samples for accuracy.